

**Nucleating quark droplets in the core of magnetars**D. Kroff<sup>\*</sup> and E. S. Fraga<sup>†</sup>*Instituto de Física, Universidade Federal do Rio de Janeiro,  
Caixa Postal 68528, 21941-972 Rio de Janeiro, RJ, Brazil  
(Received 30 September 2014; published 21 January 2015)*

To assess the possibility of homogeneous nucleation of quark matter in magnetars, we investigate the formation of chirally symmetric droplets in a cold and dense environment in the presence of an external magnetic field. As a framework, we use the one-loop effective potential of the two-flavor quark-meson model. Within the thin-wall approximation, we extract all relevant nucleation parameters and provide an estimate for the typical time scales for the chiral phase conversion in magnetized compact star matter. We show how the critical chemical potential, critical radius, correlation length and surface tension are affected, and how their combination to define the nucleation time seems to allow for nucleation of quark droplets in magnetar matter even for not so small values of the surface tension.

DOI: [10.1103/PhysRevD.91.025017](https://doi.org/10.1103/PhysRevD.91.025017)

PACS numbers: 25.75.Nq, 11.10.Wx, 12.39.Fe, 64.60.Q-

**I. INTRODUCTION**

The thermodynamics of strong interactions in cold and dense matter under the influence of strong magnetic fields is of clear relevance in the description of magnetars. These objects correspond to a class of compact stars [1] whose magnetic fields can reach up to  $10^{15}$  G at the surface [2] and even higher, yet unknown, in the core. (Reference [3] presents an upper limit of  $10^{20}$  G  $\sim 60m_\pi^2$ , which could be achieved in the core of self-bound strange stars. This value exceeds even the magnetic fields generated in peripheral heavy ion collisions at high energy [4] and would certainly affect the phase structure and phase conversion of strong interactions.)

The full description of the structure and dynamics of formation of these objects depends on the knowledge of the equation of state for the matter they are built of, including possible condensates and new phases that are energetically more favored as baryon density is increased [5]. In particular, for high enough energy densities, one expects that strongly interacting matter becomes deconfined and essentially chiral [6], so that chiral quark matter could provide the relevant degrees of freedom in the core of compact stars [7,8].

In fact, it was shown that deconfinement can happen at an early stage of a core-collapse supernova process, which could result not only in a delayed explosion but also in a neutrino signal of the presence of quark matter in compact stars [9]. However, as discussed in Ref. [10] (see also [11]), this possibility depends crucially on the time scales of phase conversion. Since one expects a first-order nature for the chiral and the deconfinement transitions in cold and dense matter, this process would be guided by bubble nucleation, which is usually slow, or spinodal

decomposition, depending on how fast the system reaches the spinodal instability as compared to the nucleation rate. It has been shown in Ref. [10] that a key ingredient is the surface tension, which was later estimated in Refs. [12–16].

The surface tension for magnetized quark matter was estimated within the Nambu–Jona-Lasinio model in Ref. [17], exhibiting an interesting nonmonotonic behavior as a function of the magnetic field. However, as has become clear in the analysis of Ref. [12], different ingredients in the nucleation process (such as the critical radius, the critical chemical potential, and the surface tension) can react very differently to variations of an external control parameter. Since the time scales for the phase conversion process are built from a nontrivial combination of these quantities, one needs to compute how each of them is affected by an external magnetic field to assess whether nucleation can be the driving mechanism for the chiral transition in the case of magnetar matter.

In this paper we assess the possibility of homogeneous nucleation of quark matter in magnetars by investigating the formation of chirally symmetric droplets in a cold and dense environment in the presence of an external magnetic field. As a framework, we use the linear sigma model coupled to quarks, also known as the two-flavor quark-meson model. From the one-loop effective potential, and within the thin-wall approximation, we extract all relevant nucleation parameters and provide an estimate for the typical time scales for the chiral phase conversion in magnetized compact star matter. We show how the critical chemical potential, the correlation length, the critical radius, the surface tension and the nucleation rate are affected. The nucleation time is obtained from a nontrivial combination of these quantities and seems to favor nucleation even for not so small values of the surface tension.

The paper is organized as follows. In Sec. II we briefly describe the effective model and the approximations used to compute the effective potential. Section III shows how we

<sup>\*</sup>kroff@if.ufrj.br  
<sup>†</sup>fraga@if.ufrj.br

proceed in order to obtain all nucleation parameters in the thin-wall approximation. Our results for the relevant quantities related to the nucleation process are presented in Sec. IV. Section V presents our summary.

