Inhomogeneous and anisotropic Universe and apparent acceleration

G. Fanizza[*] and L. Tedesco[†]

Dipartimento di Fisica, Università di Bari, Via G. Amendola 173, 70126 Bari, Italy
Istituto Nazionale di Fisica Nucleare, Sezione di Bari, 70126 Bari, Italy

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In this paper, we introduce a Lemaître-Tolman-Bondi (LTB) Bianchi type I (plane symmetric) model of the Universe. We study and solve Einstein field equations. We investigate the effects of such a model of the Universe; in particular, these results are important in understanding the effect of the combined presence of an inhomogeneous and anisotropic universe. The observational magnitude-redshift data deviated from the UNION 2 catalog have been analyzed in the framework of this LTB anisotropic universe, and the fit has been achieved without the inclusion of any dark energy.

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I. INTRODUCTION

A very important assumption of the standard model of cosmology (ΛCDM) is based on the homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker solutions of Einstein's equations. The homogeneity and the isotropy are considered on a large scale in the Universe. The Universe is not isotropic or spatially homogeneous on local scales.

The question of whether the Universe is homogeneous and isotropic is of fundamental importance to cosmology, but we have no decisive answers. On the other hand, neither observations of luminosity distance combined with galaxy number counts nor isotropic cosmic microwave background radiation is able to say if the Universe is spatially homogeneous and isotropic.

The fundamental question consists of a simple observation: is this geometry the only one that is able to explain and to be compatible with experimental data? Are we sure that the assumption of homogeneity and isotropy is a logical and comforting way of thinking, or better, is it an *a priori* assumption?

This is a pertinent question because we need more than 96% of the content of our Universe to be dark (energy and matter) in order to have a compatible model with observations. The solution of the dark energy puzzle is the keystone of modern cosmology.

Are there observables that can prove the Universe is homogeneous and isotropic on large scales? Very interesting studies have been done in this direction [1–3].

The ΛCDM model of the Universe is remarkably successful, but we have important tensions between the model and the experimental data [4,5]. On the other hand, dark energy is the biggest puzzle in cosmology. There are many papers with more detailed discussions about dark energy that are outside the scope of this paper; see, for example, Refs. [6,7] and references therein. There are many reasons that consider the ΛCDM model full of theoretical problems [8]: one is that Λ has a value that is absurdly small in quantum physics. Moreover, we cannot expect that dark energy will have locally observable effects in the future.

The cosmic microwave background (CMB) has high isotropy, and this is considered strong evidence of the homogeneity and isotropy of the Universe; that is to say, the Universe is well described by means of a Friedmann-Lemaître-Robertson-Walker (FLRW) model. The main indication for this model is due to the theorem of Ehlers, Geren, and Sachs [1] from 1968. This theorem is due to an earlier paper by Tauber and Weinberg [9] in 1961. In the Ehlers-Geren-Sachs theorem, we consider the observers in an expanding universe, the dust universe measures isotropic CMB, and this implies that the FLRW metric is valid and that the cosmological principle is also valid. This theorem is important because it permits us to have the homogeneity and isotropy not from experimental measurements of the isotropy of the Universe but from the CMB. But, as we will discuss later, CMB radiation has small anisotropies with 10^{-5} amplitude.

As regards the homogeneity of the Universe, it is important to note that the mass density of the Universe is not inhomogeneous on scales much smaller than the Hubble radius; in other terms, the homogeneity is not true at all orders, but we can assume it to be valid on distances greater that 100 Mpc. Many papers indicate this feature; see, for example, Ref. [10] (and references therein), where the author indicates evidence that galaxy distribution is spatially inhomogeneous for $r < 100$ Mpc/h.

The strong interest in inhomogeneous cosmological models, in particular, the so called Lemaître-Tolman-Bondi (LTB) model [11–13] (for more details, see Ref. [14] and references therein), which represents a spherically symmetric exact solution to the Einstein's equations with pressureless ideal fluid, is due to its simplicity, and it is very useful. In fact, it allows for

[*]giuseppe.fanizza@ba.infn.it
[†]luigi.tedesco@ba.infn.it

studies of inhomogeneities that cannot be analyzed as perturbative deviations from FLRW and it permits us to evaluate the effect of inhomogeneities. In particular, it has been studied that LTB models without dark energy can fit observed data.

The high precision cosmology is able to understand by more details about our study of the Universe. When we consider the isotropy of the CMB, we must not forget that it is not sufficient to say that our region of space is isotropic [15].

We have two very important observational pieces of evidence showing that we do not have exact isotropy [16]. Both pieces of evidence may be caused by an anisotropic phase during the evolution of our Universe; in other terms, the existence of anomalies in the CMB suggests the presence of an anomalous plane-mirroring symmetry on large scales [17,18]. The same anomalous features in seven years of WMAp data and Planck data seem to suggest that our Universe could be nonisotropic.

