Renormalizability of Yang-Mills theory with Lorentz violation and gluon mass generation

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We show that pure Yang-Mills theories with Lorentz violation are renormalizable to all orders in perturbation theory. To do this, we employ the algebraic renormalization technique. Specifically, we control the breaking terms with a suitable set of external sources, which eventually attain certain physical values. The Abelian case is also analyzed as a starting point. The main result is that the renormalizability of the usual Maxwell and Yang-Mills sectors are both left unchanged. Furthermore, in contrast to Lorentz-violating QED, the CPT-odd violation sector of Yang-Mills theories renormalizes independently. Moreover, the method induces mass terms for the gauge field in a natural way, while the photon remains massless (at least in the sense of a Proca-like term). The entire analysis is carried out in the Landau gauge.

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I. INTRODUCTION

Lorentz and gauge symmetries play an important and perhaps indispensable role in quantum field theory and particle physics [1–4]. From the classification of particles to renormalizability proofs, these symmetries are crucial. However, theories for which Lorentz symmetry is not required have received considerable attention in the last few decades [5–10]. Although direct effects of such theories would only appear beyond the Planck scale, some "cumulative" effects could arise as well [11–14]. Even though these types of theories originate as effective models from an extremely high-energy theory [15,16], they should be studied in the context of quantum field theory. And, in order to provide reliable and consistent theoretical predictions, certain attributes—such as stability, renormalizability, unitarity, and causality—are very welcome features. For example, stability requires that the Hamiltonian of the theory is bounded from below, and causality refers to the commutativity of observables at space-like intervals; see, for instance, Refs. [5,6,17–19] for more details. In this work we confine ourselves to a detailed analysis of the renormalizability of pure non-Abelian gauge theories with Lorentz violation.

Models with broken Lorentz and CPT symmetries are characterized by the presence of background tensorial fields coupled to the fundamental fields of the theory. Typically, the Lorentz-violation background fields arise in the scenario of effective field theories originating from fundamental models, such as string theories [15], noncommutative field theories [20–24], supersymmetric field theories [25–27], and loop quantum gravity [28]. In string theory, for instance, the Lorentz symmetry breaking arises from a spontaneous symmetry breaking, specifically from the nontrivial vacuum expectation value of the tensorial fields. Such background fields could contain effects of an underlying fundamental theory at the Planck mass scale $M_P \sim 10^{19}$ GeV. In fact, there is some hope of detecting possible signals for bounds of these violating coefficients, such as in high-precision experiments in atomics processes [12,29–31]. A theoretical proposal to describe the Lorentz symmetry breaking at this scale is the standard model extension (SME). In this model, the Lorentz-breaking coefficients are introduced through couplings with fundamentals fields of the standard model and the model is power-counting renormalizable [6]. Another theoretical proposal for Lorentz violation are the modified dispersion relations (MDRs) [32]. Essentially, these new dispersion relations carry extra contributions that depend on the energy scale, and which are only meaningful at ultrahigh energies and suppressed in the low-energy limit. In principle, ultrahigh-energy cosmic rays at the Planck energy scale where Lorentz and CPT symmetry breaking would take place are generated in astrophysical processes. A possible explanation for the observation of the apparent excess of cosmic rays in this region of energy [33] are the MDRs which, in this case, suggest that these cosmic rays could develop velocities faster than light.

Concerning the renormalization properties of Lorentz-violating QED, a one-loop renormalization analysis was discussed in Ref. [34] and a full algebraic study at all orders in perturbation theory was established in Ref. [35]. Another interesting study of renormalizability issues in Lorentz- and CPT-violating QED was performed in Ref. [36]. In that work, it was assumed that the fields of this model reside in a curved manifold, and the Lorent- and CPT-violating parameters were treated as classical fields rather than constants, which happens to be very similar to the approach employed in the present work.

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Until now, the non-Abelian sector of the standard model extension has received little attention from both theoretical studies and experimental tests for the bounds of the Lorentz-violating background parameters. As pointed out in Ref. [37], the ultraviolet behavior of the CPT-even coupling may give a great bound for these coefficients, in contrast to CPT-odd couplings. In what concerns the renormalization properties of pure Yang-Mills (YM) theory with Lorentz violation, it was shown in Ref. [37] that this model can be renormalized at one-loop order. It is worth mentioning that a non-Abelian Chern-Simons-like term can be induced from the Abelian Lorentz-violating term at the level of one-loop radiative corrections [38].

In the present work we focus on the non-Abelian sector of the SME, i.e., pure Yang-Mills theory with Lorentz violation. In particular, we employ the algebraic renormalization approach [39] to prove that this model is renormalizable, at least to all orders in perturbation theory. In our analysis we include all possible breaking terms. Besides Becchi-Rouet-Stora-Tyutin (BRST) quantization, we introduce a suitable set of sources that controls the Lorentz-breaking terms. Eventually, in order to regain the original action, these sources attain specific physical values. This trick is originally due to Symanzik [40] and was vastly employed in non-Abelian gauge theories in order to control a soft BRST symmetry breaking; see, for instance, Refs. [41–45]. Essentially, the broken model is embedded in a larger theory where the relevant symmetry is respected. Then, after renormalization, the theory is contracted down to the original model. We will also give attention to the Abelian theory in the presence of Lorentz violation and in the absence of fermions as starting point.[1] Adopting the Symanzik source approach, we can introduce the most general action which carries, for instance, vacuum-type terms as well as dimension-two condensate terms. The price we pay is that extra independent renormalization parameters are needed to account for the extra vacuum divergences. Remarkably, the extra condensate-type term $A^a_\mu A^a_\mu$ arises due to a coupling with the CPT-odd sector of the model and also carries an independent renormalization coefficient. We then have an induced mass term for the gluon originating from the Lorentz-violating terms. However, these terms are ruled out in the Lorentz-violating Maxwell theory due to the fact that the ghost equation is not integrated, making it stronger than its non-Abelian version. These different characteristics between the Ward identities of the Abelian and non-Abelian models will result in different renormalization properties among Maxwell and Yang-Mills Lorentz-violation coefficients. For instance, we will show that the CPT-odd breaking term in the Maxwell theory, $\epsilon_{\mu\nu\alpha\beta}v_\mu A_\nu\partial_\alpha A_\beta$, does not renormalize. Nevertheless, the CPT-odd breaking term in Yang-Mills theory renormalizes independently.

This work is organized as follows. Section II is dedicated to the renormalizability proof of the Maxwell theory with Lorentz violation. In Sec. III, we provide the definitions and conventions of the pure Yang-Mills theory with Lorentz violation and the BRST quantization of the model with the extra set of auxiliary sources. Then, in Sec. IV we study the renormalizability of the model. Our final considerations are given in Sec. V.

II. LORENTZ-VIOLATING MAXWELL THEORY

We consider the U(1) Abelian gauge theory with Lorentz violation. For convenience the scenario for this theory (and also for the non-Abelian case) is Euclidean four-dimensional spacetime.[2] The action of the model is as follows[3]: [34]

$$S_0 = S_M + S_{LVE} + S_{LVO}, \tag{1}$$

where

$$S_M = \frac{1}{4}\int d^4x F_{\mu\nu}F_{\mu\nu}, \tag{2}$$

is the Maxwell action. The field strength is defined as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, where A_μ is the gauge field. The CPT-even Lorentz-violating sector is given by

$$S_{LVE} = \frac{1}{4}\int d^4x \kappa_{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}, \tag{3}$$

while the CPT-odd Lorentz-violation term is defined as

$$S_{LVO} = \int d^4x \epsilon_{\mu\nu\alpha\beta}v_\mu A_\nu \partial_\alpha A_\beta. \tag{4}$$

The Lorentz violation is characterized by the fields v_μ, with mass dimension one, and $\kappa_{\alpha\beta\mu\nu}$, which is dimensionless. These tensors fix privileged directions in spacetime, dooming it to anisotropy. Tensorial fields with even numbers of indices preserve CPT, while tensors with odd numbers of indices do not. The tensor $\kappa_{\alpha\beta\mu\nu}$ obeys the same properties as the Riemann tensor, and is double traceless:

[1] In fact, the presence of fermions in a Lorentz-violating model (even an Abelian model) will make the study of renormalizability very difficult, at least in our approach. Thus the Abelian model is studied here in the absence of fermions in order to compare it with the non-Abelian case; the latter introduces many difficulties compared with the former, even in the absence of fermions. The study of the fermionic sector is left for future investigation [46].

[2] Besides the fact that the Euclidean metric is easier to handle, this choice is convenient in the treatment of nonperturbative effects where it is unknown if the Wick rotation is valid.

[3] We are not considering fermions in this work, as mentioned in the Introduction.

$$\kappa_{\alpha\beta\mu\nu} = \kappa_{\mu\nu\alpha\beta} = -\kappa_{\beta\alpha\mu\nu},$$
$$\kappa_{\alpha\beta\mu\nu} + \kappa_{\alpha\mu\nu\beta} + \kappa_{\alpha\nu\beta\mu} = 0,$$
$$\kappa_{\mu\nu\mu\nu} = 0. \quad (5)$$

As the reader can easily verify, the action (1) is a Lorentz scalar, which is invariant under Lorentz transformations in the observer's frame; in contrast, it also presents a violation with respect to particle Lorentz transformations.

In the present work, we employ the BRST quantization method and adopt the Landau gauge condition $\partial_\mu A_\mu = 0$. Thus, besides the photon field, we introduce the Lautrup-Nakanishi field b, and the Faddeev-Popov ghost and antighost fields c and $\bar{c}$, respectively. The respective BRST transformations are

$$sA_\mu = -\partial_\mu c, \quad sc = 0, \quad s\bar{c} = b, \quad sb = 0, \quad (6)$$

where s is the nilpotent BRST operator. The quantum numbers of the fields and background tensors are displayed in Table I. The full Landau gauge fixed action is

$$S_0 = S_M + S_{LVE} + S_{LVO} + S_{gf}, \quad (7)$$

where

$$S_{gf} = s\int d^4x \bar{c}\partial_\mu A_\mu = \int d^4x (b\partial_\mu A_\mu + \bar{c}\partial^2 c) \quad (8)$$

is the gauge-fixing action enforcing the Landau gauge condition. The Landau gauge is chosen for a few simple reasons [39]: i) it is a covariant gauge; ii) it has a rich symmetry content; iii) it is a fixed point of the renormalization group; iv) it is the simplest case, so it is a convenient starting choice; and v) it is renormalizable in the ordinary case.

Lorentz symmetry plays a fundamental role in the renormalizability of gauge theories, and thus the presence of a Lorentz-violating sector demands extra care. To deal with this obstacle we replace each of the background tensors by an external classical source and, possibly, its BRST doublet counterpart (if needed). Thus, the local composite operator (whose background tensors are coefficients) will be coupled to one of these sources. Indeed, there will be two classes of sources: BRST-invariant sources and BRST doublet sources. The first class will be coupled to the BRST-/gauge-invariant composite operators, while the second class couples to the other operators.

TABLE I. Quantum numbers of the fields and background tensors.

fields/tensors	A	b	c	$\bar{c}$	v	κ
UV dimension	1	2	0	2	1	0
Ghost number	0	0	1	−1	0	0

TABLE II. Quantum numbers of the sources.

sources	λ	J	$\bar{\kappa}$
UV dimension	1	1	0
Ghost number	−1	0	0

Thus, we define the following invariant source:

$$s\bar{\kappa}_{\alpha\beta\mu\nu} = 0. \quad (9)$$

The BRST doublet sources are given by

$$s\lambda_{\mu\nu\alpha} = J_{\mu\nu\alpha}, \quad sJ_{\mu\nu\alpha} = 0. \quad (10)$$

The quantum numbers of the sources are displayed in Table II. Eventually, these sources will attain the following physical values:

$$J_{\mu\nu\alpha}|_{\mathrm{phys}} = v_\beta \epsilon_{\beta\mu\nu\alpha},$$
$$\lambda_{\mu\nu\alpha}|_{\mathrm{phys}} = 0,$$
$$\bar{\kappa}_{\alpha\beta\mu\nu}|_{\mathrm{phys}} = \kappa_{\alpha\beta\mu\nu}. \quad (11)$$

Thus, we replace the action (7) by[4]

$$S = S_M + S_{LO} + S_{LE} + S_{gf}, \quad (12)$$

where now

$$S_{LE} = \frac{1}{4}\int d^4x \bar{\kappa}_{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu},$$
$$S_{LO} = s\int d^4x \lambda_{\mu\nu\alpha} A_\mu \partial_\nu A_\alpha$$
$$= \int d^4x (J_{\mu\nu\alpha} A_\mu \partial_\nu A_\alpha + \lambda_{\mu\nu\alpha}\partial_\mu c \partial_\nu A_\alpha) \quad (13)$$

is the embedding of the Lorentz-violating bosonic sector. The BRST symmetry demands that all possible terms (i.e., integrated local polynomials in the fields and sources with dimension four and vanishing ghost number) that respect BRST symmetry must be added to the model. Then, by using the algebraic renormalization techniques, the Ward identities will select the terms that are actually needed (see next section). Power-counting renormalizability also allows one more term to be added to the action (12), namely

$$S_V = s\int d^4x (\zeta \lambda_{\mu\nu\alpha} J_{\mu\beta\gamma} J_{\nu\beta\kappa} J_{\gamma\kappa\alpha} + \vartheta \bar{\kappa}_{\mu\nu\alpha\beta} \lambda_{\mu\rho\omega} J_{\nu\rho\sigma} J_{\alpha\omega\delta} J_{\beta\sigma\delta})$$
$$= \int d^4x (\zeta J_{\mu\nu\alpha} J_{\mu\beta\gamma} J_{\nu\beta\kappa} J_{\gamma\kappa\alpha} + \vartheta \bar{\kappa}_{\mu\nu\alpha\beta} J_{\mu\rho\omega} J_{\nu\rho\sigma} J_{\alpha\omega\delta} J_{\beta\sigma\delta}). \quad (14)$$

[4]Since the Lorentz breaking is now controlled by the external sources, we rename the original actions without the letter "V" (for violation).

