Holography from quantum cosmology

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The Weyl-Wigner-Groenewold-Moyal formalism of deformation quantization is applied to the closed Friedmann-Lemaître-Robertson-Walker cosmological model. We show that the phase space average for the surface of the apparent horizon is quantized in units of the Planck's surface, and that the total entropy of the Universe is also quantized. Taking into account these two concepts, it is shown that 't Hooft conjecture on the cosmological holographic principle in radiation and dust dominated quantum universes is satisfied as a manifestation of quantization. This suggests that the entire Universe (not only inside the apparent horizon) can be seen as a two-dimensional information structure encoded on the apparent horizon.

I. INTRODUCTION

Deformation quantization, which is presented as Weyl-Wigner-Groenewold-Moyal phase space quantization, describes a quantum system in terms of the classical number (c-number) variables [1,2]. Operators are mapped into the c-number functions so that their compositions could be obtained by the star product that is noncommutative but associative. Therefore, the observables would be classical functions of the phase space. Quantum structure is constructed by replacing pointwise products of classical observables of the phase space, by star product [3,4]. The product of two smooth functions, say f and g, on a Poisson manifold is given by

$$f * g := \sum_{n=0}^{\infty} (i\hbar)^n \mathcal{C}_n(f,g) = fg + i\hbar \mathcal{C}_1(f,g) + \mathcal{O}(\hbar^2), \quad (1)$$

where $\hbar$ plays the role of the deformation parameter. The first term denotes the common product of f and g. Also, the coefficients $\mathcal{C}_n(f,g)$ are bidifferential operators, where their product is noncommutative [5]. These coefficients satisfy the following properties:

$$\begin{cases} \mathcal{C}_0(f,g) = fg, \\ \mathcal{C}_1(f,g) - \mathcal{C}_1(g,f) = \{f,g\}, \\ \sum_{i+j=n} \mathcal{C}_i(\mathcal{C}_j(f,g),h) = \sum_{i+j=n} \mathcal{C}_i(f,(\mathcal{C}_j(g,h)), \end{cases} \quad (2)$$

where $\{f,g\}$ denotes the Poisson bracket. In Eq. (2) the first expression means that in the limit, $\hbar \to 0$, the star product of f and g agrees with the pointwise products of these two functions. The second expression shows that, at the lowest order of the deformation parameter, the commutator $[f,g]_* := f * g - g * f$ tends to the Poisson bracket:

$\lim_{\hbar \to 0} \frac{1}{i\hbar}[f,g]_* = \{f,g\}$. The last expression implies that the star product is associative: $(f * g) * h = f * (g * h)$.

One of the most important components of deformation quantization is the Wigner quasiprobability distribution function (WF) [6,7]. In fact, it is a generating function for all spatial autocorrelation functions of a given quantum mechanical wave function [8,9]. The WF in a ($2D$)-dimensional phase space is given by

$$W_n(x,p) := \frac{1}{(2\pi)^D} \int \psi_n^*\left(x - \frac{\hbar}{2}y\right) e^{-ip \cdot y} \psi_n\left(x + \frac{\hbar}{2}y\right) d^D y, \quad (3)$$

where ψ_n is the state of the system. The distribution is real and the normalization is expressed as $\int d^D x\, d^D p\, W_n(x,p) = 1$.

In flat spaces, the special star product has long been known. In this case, the components of the Poisson tensor J^{ij} can be considered constant. The coefficient $\mathcal{C}_2$ could be chosen as antisymmetric so that

$$\mathcal{C}_2(f,g) = \frac{1}{2} J^{ij} \partial_i f \partial_j g = \frac{1}{2}\{f,g\}. \quad (4)$$

In canonical coordinates, the poisson tensor J is represented by the matrix

$$J = \begin{pmatrix} 0 & -I_D \\ I_D & 0 \end{pmatrix}, \quad (5)$$

where I_D is the $D \times D$ identity matrix. The higher order coefficients may be obtained by exponentiation of $\mathcal{C}_2$. This procedure yields the following Moyal star product [1]:

$$f(x,p) *_M g(x,p) := f \exp\left(\frac{i\hbar}{2}(\overleftarrow{\partial_q}\overrightarrow{\partial_p} - \overleftarrow{\partial_p}\overrightarrow{\partial_q})\right) g$$

$$= f\left(x + \frac{i\hbar}{2}\overrightarrow{\partial_p}, p - \frac{i\hbar}{2}\overrightarrow{\partial_x}\right) g(x,p), \quad (6)$$