## II. EFFECTIVE THEORY

### A. General framework

To study the phase conversion process, we adopt the linear sigma model coupled to quarks (LSMq) [18] as our effective theory description of the chiral sector of strong interactions. The Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_f [i\gamma^\mu \partial_\mu - g(\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi})] \psi_f \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}) \\ & - \frac{\lambda}{4} (\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 + h\sigma. \end{aligned} \quad (1)$$

The model contains a fermionic SU(2) chiral doublet,  $\psi_f$ , representing the up and down constituent quarks, and four mesons—one scalar,  $\sigma$ , and three pseudoscalars,  $\boldsymbol{\pi}$ . The mesons can be grouped into a single  $O(4)$  chiral field  $\phi \equiv (\sigma, \boldsymbol{\pi})$ . It is well known that the LSMq reproduces correctly all the chiral low energy phenomenology of strong interactions, such as meson masses and the spontaneous and (small) chiral symmetry breaking, which are present in the mesonic self-interaction potential. The model parameters are fixed accordingly [18]. Moreover, it was argued [19] that both QCD with two flavors of massless quarks and the model we consider belong to the same universality class, thus exhibiting the same behavior at criticality.<sup>1</sup>

Due to spontaneous symmetry breaking, the  $\sigma$  field acquires a nonvanishing vacuum expectation value. However, for sufficiently high temperatures, the condensate melts and chiral symmetry is approximately restored. Therefore, in this context the expectation value of the  $\sigma$  field plays the role of an approximate order parameter for the chiral transition, being exact only in the limit of vanishing quark (and pion) masses, which happens for  $h = 0$ . In this limit, the model becomes truly chiral, and the pions behave as Goldstone bosons. So, to investigate the phase conversion in the LSMq, one ultimately needs to study how the expectation value  $\langle \sigma \rangle = \bar{\sigma}$  varies as a function of the relevant control parameters, such as temperature, chemical potentials and external fields. As usual in this approach, the effective potential formalism rises as the appropriate means for the description of phase transitions. In the spirit of effective theory descriptions, we will not be concerned with numerical precision, but rather in obtaining

qualitative information about the system under consideration. Moreover, in order to perform a semianalytic study, some simplifying approximations are needed.

The first regards the fermionic contribution to the effective potential. As the action is quadratic in the fermion fields, we can formally integrate over the quarks, so that their contribution to the effective potential is given by a determinant. However, as the quarks couple to  $\sigma$ , one is left to compute a fermionic determinant in the presence of an arbitrary background field, which cannot be done in closed form, unless for systems in 1 + 1 dimensions under some special circumstances [21–23]. As is customary, we consider the quark gas as a thermal bath in which the long-wavelength modes of the chiral field evolve, so that the calculation is performed considering a static and homogeneous background field. This procedure can be further improved, e.g., via a derivative expansion [24–26].

The contribution from the mesons to the effective potential is also subject to simplifying approximations. First, it has been shown that the pions do not appreciably affect the phase conversion process, so their dynamics is usually discarded and the whole analysis can be done by setting  $\boldsymbol{\pi} = \langle \boldsymbol{\pi} \rangle = 0$ . Second, since  $\lambda \approx 20$ , quantum corrections arising from the sigma self-interaction are usually ignored, and its contribution to the effective potential is taken to be classical.<sup>2</sup>

### B. Effective potential at one loop in a magnetic background

Our aim is to study the chiral transition in a cold and dense environment in the presence of an external constant and homogeneous magnetic field, as a very simplified model for the core of a magnetar. Adapting the previous setup to describe such a system is straightforward. The interaction with the magnetic field is introduced via minimal coupling; i.e., the derivatives acting on quarks are traded for  $D_\mu \equiv \partial_\mu + iqA_\mu$ . Following previous work, we use the aforementioned approximations when computing the effective potential.

In this setup, the effective potential for two flavors of quarks with  $N_c$  colors in the presence of a homogeneous and static magnetic field  $\mathbf{B}$  in the cold and dense limit can be written as the sum of three contributions [28]:

$$V_{\text{eff}}(\bar{\sigma}) = U_{\text{cl}}(\bar{\sigma}) + U_f^{\text{vac}}(\bar{\sigma}, B) + U_f^{\text{med}}(\bar{\sigma}, \mu, B). \quad (2)$$

The first term is just the classical potential for  $\sigma$ ; the second gives the fermionic vacuum contribution

<sup>1</sup>Recent lattice results seem to challenge this connection in the chiral limit [20], although further detailed studies are still necessary.