The dimensionless parameters ζ and ϑ are introduced to absorb possible vacuum divergences. A remark must be made at this point. In principle (from a power-counting analysis), we could add a series of terms of the type $\bar{\kappa}_{\alpha\beta\rho\sigma}\bar{\kappa}_{\rho\sigma\mu\nu}F_{\alpha\beta}F_{\mu\nu}$, $\bar{\kappa}_{\alpha\beta\rho\sigma}\bar{\kappa}_{\rho\sigma\omega\delta}\bar{\kappa}_{\omega\delta\mu\nu}F_{\alpha\beta}F_{\mu\nu}$, and so on. Nevertheless, all these terms could be rearranged into only one term coupled to the operator $F_{\mu\nu}F_{\mu\nu}$. This infinite series can then be recast as a single source term by means of the first equation of Eq. (13), preserving the original term. In fact, this argument is valid for all terms that mix with $\bar{\kappa}_{\mu\nu\alpha\beta}$ in the Abelian or non-Abelian cases. Formally, one can consider the infinite tower of terms (and their respective counterterms), and the redefinition only applies after the absorption of the divergences. Obviously, the classical character of $\bar{\kappa}_{\mu\nu\alpha\beta}$ is crucial to this argument (see also Refs. [47,48]).

The complete action we have is

$$\Xi = S + S_V. \tag{15}$$

Explicitly, the action (15) has the following form:

$$\begin{aligned}\Xi =& \frac{1}{4}\int d^4x F_{\mu\nu}F_{\mu\nu} + \frac{1}{4}\int d^4x \bar{\kappa}_{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu} \\ &+ \int d^4x (J_{\mu\nu\alpha}A_\mu\partial_\nu A_\alpha + \lambda_{\mu\nu\alpha}\partial_\mu c\partial_\nu A_\alpha) \\ &+ \int d^4x (b\partial_\mu A_\mu + \bar{c}\partial^2 c) \\ &+ \int d^4x (\zeta J_{\mu\nu\alpha}J_{\mu\beta\gamma}J_{\nu\beta\kappa}J_{\gamma\kappa\alpha} + \vartheta\bar{\kappa}_{\mu\nu\alpha\beta}J_{\mu\rho\omega}J_{\nu\rho\sigma}J_{\alpha\omega\delta}J_{\beta\sigma\delta}).\end{aligned} \tag{16}$$

The action (16), at the physical value of the sources (11), reduces to

$$\begin{aligned}\Xi_{\text{phys}} =& \frac{1}{4}\int d^4x F_{\mu\nu}F_{\mu\nu} + \frac{1}{4}\int d^4x \kappa_{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu} \\ &+ \int d^4x v_\beta \epsilon_{\beta\mu\nu\alpha}A_\mu\partial_\nu A_\alpha + \int d^4x (b\partial_\mu A_\mu + \bar{c}\partial^2 c) \\ &+ 2v^2\int d^4x (3\zeta v^2 - \vartheta\kappa_{\alpha\mu\sigma\mu}v_\alpha v_\sigma).\end{aligned} \tag{17}$$

A remark is now in order. The source J is introduced as a BRST doublet, where its BRST counterpart is the source λ. As a consequence, the entire term depending on J and λ is an exact BRST variation. Thus, it belongs to the nonphysical sector of the model. However, the model suffers a contraction in order to be deformed to the action of interest (the physical action). Under such a contraction, this term is moved to the physical sector of the theory. In fact, the terms depending on v_μ in the physical action can no longer be written as a BRST exact variation. Let us put this in other words. The physical action (17) is the true action (i.e., it violates Lorentz symmetry) and the violating terms cannot be written as a BRST exact variation. Thus, in order to study its renormalizability, the theory is embedded into a larger theory which displays full Lorentz and BRST symmetries. The embedding is characterized by the auxiliary sources which appear in place of the violating parameters. The physical theory is recovered from a specific choice of these sources (the physical values). These values are attained by contracting the functional space of the sources into the $\mathbb{R}^4$ space of the vector v_μ. The main idea of the method is that the model is renormalized in its embedded form, and only after the renormalization is the model contracted to the physical sector.

For completeness, we compute the propagator for the photon, in the Landau gauge, taking[5] $\kappa_{\alpha\beta\mu\nu} = 0$. The result is

$$\begin{aligned}\langle A_\mu(k)A_\nu(-k)\rangle = \frac{1}{Q}\Big[&k^2\theta_{\mu\nu} - \frac{4(v_\alpha k_\alpha)^2}{k^2}\omega_{\mu\nu} + 2S_{\mu\nu} \\ &+ \frac{4(v_\alpha k_\alpha)}{k^2}\Sigma_{\mu\nu} - 4\Lambda_{\mu\nu}\Big],\end{aligned} \tag{18}$$

where $Q = k^4 - 4[v^2k^2 - (v_\alpha k_\alpha)^2]$, and the operators

$$\begin{aligned}\theta_{\mu\nu} &= \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \\ \omega_{\mu\nu} &= \frac{k_\mu k_\nu}{k^2}, \\ S_{\mu\nu} &= i\epsilon_{\mu\nu\alpha\beta}v_\alpha k_\beta, \\ \Sigma_{\mu\nu} &= v_\mu k_\nu + v_\nu k_\mu, \\ \Lambda_{\mu\nu} &= v_\mu v_\nu\end{aligned} \tag{19}$$

form a closed algebra (see, for instance, Ref. [50] for more details). It is worth mentioning here that the physical modes of the gauge field, i.e., the photon, do not change with respect to the usual Maxwell theory with Lorentz violation; our approach does not change the kinetic part of this model and does not generate any Proca-like terms. Thus, the causality and unitarity of the model are maintained [17]. However, as is clear from Eq. (17), the vacuum of the model changes when we take the physical limit of the sources in the action (14). We will discuss these points again in the non-Abelian case.

A. Renormalizability

In order to prove that this model is renormalizable to all orders in perturbation theory, let us now display the full set of Ward identities obeyed by the action (16).

[5]The presence of a general $\kappa_{\alpha\beta\mu\nu}$ makes the computation highly nontrivial. For a detailed study of this sector see, for instance, Ref. [49].

(i) Slavnov-Taylor identity:

$$\mathcal{S}(\Xi) = \int d^4x \left(-\partial_\mu c \frac{\delta \Xi}{\delta A_\mu} + b \frac{\delta \Xi}{\delta \bar{c}} + J_{\mu\nu\alpha} \frac{\delta \Xi}{\delta \lambda_{\mu\nu\alpha}} \right) = 0. \tag{20}$$

(ii) Gauge-fixing and antighost equations:

$$\frac{\delta \Xi}{\delta b} = \partial_\mu A_\mu, \qquad \frac{\delta \Xi}{\delta \bar{c}} = \partial^2 c. \tag{21}$$

(iii) Ghost equation:

$$\frac{\delta \Xi}{\delta c} = \partial_\mu (\lambda_{\mu\nu\alpha} \partial_\nu A_\alpha) - \partial^2 \bar{c}. \tag{22}$$

In Eqs. (21) and (22), the breaking terms are linear in the fields, and thus they will remain at the classical level [39]. From Eq. (22) it is possible to predict that the CPT-odd Lorentz-violating sector of the Maxwell theory will not suffer renormalization. This is due to the fact that this term induces a violation of the ghost equation. As a consequence, a counterterm associated with the CPT-odd Lorentz-violating sector will be eliminated by the Ward identity (22).

In order to obtain the most general counterterm that can be freely added to the classical action Ξ at any order in perturbation theory, we define a general local integrated polynomial Ξ^c with dimension bounded by four and vanishing ghost number. Thus, by applying the Ward identities (20)–(22) to the perturbed action $\Xi + \varepsilon \Xi^c$, where ε is a small parameter, it is easy to find that the counterterm must obey the following constraints:

$$\mathcal{B}_\Xi \Xi^c = 0, \qquad \frac{\delta \Xi^c}{\delta b} = 0, \qquad \frac{\delta \Xi^c}{\delta \bar{c}} = 0, \qquad \frac{\delta \Xi^c}{\delta c} = 0, \tag{23}$$

where the operator $\mathcal{B}_\Xi$ is the nilpotent Slavnov-Taylor operator,

$$\mathcal{B}_\Xi = \int d^4x \left(-\partial_\mu c \frac{\delta}{\delta A_\mu} + b \frac{\delta}{\delta \bar{c}} + J_{\mu\nu\alpha} \frac{\delta}{\delta \lambda_{\mu\nu\alpha}} \right). \tag{24}$$

The first constraint of Eq. (23) states that finding the invariant counterterm is a cohomology problem for the operator $\mathcal{B}_\Xi$ in the space of the integrated local field polynomials of dimension four. From the general results of algebraic renormalization [39], it is an easy task to find

$$\Xi^c = \frac{1}{4} \int d^4x a_0 F_{\mu\nu} F_{\mu\nu} + \frac{1}{4} \int d^4x a_1 \bar{\kappa}_{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} + \mathcal{B}_\Xi \Delta^{(-1)}, \tag{25}$$

where $\Delta^{(-1)}$ is the most general local polynomial counterterm with dimension bounded by four and ghost number -1, given by

$$\Delta^{(-1)} = \int d^4x (a_2 \bar{c} \partial_\mu A_\mu + a_3 \bar{c} b + a_4 \lambda_{\mu\nu\alpha} A_\mu \partial_\nu A_\alpha + a_5 \zeta \lambda_{\mu\nu\alpha} J_{\mu\beta\gamma} J_{\nu\beta\kappa} J_{\gamma\kappa\alpha} + a_6 \vartheta \bar{\kappa}_{\mu\nu\alpha\beta} \lambda_{\mu\rho\omega} J_{\nu\rho\sigma} J_{\alpha\omega\delta} J_{\beta\sigma\delta}), \tag{26}$$

where the parameters a_i are free coefficients. Defining $\hat{\Xi} = \mathcal{B}_\Xi \Delta^{(-1)}$, one finds

$$\begin{aligned} \hat{\Xi} = {} & a_2 \int d^4x (b \partial_\mu A_\mu + \bar{c} \partial^2 c) + a_3 \int d^4x b^2 \\ & + a_4 \int d^4x (J_{\mu\nu\alpha} A_\mu \partial_\nu A_\alpha + \lambda_{\mu\nu\alpha} \partial_\mu c \partial_\nu A_\alpha) \\ & + a_5 \int d^4x \zeta J_{\mu\nu\alpha} J_{\mu\beta\gamma} J_{\nu\beta\kappa} J_{\gamma\kappa\alpha} \\ & + a_6 \int d^4x \vartheta \bar{\kappa}_{\mu\nu\alpha\beta} J_{\mu\rho\omega} J_{\nu\rho\sigma} J_{\alpha\omega\delta} J_{\beta\sigma\delta}. \end{aligned} \tag{27}$$

From the second or third constraints in Eq. (23), it follows that $a_2 = a_3 = 0$. Moreover, from the ghost equation, $a_4 = 0$. It then follows that the most general counterterm allowed by the Ward identities is given by

$$\begin{aligned} \Xi^c = {} & \frac{1}{4} \int d^4x a_0 F_{\mu\nu} F_{\mu\nu} + \frac{1}{4} \int d^4x a_1 \bar{\kappa}_{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \\ & + a_5 \int d^4x \zeta J_{\mu\nu\alpha} J_{\mu\beta\gamma} J_{\nu\beta\kappa} J_{\gamma\kappa\alpha} \\ & + a_6 \int d^4x \vartheta \bar{\kappa}_{\mu\nu\alpha\beta} J_{\mu\rho\omega} J_{\nu\rho\sigma} J_{\alpha\omega\delta} J_{\beta\sigma\delta}. \end{aligned} \tag{28}$$

It remains to be inferred whether the counterterm Ξ^c can be reabsorbed by the original action Ξ by means of the redefinition of the fields, sources, and parameters of the theory through

$$\Xi(\Phi, J, \xi) + \varepsilon \Xi^c(\Phi, J, \xi) = \Xi(\Phi_0, J_0, \xi_0) + \mathcal{O}(\varepsilon^2), \tag{29}$$

where the bare fields, sources, and parameters are defined as

$$\begin{aligned} \Phi_0 &= Z_\Phi^{1/2} \Phi, && \Phi \in \{A, b, \bar{c}, c\}, \\ J_0 &= Z_J J, && J \in \{J, \lambda, \bar{\kappa}\}, \\ \xi_0 &= Z_\xi \xi, && \xi \in \{\vartheta, \zeta\}. \end{aligned} \tag{30}$$

It is not difficult to check that this can be performed, which provides the multiplicative renormalizability proof of the theory to all orders in perturbation theory. In fact, for the independent renormalization factor of the photon, we have

$$Z_A^{1/2} = 1 + \frac{1}{2}\varepsilon a_0. \tag{31}$$

The ghost fields do not renormalize,

$$Z_c^{1/2} = Z_{\bar{c}}^{1/2} = 1, \tag{32}$$

and the Lautrup-Nakanishi field renormalization is not independent,

$$Z_b^{1/2} = Z_A^{-1/2}. \tag{33}$$

Thus, the standard QED sector remains unchanged with respect to the ordinary case. For the violating sector we have

$$\begin{aligned} Z_J &= Z_\lambda^2 = Z_A^{-1}, \\ Z_\zeta &= 1 + \varepsilon(a_5 + 4a_0), \\ Z_{\bar{\kappa}} &= 1 + \varepsilon(a_1 - a_0), \\ Z_\vartheta &= 1 + \varepsilon(a_6 - a_1 + 5a_0). \end{aligned} \tag{34}$$

From the first equation in Eq. (34), as we have pointed out before, we see that the CPT-odd Lorentz-violating coefficient v_μ does not renormalize independently, namely, its renormalization depends only on the photon renormalization. This is also clear from the final counterterm (28), where the CPT-odd part is not present and thus does not renormalize. This ends the renormalizability of the Lorentz-violating Abelian gauge theory, at least to all orders in perturbation theory.