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where in the last step we used the Bopp shift argument. An alternative integral representation of the Moyal star product is given by [10]

$$f *_{\mathrm{M}} g = \frac{1}{(2\pi\hbar)^{4D}} \int d^{2D}v \, d^{2D}v' f(v) g(v') e^{\frac{i}{\hbar}(u^t Jv + v^t Jv' + v'^t Ju)},$$

(7)

where $u = (x, p)^t$, $v = (x', p')^t$ and $v' = (x'', p'')^t$. As a direct consequence, the Moyal star product is a nonlocal product. As a result, we have

$$\int d^D x \, d^D p \, f * g = \int d^D x \, d^D p \, g * f = \int d^D x \, d^D p \, fg.$$

(8)

The WF is closely tied to the wave function. Therefore, it is necessary to define the phase space integrals corresponding to the expectation values of the operator formalism. The expectation value or "phase space average" of a phase space function, say $A(x, p)$, is given by

$$\langle A(x, p) \rangle = \int W(x, p) * A(x, p) d^D x \, d^D p$$

$$= \int A(x, p) * W(x, p) d^D x \, d^D p,$$

(9)

where in the last step we have used the property expressed by Eq. (8). The $*_{\mathrm{M}}$-genvalue equation for WF is given by [8]

$$H *_{\mathrm{M}} W_n(x, p) = E_n W_n(x, p),$$

(10)

or equivalently

$$H\left(x + \frac{i\hbar}{2}\vec{\partial}_p, p - \frac{i\hbar}{2}\vec{\partial}_x\right) W_n(x, p) = E_n W_n(x, p),$$

(11)

where H is the Weyl correspondence to the Hamiltonian and E_n is the spectrum of energy. The dynamical equations in this picture are given by Moyal's equation

$$\frac{\partial f}{\partial t} = \frac{1}{i\hbar}[H, f]_{*_{\mathrm{M}}}.$$

(12)

In fact, it is the generalization of Liouville's theorem of classical mechanics. The Moyal dynamical equation is similar to Heisenberg's equation of motion for operators. But here, H and f, as was said previously, are phase space functions, not operators. Another point in this formulation of quantum mechanics is the absence of the wave function. This plays an important role in the construction of quantum cosmology. In quantum cosmology, problems occur in two ways. First, when the Copenhagen interpretation is implemented, and second when the working tool is the wave function. In the former, the observer itself is also an element

of the quantum cosmology, where the Copenhagen interpretation requires an external observer, while the whole Universe has nothing external to it. For the latter, we must ask how it is possible to construct a wave packet that would peak around the classical trajectories in the configuration space; the wave function describing this universe must approach a wave packet that characterizes the presently observed cosmological data. The advantage of deformation quantization is that it makes quantum cosmology look like the Hamiltonian formalism of cosmology. This is done by avoiding the operator formalism.

The holographic principle is a feature of string theory and, in principle, implies that the degrees of freedom in a spatial region can all be encoded on its boundary. Note that the holographic principle was first proposed by Gerard 't Hooft [11], where it is worth seeing [12] if the reader is interested in a string theory interpretation. The holographic principle has since been applied in the context of pre–big bang scenarios [13], the singularity problem [14], and inflation [15], typically for a flat universe. Also, it is investigated regarding the standard big bang cosmology by Fischler and Susskind (FS) [16]. They have found that if our Universe is flat or open, it obeys this principle. This FS version of cosmological holographic principle (CHP) demands that the entropy contained in a volume of particle horizon should not exceed the area of the horizon in Planck units. Lately, there have been two further proposals for the completion of the holographic principle by Easther and Lowe, based on the second law of thermodynamics [17], and by Bak and Rey, using the cosmological apparent horizon instead of the particle horizon [18]. In both of these completions, the closed Universe also obeys the holographic principle naturally. Therefore, these proposals are perhaps more natural compared to the FS proposal.