<sup>2</sup>See, however, Ref. [27], where the authors consider thermal meson fluctuations using resummations, and Ref. [12], where the authors compute the one-loop correction to the classical potential and systematically treat vacuum terms.

$$U_f^{\text{vac}} = -\frac{N_c}{2\pi^2} \sum_f (q_f B)^2 \left[ \zeta'_H(-1, x_f) + \frac{x_f^2 - x_f}{2} \log x_f + \frac{x_f^2}{4} \right], \quad (3)$$

where  $x_f = M_q^2/(2|q_f|B)$ ,  $M_q = g\bar{\sigma}$  is the quark dynamically generated mass,  $q_f$  is the electric charge of quark species  $f$  and  $\zeta'_H$  denotes the derivative with respect to the first argument of the Hurwitz  $\zeta$  function. Finally, the last term of Eq. (2) is the medium contribution due to the quarks (see, e.g., Ref. [29]):

$$U_f^{\text{med}} = -\frac{N_c}{4\pi^2} \sum_f \sum_{\nu=0}^{\nu_{\text{max}}} (2 - \delta_{\nu 0}) |q_f| B \left[ \mu \sqrt{\mu^2 - M_{fB}^2} + M_{fB}^2 \log \left( \frac{\mu + \sqrt{\mu^2 - M_{fB}^2}}{M_{fB}} \right) \right]. \quad (4)$$

In this last expression we assume that both fermion species have the same chemical potential  $\mu$ . In addition,  $M_{qB}^2 = M_q^2 + 2\nu|q_f|B$  denotes the magnetic correction to the quark mass and  $\nu$  is an integer value that labels Landau levels. The last occupied level is given by

$$\nu_{\text{max}} = \left\lfloor \frac{\mu^2 - M_f^2}{2|q_f|B} \right\rfloor. \quad (5)$$

This effective potential exhibits a first-order phase transition for a critical value  $\mu_c(B)$  of the chemical potential.

Recent lattice QCD results have shown that the critical temperature for the chiral and deconfinement transitions decreases as the external magnetic field is increased. However, models like the one we adopt are not able to reproduce this behavior. It is true that we are considering the cold and dense scenario, which for the time being is not accessible to lattice simulations; nevertheless, if the mechanism behind inverse magnetic catalysis is related to the running of the coupling constant, as argued in Ref. [30] for instance, one should expect to see the same behavior when looking at the critical chemical potential. Our analysis will not take this into consideration, as our point is not to give numerically accurate results, but to discuss how different quantities compete when one computes the typical time scales.

### III. SURFACE TENSION AND NUCLEATION

Given the effective potential, we can proceed to the study of the phase conversion process driven by the chiral transition. The physical setup we have in mind is that of a collapsing star and, more specifically, the scenario of magnetar formation. Thus, we investigate whether chirally symmetric matter can be nucleated as the density increases in the presence of a strong magnetic field.

In our analysis, we focus on homogeneous nucleation. Dynamically, there are two ways by which nucleation can occur: thermal activation and quantum tunneling. At the temperatures that correspond to the scenario at hand, of the order of 10–30 MeV, and in the presence of a barrier in the effective potential, thermal activation is by far the dominant way [10]. Once the barrier disappears, the initial state of the system is no longer in a metastable vacuum, so that spinodal decomposition takes place and the phase conversion is explosive [31].

It is important to state that there is no contradiction in considering thermal activation of bubbles and taking the cold, i.e.,  $T \sim 0$ , limit to compute the effective potential [see Eq. (4)]. When we focus on thermal nucleation, we are ultimately comparing temperature with the height of barrier separating true and false vacua, whereas when we consider the cold limit we compare it with the quark chemical potential. Indeed, in our setup the temperature is high enough to enable thermal activation and low enough to justify the use of the zero-temperature effective potential.

Furthermore, thermal corrections were computed in Ref. [12]; the results vary within  $\sim 10\%$  when compared to the zero-temperature approximations. Thus, the qualitative behavior and the order of magnitude of our estimates will not change drastically by including or not the thermal corrections.<sup>3</sup>

Our aim is to estimate typical times scales for the nucleation process and to understand under which conditions it is favored. In other words, which are the features that can make nucleation happen effectively in magnetar matter, producing chirally symmetric matter in the core of such stars? As mentioned previously, a key quantity seems to be the surface tension, since it is the amount of energy needed to build up a barrier separating the two phases. In other words, the surface tension is the energetic cost to create a bubble.

#### A. Extracting nucleation parameters from the effective potential

Since we are not concerned with numerical precision, but rather with obtaining reasonable estimates and the qualitative functional behavior, it is convenient to work with approximate analytic relations by fitting the effective potential in the relevant region. This can be done conveniently by using a quartic polynomial and imposing the thin-wall limit. In the range between the critical chemical potential  $\mu_c$  and the spinodal  $\mu_{\text{sp}}$ , the effective potential can be written in the following Landau-Ginzburg form [25,32]:

$$V_{\text{eff}} \approx \sum_{n=0}^4 a_n \phi^n. \quad (6)$$

<sup>3</sup>It has also been shown that thermal fluctuations and quantum vacuum corrections compete when they are included in  $V_{\text{eff}}$  [12].