The study of the renormalizability of pure QED might be seen as an unnecessary effort since the theory is free (we are not considering fermions at this point). In fact, no interaction terms would be generated from the analysis of quantum stability and no parameters would be renormalized; only the fields would be renormalized. Nevertheless, the study of the quantum stability of Maxwell theory with Lorentz violation using the method of external auxiliary sources can establish whether the model accepts other quadratic terms involving the sources (for instance, a mass term of the type $v^2 A_\mu A_\mu$ could appear in the physical limit). Thus, the study of the free Abelian case can be used as a first consistency check of the method. Nevertheless, the presence of the quartic J-source terms generate independent renormalizations of the vacuum energy. Moreover, the study of the free theory is always a first step before considering interacting theories and the respective violating terms, which is the case for non-Abelian theories as well as the Abelian theory with fermions.

III. PURE YANG-MILLS THEORY WITH LORENTZ VIOLATION

From now on (unless otherwise stated), we consider pure[6] Yang-Mills theory for the $SU(N)$ symmetry group with Lorentz violation. The gauge fields are algebra valued, $A_\mu = A_\mu^a T^a$, where T^a are the generators of the $SU(N)$ algebra. They are chosen to be anti-Hermitian and have vanishing trace. The typical Lie algebra is given by $[T^a, T^b] = f^{abc}T^c$, where f^{abc} are the skew-symmetric structure constants. The latin indices run as $\{a, b, c, \ldots\} \in \{1, 2, \ldots, N^2 - 1\}$.

The model is described by the following action[7] [37]:

$$\Sigma_0 = S_{YM} + \Sigma_{LVE} + \Sigma_{LVO}, \tag{35}$$

where

$$S_{YM} = \frac{1}{4}\int d^4x F_{\mu\nu}^a F_{\mu\nu}^a \tag{36}$$

is the classical Yang-Mills action. The field strength is defined as $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$. The CPT-even Lorentz-violating sector is

$$\Sigma_{LVE} = \frac{1}{4}\int d^4x \kappa_{\alpha\beta\mu\nu} F_{\alpha\beta}^a F_{\mu\nu}^a, \tag{37}$$

and the CPT-odd Lorentz-violation term is

$$\Sigma_{LVO} = \int d^4x \epsilon_{\mu\nu\alpha\beta} v_\mu \left(A_\nu^a \partial_\alpha A_\beta^a + \frac{g}{3} f^{abc} A_\nu^a A_\alpha^b A_\beta^c \right). \tag{38}$$

The Lorentz violation is characterized by the fields v_μ, with mass dimension one, and $\kappa_{\alpha\beta\mu\nu}$, which is dimensionless. These tensors have the same symmetry properties as those described in Sec. II for the Abelian case.

A. BRST quantization and the restoration of Lorentz symmetry

Gauge fixing is also required in the process of quantizing the pure Yang-Mills theory with Lorentz violation. In the following, we employ the BRST quantization method and adopt the Landau gauge condition $\partial_\mu A_\mu^a = 0$. Thus, besides the gluon field, we also need the Lautrup-Nakanishi field b^a and the Faddeev-Popov ghost and antighost fields c^a and $\bar{c}^a$, respectively. The BRST transformations of the fields are

[6]Just like the Abelian case, we are not considering fermions.
[7]No confusion is expected with the Abelian case.

$$sA^a_\mu = -D^{ab}_\mu c^b, \quad sc^a = \frac{g}{2} f^{abc} c^b c^c, \quad s\bar{c}^a = b^a, \quad sb^a = 0, \tag{39}$$

where $D^{ab}_\mu = \delta^{ab}\partial_\mu - g f^{abc} A^c_\mu$ is the covariant derivative. Thus, the Landau gauge fixed action is

$$\Sigma_0 = S_{YM} + \Sigma_{LVE} + \Sigma_{LVO} + \Sigma_{gf}, \tag{40}$$

where

$$\Sigma_{gf} = s \int d^4x \bar{c}^a \partial_\mu A^a_\mu = \int d^4x (b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b) \tag{41}$$

is the gauge-fixing action enforcing the Landau gauge condition. The Landau gauge is chosen here for the same reasons as in the Abelian case.[8] The quantum numbers of the fields and background tensors are the same as those in the Abelian case (see Table I).

To deal with the renormalizability issue we will proceed in the same way as in Sec. II, namely, we replace each background tensor by an external source and (possibly) its BRST doublet counterpart. However, the non-Abelian case is a bit more subtle than the Abelian case. For instance, let us take the Chern-Simons term. To ensure the renormalizability of the model we need two BRST doublets: one coupled to the bilinear term and another coupled to the trilinear term in the gauge field. Both terms have to be treated separately since they are independent composite operators (in the Abelian case the Chern-Simons term has only one composite operator); see Eq. (45) below. The set of sources are characterized by

$$\begin{aligned} s\bar{\kappa}_{\alpha\beta\mu\nu} &= 0, \quad s\lambda_{\mu\nu\alpha} = J_{\mu\nu\alpha}, \quad sJ_{\mu\nu\alpha} = 0, \\ s\eta_{\mu\nu\alpha} &= \tau_{\mu\nu\alpha}, \quad s\tau_{\mu\nu\alpha} = 0. \end{aligned} \tag{42}$$

Eventually, these sources will attain the following physical values:

TABLE III. Quantum numbers of the sources.

sources	Ω	L	λ	J	η	τ	$\bar{\kappa}$
UV dimension	3	4	1	1	1	1	0
Ghost number	−1	−2	−1	0	−1	0	0

$$\begin{aligned} J_{\mu\nu\alpha}|_{\text{phys}} &= \tau_{\mu\nu\alpha}|_{\text{phys}} = v_\beta \epsilon_{\beta\mu\nu\alpha}, \\ \lambda_{\mu\nu\alpha}|_{\text{phys}} &= \eta_{\mu\nu\alpha}|_{\text{phys}} = 0, \\ \bar{\kappa}_{\alpha\beta\mu\nu}|_{\text{phys}} &= \kappa_{\alpha\beta\mu\nu}. \end{aligned} \tag{43}$$

Thus, we replace the action (40) by[9]

$$\Sigma' = S_{YM} + \Sigma_{LO} + \Sigma_{LE} + \Sigma_{gf}, \tag{44}$$

where now

$$\begin{aligned} \Sigma_{LE} &= \frac{1}{4} \int d^4x \bar{\kappa}_{\alpha\beta\mu\nu} F^a_{\alpha\beta} F^a_{\mu\nu}, \\ \Sigma_{LO} &= s \int d^4x \left(\lambda_{\mu\nu\alpha} A^a_\mu \partial_\nu A^a_\alpha + \frac{g}{3} \eta_{\mu\nu\alpha} f^{abc} A^a_\mu A^b_\nu A^c_\alpha \right) \\ &= \int d^4x \Big[J_{\mu\nu\alpha} A^a_\mu \partial_\nu A^a_\alpha + \frac{g}{3} \tau_{\mu\nu\alpha} f^{abc} A^a_\mu A^b_\nu A^c_\alpha \\ &\quad + \lambda_{\mu\nu\alpha} \partial_\mu c^a \partial_\nu A^a_\alpha + g(\eta_{\mu\nu\alpha} - \lambda_{\mu\nu\alpha}) f^{abc} A^a_\mu A^b_\nu \partial_\alpha c^c \Big] \end{aligned} \tag{45}$$

is the embedding of the Lorentz-violating bosonic sector. It is a trivial exercise to check that the new action is BRST invariant. The quantum numbers of the auxiliary sources follow the quantum numbers of the background fields, as displayed in Table III.

To face the issue of the renormalizability of the model, we need one last set of external BRST-invariant sources, namely, Ω and L, in order to control the nonlinear BRST transformations of the original fields,

$$\begin{aligned} \Sigma_{\text{ext}} &= s \int d^4x (-\Omega^a_\mu A^a_\mu + L^a c^a) \\ &= \int d^4x \left(-\Omega^a_\mu D^{ab}_\mu c^b + \frac{g}{2} f^{abc} L^a c^b c^c \right). \end{aligned} \tag{46}$$

However, from a power-counting analysis and BRST symmetry, extra bilinear terms in the gauge fields coupled to the auxiliary sources can still be added to the action, namely,

[8]Nevertheless, the renormalizability of YM theories with Lorentz violation could also be analyzed in other renormalizable gauges, e.g., the linear covariant ξ gauges, the maximal Abelian gauge, and the Curci-Ferrari gauge. All of these are very important in nonperturbative QCD studies. However, the last two cases consist of nonlinear gauges, a fact that demands the introduction of quartic ghost interacting terms for renormalizability and generates a large amount of extra counterterms, making the whole analysis much less interesting and much more technical. The linear covariant gauges could be easily implemented, although extra terms depending on the gauge parameter would appear. However, as mentioned above, the Landau gauge is a natural fixed point of the linear covariant gauges, making them equivalent on some level.

[9]Since the Lorentz breaking is controlled by the external sources, we rename the original actions without the letter "V" (for violation).

$$
\begin{aligned}
\Sigma_{LCO} = s\int d^4x\Big\{&(\alpha_1\lambda_{\mu\nu\alpha}J_{\mu\nu\alpha}+\alpha_2\lambda_{\mu\nu\alpha}\tau_{\mu\nu\alpha}+\alpha_3\eta_{\mu\nu\alpha}J_{\mu\nu\alpha}+\alpha_4\eta_{\mu\nu\alpha}\tau_{\mu\nu\alpha})\frac{1}{2}A^a_\beta A^a_\beta\\
&+(\beta_1\lambda_{\mu\alpha\beta}J_{\nu\alpha\beta}+\beta_2\lambda_{\mu\alpha\beta}\tau_{\nu\alpha\beta}+\beta_3\eta_{\mu\alpha\beta}J_{\nu\alpha\beta}+\beta_4\eta_{\mu\alpha\beta}\tau_{\nu\alpha\beta})A^a_\mu A^a_\nu\\
&+\bar\kappa_{\alpha\beta\mu\nu}(\gamma_1\lambda_{\alpha\beta\rho}J_{\mu\nu\rho}+\gamma_2\lambda_{\alpha\beta\rho}\tau_{\mu\nu\rho}+\gamma_3\eta_{\alpha\beta\rho}J_{\mu\nu\rho}+\gamma_4\eta_{\alpha\beta\rho}\tau_{\mu\nu\rho})\frac{1}{2}A^a_\sigma A^a_\sigma\\
&+\bar\kappa_{\alpha\beta\mu\nu}(\chi_1\lambda_{\beta\rho\sigma}J_{\nu\rho\sigma}+\chi_2\lambda_{\beta\rho\sigma}\tau_{\nu\rho\sigma}+\chi_3\eta_{\beta\rho\sigma}J_{\nu\rho\sigma}+\chi_4\eta_{\beta\rho\sigma}\tau_{\nu\rho\sigma})A^a_\alpha A^a_\mu\\
&+\bar\kappa_{\alpha\rho\sigma\delta}(\varrho_1\lambda_{\nu\rho\delta}J_{\mu\alpha\sigma}+\varrho_2\lambda_{\nu\rho\delta}\tau_{\mu\alpha\sigma}+\varrho_3\eta_{\nu\rho\delta}\tau_{\mu\alpha\sigma}+\varrho_4\eta_{\nu\rho\delta}\tau_{\mu\alpha\sigma})A^a_\mu A^a_\nu\Big\}\\
=\int d^4x\Big\{&(\alpha_1 J_{\mu\nu\alpha}J_{\mu\nu\alpha}+\alpha_2 J_{\mu\nu\alpha}\tau_{\mu\nu\alpha}+\alpha_3\tau_{\mu\nu\alpha}J_{\mu\nu\alpha}+\alpha_4\tau_{\mu\nu\alpha}\tau_{\mu\nu\alpha})\frac{1}{2}A^a_\beta A^a_\beta\\
&+(\beta_1 J_{\mu\alpha\beta}J_{\nu\alpha\beta}+\beta_2 J_{\mu\alpha\beta}\tau_{\nu\alpha\beta}+\beta_3\tau_{\mu\alpha\beta}J_{\nu\alpha\beta}+\beta_4\tau_{\mu\alpha\beta}\tau_{\nu\alpha\beta})A^a_\mu A^a_\nu\\
&+\bar\kappa_{\alpha\beta\mu\nu}(\gamma_1 J_{\alpha\beta\rho}J_{\mu\nu\rho}+\gamma_2 J_{\alpha\beta\rho}\tau_{\mu\nu\rho}+\gamma_3\tau_{\alpha\beta\rho}J_{\mu\nu\rho}+\gamma_4\tau_{\alpha\beta\rho}\tau_{\mu\nu\rho})\frac{1}{2}A^a_\sigma A^a_\sigma\\
&+\bar\kappa_{\alpha\beta\mu\nu}(\chi_1 J_{\beta\rho\sigma}J_{\nu\rho\sigma}+\chi_2 J_{\beta\rho\sigma}\tau_{\nu\rho\sigma}+\chi_3\tau_{\beta\rho\sigma}J_{\nu\rho\sigma}+\chi_4\tau_{\beta\rho\sigma}\tau_{\nu\rho\sigma})A^a_\alpha A^a_\mu\\
&+\bar\kappa_{\alpha\rho\sigma\delta}(\varrho_1 J_{\nu\rho\delta}J_{\mu\alpha\sigma}+\varrho_2 J_{\nu\rho\delta}J_{\mu\alpha\sigma}+\varrho_3\tau_{\nu\rho\delta}J_{\mu\alpha\sigma}+\varrho_4\tau_{\nu\rho\delta}J_{\mu\alpha\sigma})A^a_\mu A^a_\nu\\
&+(\alpha_1\lambda_{\mu\nu\alpha}J_{\mu\nu\alpha}+\alpha_2\lambda_{\mu\nu\alpha}\tau_{\mu\nu\alpha}+\alpha_3\eta_{\mu\nu\alpha}J_{\mu\nu\alpha}+\alpha_4\eta_{\mu\nu\alpha}\tau_{\mu\nu\alpha})A^a_\beta\partial_\beta c^a\\
&+(\beta_1\lambda_{\mu\alpha\beta}J_{\nu\alpha\beta}+\beta_2\lambda_{\mu\alpha\beta}\tau_{\nu\alpha\beta}+\beta_3\eta_{\mu\alpha\beta}J_{\nu\alpha\beta}+\beta_4\eta_{\mu\alpha\beta}\tau_{\nu\alpha\beta})(A^a_\mu\partial_\nu c^a+\partial_\mu c^a A^a_\nu)\\
&+\bar\kappa_{\alpha\beta\mu\nu}(\gamma_1\lambda_{\alpha\beta\rho}J_{\mu\nu\rho}+\gamma_2\lambda_{\alpha\beta\rho}\tau_{\mu\nu\rho}+\gamma_3\eta_{\alpha\beta\rho}J_{\mu\nu\rho}+\gamma_4\eta_{\alpha\beta\rho}\tau_{\mu\nu\rho})A^a_\sigma\partial_\sigma c^a\\
&+\bar\kappa_{\alpha\beta\mu\nu}(\chi_1\lambda_{\beta\rho\sigma}J_{\nu\rho\sigma}+\chi_2\lambda_{\beta\rho\sigma}\tau_{\nu\rho\sigma}+\chi_3\eta_{\beta\rho\sigma}J_{\nu\rho\sigma}+\chi_4\eta_{\beta\rho\sigma}\tau_{\nu\rho\sigma})(A^a_\alpha\partial_\mu c^a+\partial_\alpha c^a A^a_\mu)\\
&+\bar\kappa_{\alpha\rho\sigma\delta}(\varrho_1\lambda_{\nu\rho\delta}J_{\mu\alpha\sigma}+\varrho_2\lambda_{\nu\rho\delta}\tau_{\mu\alpha\sigma}+\varrho_3\eta_{\nu\rho\delta}J_{\mu\alpha\sigma}+\varrho_4\eta_{\nu\rho\delta}\tau_{\mu\alpha\sigma})(A^a_\mu\partial_\nu c^a+\partial_\mu c^a A^a_\nu)\Big\}.
\end{aligned}
\tag{47}
$$