In this paper we investigate the quantum cosmology of a closed Friedmann-Lemaître-Robertson-Walker (FLRW) universe, filled with radiation or dust. In the first step, we investigate the deformation quantization of the model. Using WF we show that the deformed cosmology predicts a good agreement with the corresponding classical cosmology. Also, we demonstrate that the phase space average of apparent horizon is quantized. This leads us to conclude that the total entropy of radiation or a dust dominated quantum universe satisfies 't Hooft conjecture. The paper consists of the following sections. In Sec. II we present the classical model. Section III provides quantum cosmological description of the model and quantization rules. In Sec. IV, we summarize our results.

II. THE CLASSICAL MODEL

A useful cosmological model that agrees well with observations is the homogeneous and isotropic FLRW universe. In this model the line element for a closed universe is given by

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\Omega^2_{(3)}, \qquad (13)$$

where $N(t)$ is the lapse function, $a(t)$ is the scale factor, and $d\Omega^2_{(3)}$ is the standard line element of the unit three-sphere. The action functional that consists of a gravitational part and a matter part when the matter field is considered as a perfect fluid is given by [19]

$$\mathcal{I} = \frac{M_P^2}{2}\int_{\mathcal{M}}\sqrt{-g}R d^4x + M_P^2\int_{\partial\mathcal{M}}\sqrt{g^{(3)}}K d^3x$$
$$- \int_{\mathcal{M}}\sqrt{-g}\rho d^4x, \qquad (14)$$

where $M_P = \frac{1}{\sqrt{8\pi G}} = \frac{1}{L_P}$ is the reduced Planck mass in natural units ($c = \hbar = k_B = 1$), $\mathcal{M} = I \times S^3$ is the space-time manifold, $\partial\mathcal{M}$ is equal to S^3, K is the trace of extrinsic curvature of the spacetime boundary, and the overdot denotes differentiation with respect to t. If we assume a universe filled with noninteracting dust $\rho = \rho_{0m}(a/a_0)^{-3}$ and radiation $\rho = \rho_{0\gamma}(a/a_0)^{-4}$, and redefining the scale factor and the lapse function as

$$\begin{cases} N(t) = 12\pi^2 M_P a(t)\tilde{N}(t), \\ a(t) = x(t) + \frac{M}{12\pi^2 M_P^2} = x - x_0, \end{cases} \qquad (15)$$

the total Lagrangian will be [20]

$$\mathcal{L} = -\frac{1}{2\tilde{N}}M_P\dot{x}^2 + \frac{\tilde{N}}{2}M_P\omega^2 x^2 - \mathcal{E}\tilde{N}, \qquad (16)$$

where we have defined

$$\mathcal{E} = 12\pi^2 \mathcal{N}_\gamma M_P + \frac{M^2}{2M_P}, \qquad \omega = 12\pi^2 M_P. \quad (17)$$

Besides, we introduce M and $\mathcal{N}_\gamma$ as

$$\begin{cases} M = \int_{\partial M}\sqrt{g^{(3)}}\rho_{0m}a_0^3 d^3x = 2\pi^2\rho_{0m}a_0^3, \\ \mathcal{N}_\gamma = \int_{\partial M}\sqrt{g^{(3)}}\rho_{0\gamma}a_0^4 d^3x = 2\pi^2\rho_{0\gamma}a_0^4, \end{cases} \qquad (18)$$

where M is the total mass of the dust content of the Universe and $\mathcal{N}_\gamma$ could be related to the total entropy of radiation; see Eq. (38). The conjugate momentum to the shifted scale factor x and the primary constraint are given by

$$\begin{cases} \Pi_x = \frac{\partial\mathcal{L}}{\partial\dot{x}} = -\frac{M_P}{\tilde{N}}\dot{x}, \\ \Pi_{\tilde{N}} = \frac{\partial\mathcal{L}}{\partial\dot{\tilde{N}}} = 0. \end{cases} \qquad (19)$$

Consequently, the Hamiltonian corresponding to Lagrangian (16) will be

$$H := -\tilde{N}\mathcal{H} = -\tilde{N}\left[\frac{1}{2M_P}\Pi_x^2 + \frac{1}{2}M_P\omega^2 x^2 - \mathcal{E}\right]. \quad (20)$$