Although this approximation is not able to reproduce the three minima of  $V_{\text{eff}}$ , the polynomial form gives a good quantitative description of the function in the region containing the two minima representing the symmetric and broken phases as well as the barrier between them.

A quartic potential such as Eq. (6) can always be written in the form

$$\mathcal{V}(\varphi) = \alpha(\varphi^2 - a^2)^2 + j\varphi, \quad (7)$$

with the coefficients above defined in terms of the  $a_n$  as follows [25,32]:

$$\alpha = a_4, \quad (8a)$$

$$a^2 = \frac{1}{2} \left[ -\frac{a_2}{a_4} + \frac{3}{8} \left( \frac{a_3}{a_4} \right)^2 \right], \quad (8b)$$

$$j = a_4 \left[ \frac{a_1}{a_4} - \frac{1}{2} \frac{a_2}{a_3} + \frac{1}{8} \left( \frac{a_3}{a_4} \right)^3 \right], \quad (8c)$$

$$\varphi = \phi + \frac{1}{4} \frac{a_3}{a_4}. \quad (8d)$$

The new potential  $\mathcal{V}(\varphi)$  reproduces the original  $V_{\text{eff}}(\phi)$  up to a shift in the zero of energy. We are interested in the effective potential only between  $\mu_c$  and  $\mu_{\text{sp}}$ . At  $\mu_c$ , we will have two distinct minima of equal depth. This clearly corresponds to the choice  $j = 0$  in Eq. (7), so that  $\mathcal{V}$  has minima at  $\varphi = \pm a$  and a maximum at  $\varphi = 0$ . The minimum at  $\varphi = -a$  and the maximum move closer together as the chemical potential is shifted, and they merge at  $\mu_{\text{sp}}$ . Thus, the spinodal requires  $j/\alpha a^3 = -8/3\sqrt{3}$  in Eq. (7). The parameter  $j/\alpha a^3$  falls roughly linearly from 0, at  $\mu = \mu_c$ , to  $-8/3\sqrt{3}$  at the spinodal.

In the thin-wall limit the explicit form of the critical bubble is given by [22]

$$\varphi_b(r, \xi, R_c) = \varphi_f + \frac{1}{\xi\sqrt{2\alpha}} \left[ 1 - \tanh\left(\frac{r - R_c}{\xi}\right) \right], \quad (9)$$

where  $\varphi_f$  is the new false vacuum,  $R_c$  is the radius of the critical bubble, and  $\xi = 2/m$ , with  $m^2 \equiv \mathcal{V}''(\varphi_f)$ , is a measure of the wall thickness. The thin-wall limit corresponds to  $\xi/R_c \ll 1$  [22], which can be rewritten as  $(3|j|/8\alpha a^3) \ll 1$ . Nevertheless, it was shown in [32,33], for the case of zero density and finite temperature, that the thin-wall limit becomes very inaccurate as one approaches the spinodal. (This is actually a very general feature of this description [31].) In this vein, the analysis presented below is to be regarded as semiquantitative, and it provides estimates, not accurate results.

In terms of the parameters  $\alpha$ ,  $a$ , and  $j$  defined above, one finds [25,32]

$$\varphi_{t,f} \approx \pm a - \frac{j}{8\alpha a^2}, \quad (10)$$

$$\xi = \left[ \frac{1}{\alpha(3\varphi_f^2 - a^2)} \right]^{1/2} \quad (11)$$

in the thin-wall limit. The surface tension  $\Sigma$  is given by

$$\Sigma \equiv \int_0^\infty dr \left( \frac{d\varphi_b}{dr} \right)^2 \approx \frac{2}{3\alpha\xi^3}, \quad (12)$$

and the critical radius is obtained from  $R_c = (2\Sigma/\Delta V)$ , where  $\Delta V \equiv V(\phi_f) - V(\phi_t) \approx 2a|j|$ . Finally, the free energy of a critical bubble is given by  $F_b = (4\pi\Sigma/3)R_c^2$ , and from knowledge of  $F_b$  one can evaluate the nucleation rate  $\Gamma \sim e^{-F_b/T}$ . In calculating thin-wall properties, we shall use the approximate forms for  $\phi_t$ ,  $\phi_f$ ,  $\Sigma$ , and  $\Delta V$  for all values of the potential parameters.