Clearly, a term of this type does not arise in the Abelian model. This property is due to the fact that the Abelian ghost equation is a nonintegrated identity, making it stronger than its non-Abelian version [we will discuss this issue after we define the physical action (50)]. Just like the Abelian case, a vacuum action, i.e., a term that only depends on the sources, is also allowed,

$$
\begin{aligned}
\Sigma_V = s\int d^4x\{&\zeta_1\lambda_{\mu\nu\alpha}J_{\mu\beta\gamma}J_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_2\lambda_{\mu\nu\alpha}J_{\mu\beta\gamma}J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+\zeta_3\lambda_{\mu\nu\alpha}J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_4\lambda_{\mu\nu\alpha}J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}\\
&+\zeta_5\lambda_{\mu\nu\alpha}\tau_{\mu\beta\gamma}J_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_6\lambda_{\mu\nu\alpha}\tau_{\mu\beta\gamma}J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+\zeta_7\lambda_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_8\lambda_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}\\
&+\zeta_9\eta_{\mu\nu\alpha}J_{\mu\beta\gamma}J_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_{10}\eta_{\mu\nu\alpha}J_{\mu\beta\gamma}J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+\zeta_{11}\eta_{\mu\nu\alpha}J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_{12}\eta_{\mu\nu\alpha}J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}\\
&+\zeta_{13}\eta_{\mu\nu\alpha}\tau_{\mu\beta\gamma}J_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_{14}\eta_{\mu\nu\alpha}\tau_{\mu\beta\gamma}J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+\zeta_{15}\eta_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_{16}\eta_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}\\
&+\bar\kappa_{\mu\nu\alpha\beta}(\vartheta_1\lambda_{\mu\rho\omega}J_{\nu\rho\sigma}J_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_2\lambda_{\mu\rho\omega}J_{\nu\rho\sigma}J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}+\vartheta_3\lambda_{\mu\rho\omega}J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_4\lambda_{\mu\rho\omega}J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta}\\
&+\vartheta_5\lambda_{\mu\rho\omega}\tau_{\nu\rho\sigma}J_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_6\lambda_{\mu\rho\omega}\tau_{\nu\rho\sigma}J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}+\vartheta_7\lambda_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_8\lambda_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta}\\
&+\vartheta_9\eta_{\mu\rho\omega}J_{\nu\rho\sigma}J_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_{10}\eta_{\mu\rho\omega}J_{\nu\rho\sigma}J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}+\vartheta_{11}\eta_{\mu\rho\omega}J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_{12}\eta_{\mu\rho\omega}J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta}\\
&+\vartheta_{13}\eta_{\mu\rho\omega}\tau_{\nu\rho\sigma}J_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_{14}\eta_{\mu\rho\omega}\tau_{\nu\rho\sigma}J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}+\vartheta_{15}\eta_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_{16}\eta_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta})\}\\
=\int d^4x\{&\zeta_1 J_{\mu\nu\alpha}J_{\mu\beta\gamma}J_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_2 J_{\mu\nu\alpha}J_{\mu\beta\gamma}J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+\zeta_3 J_{\mu\nu\alpha}J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_4 J_{\mu\nu\alpha}J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}\\
&+\zeta_5 J_{\mu\nu\alpha}\tau_{\mu\beta\gamma}J_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_6 J_{\mu\nu\alpha}\tau_{\mu\beta\gamma}J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+\zeta_7 J_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_8 J_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}\\
&+\zeta_9\tau_{\mu\nu\alpha}J_{\mu\beta\gamma}J_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_{10}\tau_{\mu\nu\alpha}J_{\mu\beta\gamma}J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+\zeta_{11}\tau_{\mu\nu\alpha}J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_{12}\tau_{\mu\nu\alpha}J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}\\
&+\zeta_{13}\tau_{\mu\nu\alpha}\tau_{\mu\beta\gamma}J_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_{14}\tau_{\mu\nu\alpha}\tau_{\mu\beta\gamma}J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+\zeta_{15}\tau_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+\zeta_{16}\tau_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}\\
&+\bar\kappa_{\mu\nu\alpha\beta}(\vartheta_1 J_{\mu\rho\omega}J_{\nu\rho\sigma}J_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_2 J_{\mu\rho\omega}J_{\nu\rho\sigma}J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}+\vartheta_3 J_{\mu\rho\omega}J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_4 J_{\mu\rho\omega}J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta}
\end{aligned}
$$

$$+\vartheta_5 J_{\mu\rho\omega}\tau_{\nu\rho\sigma}J_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_6 J_{\mu\rho\omega}\tau_{\nu\rho\sigma}J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}+\vartheta_7 J_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_8 J_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta}$$
$$+\vartheta_9 \tau_{\mu\rho\omega}J_{\nu\rho\sigma}J_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_{10}\tau_{\mu\rho\omega}J_{\nu\rho\sigma}J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}+\vartheta_{11}\tau_{\mu\rho\omega}J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_{12}\tau_{\mu\rho\omega}J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta}$$
$$+\vartheta_{13}\tau_{\mu\rho\omega}\tau_{\nu\rho\sigma}J_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_{14}\tau_{\mu\rho\omega}\tau_{\nu\rho\sigma}J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}+\vartheta_{15}\tau_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}J_{\beta\sigma\delta}+\vartheta_{16}\tau_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta})\}. \tag{48}$$

Nevertheless, this action is larger than the Abelian action due to the number of auxiliary sources and their quantum numbers (see Table III). The dimensionless parameters $\alpha_i, \beta_i, \gamma_i, \chi_i$, and ϱ_i (with $i=\{1,\ldots,4\}$), and ζ_j and ϑ_j (with $j=\{1,\ldots,16\}$) are required in order to absorb possible vacuum divergences. This extra term is inevitable due to the quantum numbers of the sources and the symmetries of the full action (see next section). Moreover, some of the terms appearing in the actions (47) and (48) (as we will see) always survive at the physical value of the sources. Thus, the vacuum of the model is directly affected. Just like the Abelian case, all infinite towers of the dimensionless source can be rearranged and redefined as the same original terms. The full action is then

$$\Sigma=\Sigma'+\Sigma_{\text{ext}}+\Sigma_{LCO}+\Sigma_V. \tag{49}$$

At the physical value of the sources (43), the action (49) reduces to

$$\Sigma_{\text{phys}}=\frac{1}{4}\int d^4x F^a_{\mu\nu}F^a_{\mu\nu}+\frac{1}{4}\int d^4x\kappa_{\alpha\beta\mu\nu}F^a_{\alpha\beta}F^a_{\mu\nu}+\int d^4x v_\beta\epsilon_{\beta\mu\nu\alpha}\left(A^a_\mu\partial_\nu A^a_\alpha+\frac{g}{3}f^{abc}A^a_\mu A^b_\nu A^c_\alpha\right)$$
$$+\int d^4x(b^a\partial_\mu A^a_\mu+\bar{c}^a\partial_\mu D^{ab}_\mu c^b)+\int d^4x\{((3\alpha+2\beta)v^2-2(\gamma+\varrho)\kappa_{\alpha\sigma\rho\sigma}v_\alpha v_\rho)A^a_\mu A^a_\mu$$
$$-2(\beta v_\mu v_\nu+(\chi-\varrho)\kappa_{\sigma\mu\beta\nu}v_\sigma v_\beta-(\chi-\varrho)\kappa_{\mu\alpha\nu\alpha}v^2-2\varrho\kappa_{\rho\alpha\nu\alpha}v_\rho v_\mu)A^a_\mu A^a_\nu+6\zeta v^4-2\vartheta\kappa_{\alpha\mu\sigma\mu}v_\alpha v_\sigma v^2\}, \tag{50}$$

where

$$\alpha=\sum_{i=1}^4\alpha_i,\qquad \beta=\sum_{i=1}^4\beta_i,\qquad \chi=\sum_{i=1}^4\chi_i,\qquad \gamma=\sum_{i=1}^4\gamma_i,\qquad \varrho=\sum_{i=1}^4\varrho_i,\qquad \zeta=\sum_{j=1}^{16}\zeta_j,\qquad \vartheta=\sum_{j=1}^{16}\vartheta_j, \tag{51}$$

from where it is evident that the vacuum is modified by the last two terms. Moreover, a typical Proca term is also generated, as well as a quadratic gauge-field term with mixed indices. These two quadratic terms will change the tree-level propagator in a more dramatic way than the usual Lorentz-violating Yang-Mills models. It is worth mentioning here that, in contrast to the Abelian case, the physical content of the gauge field will change drastically when the physical limit of the sources is taken. The only similarity with the Abelian case is the emergence of a vacuum term. More specifically, by deforming the theory into a larger one and contracting it back down, the theory returns with extra terms (massive terms) that were not present before. We interpret this as a kind of mass (parameter) generation. Then, the field equations are indeed affected. This can also be seen from the propagators (see below), which are different from the typical non-Abelian Lorentz-violating theories. The second point is that the pure source term Σ_V also generates extra terms in the physical limit. These terms are constants and have no dependence on the quantum fields. They are pure vacuum terms, i.e., they do not affect the field equations, but they do affect the vacuum of the theory.

It is important to emphasize once again the fact that the mass terms do not necessarily define a mass *per se.* We refer to these terms as "mass terms" only because they appear as typical terms of massive theories. However, determining whether these masses are actually physical poles of the model is a task that goes beyond the scope of this work. Strictly speaking, these terms are related to mass parameters and not actual masses of the physical spectrum; thus, the task can be rephrased as determining whether these mass parameters correspond to the propagation of massive physical modes, i.e., that they are not tachyons or ghosts. In QCD, the appearance of many mass parameters is quite typical; however, they do not necessarily describe physical poles of the gluonic field (see, for instance, Refs. [42,45]). Nevertheless, we have and will refer to these terms as mass terms.