In Hamiltonian (20), $\tilde{N}$ is a Lagrange multiplier; therefore, it enforces the Hamiltonian constraint

$$\mathcal{H} = \frac{1}{2M_P}\Pi_x^2 + \frac{1}{2}M_P\omega^2 x^2 - \mathcal{E} = 0. \qquad (21)$$

Equation (21) for any value of $\mathcal{E}$ shows the elliptical patterns in two-dimensional phase space. By choosing the gauge $\tilde{N} = \frac{1}{\omega}$ the Hamiltonian equations of motion will be

$$\dot{x} = \{x, H\} = \frac{1}{\omega M_P}\Pi_x, \qquad \dot{\Pi}_x = \{\Pi_x, H\} = -\omega M_P x, \qquad (22)$$

which leads us to

$$\begin{cases} x(t) = x(t_0)\cos(t) + \frac{1}{\omega M_P}\Pi_x(t_0)\sin(t), \\ \Pi(t) = \Pi(t_0)\cos(t) - \omega M_P x(t_0)\sin(t). \end{cases} \qquad (23)$$

If we assume that the origin of cosmic time is $t_0 = 0$ and $x(0) = x_0$, where x_0 is defined in (15), we obtain the well-known classical solution

$$\begin{cases} a(t) = \frac{a_{max}}{1+\sec\phi}[1 - \sec\phi\cos(t+\phi)], \\ a_{max} := \frac{M}{12\pi^2 M_P^2} + \left(\frac{2\mathcal{E}}{M_P\omega^2}\right)^{\frac{1}{2}}, \\ \cos(\phi) := \frac{M}{\sqrt{2\mathcal{E}M_P}}, \end{cases} \qquad (24)$$

where a_{max} is the maximum radius of the closed Universe.

III. DEFORMATION QUANTIZATION

The deformation quantization of this simple model is accomplished straightforwardly by replacing the ordinary products of the observables in phase space by the Moyal product. Therefore, Hamiltonian constraint (21) becomes the Moyal-Wheeler-DeWitt (MWDW) equation by replacing the classical Hamiltonian (21) with its deformed counterpart [21]

$$\mathcal{H} *_M W_n(x, \Pi_x) = \mathcal{H}\left(x + \frac{i\hbar}{2}\vec{\partial}_{\Pi_x}, \Pi_x - \frac{i\hbar}{2}\vec{\partial}_x\right)W_n(x, \Pi_x)$$
$$= 0. \qquad (25)$$

For the simple Hamiltonian defined in (21), this equation has turned into two simple partial differential equations [5,7]. The imaginary part of this equation restricts WF to depend on $\frac{1}{2}(\frac{1}{M_P}\Pi_x^2 + M_P\omega^2 x^2)$. The real part yields Laguerre's equation. Hence, one can easily find the following solution of the MWDW equation for the closed FLRW cosmology

$$W_n(x, \Pi_x) = \frac{(-1)^n}{\pi} \exp\left(-\frac{\Pi_x^2}{M_P \omega} - M_P \omega x^2\right) L_n$$

$$\times \left(\frac{2\Pi_x^2}{\omega M_P} + 2M_P \omega x^2\right), \quad (26)$$

where $L_n(z)$ represents the Laguerre polynomials. Figure 1 shows the WF of the model for the third excited state. It will be observed that there exists a pattern for the extrema in the vicinity of classical loci defined in Eq. (21). Also, the Moyal evolution equations (12) will be

$$\begin{cases} \dot{x} = \frac{1}{i}(x *_M H - H *_M x) = \frac{1}{\omega M_P} \Pi_x, \\ \dot{\Pi}_x = \frac{1}{i}(\Pi_x *_M H - H *_M \Pi_x) = -\omega M_P x. \end{cases} \quad (27)$$

The solutions of the above deformed cosmology are

$$\begin{cases} x(t) = x(t_0) \cos(t) + \frac{1}{\omega M_P} \Pi_x(t_0) \sin(t), \\ \Pi(t) = \Pi(t_0) \cos(t) - \omega M_P x(t_0) \sin(t). \end{cases} \quad (28)$$

These look similar to the classical equations of motion (23). These equations of motions show that the functional form of WF is preserved along classical phase space trajectories.