## IV. RESULTS

In this section we use the method described above to describe quantitatively the nucleation process in the LSMq in the presence of a magnetic background field. We compute different nucleation parameters for the formation of chirally symmetric droplets in a chirally asymmetric medium for values of the external magnetic field that are compatible with what one expects to be relevant to magnetar matter. As an initial step, we analyze how the critical chemical potential depends on  $B$ .

### A. Landau level filling and oscillations

When studying the critical behavior of the LSMq in the presence of an external magnetic field, the first question we should consider is how the position of the critical line is affected by  $B$ . The plot in Fig. 1 shows the behavior of the critical chemical potential  $\mu_c(B)$  normalized by the critical chemical potential in the absence of the external field,  $\mu_c(0) = \mu_c^0 \approx 305$  MeV.

From the plot it is clear that  $\mu_c$  has a nonmonotonic dependence on  $B$ ; it oscillates and reaches a minimum value for  $eB \approx 10m_\pi^2$ . The results show clearly that the presence of a moderate external magnetic field can reduce the value of  $\mu_c$  up to 15%.

The small oscillations observed for  $eB \lesssim 4m_\pi^2$  are analogous to the de Haas–van Alphen oscillations in metallic crystals. They are related to the fact that, as we vary the magnetic field, the degeneracy of the Landau levels and the spacing between them are modified, so that the level filling varies with  $B$ . On the other hand, the behavior for  $eB \gtrsim 4m_\pi^2$  is purely due to the lowest Landau level filling. For a detailed discussion see Ref. [34].



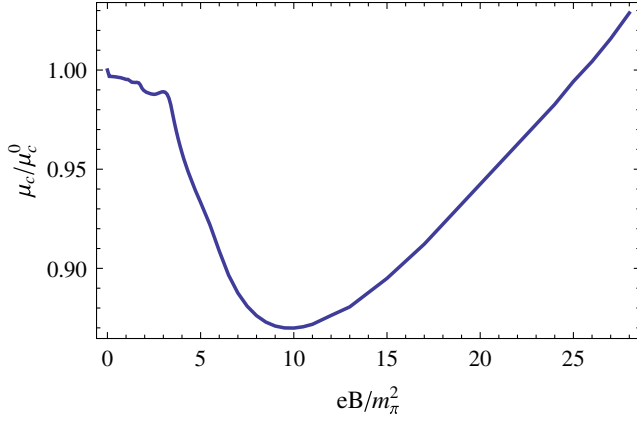


FIG. 1 (color online). Critical chemical potential  $\mu_c$  as a function of  $B$ .

### B. Nucleation parameters

Oscillations are not only seen in the behavior of  $\mu_c(B)$ . In fact, as the following plots show, all the nucleation parameters have a nontrivial oscillatory dependence on the magnetic field.

Recall that whenever a bubble is formed, its interior tends to lower the free energy of the system, since the field within it sits on the true vacuum. On the other hand, the surface of the bubble tends to increase it, as discussed previously. The critical bubble is the one whose energetic gain due to the volume exactly compensates the cost of the surface. Thus, to minimize the energy, any bubble smaller than the critical one will shrink, and the ones that are bigger will expand. Therefore, the radius of the critical bubble, or critical radius, sets the threshold between suppressed and favored bubbles.

In Fig. 2 we show this quantity as a function of the quark chemical potential for different values of magnetic field. It is interesting to notice that, as a consequence of the critical

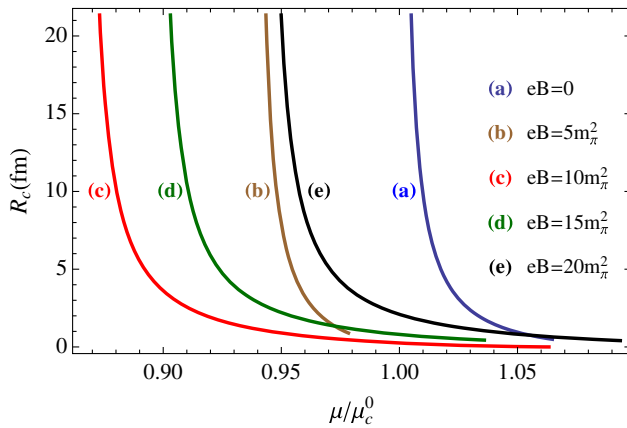


FIG. 2 (color online). Critical radius of chirally symmetric droplets as a function of the quark chemical potential for different values of  $eB$ .

chemical potential oscillation, the metastable region shifts when the magnetic field varies: first to lower values of  $\mu$  and then in the opposite direction.