For the propagator in the Landau gauge, a straightforward computation leads to (again, for technical reasons, we set $\kappa_{\alpha\beta\mu\nu}=0$)

$$\langle A^a_\mu(k)A^b_\nu(-k)\rangle=\delta^{ab}(A\theta_{\mu\nu}+B\omega_{\mu\nu}+CS_{\mu\nu}+D\Sigma_{\mu\nu}+E\Lambda_{\mu\nu}), \tag{52}$$

where

$$
\begin{aligned}
A &= \frac{k^2+\Delta v^2}{(k^2+\Delta v^2)^2-4[v^2k^2-(v_\alpha k_\alpha)^2]},\\
B &= -(v_\alpha k_\alpha)D,\\
C &= \frac{2}{(k^2+\Delta v^2)^2-4[v^2k^2-(v_\alpha k_\alpha)^2]},\\
D &= \frac{(v_\alpha k_\alpha)[\Omega(k^2+\Delta v^2)+4k^2]}{[k^2(k^2+\Delta v^2+\Omega v^2)-\Omega(v_\alpha k_\alpha)^2][(k^2+\Delta v^2)^2-4(v^2k^2-(v_\alpha k_\alpha)^2)]},\\
E &= -\frac{k^2[\Omega(k^2+\Delta v^2)+4k^2]}{[k^2(k^2+\Delta v^2+\Omega v^2)-\Omega(v_\alpha k_\alpha)^2][(k^2+\Delta v^2)^2-4(v^2k^2-(v_\alpha k_\alpha)^2)]},
\end{aligned}
\tag{53}
$$

where $\Delta = 6\alpha + 4\beta$ and $\Omega = -4\beta$.

IV. RENORMALIZABILITY

A. Ward identities

In order to prove the renormalizability of the model, we start by displaying the full set of Ward identities enjoyed by the action (49).

(i) Slavnov-Taylor identity:

$$
\mathcal{S}(\Sigma) = \int d^4x\left(\frac{\delta\Sigma}{\delta\Omega^a_\mu}\frac{\delta\Sigma}{\delta A^a_\mu}+\frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta c^a}+b^a\frac{\delta\Sigma}{\delta\bar{c}^a} + J_{\mu\nu\alpha}\frac{\delta\Sigma}{\delta\lambda_{\mu\nu\alpha}}+\tau_{\mu\nu\alpha}\frac{\delta\Sigma}{\delta\eta_{\mu\nu\alpha}}\right)=0. \tag{54}
$$

(ii) Gauge-fixing equation and antighost equation:

$$
\frac{\delta\Sigma}{\delta b^a}=\partial_\mu A^a_\mu,\qquad \frac{\delta\Sigma}{\delta\bar{c}^a}+\partial_\mu\frac{\delta\Sigma}{\delta\Omega^a_\mu}=0. \tag{55}
$$

(iii) Ghost equation:

$$
\mathcal{G}^a\Sigma=\Delta^a_{cl}, \tag{56}
$$

with

$$
\mathcal{G}^a=\int d^4x\left(\frac{\delta}{\delta c^a}+gf^{abc}\bar{c}^b\frac{\delta}{\delta b^c}\right) \tag{57}
$$

and

$$
\Delta^a_{cl}=\int d^4x gf^{abc}(\Omega^b_\mu A^c_\mu - L^b c^c). \tag{58}
$$

In Eqs. (55) and (56), the breaking terms are linear in the fields, and thus they will remain at the classical level [39].

B. The most general counterterm

In order to obtain the most general counterterm that can be freely added to the classical action Σ at any order in perturbation theory, we define the most general local integrated polynomial Σ^c with dimension bounded by four and vanishing ghost number. As usual, we apply the Ward identities (54)–(56) to the perturbed action $\Sigma+\varepsilon\Sigma^c$, where ε is a small parameter. It is easy to find that the counterterm must obey the following constraints:

$$
\mathcal{S}_\Sigma\Sigma^c=0,\qquad \frac{\delta\Sigma^c}{\delta b^a}=0,
$$
$$
\left(\frac{\delta}{\delta\bar{c}^a}+\partial_\mu\frac{\delta}{\delta\Omega^a_\mu}\right)\Sigma^c=0,\qquad \mathcal{G}^a\Sigma^c=0, \tag{59}
$$

where the operator $\mathcal{S}_\Sigma$ is the nilpotent linearized Slavnov-Taylor operator,

$$
\mathcal{S}_\Sigma=\int d^4x\left(\frac{\delta\Sigma}{\delta\Omega^a_\mu}\frac{\delta}{\delta A^a_\mu}+\frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta}{\delta\Omega^a_\mu}+\frac{\delta\Sigma}{\delta L^a}\frac{\delta}{\delta c^a}+\frac{\delta\Sigma}{\delta c^a}\frac{\delta}{\delta L^a} + b^a\frac{\delta}{\delta\bar{c}^a}+J_{\mu\nu\alpha}\frac{\delta}{\delta\lambda_{\mu\nu\alpha}}+\tau_{\mu\nu\alpha}\frac{\delta}{\delta\eta_{\mu\nu\alpha}}\right). \tag{60}
$$

The first constraint of Eq. (59) identifies the invariant counterterm as the solution of the cohomology problem for the operator $\mathcal{S}_\Sigma$ in the space of the integrated local field polynomials of dimension four and vanishing ghost number [39]. It follows that Σ^c can be written as

$$
\Sigma^c=\frac{1}{4}\int d^4x a_0 F^a_{\mu\nu}F^a_{\mu\nu}+\frac{1}{4}\int d^4x a_1\bar{\kappa}_{\alpha\beta\mu\nu}F^a_{\alpha\beta}F^a_{\mu\nu}+\mathcal{S}_\Sigma\Delta^{(-1)}, \tag{61}
$$

where $\Delta^{(-1)}$ is the most general local polynomial counterterm with dimension bounded by four and ghost number -1, given by[10]

[10]Just like the Abelian case, any infinite series in $\bar{\kappa}_{\mu\nu\alpha\beta}$ can be redefined as a single term linear in $\bar{\kappa}_{\mu\nu\alpha\beta}$.

$$\Delta^{(-1)} = \int d^4x \Big\{ a_2 \Omega^a_\mu A^a_\mu + a_3 \partial_\mu \bar{c}^a A^a_\mu + a_4 L^a c^a + a_5 \frac{1}{2} \bar{c}^a b^a + a_6 \frac{g}{2} f^{abc} \bar{c}^a \bar{c}^b c^c + (a_7 \lambda_{\mu\nu\alpha} + a_8 \eta_{\mu\nu\alpha}) A^a_\mu \partial_\nu A^a_\alpha$$
$$+ \frac{g}{3}(a_9 \lambda_{\mu\nu\alpha} + a_{10}\eta_{\mu\nu\alpha}) f^{abc} A^a_\mu A^b_\nu A^c_\alpha + (a_{11}\alpha_1 \lambda_{\mu\nu\alpha} J_{\mu\nu\alpha} + a_{12}\alpha_2 \lambda_{\mu\nu\alpha}\tau_{\mu\nu\alpha} + a_{13}\alpha_3 \eta_{\mu\nu\alpha} J_{\mu\nu\alpha} + a_{14}\alpha_4 \eta_{\mu\nu\alpha}\tau_{\mu\nu\alpha}) \frac{1}{2} A^a_\beta A^a_\beta$$
$$+ (a_{15}\beta_1 \lambda_{\mu\alpha\beta} J_{\nu\alpha\beta} + a_{16}\beta_2 \lambda_{\mu\alpha\beta}\tau_{\nu\alpha\beta} + a_{17}\beta_3 \eta_{\mu\alpha\beta} J_{\nu\alpha\beta} + a_{18}\beta_4 \eta_{\mu\alpha\beta}\tau_{\nu\alpha\beta}) A^a_\mu A^a_\nu$$
$$+ \bar{\kappa}_{\alpha\beta\mu\nu}(a_{19}\gamma_1 \lambda_{\alpha\beta\rho} J_{\mu\nu\rho} + a_{20}\gamma_2 \lambda_{\alpha\beta\rho}\tau_{\mu\nu\rho} + a_{21}\gamma_3 \eta_{\alpha\beta\rho} J_{\mu\nu\rho} + a_{22}\gamma_4 \eta_{\alpha\beta\rho}\tau_{\mu\nu\rho}) \frac{1}{2} A^a_\sigma A^a_\sigma$$
$$+ \bar{\kappa}_{\alpha\beta\mu\nu}(a_{23}\chi_1 \lambda_{\beta\rho\sigma} J_{\nu\rho\sigma} + a_{24}\chi_2 \lambda_{\beta\rho\sigma}\tau_{\nu\rho\sigma} + a_{25}\chi_3 \eta_{\beta\rho\sigma} J_{\nu\rho\sigma} + a_{26}\chi_4 \eta_{\beta\rho\sigma}\tau_{\nu\rho\sigma}) A^a_\alpha A^a_\mu$$
$$+ \bar{\kappa}_{\alpha\rho\sigma\delta}(a_{27}\varrho_1 \lambda_{\nu\rho\delta} J_{\mu\alpha\sigma} + a_{28}\varrho_2 \lambda_{\nu\rho\delta}\tau_{\mu\alpha\sigma} + a_{29}\varrho_3 \eta_{\nu\rho\delta}\tau_{\mu\alpha\sigma} + a_{30}\varrho_4 \eta_{\nu\rho\delta}\tau_{\mu\alpha\sigma}) A^a_\mu A^a_\nu + a_{31}\zeta_1 \lambda_{\mu\nu\alpha} J_{\mu\beta\gamma} J_{\nu\beta\kappa} J_{\gamma\kappa\alpha}$$
$$+ a_{32}\zeta_2 \lambda_{\mu\nu\alpha} J_{\mu\beta\gamma} J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha} + a_{33}\zeta_3 \lambda_{\mu\nu\alpha} J_{\mu\beta\gamma}\tau_{\nu\beta\kappa} J_{\gamma\kappa\alpha} + a_{34}\zeta_4 \lambda_{\mu\nu\alpha} J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha} + a_{35}\zeta_5 \lambda_{\mu\nu\alpha}\tau_{\mu\beta\gamma} J_{\nu\beta\kappa} J_{\gamma\kappa\alpha}$$
$$+ a_{36}\zeta_6 \lambda_{\mu\nu\alpha}\tau_{\mu\beta\gamma} J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha} + a_{37}\zeta_7 \lambda_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa} J_{\gamma\kappa\alpha} + a_{38}\zeta_8 \lambda_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha} + a_{39}\zeta_9 \eta_{\mu\nu\alpha} J_{\mu\beta\gamma} J_{\nu\beta\kappa} J_{\gamma\kappa\alpha}$$
$$+ a_{40}\zeta_{10} \eta_{\mu\nu\alpha} J_{\mu\beta\gamma} J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha} + a_{41}\zeta_{11} \eta_{\mu\nu\alpha} J_{\mu\beta\gamma}\tau_{\nu\beta\kappa} J_{\gamma\kappa\alpha} + a_{42}\zeta_{12} \eta_{\mu\nu\alpha} J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}$$
$$+ a_{43}\zeta_{13} \eta_{\mu\nu\alpha}\tau_{\mu\beta\gamma} J_{\nu\beta\kappa} J_{\gamma\kappa\alpha} + a_{44}\zeta_{14} \eta_{\mu\nu\alpha}\tau_{\mu\beta\gamma} J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha} + a_{45}\zeta_{15} \eta_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa} J_{\gamma\kappa\alpha}$$
$$+ a_{46}\zeta_{16} \eta_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha} + \bar{\kappa}_{\mu\nu\alpha\beta}(a_{47}\vartheta_1 \lambda_{\mu\rho\omega} J_{\nu\rho\sigma} J_{\alpha\omega\delta} J_{\beta\sigma\delta} + a_{48}\vartheta_2 \lambda_{\mu\rho\omega} J_{\nu\rho\sigma} J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}$$
$$+ a_{49}\vartheta_3 \lambda_{\mu\rho\omega} J_{\nu\rho\sigma}\tau_{\alpha\omega\delta} J_{\beta\sigma\delta} + a_{50}\vartheta_4 \lambda_{\mu\rho\omega} J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta} + a_{51}\vartheta_5 \lambda_{\mu\rho\omega}\tau_{\nu\rho\sigma} J_{\alpha\omega\delta} J_{\beta\sigma\delta}$$
$$+ a_{52}\vartheta_6 \lambda_{\mu\rho\omega}\tau_{\nu\rho\sigma} J_{\alpha\omega\delta}\tau_{\beta\sigma\delta} + a_{53}\vartheta_7 \lambda_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta} J_{\beta\sigma\delta} + a_{54}\vartheta_8 \lambda_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta}$$
$$+ a_{55}\vartheta_9 \eta_{\mu\rho\omega} J_{\nu\rho\sigma} J_{\alpha\omega\delta} J_{\beta\sigma\delta} + a_{56}\vartheta_{10} \eta_{\mu\rho\omega} J_{\nu\rho\sigma} J_{\alpha\omega\delta}\tau_{\beta\sigma\delta} + a_{57}\vartheta_{11} \eta_{\mu\rho\omega} J_{\nu\rho\sigma}\tau_{\alpha\omega\delta} J_{\beta\sigma\delta}$$
$$+ a_{58}\vartheta_{12} \eta_{\mu\rho\omega} J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta} + a_{59}\vartheta_{13} \eta_{\mu\rho\omega}\tau_{\nu\rho\sigma} J_{\alpha\omega\delta} J_{\beta\sigma\delta} + a_{60}\vartheta_{14} \eta_{\mu\rho\omega}\tau_{\nu\rho\sigma} J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}$$
$$+ a_{61}\vartheta_{15} \eta_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta} J_{\beta\sigma\delta} + a_{62}\vartheta_{16} \eta_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta}) \Big\}, \quad (62)$$

where the parameters a_i are free coefficients. The ghost equation implies $a_4 = 0$. Moreover, from the second or third equations in Eq. (59), it follows that $a_2 = a_3$. Still, from the second equation in Eq. (59) one finds that $a_5 = a_6 = 0$. Then, it is straightforward to verify that the explicit form of the most general counterterm allowed by the Ward identities is the one given by Eq. (A1) in Appendix A.