The first is the presence of small anisotropy deviations as regards the isotropy of the CMB. In fact, we have small anisotropies with 10^{-5} amplitude.

The second is connected with the presence of large angle anomalies [19]. These anomalies can be considered in four families: (1) the alignment of quadrupole and octupole moments [20–23], (2) the large-scale asymmetry [24,25], (3) the very strange cold spot [26], and (4) the low quadrupole moment of the CMB that is very important because it may indicate an ellipsoidal Bianchi type I anisotropic evolution of the Universe [27–30]. This is due to the fact that the low quadrupole moment is suppressed at large scales and this suppression cannot be explained by the common cosmological model.

Some years ago, it has been shown [31] that if we start with a FLRW universe, it is possible to have small deviations from homogeneity and isotropy taking into account small deviations in the CMB. In particular, if we consider a homogeneous and anisotropic universe, the small quadrupole anisotropy in CMB implies a very small anisotropy in the Universe. Next, general results have been established [32,33], in which the authors do not assume *a priori* homogeneity, and they found that small anisotropies in CMB imply that the cosmos is not exactly FLRW but it is almost FLRW. Limits on anisotropy and inhomogeneity can be found starting from CMB.

The cosmological model that takes into account all these and stimulates much interest is the "anisotropic Bianchi type I model" that can be an intriguing alternative to the standard model FLRW, in which small deviations from the isotropy are able to explain the anisotropies and the anomalies in the CMB.

The anisotropy considered in this work might be interpreted as an imprinting, a primordial relic of a early anisotropy that appears in the context of a multidimensional cosmological model of unified string theories.

In this paper, our goals are to study an anisotropic and inhomogeneous model of the Universe. In particular, we introduce a new approach to a universe in which inhomogeneities and anisotropies coexist; therefore, we study in order to obtain the relative Einstein's equations. These models of an inhomogeneous and anisotropic universe have been studied in different physical situations as the role of the diffusion forces in governing the large-scale dynamics of an inhomogeneous and anisotropic universe [34].

The supernova observations are good tests about the structure of the space-time on different scales. This is a very important point; in fact, some years ago, Zel'dovich [35] studied the importance of the effects of the inhomogeneities on light propagation, which continued in later years [36–42]. To check this model, we calculate the luminosity distance in order to compare the theoretical approach with experimental data. We explain the acceleration of the Universe without invoking the presence of a cosmological constant or dark energy.

The structure of this paper is the following. In the next section, we calculate the metric for this LTB Bianchi type I model of the Universe. In Sec. III, after providing the calculation of various symbols, we write the Einstein's equations taking into account this geometry. Section IV is dedicated to calculating the luminosity distance, and in Sec. V, we compare the theoretical data with experimental data. Finally, the discussion and conclusion are summarized in Sec. VI.

II. LTB BIANCHI TYPE I METRIC

In order to find the anisotropic LTB metric, let us start with a Bianchi type I space-time metric, spatially homogeneous, described by the metric

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2) - b^2(t)dz^2 \qquad (1)$$

with two expansion parameters a and b that are the scale factors normalized in order that $a(t_0) = b(t_0) = 1$ and t_0 present cosmic time. The metric (1) considers the xy plane as a symmetry plane. To our aim, we write the Bianchi type I metric in polar coordinates $(x = r\sin\theta\cos\phi,$ $y = r\sin\theta\sin\phi, z = r\cos\theta)$:

$$\begin{aligned} ds^2 = dt^2 &- [a^2(t)\sin^2\theta + b^2(t)\cos^2\theta]dr^2 \\ &- r^2[a^2(t)\cos^2\theta + b^2(t)\sin^2\theta]d\theta^2 \\ &- 2r[a^2(t) - b^2(t)]\sin\theta\cos\theta dr d\theta \\ &- r^2a^2(t)\sin^2\theta d\phi^2. \end{aligned} \qquad (2)$$

In order to have a LTB Bianchi type I metric, we make the following substitutions:

$$ra(t) \to A_{\parallel}(r,t) \equiv A_{\parallel}, \qquad (3)$$

$$rb(t) \to A_{\perp}(r,t) \equiv A_{\perp}. \qquad (4)$$

In this way, it is possible to obtain the general LTB Bianchi type I metric in polar coordinates, observing that $a(t) = A'$ and $2r'a^2(t) = (A_\parallel^2)'$, where $\prime \equiv \partial/\partial r$; we have

$$\begin{aligned} ds^2 = dt^2 &- (A'^2_\parallel \sin^2\theta + A'^2_\perp \cos^2\theta) dr^2 \\ &- (A_\parallel^2 \cos^2\theta + A_\perp^2 \sin^2) d\theta^2 \\ &- (A_\parallel^{2\prime} - A_\perp^{2\prime}) \sin\theta\cos\theta dr d\theta + -A_\parallel^2 \sin^2\theta d\phi^2. \end{aligned} \tag{5}$$