Let us define in the unconstrained phase space the complex-valued holomorphic functions

$$\begin{cases} A = \sqrt{\frac{\omega M_P}{2}}\left(x + \frac{i\Pi_x}{\omega M_P}\right), \\ \bar{A} = \sqrt{\frac{\omega M_P}{2}}\left(x - \frac{i\Pi_x}{\omega M_P}\right). \end{cases} \quad (29)$$
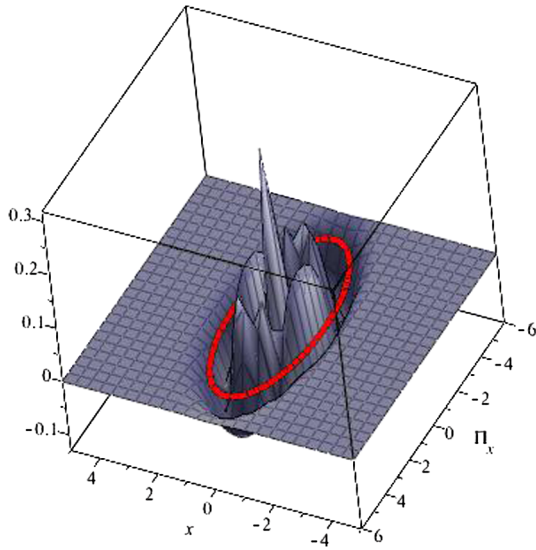
Then the classical Hamiltonian (21) will be

FIG. 1 (color online). Wigner function for the third excited state, $n = 3$, $(M_p = \frac{1}{2\pi})$. The corresponding classical trajectory is denoted by redline loci.

$$\mathcal{H} = \omega A \bar{A} - \mathcal{E}. \quad (30)$$

On the other hand, the Moyal commutation relation between these new variables is

$$[A, \bar{A}]_{*_M} = A *_M \bar{A} - \bar{A} *_M A = 1, \quad (31)$$

where the Moyal star product is redefined as $*_M :=$ $e^{\frac{1}{2}(\overleftarrow{\partial}_A \overrightarrow{\partial}_{\bar{A}} - \overleftarrow{\partial}_{\bar{A}} \overrightarrow{\partial}_A)}$ [5]. The Moyal star product between A and $\bar{A}$ leads us to the following relation between star and ordinary products of holomorphic variables:

$$\bar{A} *_M A = \bar{A} e^{\frac{1}{2}(\overleftarrow{\partial}_A \overrightarrow{\partial}_{\bar{A}} - \overleftarrow{\partial}_{\bar{A}} \overrightarrow{\partial}_A)} A = \bar{A}A - \frac{1}{2}. \quad (32)$$

Consequently, by combining Eqs. (32) and (30) we obtain the Hamiltonian for the model as

$$\mathcal{H} = \omega\left(\bar{A} *_M A + \frac{1}{2}\right) - \mathcal{E}. \quad (33)$$

In addition, the Wigner function (26), in terms of the holomorphic variables, will be

$$W_n(A, \bar{A}) = \frac{1}{n!}(\bar{A})^n *_M W_0 *_M (A)^n, \quad (34)$$

where $W_0 = 2e^{-2A\bar{A}}$ denotes the ground state of the WF. Note that for the ground state we have $A *_M W_0 = 0 = W_0 *_M \bar{A}$. Now, the MWDW Eq. (25) will be

$$\mathcal{H} *_M W_n = \omega\left(\bar{A} *_M A + \frac{1}{2} - \mathcal{E}\right) *_M W_n$$

$$= \left[\omega\left(n + \frac{1}{2}\right) - \mathcal{E}\right] W_n = 0, \quad (35)$$

which leads to

$$\mathcal{E}_n = \omega\left(n + \frac{1}{2}\right). \quad (36)$$

A. Cosmological holographic principle in a radiation dominated universe

Let us first assume that the Universe is radiation dominated, where $M = 0$. In this case, Eq. (36) and the definition of $\mathcal{E}$ in (17) give

$$\mathcal{N}_\gamma = n + \frac{1}{2}. \quad (37)$$

As was mentioned at the beginning of this section, $\mathcal{N}_\gamma$ could be related to the total entropy of radiation. Recalling the relation of the energy density of radiation ρ_γ, the entropy density s_γ, and the scale factor a with temperature,