C. Stability

Finally, to prove the renormalizability of the model we need to show that the counterterm Σ^c can be reabsorbed by the original action Σ by means of the redefinition of the fields, sources, and parameters of the theory. Thus,

$$\Sigma(\Phi, J, \xi) + \varepsilon \Sigma^c(\Phi, J, \xi) = \Sigma(\Phi_0, J_0, \xi_0) + \mathcal{O}(\varepsilon^2), \quad (63)$$

where the bare fields, sources, and parameters are defined as

$$\begin{aligned} \Phi_0 &= Z_\Phi^{1/2}\Phi, & \Phi &\in \{A, b, \bar{c}, c\}, \\ J_0 &= Z_J J, & J &\in \{J, \lambda, \tau, \eta, \bar{\kappa}, \Omega, L\}, \\ \xi_0 &= Z_\xi \xi, & \xi &\in \{g, \alpha_i, \beta_i, \chi_i, \gamma_i, \varrho_i, \zeta_j, \vartheta_j\}. \end{aligned} \quad (64)$$

It is not difficult to check that this can be performed, which proves that the theory is renormalizable to all orders in perturbation theory. Explicitly, the renormalization factors are listed below.

For the independent renormalization factors of the gluon and coupling parameter, we have

$$\begin{aligned} Z_A^{1/2} &= 1 + \varepsilon\left(\frac{a_0}{2} + a_2\right), \\ Z_g &= 1 - \varepsilon\frac{a_0}{2}, \end{aligned} \quad (65)$$

while the renormalization factors of the ghosts, the Lautrup-Nakanishi field, and the Ω and L sources are not independent:

$$\begin{aligned} Z_c &= Z_{\bar{c}} = Z_A^{-1/2} Z_g^{-1}, \\ Z_\Omega &= Z_A^{-1/4} Z_g^{-1/2}, \\ Z_L &= Z_b^{-1/2} = Z_A^{1/2}. \end{aligned} \quad (66)$$

Thus, the renormalization of the standard Yang-Mills sector remains unchanged. For the sector associated with the vector v_μ, i.e., the CPT-odd breaking term, there is a mixing between their respective operators, i.e., $A^a_\mu \partial_\nu A^a_\alpha$ and $gf^{abc} A^a_\mu A^b_\nu A^c_\alpha$ (due to the quantum numbers of $J_{\mu\nu\alpha}$ and $\tau_{\mu\nu\alpha}$). Thus, matrix renormalization is required, namely,

$$\mathcal{J}_0 = \mathcal{Z}_{\mathcal{J}}\mathcal{J}, \tag{67}$$

where $\mathcal{J}$ is a column matrix of sources that share the same quantum numbers. The quantity $Z_{\mathcal{J}}$ is a square matrix with the associated renormalization factors. In this case,

$$\mathcal{J}_1 = \begin{pmatrix} J_{\mu\nu\alpha} \\ \tau_{\mu\nu\alpha} \end{pmatrix} \quad \text{and} \quad \mathcal{Z}_1 = \begin{pmatrix} Z_{JJ} & Z_{J\tau} \\ Z_{\tau J} & Z_{\tau\tau} \end{pmatrix} = \mathbb{1} + \varepsilon\mathbb{A}, \tag{68}$$

where $\mathbb{A}$ is a matrix depending on a_i. It is found that

$$\mathcal{Z}_1 = \mathbb{1} + \varepsilon\begin{pmatrix} a_7 - a_0 & a_8 \\ a_9 & a_{10} - a_0 \end{pmatrix}. \tag{69}$$

The same rule will be used for the sources $\lambda_{\mu\nu\alpha}$ and $\eta_{\mu\nu\alpha}$, namely,

$$\mathcal{J}_2 = \begin{pmatrix} \lambda_{\mu\nu\alpha} \\ \eta_{\mu\nu\alpha} \end{pmatrix} \quad \text{and} \quad \mathcal{Z}_2 = \begin{pmatrix} Z_{\lambda\lambda} & Z_{\lambda\eta} \\ Z_{\eta\lambda} & Z_{\eta\eta} \end{pmatrix} = \mathbb{1} + \varepsilon\mathbb{A}, \tag{70}$$

where we find

$$\mathcal{Z}_2 = \mathbb{1} + \varepsilon\begin{pmatrix} \frac{a_2}{2} - \frac{a_0}{2} + a_7 & a_8 \\ a_9 & \frac{a_2}{2} - \frac{a_0}{2} + a_{10} \end{pmatrix}. \tag{71}$$

For the CPT-even breaking sector, the tensor $\kappa_{\mu\nu\alpha\beta}$ renormalizes through the factor

$$Z_{\tilde{\kappa}} = 1 + \varepsilon(a_1 - a_0), \tag{72}$$

while the renormalization factors of the corresponding parameters are given in Appendix B. This ends the multiplicative renormalizability proof of the Lorentz-violating pure Yang-Mills theory. An alternative (but equivalent) way of presenting the renormalization coefficients of the massless parameters is briefly displayed in Appendix C.

V. CONCLUSIONS

In this work we have shown the multiplicative renormalizability of the Lorentz-violating pure Yang-Mills theories, at least to all orders in perturbation theory. We have considered the Abelian and non-Abelian cases separately. In Ref. [37], using the analytical renormalization technique (i.e., the explicit one-loop computation of the renormalization factors), the authors have already discussed the renormalizability of the non-Abelian case. In our prescription we employ only the algebraic technique [39]. This method allows for an all-order analysis in perturbation theory. Remarkably, we have found that the CPT-odd term induces mass terms for the non-Abelian gauge field, while no mass is generated for the photon. It is known that massive parameters are already present due to the background v_μ. However, the induced mass parameters come from the typical mass term of the action, namely $\nu^2 A^a_\mu A^a_\mu$, and a mixing mass term $V_{\mu\nu}A^a_\mu A^a_\nu$, where $V_{\mu\nu}$ is a constant tensor [see Eq. (50)]. Furthermore, it was found that the renormalization properties of the usual sector of these theories remain unaffected. The violating terms, however, have new renormalizations properties, except for the Abelian Chern-Simons-like term, which does not renormalize.

In fact, in the Abelian case, there are only three new renormalizations: one is associated with the even sector of the breaking, and the other two are associated with pure vacuum terms. On the other hand, the odd sector of the Abelian breaking does not renormalize. In the non-Abelian case, however, 59 independent renormalizations are present. Besides the typical two renormalizations, the theory presents five independent renormalizations for the odd and even violating terms and 32 parameters associated with a pure vacuum term. It is exactly the odd-sector parameter that induces the extra mass terms, which also renormalizes independently with 20 more parameters.

In Ref. [47], the authors argued that quantum corrections in Lorentz- and CPT-violating QED in a curved manifold can induce, in a natural way, an effective action for gravity. In addition, as was shown in Ref. [48], the original vacuum of the model is affected as well. It is worth mentioning that in the latter work the non-Abelian case was included. However, there exist some differences between Refs. [47,48] and the method presented here: the main one is that here we work in a flat manifold, i.e., Euclidean spacetime. Furthermore, besides the fact that the Lorentz-violating coefficients have been treated here as local sources, their physical values are simply constant coefficients, in contrast with Refs. [47,48]. Moreover, in those works the CPT-even Lorentz-violating coefficient does not have double vanishing trace. A non-vanishing double trace of the CPT-even Lorentz-violating coefficient could generate important consequences in a non-Abelian model (such as the presence of dimension-four operators [51]) and for the ghost sector of the model. A common assumption between our work and Refs. [36,47,48] was that higher towers in the dimensionless parameters (sources) are suppressed assuming that they behave classically. In our case, however, nothing can be said about whether the vacuum terms presented here could generate cosmological effects, at least in a phenomenological way, in contrast to what was discussed in Refs. [47,48].

An interesting point to be studied is the explicit computation of the background tensors by applying the renormalization group equations combined with some extra condition to each of the tensors. For instance, following the Gribov-Zwanziger method, a minimal sensitivity principle could be applied. Such a condition may also be combined with phenomenological information in order to provide reliable bounds for these tensors. In this context, it will be important to choose a renormalization scheme that works in the present approach. The first reliable choice would be a minimal subtraction scheme because it works nicely in similar contexts such as the Gribov-Zwanziger analysis; see Refs. [42,45].

Another interesting point would be the all-orders proof of the renormalizability of the electroweak theory and QCD theory with Lorentz violation considering the fermionic and bosonic sectors [52,53]. Moreover, the Gribov ambiguity problem [54–57] is also manifest in the Lorentz-violating Yang-Mills action. Thus, the inclusion of the refined Gribov-Zwanziger terms could also lead to nontrivial effects that could be visualized through the propagators. In fact, the analysis of the poles of the propagators (18) and (52) and the respective restrictions on the backgrounds is currently under investigation [58]. Nevertheless, all these analyses might be very difficult and tricky and, for this reason, we leave them to future investigation.