It is important to observe that Eq. (5) brings us back to the known cases:

$$\begin{cases} A_\parallel(r,t) = ra(t) \,\text{and}\, A_\perp(r,t) = rb(t) & \text{Bianchi type I} \\ A_\parallel(r,t) = A_\perp(r,t) & \text{LTB} \\ A_\parallel(r,t) = A_\perp(r,t) = ra(t) & \text{FLRW.} \end{cases} \tag{6}$$

Therefore, the metric (5) is a nonhomogeneous metric with axial symmetry that is simply referable to the pure homogeneous or the pure isotropic case.

Let us define the following quantity

$$\epsilon(r,t) = A_\perp - A_\parallel \tag{7}$$

that represents the degree of anisotropy of the Universe. From the definition of ϵ, we obtain

$$A'_\perp = A'_\parallel + \epsilon', \tag{8a}$$

$$A'^2_\perp = A'^2_\parallel + \epsilon'^2 + 2A'_\parallel \epsilon', \tag{8b}$$

$$A_\perp^2 = A_\parallel^2 + \epsilon^2 + 2A_\parallel \epsilon, \tag{8c}$$

$$(A_\perp^2)' = (A_\parallel^2)' + (\epsilon^2)' + 2A'_\parallel \epsilon + 2A_\parallel \epsilon'. \tag{8d}$$

Let us introduce these relations in the metric (5) in order to show it as a function of ϵ and $A_\parallel$ (or ϵ and $A_\perp$). Putting it all together, we have

$$\begin{aligned} ds^2 = dt^2 &- [A'^2_\parallel + (\epsilon'^2 + 2A'_\parallel \epsilon') \cos^2\theta] dr^2 \\ &- [A_\parallel^2 + (\epsilon^2 + 2A_\parallel \epsilon) \sin^2\theta] d\theta^2 \\ &+ [(\epsilon^2)' + 2A'_\parallel \epsilon + 2A_\parallel \epsilon'] \sin\theta\cos\theta dr d\theta \\ &- A_\parallel^2 \sin^2\theta d\phi^2 \\ &\equiv (g^{(\mathrm{LTB})}_{\parallel\mu\nu} + \Delta g^{(\mathrm{AN})}_{\parallel\mu\nu}) dx^\mu dx^\nu, \end{aligned} \tag{9}$$

with our metric given by

$$g_{\mu\nu} \equiv g^{(\mathrm{LTB})}_{\parallel\mu\nu} + \Delta g^{(\mathrm{AN})}_{\parallel\mu\nu}, \tag{10}$$

where

$$g^{(\mathrm{LTB})}_{\parallel\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -A'^2_\parallel & 0 & 0 \\ 0 & 0 & -A_\parallel^2 & 0 \\ 0 & 0 & 0 & -A_\parallel^2 \sin^2\theta \end{pmatrix}, \tag{11a}$$

$$\Delta g^{(\mathrm{AN})}_{\parallel 11} = -\cos^2\theta(\epsilon'^2 + 2A'_\parallel \epsilon'), \tag{11b}$$

$$\Delta g^{(\mathrm{AN})}_{\parallel 12} = \frac{\sin 2\theta}{2} [(\epsilon^2)' + 2A'_\parallel \epsilon + 2A_\parallel \epsilon'], \tag{11c}$$

$$\Delta g^{(\mathrm{AN})}_{\parallel 22} = -\sin^2\theta(\epsilon^2 + 2A_\parallel \epsilon). \tag{11d}$$

The script "(LTB)" (super- or subscripts are the same) means that the quantity refers to a Lemaître-Tolman-Bondi universe, while "(AN)" refers to an anisotropic universe. In other words, the metric (9) is able to describe the inhomogeneity and axial anisotropy of the Universe; on the other hand, a very interesting thing is that it has been decomposed in the sum of a LTB metric with null curve and a term that contains all information about the anisotropy $\epsilon(r,t)$.