As mentioned in the previous section, the correlation length  $\xi$  provides a measure of the thickness of the bubble wall. The thin-wall approximation relies on the assumption that  $\xi/R_c \ll 1$  or, equivalently, that the free energy difference between both vacua is small compared to the barrier between them. In Fig. 3 we plot this quantity as a function of quark chemical potential. As one should expect, this assumption is reasonable far from the spinodal, in the vicinity of the critical line. Nevertheless, in the spirit of providing estimates and the qualitative behavior, we apply the thin-wall limit in the whole range of chemical potentials between  $\mu_c$  and  $\mu_{sp}$ .

Finally, in Fig. 4 we present the results for the surface tension as a function of quark chemical potential for different magnetic fields. This plot shows clearly that for  $B \lesssim 5m_\pi^2$  the presence of an external magnetic field can actually reduce the energetic cost to build up the bubble wall, which would, in principle, favor nucleation in this scenario. However, the behavior of  $\mu_c$  as a function of the magnetic field already gives a hint that the situation is not so straightforward.

### C. Estimating typical time scales

To obtain an estimate of the typical time scales involved in the nucleation of chirally symmetric matter in a cold and dense medium under the influence of an external magnetic field, we first need an estimate of the nucleation rate per unit volume, which can be written as  $\Gamma \sim T_f^4 e^{-F_b/T_f}$ , where  $F_b$  is the free energy of the critical bubble and the prefactor just gives an upper limit with the correct dimensions [31]. Here, we take  $T_f = 30$  MeV as a typical temperature for protostars. In doing so, we are neglecting the temperature dependence of the critical-bubble free energy or, as we

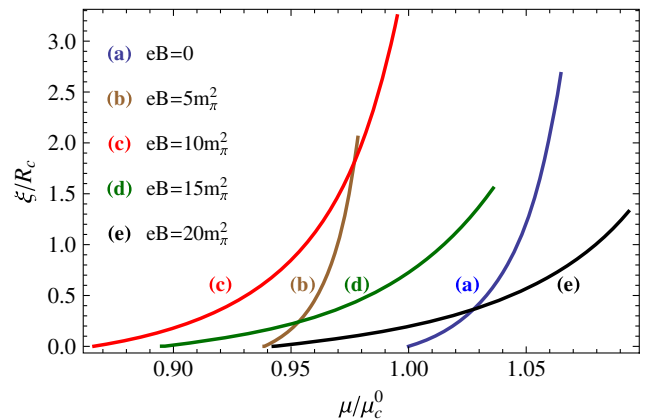


FIG. 3 (color online). Ratio between the correlation length  $\xi$  and the critical radius as a function of quark chemical potential for different values of  $eB$ .

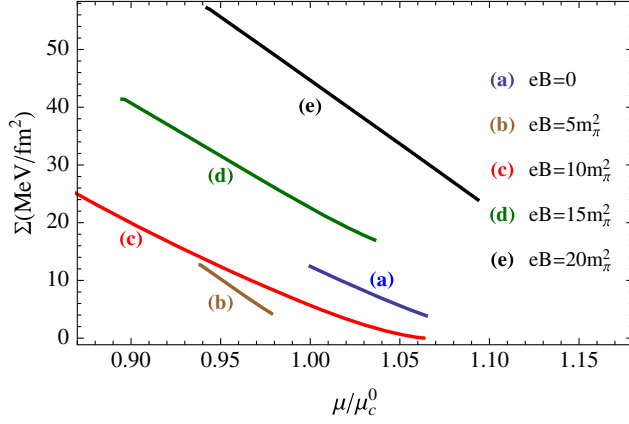


FIG. 4 (color online). Surface tension as a function of quark chemical potential for different values of  $eB$ .

discussed before, using the cold and dense effective potential since the difference scales justify this procedure.

In Fig. 5 we show the results for  $\Gamma$  as a function of the chemical potential for the same values of magnetic field adopted before. Again, a nontrivial oscillation with the magnetic field can be detected.

In order to estimate the typical time scales for the phase conversion process, i.e., the formation of chiral quark matter in the core of magnetars, we follow Ref. [10] and define the nucleation time as being the time it takes for the nucleation of a single critical bubble inside a volume of  $1 \text{ km}^3$ , which is typical of the core of a protoneutron star, i.e.,

$$\tau \equiv \left( \frac{1}{1 \text{ km}^3} \right) \frac{1}{\Gamma}. \quad (13)$$

Figure 6 exhibits this quantity as a function of the chemical potential for different values of  $eB$ . The relevant time scale to compare is the time interval the system takes from the critical chemical potential to the spinodal during the star collapse. Implicitly, in the expression above we are using an