$\rho_\gamma = \frac{\pi^2}{30} g T^4$, $s_\gamma = \frac{4}{3}\frac{\rho}{T}$, $a(t) \sim \frac{1}{T}$ [22], and using these relations in the definition of $\mathcal{N}_\gamma$ in (18), we find

$$\mathcal{N}_\gamma = \left(\frac{5 \times 3^5}{2^8 \pi^4 g}\right)^{\frac{1}{3}} (S_\gamma)^{\frac{4}{3}}, \tag{38}$$

where $S_\gamma = 2\pi^2 a^3 s_\gamma$ denotes the total entropy and g is the internal degrees of freedom. Now, by inserting (37) into Eq. (38), we obtain

$$S_\gamma = \left(\frac{2^8 \pi^4 g}{5 \times 3^5}\right)^{\frac{1}{4}} \left(n + \frac{1}{2}\right)^{\frac{3}{4}}, \tag{39}$$

which shows that the total entropy of radiation is quantized. Let us now deal with the relation between the total entropy and the phase space average of the apparent horizon. First, note that in definition (15), for a radiation dominated universe, we have $x(t) = a(t)$. Hence, Eq. (29) leads us to obtain the scale factor in terms of holomorphic variables $a(t) = \frac{1}{\sqrt{2M_P}\omega}(A + \bar{A})$. One can easily show that the phase space average of the biquadratic scale factor is

$$\langle a^4 \rangle = \frac{1}{4M_P^2\omega^2}\langle(A + \bar{A})^4\rangle = \frac{3}{2M_P^2\omega^2}\left(n^2 + n + \frac{1}{2}\right). \tag{40}$$

On the other hand, the apparent horizon of a radiation dominated universe is given by $R_{\mathrm{ah}}^2 = (\mathrm{H}^2 + \frac{1}{a^2})^{-1} = \frac{6\pi^2 M_P^2}{\mathcal{N}_\gamma} a^4$, where H is the Hubble parameter. Therefore, the phase space average of the area for the apparent horizon becomes

$$\langle \mathcal{A}_{\mathrm{ah}} \rangle := 4\pi\langle R_{\mathrm{ah}}^2 \rangle = \frac{L_P^2}{4\pi}\left(1 + \frac{n^2}{n + \frac{1}{2}}\right), \tag{41}$$

where L_P is the reduced Planck length. Hence, the phase space average of the apparent horizon is quantized. By comparing Eqs. (39) and (41) for large values of the quantum number n, we obtain

$$S_\gamma \simeq g^{\frac{1}{4}}\left(\frac{\langle \mathcal{A}_{\mathrm{ah}} \rangle}{4G}\right)^{\frac{3}{4}}. \tag{42}$$

The above equation is in the form conjectured by 't Hooft [11].

B. 't Hooft conjecture in a dust dominated universe

Let us now return to a universe filled only with dust, ($\mathcal{N}_\gamma = 0$). In this case, comparing Eqs. (17) and (36) implies the following quantization rule for the total mass of the Universe:

$$M = \sqrt{24\pi^2\left(n + \frac{1}{2}\right)}M_P. \tag{43}$$

We now estimate the total entropy of the dust dominated universe. Consider the case where a system has a total of Ω states of equal likelihood. Then the entropy will be

$$S = \ln(\Omega), \qquad (k_B = 1). \tag{44}$$

Further, let us assume that all the particles are identical. Then $\Omega = \tilde{n}^N$, where $\tilde{n}$ is the number of states accessible to a single particle; hence,

$$S = N\ln(\tilde{n}). \tag{45}$$

Evaluating the one particle phase space, one finds [23] for an ideal gas with N free particles

$$S_{(\mathrm{ideal})} = N\ln\left(\frac{V}{N}\left(\frac{mT}{2\pi}\right)^{\frac{3}{2}}e^{\frac{5}{2}}\right), \tag{46}$$

where V is the volume and m denotes the mass of particles. For the case of a continuous fluid, let us rewrite Eq. (46). To this end, we consider an ideal gas contained within a small volume element dV. The number of particles inside dV is