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APPENDIX A: COUNTERTERM

The counterterm of the non-Abelian theory is found to be

$$
\begin{aligned}
\Sigma^c = {} & a_0 S_{YM} + a_1\Sigma_{LE} + a_2\int d^4x\Bigg[\frac{\delta S_{YM}}{\delta A^a_\mu}A^a_\mu + \frac{\delta\Sigma_{LE}}{\delta A^a_\mu}A^a_\mu + \frac{\delta\Sigma_{LO}}{\delta A^a_\mu}A^a_\mu + (\alpha_1 J_{\mu\nu\alpha}J_{\mu\nu\alpha} + \alpha_2 J_{\mu\nu\alpha}\tau_{\mu\nu\alpha} + \alpha_3\tau_{\mu\nu\alpha}J_{\mu\nu\alpha} + \alpha_4\tau_{\mu\nu\alpha}\tau_{\mu\nu\alpha})A^a_\beta A^a_\beta \\
& + 2(\beta_1 J_{\mu\alpha\beta}J_{\nu\alpha\beta} + \beta_2 J_{\mu\alpha\beta}\tau_{\nu\alpha\beta} + \beta_3\tau_{\mu\alpha\beta}J_{\nu\alpha\beta} + \beta_4\tau_{\mu\alpha\beta}\tau_{\nu\alpha\beta})A^a_\mu A^a_\nu \\
& + \bar\kappa_{\alpha\beta\mu\nu}(\gamma_1 J_{\alpha\beta\rho}J_{\mu\nu\rho} + \gamma_2 J_{\alpha\beta\rho}\tau_{\mu\nu\rho} + \gamma_3\tau_{\alpha\beta\rho}J_{\mu\nu\rho} + \gamma_4\tau_{\alpha\beta\rho}\tau_{\mu\nu\rho})A^a_\sigma A^a_\sigma \\
& + 2\bar\kappa_{\alpha\beta\mu\nu}(\chi_1 J_{\beta\rho\sigma}J_{\nu\rho\sigma} + \chi_2 J_{\beta\rho\sigma}\tau_{\nu\rho\sigma} + \chi_3\tau_{\beta\rho\sigma}J_{\nu\rho\sigma} + \chi_4\tau_{\beta\rho\sigma}\tau_{\nu\rho\sigma})A^a_\alpha A^a_\mu \\
& + 2\bar\kappa_{\alpha\rho\sigma\delta}(\varrho_1 J_{\nu\rho\delta}J_{\mu\alpha\sigma} + \varrho_2 J_{\nu\rho\delta}J_{\mu\alpha\sigma} + \varrho_3\tau_{\nu\rho\delta}J_{\mu\alpha\sigma} + \varrho_4\tau_{\nu\rho\delta}J_{\mu\alpha\sigma})A^a_\mu A^a_\nu \\
& + (\alpha_1\lambda_{\mu\nu\alpha}J_{\mu\nu\alpha} + \alpha_2\lambda_{\mu\nu\alpha}\tau_{\mu\nu\alpha} + \alpha_3\eta_{\mu\nu\alpha}J_{\mu\nu\alpha} + \alpha_4\eta_{\mu\nu\alpha}\tau_{\mu\nu\alpha})A^a_\beta\partial_\beta c^a \\
& + (\beta_1\lambda_{\mu\alpha\beta}J_{\nu\alpha\beta} + \beta_2\lambda_{\mu\alpha\beta}\tau_{\nu\alpha\beta} + \beta_3\eta_{\mu\alpha\beta}J_{\nu\alpha\beta} + \beta_4\eta_{\mu\alpha\beta}\tau_{\nu\alpha\beta})(A^a_\mu\partial_\nu c^a + \partial_\mu c^a A^a_\nu) \\
& + \bar\kappa_{\alpha\beta\mu\nu}(\gamma_1\lambda_{\alpha\beta\rho}J_{\mu\nu\rho} + \gamma_2\lambda_{\alpha\beta\rho}\tau_{\mu\nu\rho} + \gamma_3\eta_{\alpha\beta\rho}J_{\mu\nu\rho} + \gamma_4\eta_{\alpha\beta\rho}\tau_{\mu\nu\rho})A^a_\sigma\partial_\sigma c^a \\
& + \bar\kappa_{\alpha\beta\mu\nu}(\chi_1\lambda_{\beta\rho\sigma}J_{\nu\rho\sigma} + \chi_2\lambda_{\beta\rho\sigma}\tau_{\nu\rho\sigma} + \chi_3\eta_{\beta\rho\sigma}J_{\nu\rho\sigma} + \chi_4\eta_{\beta\rho\sigma}\tau_{\nu\rho\sigma})(A^a_\alpha\partial_\mu c^a + \partial_\alpha c^a A^a_\mu) \\
& + \bar\kappa_{\alpha\rho\sigma\delta}(\varrho_1\lambda_{\nu\rho\delta}J_{\mu\alpha\sigma} + \varrho_2\lambda_{\nu\rho\delta}\tau_{\mu\alpha\sigma} + \varrho_3\eta_{\nu\rho\delta}J_{\mu\alpha\sigma} + \varrho_4\eta_{\nu\rho\delta}\tau_{\mu\alpha\sigma})(A^a_\mu\partial_\nu c^a + \partial_\mu c^a A^a_\nu)\Bigg] \\
& + \int d^4x\Bigg\{J_{\mu\nu\alpha}\left(a_7 A^a_\mu\partial_\nu A^a_\alpha + a_9\frac{g}{3}f^{abc}A^a_\mu A^b_\nu A^c_\alpha\right) + a_7\lambda_{\mu\nu\alpha}\partial_\mu c^a\partial_\nu A^a_\alpha + (a_9 - a_7)g\lambda_{\mu\nu\alpha}f^{abc}A^a_\mu A^c_\alpha\partial_\nu c^b \\
& + \tau_{\mu\nu\alpha}\left(a_8 A^a_\mu\partial_\nu A^a_\alpha + a_{10}\frac{g}{3}f^{abc}A^a_\mu A^b_\nu A^c_\alpha\right) + a_8\eta_{\mu\nu\alpha}\partial_\mu c^a\partial_\nu A^a_\alpha + (a_{10} - a_8)g\eta_{\mu\nu\alpha}f^{abc}A^a_\mu A^c_\alpha\partial_\nu c^b \\
& + (a_{11}\alpha_1 J_{\mu\nu\alpha}J_{\mu\nu\alpha} + a_{12}\alpha_2 J_{\mu\nu\alpha}\tau_{\mu\nu\alpha} + a_{13}\alpha_3\tau_{\mu\nu\alpha}J_{\mu\nu\alpha} + a_{14}\alpha_4\tau_{\mu\nu\alpha}\tau_{\mu\nu\alpha})\frac{1}{2}A^a_\beta A^a_\beta \\
& + (a_{11}\alpha_1\lambda_{\mu\nu\alpha}J_{\mu\nu\alpha} + a_{12}\alpha_2\lambda_{\mu\nu\alpha}\tau_{\mu\nu\alpha} + a_{13}\alpha_3\eta_{\mu\nu\alpha}J_{\mu\nu\alpha} + a_{14}\alpha_4\eta_{\mu\nu\alpha}\tau_{\mu\nu\alpha})A^a_\beta\partial_\beta c^a \\
& + (a_{15}\beta_1 J_{\mu\alpha\beta}J_{\nu\alpha\beta} + a_{16}\beta_2 J_{\mu\alpha\beta}\tau_{\nu\alpha\beta} + a_{17}\beta_3\tau_{\mu\alpha\beta}J_{\nu\alpha\beta} + a_{18}\beta_4\tau_{\mu\alpha\beta}\tau_{\nu\alpha\beta})A^a_\mu A^a_\nu \\
& + (a_{15}\beta_1\lambda_{\mu\alpha\beta}J_{\nu\alpha\beta} + a_{16}\beta_2\lambda_{\mu\alpha\beta}\tau_{\nu\alpha\beta} + a_{17}\beta_3\eta_{\mu\alpha\beta}J_{\nu\alpha\beta} + a_{18}\beta_4\eta_{\mu\alpha\beta}\tau_{\nu\alpha\beta})(A^a_\mu\partial_\nu c^a + \partial_\mu c^a A^a_\nu) \\
& + \bar\kappa_{\alpha\beta\mu\nu}(a_{19}\gamma_1 J_{\alpha\beta\rho}J_{\mu\nu\rho} + a_{20}\gamma_2 J_{\alpha\beta\rho}\tau_{\mu\nu\rho} + a_{21}\gamma_3\tau_{\alpha\beta\rho}J_{\mu\nu\rho} + a_{22}\gamma_4\tau_{\alpha\beta\rho}\tau_{\mu\nu\rho})\frac{1}{2}A^a_\sigma A^a_\sigma \\
& + \bar\kappa_{\alpha\beta\mu\nu}(a_{19}\gamma_1\lambda_{\alpha\beta\rho}J_{\mu\nu\rho} + a_{20}\gamma_2\lambda_{\alpha\beta\rho}\tau_{\mu\nu\rho} + a_{21}\gamma_3\eta_{\alpha\beta\rho}J_{\mu\nu\rho} + a_{22}\gamma_4\eta_{\alpha\beta\rho}\tau_{\mu\nu\rho})A^a_\sigma\partial_\sigma c^a \\
& + \bar\kappa_{\alpha\beta\mu\nu}(a_{23}\chi_1 J_{\beta\rho\sigma}J_{\nu\rho\sigma} + a_{24}\chi_2 J_{\beta\rho\sigma}\tau_{\nu\rho\sigma} + a_{25}\chi_3\tau_{\beta\rho\sigma}J_{\nu\rho\sigma} + a_{26}\chi_4\tau_{\beta\rho\sigma}\tau_{\nu\rho\sigma})A^a_\alpha A^a_\mu \\
& + \bar\kappa_{\alpha\beta\mu\nu}(a_{23}\chi_1\lambda_{\beta\rho\sigma}J_{\nu\rho\sigma} + a_{24}\chi_2\lambda_{\beta\rho\sigma}\tau_{\nu\rho\sigma} + a_{25}\chi_3\eta_{\beta\rho\sigma}J_{\nu\rho\sigma} + a_{26}\chi_4\eta_{\beta\rho\sigma}\tau_{\nu\rho\sigma})(A^a_\alpha\partial_\mu c^a + \partial_\mu c^a A^a_\alpha)
\end{aligned}
$$

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$$
\begin{aligned}
&+\bar{\kappa}_{\alpha\rho\sigma\delta}(a_{27}\varrho_1 J_{\nu\rho\delta}J_{\mu\alpha\sigma}+a_{28}\varrho_2 J_{\nu\rho\delta}\tau_{\mu\alpha\sigma}+a_{29}\varrho_3\tau_{\nu\rho\delta}\tau_{\mu\alpha\sigma}+a_{30}\varrho_4\tau_{\nu\rho\delta}\tau_{\mu\alpha\sigma})A^a_\mu A^a_\nu\\
&+\bar{\kappa}_{\alpha\rho\sigma\delta}(a_{27}\varrho_1\lambda_{\nu\rho\delta}J_{\mu\alpha\sigma}+a_{28}\varrho_2\lambda_{\nu\rho\delta}\tau_{\mu\alpha\sigma}+a_{29}\varrho_3\eta_{\nu\rho\delta}\tau_{\mu\alpha\sigma}+a_{30}\varrho_4\eta_{\nu\rho\delta}\tau_{\mu\alpha\sigma})(A^a_\mu\partial_\nu c^a+\partial_\mu c^a A^a_\nu)\\
&+a_{31}\zeta_1 J_{\mu\nu\alpha}J_{\mu\beta\gamma}J_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+a_{32}\zeta_2 J_{\mu\nu\alpha}J_{\mu\beta\gamma}J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+a_{33}\zeta_3 J_{\mu\nu\alpha}J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}J_{\gamma\kappa\alpha}\\
&+a_{34}\zeta_4 J_{\mu\nu\alpha}J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+a_{35}\zeta_5 J_{\mu\nu\alpha}\tau_{\mu\beta\gamma}J_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+a_{36}\zeta_6 J_{\mu\nu\alpha}\tau_{\mu\beta\gamma}J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}\\
&+a_{37}\zeta_7 J_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+a_{38}\zeta_8 J_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+a_{39}\zeta_9\tau_{\mu\nu\alpha}J_{\mu\beta\gamma}J_{\nu\beta\kappa}J_{\gamma\kappa\alpha}\\
&+a_{40}\zeta_{10}\tau_{\mu\nu\alpha}J_{\mu\beta\gamma}J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+a_{41}\zeta_{11}\tau_{\mu\nu\alpha}J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+a_{42}\zeta_{12}\tau_{\mu\nu\alpha}J_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}\\
&+a_{43}\zeta_{13}\tau_{\mu\nu\alpha}\tau_{\mu\beta\gamma}J_{\nu\beta\kappa}J_{\gamma\kappa\alpha}+a_{44}\zeta_{14}\tau_{\mu\nu\alpha}\tau_{\mu\beta\gamma}J_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+a_{45}\zeta_{15}\tau_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}J_{\gamma\kappa\alpha}\\
&+a_{46}\zeta_{16}\tau_{\mu\nu\alpha}\tau_{\mu\beta\gamma}\tau_{\nu\beta\kappa}\tau_{\gamma\kappa\alpha}+\bar{\kappa}_{\mu\nu\alpha\beta}(a_{47}\vartheta_1 J_{\mu\rho\omega}J_{\nu\rho\sigma}J_{\alpha\omega\delta}J_{\beta\sigma\delta}+a_{48}\vartheta_2 J_{\mu\rho\omega}J_{\nu\rho\sigma}J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}\\
&+a_{49}\vartheta_3 J_{\mu\rho\omega}J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}J_{\beta\sigma\delta}+a_{50}\vartheta_4 J_{\mu\rho\omega}J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta}+a_{51}\vartheta_5 J_{\mu\rho\omega}\tau_{\nu\rho\sigma}J_{\alpha\omega\delta}J_{\beta\sigma\delta}\\
&+a_{52}\vartheta_6 J_{\mu\rho\omega}\tau_{\nu\rho\sigma}J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}+a_{53}\vartheta_7 J_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}J_{\beta\sigma\delta}+a_{54}\vartheta_8 J_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta}\\
&+a_{55}\vartheta_9\tau_{\mu\rho\omega}J_{\nu\rho\sigma}J_{\alpha\omega\delta}J_{\beta\sigma\delta}+a_{56}\vartheta_{10}\tau_{\mu\rho\omega}J_{\nu\rho\sigma}J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}+a_{57}\vartheta_{11}\tau_{\mu\rho\omega}J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}J_{\beta\sigma\delta}\\
&+a_{58}\vartheta_{12}\tau_{\mu\rho\omega}J_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta}+a_{59}\vartheta_{13}\tau_{\mu\rho\omega}\tau_{\nu\rho\sigma}J_{\alpha\omega\delta}J_{\beta\sigma\delta}+a_{60}\vartheta_{14}\tau_{\mu\rho\omega}\tau_{\nu\rho\sigma}J_{\alpha\omega\delta}\tau_{\beta\sigma\delta}\\
&+a_{61}\vartheta_{15}\tau_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}J_{\beta\sigma\delta}+a_{62}\vartheta_{16}\tau_{\mu\rho\omega}\tau_{\nu\rho\sigma}\tau_{\alpha\omega\delta}\tau_{\beta\sigma\delta})\Big\}.
\end{aligned}
\tag{A1}
$$

APPENDIX B: RENORMALIZATION FACTORS OF THE PARAMETERS

The renormalization factors of the dimensionless parameters are found to be

$$
\begin{aligned}
Z_{\alpha_1}&=1+\varepsilon\left(a_{11}-2a_7+a_0-\frac{\alpha_2+\alpha_3}{\alpha_1}a_9\right),\\
Z_{\alpha_2}&=1+\varepsilon\left(a_{12}-a_7-a_{10}+a_0-\left(\frac{\alpha_1}{\alpha_2}a_8+\frac{\alpha_4}{\alpha_2}a_9\right)\right),\\
Z_{\alpha_3}&=1+\varepsilon\left(a_{13}-a_7-a_{10}+a_0-\left(\frac{\alpha_1}{\alpha_3}a_8+\frac{\alpha_4}{\alpha_3}a_9\right)\right),\\
Z_{\alpha_4}&=1+\varepsilon\left(a_{14}-2a_{10}+a_0-\frac{\alpha_2+\alpha_3}{\alpha_4}a_8\right),\\
Z_{\beta_1}&=1+\varepsilon\left(a_{15}-2a_7+a_0-\frac{\beta_2+\beta_3}{\beta_1}a_9\right),\\
Z_{\beta_2}&=1+\varepsilon\left(a_{16}-a_7-a_{10}+a_0-\left(\frac{\beta_1}{\beta_2}a_8+\frac{\beta_4}{\beta_2}a_9\right)\right),\\
Z_{\beta_3}&=1+\varepsilon\left(a_{17}-a_7-a_{10}+a_0-\left(\frac{\beta_1}{\beta_3}a_8+\frac{\beta_4}{\beta_3}a_9\right)\right),\\
Z_{\beta_4}&=1+\varepsilon\left(a_{18}-2a_{10}+a_0-\frac{\beta_2+\beta_3}{\beta_4}a_8\right),\\
Z_{\gamma_1}&=1+\varepsilon\left(a_{19}-a_1-2a_7+2a_0-\frac{\gamma_2+\gamma_3}{\gamma_1}a_9\right),\\
Z_{\gamma_2}&=1+\varepsilon\left(a_{20}-a_1-a_7-a_{10}+2a_0-\left(\frac{\gamma_1}{\gamma_2}a_8+\frac{\gamma_4}{\gamma_2}a_9\right)\right),\\
Z_{\gamma_3}&=1+\varepsilon\left(a_{21}-a_1-a_7-a_{10}+2a_0-\left(\frac{\gamma_1}{\gamma_3}a_8+\frac{\gamma_4}{\gamma_3}a_9\right)\right),
\end{aligned}
$$