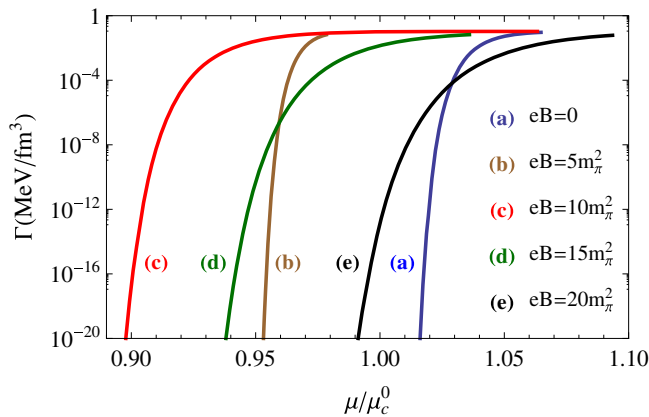


FIG. 5 (color online). Nucleation rate as a function of the quark chemical potential for different values of  $eB$ .

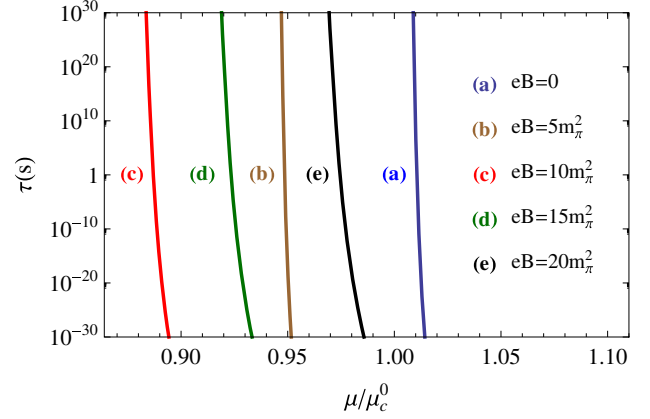


FIG. 6 (color online). Nucleation time as a function of quark chemical potential for different values of  $eB$ .

approximation of constant density and temperature over the core, which should give a good estimate as the density profile in this region of the star is quite flat [1]. The plot shows that moderate magnetic fields,  $B \lesssim 20m_\pi^2$ , can actually favor nucleation, as a given nucleation time is achieved for lower values of chemical potential.

## V. SUMMARY AND FINAL REMARKS

In this paper we have used the LSMq minimally coupled to an external classical magnetic field in a cold and dense environment as a simple model to describe critical properties of strongly interacting matter in the core of a magnetar, in particular, the likelihood of nucleating approximately chiral quark droplets. Using the one-loop effective potential we computed all relevant nucleation parameters within the thin-wall approximation and obtained an estimate for the typical time scales. Our findings indicate that nucleation may be present in the phenomenologically interesting range of magnetic fields. Of course, one also has to simulate in detail the evolution of the density profile of the protostar to make any stronger assertion.

The results obtained for the surface tension and nucleation time are very interesting, showing that many different effects sum up in a nontrivial fashion yielding a small nucleation time for cases whose surface tension are not so small. Specifically, the  $B$  dependence of  $\mu_c$  and the fact that the difference between the free energy of the vacua increases faster for higher values of magnetic field can combine in such a way that cases with a higher surface tension could have a smaller critical radius, ultimately favoring the nucleation picture. Therefore, for magnetars it is not enough to consider the behavior (and value) of the surface tension to address the competition between relevant time scales.

Despite its content of quarks and mesons, the linear sigma model provides essentially a chiral description; i.e., it does not contain essential ingredients to describe nuclear matter, such as the saturation density and the binding

energy. Nevertheless, this analysis has unveiled how the process of Landau level filling affects the nucleation parameters in a nontrivial way, bringing new forms of competition between them and affecting qualitatively the dynamics of quark matter formation in compact stars.

## ACKNOWLEDGMENTS

The authors would like to thank A. Schmitt for discussion. This work was partially supported by CAPES, CNPq and FAPERJ.