$$dN = \frac{\rho}{m}dV. \tag{47}$$

Inserting expression (47) into Eq. (46), the entropy associated with the volume element, in terms of the density of the fluid, can be written as

$$dS^{(\mathrm{dust})} = \frac{\rho}{m}\ln\left(\frac{KT^{\frac{3}{2}}}{\rho}\right)dV, \tag{48}$$

where $K = (\frac{m^5 e^5}{2\pi})^{\frac{1}{2}}$ [24]. For a dust dominated universe, the density and temperature are $\rho = \rho_0(\frac{a}{a_0})^{-3}$ and $T = T_0(\frac{a}{a_0})^{-2}$. Hence, we have $S^{(\mathrm{dust})} = \ln(KT_0^{\frac{3}{2}}/\rho_0)N$. We use the simple approximation

$$S^{(\mathrm{dust})} \simeq N = \frac{M}{m}, \tag{49}$$

which is accurate within two orders of magnitude because, as noted by Fermi, all large logs are less than a thousand even in cosmology. Therefore, from Eqs. (43) and (49) we obtain

$$S_n^{(\mathrm{dust})} \simeq \frac{M_P}{m}\pi\sqrt{24\left(n + \frac{1}{2}\right)}. \tag{50}$$

Let us investigate the 't Hooft conjecture for this model. The apparent horizon of a dust dominated universe using

the definition of total mass in Eq. (18) and the Friedmann equation is given by

$$R_{\text{ah}}^2 = \frac{6\pi^2 M_{\text{P}}^2}{M} a^3.$$ (51)

Moreover, using definition (10), the phase space average of the cubic scale factor will be

$$\langle a^3 \rangle = \langle (x - x_0)^3 \rangle = \langle x^3 \rangle - 3x_0 \langle x^2 \rangle + 3x_0^2 \langle x \rangle - x_0^3,$$ (52)

where from definition (29) we have $x = \frac{1}{\sqrt{2\omega M_{\text{P}}}}(A + \bar{A})$. Hence, with an eye on the definition of x_0 in (15), we obtain

$$\langle a^3 \rangle = \frac{3M(n + \frac{1}{2})}{M_{\text{P}}^2 \omega^2} + \left(\frac{M}{M_{\text{P}}\omega}\right)^3.$$ (53)

Eqs. (43) and (53) lead us to

$$\langle a^3 \rangle = \frac{5\sqrt{2}}{(M_{\text{P}}\omega)^{\frac{3}{2}}}\left(n + \frac{1}{2}\right)^{\frac{3}{2}}.$$ (54)

Therefore, the phase space average of squared apparent horizon becomes $\langle R_{\text{ah}}^2 \rangle = (\frac{30\pi^2}{\omega^2})(n + \frac{1}{2})$, which shows that the area of the apparent horizon is quantized,

$$\langle \mathcal{A}_{\text{ah}} \rangle = 4\pi \langle R_{\text{ah}}^2 \rangle = \left(\frac{5L_{\text{P}}^2}{6\pi}\right)\left(n + \frac{1}{2}\right).$$ (55)

Furthermore, from Eqs. (43) and (55) the total mass of the Universe is

$$M = M_{\text{P}} \frac{12\pi}{\sqrt{10}} \left(\frac{\langle \mathcal{A}_{\text{ah}} \rangle}{4G}\right)^{\frac{1}{2}}.$$ (56)

Substituting (56) into (49), the entropy of dust will be

$$S^{(\text{dust})} \simeq \frac{M_{\text{P}}}{m}\left(\frac{\langle \mathcal{A}_{\text{ah}} \rangle}{4G}\right)^{\frac{1}{2}}.$$ (57)

For further simplification, we use the well-known relation between the radius of the Universe (herein the radius of apparent horizon defined via $L_{\text{ah}} := \sqrt{\langle \mathcal{A}_{\text{ah}} \rangle / 4\pi}$) and mass of nucleons, m, as a result of the uncertainty principle [25]

$$L_{\text{ah}} \simeq \sqrt{N}\frac{1}{m} = \sqrt{\frac{M}{m}}\frac{1}{m}.$$ (58)