$$Z_{\gamma_4} = 1 + \varepsilon\left(a_{22} - a_1 - 2a_{10} + 2a_0 - \frac{\gamma_2 + \gamma_3}{\gamma_4} a_8\right),$$
$$Z_{\chi_1} = 1 + \varepsilon\left(a_{23} - a_1 - 2a_7 + 2a_0 - \frac{\chi_2 + \chi_3}{\chi_1} a_9\right),$$
$$Z_{\chi_2} = 1 + \varepsilon\left(a_{24} - a_1 - a_7 - a_{10} + 2a_0 - \left(\frac{\chi_1}{\chi_2} a_8 + \frac{\chi_4}{\chi_2} a_9\right)\right),$$
$$Z_{\chi_3} = 1 + \varepsilon\left(a_{25} - a_1 - a_7 - a_{10} + 2a_0 - \left(\frac{\chi_1}{\chi_3} a_8 + \frac{\chi_4}{\chi_3} a_9\right)\right),$$
$$Z_{\chi_4} = 1 + \varepsilon\left(a_{26} - a_1 - 2a_{10} + 2a_0 - \frac{\chi_2 + \chi_3}{\chi_4} a_8\right),$$
$$Z_{\varrho_1} = 1 + \varepsilon\left(a_{27} - a_1 - 2a_7 + 2a_0 - \frac{\varrho_2 + \varrho_3}{\varrho_1} a_9\right),$$
$$Z_{\varrho_2} = 1 + \varepsilon\left(a_{28} - a_1 - a_7 - a_{10} + 2a_0 - \left(\frac{\varrho_1}{\varrho_2} a_8 + \frac{\varrho_4}{\varrho_2} a_9\right)\right),$$
$$Z_{\varrho_3} = 1 + \varepsilon\left(a_{29} - a_1 - a_7 - a_{10} + 2a_0 - \left(\frac{\varrho_1}{\varrho_3} a_8 + \frac{\varrho_4}{\varrho_3} a_9\right)\right),$$
$$Z_{\varrho_4} = 1 + \varepsilon\left(a_{30} - a_1 - 2a_{10} + 2a_0 - \frac{\varrho_2 + \varrho_3}{\varrho_4} a_8\right),$$
$$Z_{\zeta_1} = 1 + \varepsilon\left(a_{31} - 4a_7 + 4a_0 - \frac{\zeta_2 + \zeta_3 + \zeta_5 + \zeta_9}{\zeta_1} a_9\right),$$
$$Z_{\zeta_2} = 1 + \varepsilon\left(a_{32} - 3a_7 - a_{10} + 4a_0 - \left(\frac{\zeta_1}{\zeta_2} a_8 + \frac{\zeta_4 + \zeta_6 + \zeta_{10}}{\zeta_2} a_9\right)\right),$$
$$Z_{\zeta_3} = 1 + \varepsilon\left(a_{33} - 3a_7 - a_{10} + 4a_0 - \left(\frac{\zeta_1}{\zeta_3} a_8 + \frac{\zeta_4 + \zeta_7 + \zeta_{11}}{\zeta_3} a_9\right)\right),$$
$$Z_{\zeta_4} = 1 + \varepsilon\left(a_{34} - 2a_7 - 2a_{10} + 4a_0 - \left(\frac{\zeta_2 + \zeta_3}{\zeta_4} a_8 + \frac{\zeta_8 + \zeta_{12}}{\zeta_4} a_9\right)\right),$$
$$Z_{\zeta_5} = 1 + \varepsilon\left(a_{35} - 2a_7 - 2a_{10} + 4a_0 - \left(\frac{\zeta_1}{\zeta_5} a_8 + \frac{\zeta_6 + \zeta_7 + \zeta_{13}}{\zeta_5} a_9\right)\right),$$
$$Z_{\zeta_6} = 1 + \varepsilon\left(a_{36} - 2a_7 - 2a_{10} + 4a_0 - \left(\frac{\zeta_2 + \zeta_5}{\zeta_6} a_8 + \frac{\zeta_8 + \zeta_{14}}{\zeta_6} a_9\right)\right),$$
$$Z_{\zeta_7} = 1 + \varepsilon\left(a_{37} - 2a_7 - 2a_{10} + 4a_0 - \left(\frac{\zeta_3 + \zeta_5}{\zeta_7} a_8 + \frac{\zeta_8 + \zeta_{15}}{\zeta_7} a_9\right)\right),$$
$$Z_{\zeta_8} = 1 + \varepsilon\left(a_{38} - a_7 - 3a_{10} + 4a_0 - \left(\frac{\zeta_4 + \zeta_6 + \zeta_7}{\zeta_8} a_8 + \frac{\zeta_{16}}{\zeta_8} a_9\right)\right),$$
$$Z_{\zeta_9} = 1 + \varepsilon\left(a_{39} - 3a_7 - a_{10} + 4a_0 - \left(\frac{\zeta_1}{\zeta_9} a_8 + \frac{\zeta_{10} + \zeta_{11} + \zeta_{13}}{\zeta_9} a_9\right)\right),$$
$$Z_{\zeta_{10}} = 1 + \varepsilon\left(a_{40} - 2a_7 - 2a_{10} + 4a_0 - \left(\frac{\zeta_2 + \zeta_9}{\zeta_{10}} a_8 + \frac{\zeta_{12} + \zeta_{14}}{\zeta_{10}} a_9\right)\right),$$
$$Z_{\zeta_{11}} = 1 + \varepsilon\left(a_{41} - 2a_7 - 2a_{10} + 4a_0 - \left(\frac{\zeta_3 + \zeta_9}{\zeta_{11}} a_8 + \frac{\zeta_{12} + \zeta_{15}}{\zeta_{11}} a_9\right)\right),$$
$$Z_{\zeta_{12}} = 1 + \varepsilon\left(a_{42} - a_7 - 3a_{10} + 4a_0 - \left(\frac{\zeta_{14} + \zeta_{10} + \zeta_{11}}{\zeta_{12}} a_8 + \frac{\zeta_{16}}{\zeta_{12}} a_9\right)\right),$$

$$
\begin{aligned}
Z_{\zeta_{13}} &= 1+\varepsilon\left(a_{43}-2a_7-2a_{10}+4a_0-\left(\frac{\zeta_5+\zeta_9}{\zeta_{13}}a_8+\frac{\zeta_{14}+\zeta_{15}}{\zeta_{13}}a_9\right)\right),\\
Z_{\zeta_{14}} &= 1+\varepsilon\left(a_{44}-a_7-3a_{10}+4a_0-\left(\frac{\zeta_6+\zeta_{10}+\zeta_{13}}{\zeta_{14}}a_8+\frac{\zeta_{16}}{\zeta_{14}}a_9\right)\right),\\
Z_{\zeta_{15}} &= 1+\varepsilon\left(a_{45}-a_7-3a_{10}+4a_0-\left(\frac{\zeta_7+\zeta_{11}+\zeta_{13}}{\zeta_{15}}a_8+\frac{\zeta_{16}}{\zeta_{15}}a_9\right)\right),\\
Z_{\zeta_{16}} &= 1+\varepsilon\left(a_{46}-4a_{10}+4a_0-\frac{\zeta_8+\zeta_{12}+\zeta_{14}+\zeta_{15}}{\zeta_{16}}a_8\right),\\
Z_{\vartheta_1} &= 1+\varepsilon\left(a_{47}-a_1-4a_7+5a_0-\frac{\vartheta_2+\vartheta_3+\vartheta_5+\vartheta_9}{\vartheta_1}a_9\right),\\
Z_{\vartheta_2} &= 1+\varepsilon\left(a_{48}-a_1-3a_7-a_{10}+5a_0-\left(\frac{\vartheta_1}{\vartheta_2}a_8+\frac{\vartheta_4+\vartheta_6+\vartheta_{10}}{\vartheta_2}a_9\right)\right),\\
Z_{\vartheta_3} &= 1+\varepsilon\left(a_{49}-a_1-3a_7-a_{10}+5a_0-\left(\frac{\vartheta_1}{\vartheta_3}a_8+\frac{\vartheta_4+\vartheta_7+\vartheta_{11}}{\vartheta_3}a_9\right)\right),\\
Z_{\vartheta_4} &= 1+\varepsilon\left(a_{50}-a_1-2a_7-2a_{10}+5a_0-\left(\frac{\vartheta_2+\vartheta_3}{\vartheta_4}a_8+\frac{\vartheta_8+\vartheta_{12}}{\vartheta_4}a_9\right)\right),\\
Z_{\vartheta_5} &= 1+\varepsilon\left(a_{51}-a_1-2a_7-2a_{10}+5a_0-\left(\frac{\vartheta_1}{\vartheta_5}a_8+\frac{\vartheta_6+\vartheta_7+\vartheta_{13}}{\vartheta_5}a_9\right)\right),\\
Z_{\vartheta_6} &= 1+\varepsilon\left(a_{52}-a_1-2a_7-2a_{10}+5a_0-\left(\frac{\vartheta_2+\vartheta_5}{\vartheta_6}a_8+\frac{\vartheta_8+\vartheta_{14}}{\vartheta_6}a_9\right)\right),\\
Z_{\vartheta_7} &= 1+\varepsilon\left(a_{53}-a_1-2a_7-2a_{10}+5a_0-\left(\frac{\vartheta_3+\vartheta_5}{\vartheta_7}a_8+\frac{\vartheta_8+\vartheta_{15}}{\vartheta_7}a_9\right)\right),\\
Z_{\vartheta_8} &= 1+\varepsilon\left(a_{54}-a_1-a_7-3a_{10}+5a_0-\left(\frac{\vartheta_4+\vartheta_6+\vartheta_7}{\vartheta_8}a_8+\frac{\vartheta_{16}}{\vartheta_8}a_9\right)\right),\\
Z_{\vartheta_9} &= 1+\varepsilon\left(a_{55}-a_1-3a_7-a_{10}+5a_0-\left(\frac{\vartheta_1}{\vartheta_9}a_8+\frac{\vartheta_{10}+\vartheta_{11}+\vartheta_{13}}{\vartheta_9}a_9\right)\right),\\
Z_{\vartheta_{10}} &= 1+\varepsilon\left(a_{56}-a_1-2a_7-2a_{10}+5a_0-\left(\frac{\vartheta_2+\vartheta_9}{\vartheta_{10}}a_8+\frac{\vartheta_{12}+\vartheta_{14}}{\vartheta_{10}}a_9\right)\right),\\
Z_{\vartheta_{11}} &= 1+\varepsilon\left(a_{57}-a_1-2a_7-2a_{10}+5a_0-\left(\frac{\vartheta_3+\vartheta_9}{\zeta_{11}}a_8+\frac{\vartheta_{12}+\vartheta_{15}}{\vartheta_{11}}a_9\right)\right),\\
Z_{\vartheta_{12}} &= 1+\varepsilon\left(a_{58}-a_1-a_7-3a_{10}+5a_0-\left(\frac{\vartheta_{14}+\vartheta_{10}+\vartheta_{11}}{\vartheta_{12}}a_8+\frac{\vartheta_{16}}{\vartheta_{12}}a_9\right)\right),\\
Z_{\vartheta_{13}} &= 1+\varepsilon\left(a_{59}-a_1-2a_7-2a_{10}+5a_0-\left(\frac{\vartheta_5+\vartheta_9}{\vartheta_{13}}a_8+\frac{\vartheta_{14}+\vartheta_{15}}{\vartheta_{13}}a_9\right)\right),\\
Z_{\vartheta_{14}} &= 1+\varepsilon\left(a_{60}-a_1-a_7-3a_{10}+5a_0-\left(\frac{\vartheta_6+\vartheta_{10}+\vartheta_{13}}{\vartheta_{14}}a_8+\frac{\vartheta_{16}}{\vartheta_{14}}a_9\right)\right),\\
Z_{\vartheta_{15}} &= 1+\varepsilon\left(a_{61}-a_1-a_7-3a_{10}+5a_0-\left(\frac{\vartheta_7+\vartheta_{11}+\vartheta_{13}}{\vartheta_{15}}a_8+\frac{\vartheta_{16}}{\vartheta_{15}}a_9\right)\right),\\
Z_{\vartheta_{16}} &= 1+\varepsilon\left(a_{62}-a_1-4a_{10}+5a_0-\frac{\vartheta_8+\vartheta_{12}+\vartheta_{14}+\vartheta_{15}}{\vartheta_{16}}a_8\right).
\end{aligned}
\tag{B1}
$$

APPENDIX C: ALTERNATIVE RENORMALIZATION OF THE PARAMETERS

The renormalization of the coefficients related to the mass parameters, vertices, and vacuum terms was presented in Sec. IV C. An alternative (but equivalent) way of presenting the renormalization of the dimensionless coefficients (Appendix B) can be performed by using the matrix renormalization. This happens due to the fact that the mixing between the quantum sources induces, in a natural way, a mixing between their respective parameters. Thus, we can simply write

$$\begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \\ \alpha_{04} \\ \alpha_{05} \end{pmatrix} = \mathcal{Z}_\alpha \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{pmatrix}. \tag{C1}$$

It is found that

$$\mathcal{Z}_\alpha = \mathbb{1} + \varepsilon \begin{pmatrix} a_{11} - 2a_7 + a_0 & -a_9 & -a_9 & 0 \\ -a_8 & a_{12} - a_7 - a_{10} + a_0 & 0 & -a_9 \\ -a_8 & 0 & a_{13} - a_7 - a_{10} + a_0 - & -a_9 \\ 0 & -a_8 & -a_8 & a_{14} - 2a_{10} + a_0 \end{pmatrix}, \tag{C2}$$

and it is a straightforward exercise to generalize the method to the other classes of parameters.

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