- 
- [1] N. Glendenning, *Compact Stars: Nuclear Physics, Particle Physics, and General Relativity* (Springer, London, 2012).
- [2] R. C. Duncan and C. Thompson, *Astrophys. J.* **392**, L9 (1992); C. Thompson and R. C. Duncan, *Astrophys. J.* **408**, 194 (1993).
- [3] E. J. Ferrer, V. de la Incera, J. P. Keith, I. Portillo, and P. L. Springsteen, *Phys. Rev. C* **82**, 065802 (2010).
- [4] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, *Nucl. Phys. A* **803**, 227 (2008); V. Skokov, A. Y. Illarionov, and V. Toneev, *Int. J. Mod. Phys. A* **24**, 5925 (2009); V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya, V. P. Konchakovski, and S. A. Voloshin, *Phys. Rev. C* **83**, 054911 (2011).
- [5] H. Heiselberg and M. Hjorth-Jensen, *Phys. Rep.* **328**, 237 (2000).
- [6] D. H. Rischke, *Prog. Part. Nucl. Phys.* **52**, 197 (2004).
- [7] F. Weber, *Prog. Part. Nucl. Phys.* **54**, 193 (2005).
- [8] M. Alford, D. Blaschke, A. Drago, T. Klähn, G. Pagliara, and J. Schaffner-Bielich, *Nature (London)* **445**, E7 (2007).
- [9] I. Sagert, T. Fischer, M. Hempel, G. Pagliara, J. Schaffner-Bielich, A. Mezzacappa, F.-K. Thielemann, and M. Liebendörfer, *Phys. Rev. Lett.* **102**, 081101 (2009).
- [10] B. W. Mintz, E. S. Fraga, G. Pagliara, and J. Schaffner-Bielich, *Phys. Rev. D* **81**, 123012 (2010); B. W. Mintz, E. S. Fraga, J. Schaffner-Bielich, and G. Pagliara, *J. Phys. G* **37**, 094066 (2010).
- [11] I. Bombaci, D. Logoteta, P. K. Panda, C. Providencia, and I. Vidana, *Phys. Lett. B* **680**, 448 (2009).
- [12] L. F. Palhares and E. S. Fraga, *Phys. Rev. D* **82**, 125018 (2010).
- [13] M. B. Pinto, V. Koch, and J. Randrup, *Phys. Rev. C* **86**, 025203 (2012).
- [14] B. W. Mintz, R. Stiele, R. O. Ramos, and J. Schaffner-Bielich, *Phys. Rev. D* **87**, 036004 (2013).
- [15] G. Lugones, A. G. Grunfeld, and M. Al Ajmi, *Phys. Rev. C* **88**, 045803 (2013).
- [16] W. y. Ke and Y. x. Liu, *Phys. Rev. D* **89**, 074041 (2014).
- [17] A. F. Garcia and M. B. Pinto, *Phys. Rev. C* **88**, 025207 (2013).
- [18] O. Scavenius, A. Mocsy, I. N. Mishustin, and D. H. Rischke, *Phys. Rev. C* **64**, 045202 (2001).
- [19] R. D. Pisarski and F. Wilczek, *Phys. Rev. D* **29**, 338 (1984).
- [20] C. Bonati, P. de Forcrand, M. D’Elia, O. Philipsen, and F. Sanfilippo, *Phys. Rev. D* **90**, 074030 (2014).
- [21] C. Aragao De Carvalho, *Nucl. Phys. B* **324**, 729 (1989).
- [22] E. S. Fraga and C. A. A. de Carvalho, *Phys. Rev. B* **52**, 7448 (1995).
- [23] S. Novikov, S. V. Manakov, L. P. Pitaevskii, and V. E. Zakharov, *Theory of Solitons* (Consultants Bureau, New York, 1984).
- [24] D. G. Barci, E. S. Fraga, and C. A. A. de Carvalho, *Phys. Rev. D* **55**, 4947 (1997).
- [25] B. G. Taketani and E. S. Fraga, *Phys. Rev. D* **74**, 085013 (2006).
- [26] D. Nickel, *Phys. Rev. D* **80**, 074025 (2009).
- [27] G. W. Carter, P. J. Ellis, and S. Rudaz, *Nucl. Phys. A* **618**, 317 (1997).
- [28] E. S. Fraga, *Lect. Notes Phys.* **871**, 121 (2013).
- [29] D. P. Menezes, M. Benghi Pinto, S. S. Avancini, A. Perez Martinez, and C. Providencia, *Phys. Rev. C* **79**, 035807 (2009).
- [30] R. L. S. Farias, K. P. Gomes, G. I. Krein, and M. B. Pinto, *Phys. Rev. C* **90**, 025203 (2014).
- [31] J. D. Gunton, M. San Miguel, and P. S. Sahni, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. L. Lebowitz (Academic Press, New York, 1983).
- [32] O. Scavenius, A. Dumitru, E. S. Fraga, J. T. Lenaghan, and A. D. Jackson, *Phys. Rev. D* **63**, 116003 (2001).
- [33] A. Bessa, E. S. Fraga, and B. W. Mintz, *Phys. Rev. D* **79**, 034012 (2009).
- [34] F. Preis, A. Rebhan, and A. Schmitt, *J. High Energy Phys.* **03** (2011) 033; , *Lect. Notes Phys.* **871**, 51 (2013); A. Haber, F. Preis, and A. Schmitt, *Phys. Rev. D* **90**, 125036 (2014).