By substituting Eqs. (43) and (55) in Eq. (58) we obtain

$$m \simeq M_{\text{P}}\left(n + \frac{1}{2}\right)^{-\frac{1}{6}}.$$ (59)

Also combining Eqs. (55), (57), and (59) we obtain

$$S^{(\text{dust})} \simeq \left(\frac{\langle \mathcal{A}_{\text{ah}} \rangle}{4G}\right)^{\frac{2}{3}},$$ (60)

which again is in agreement with 't Hooft conjecture. Let us investigate this result for large values of quantum number n; according to the correspondence principle, the behavior of the model should reduce to its corresponding classical region. For very large values of n, we can estimate from relation (39) the following value for the entropy of radiation:

$$S_\gamma \simeq n^{\frac{3}{4}}.$$ (61)

On the other hand, the entropy of the dust content of the Universe will be

$$S^{(\text{dust})} \simeq n^{\frac{2}{3}}.$$ (62)

Let us examine our model for the present epoch of the Universe. The current entropy density of radiation in the Universe is $s_{0\gamma} = 2970(\frac{T_0}{2.5K})^3 \frac{1}{\text{cm}^3}$. Therefore, the entropy of radiation is $S_{0\gamma} \simeq 10^{88}$. This estimation leads us to obtain the approximate value of the quantum number n as $n \simeq 10^{117}$. Hence, by inserting the obtained value of the quantum number n in Eq. (62), we obtain $S^{(\text{dust})} \simeq 10^{79}$. This is in agreement with the classical estimation of the entropy of dust in the Universe [26]. At the end of this section, let us concentrate on the relation of our simple quantum cosmology model with the large number hypothesis (LNH). For very large values of quantum number n, Eqs. (43), (55), and (59) simplify to the following well-known scaling relations:

$$\begin{cases} M \simeq \beta^3 M_{\text{P}}, \\ L_{\text{ah}} \simeq \beta^3 L_{\text{P}}, \\ m \simeq \beta^{-1} M_{\text{P}}, \end{cases}$$ (63)

where $\beta := n^{1/6} \simeq 10^{19}$. As showed by Marugan and Carneiro [27], the scaling relations that lie behind the LNH can be expressed in the same way as the above relations. Also, they have shown that if one assumes a flat universe dominated by the cosmological constant Λ, then Dirac's LNH can be explained in terms of the holographic conjecture. On the other hand, our results show that the CHP could be the result of the quantum nature of the Universe. Consequently, it seems to be natural that the LNH could be embedded in quantum cosmology as one can see in relations (63). Eliminating β from the two last scaling relations in (63), we obtain

$$m \simeq \left(\frac{1}{GL_{\text{ah}}}\right)^{\frac{1}{3}}.$$ (64)

This equation is equivalent to the empirical Weinberg formula for the mass of the pion [28].

IV. CONCLUSION

In this paper we studied the deformation quantization or phase space quantization of a closed quantum FLRW model, whose matter is either a fluid of radiation or dust. Our results show that the peaks of the WF coincide with the classical trajectory of the Universe. Our main upshot is that the CHP can be achieved by means of quantization of cosmological models. According to the CHP the entropy of nonblack hole configurations is given by relation $S \simeq (\frac{\mathcal{A}}{4G})^{\frac{3}{4}}$, where $\mathcal{A}$ denotes the area of containing volume. We showed that the same result is maintained for a radiation dominated universe, where $\mathcal{A}$ is replaced by the phase space average of apparent horizon $\langle \mathcal{A}_{ah} \rangle$, and S is the total entropy (inside and outside). On the other hand, for a dust dominated universe, we obtained $S \simeq (\frac{\langle \mathcal{A}_{ah} \rangle}{4G})^{\frac{2}{3}}$. It seems that the power of apparent horizon in units of Planck's surface is different for various matter configurations: for black holes this value is equal to 1, for radiation it is equal to 3/4, and for dust it is equal to 2/3. We are aware that our results are obtained within a very simple cosmological model. Nevertheless, we think they are intriguing and provide motivation for subsequent research works. Possible extensions to test the CHP may include the following:

(i) Considering various Bianchi cosmological models.
(ii) Considering other perfect fluids besides radiation and dust.
(iii) Exploring the modified theories of gravity, like string cosmology and $f(R)$ theories.

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