For reasons of completeness, it is possible to rewrite the metric in the symmetric way as

$$g_{\mu\nu} \equiv g^{(\mathrm{LTB})}_{\perp\mu\nu} + \Delta g^{(\mathrm{AN})}_{\perp\mu\nu}, \tag{12}$$

where $g^{(\mathrm{LTB})}_{\perp\mu\nu}$ is obtained by Eq. (11a) with the substitution of $A_\perp$ instead of $A_\parallel$ and

$$\Delta g^{(\mathrm{AN})}_{\perp 11} = \sin^2\theta(\epsilon'^2 - 2A'_\perp \epsilon'), \tag{13a}$$

$$\Delta g^{(\mathrm{AN})}_{\perp 12} = \frac{\sin 2\theta}{2} [(\epsilon^2)' - 2A'_\perp \epsilon - 2A_\perp \epsilon'], \tag{13b}$$

$$\Delta g^{(\mathrm{AN})}_{\perp 22} = \cos^2\theta(\epsilon^2 - 2A_\perp \epsilon), \tag{13c}$$

$$\Delta g^{(\mathrm{AN})}_{\perp 33} = \sin^2\theta(\epsilon^2 - 2A_\perp \epsilon). \tag{13d}$$

III. EINSTEIN'S EQUATIONS IN LTB BIANCHI TYPE I UNIVERSE

In this section, we want to write the Einstein's equations taking into account the LTB Bianchi type I metric. To this end, we suppose a very small anisotropy of the Universe, in order to have

$$\epsilon(r,t) \ll A_\parallel(r,t), \tag{14a}$$

$$\epsilon'(r,t) \ll A'_\parallel(r,t). \tag{14b}$$

These positions permit us to expand our results to the first order in ϵ, in other terms:

$$\Delta g_{\mu\nu}^{(\mathrm{AN})} \to \delta g_{\mu\nu}^{(\mathrm{AN})} \tag{15}$$

with

$$\delta g_{\|11}^{(\mathrm{AN})} = -2A_{\|}' \epsilon' \cos^2\theta, \tag{16}$$

$$\delta g_{\|22}^{(\mathrm{AN})} = -2A_{\|} \epsilon \sin^2\theta, \tag{17}$$

$$\delta g_{\|12}^{(\mathrm{AN})} = 2A_{\|} \epsilon' \sin\theta\cos\theta. \tag{18}$$

At this point, we can calculate the Christoffel connection to the first order in ϵ [we repeat that the scripts (LTB) and (AN) are indifferently written as super- or subscripts]:

$$\begin{aligned}\Gamma^{\alpha}_{\mu\nu} &= \frac{1}{2} g^{\alpha\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}) \\ &\simeq \frac{1}{2}(g^{\alpha\rho}_{(\mathrm{LTB})} + \delta g^{\alpha\rho}_{(\mathrm{AN})})[\partial_\mu(g^{(\mathrm{LTB})}_{\nu\rho} + \delta g^{(\mathrm{AN})}_{\nu\rho}) \\ &\quad + \partial_\nu(g^{(\mathrm{LTB})}_{\rho\mu} + \delta g^{(\mathrm{AN})}_{\rho\mu}) - \partial_\rho(g^{(\mathrm{LTB})}_{\mu\nu} + \delta g^{(\mathrm{AN})}_{\mu\nu})]\end{aligned} \tag{19}$$

that, neglecting the second order terms in ϵ^2, becomes

$$\begin{aligned}\Gamma^{\alpha}_{\mu\nu} &\simeq \frac{1}{2} g^{\alpha\rho}_{(\mathrm{LTB})}(\partial_\mu g^{(\mathrm{LTB})}_{\nu\rho} + \partial_\nu g^{(\mathrm{LTB})}_{\rho\mu} - \partial_\rho g^{(\mathrm{LTB})}_{\mu\nu}) \\ &\quad + \frac{1}{2} g^{\alpha\rho}_{(\mathrm{LTB})}(\partial_\mu \delta g^{(\mathrm{AN})}_{\nu\rho} + \partial_\nu \delta g^{(\mathrm{AN})}_{\rho\mu} - \partial_\rho \delta g^{(\mathrm{AN})}_{\mu\nu}) \\ &\quad + \frac{1}{2} \delta g^{\alpha\rho}_{(\mathrm{AN})}(\partial_\mu g^{(\mathrm{LTB})}_{\nu\rho} + \partial_\nu g^{(\mathrm{LTB})}_{\rho\mu} - \partial_\rho g^{(\mathrm{LTB})}_{\mu\nu}).\end{aligned} \tag{20}$$

The first term in Eq. (20) is just $\Gamma^{\alpha(\mathrm{LTB})}_{\mu\nu}$, the Christoffel connection with the metric tensor $g^{(\mathrm{LTB})}_{\mu\nu}$, and putting

$$\Sigma^{\alpha}_{\mu\nu} \equiv \frac{1}{2} g^{\alpha\rho(\mathrm{LTB})}(\partial_\mu \delta g^{(\mathrm{AN})}_{\nu\rho} + \partial_\nu \delta g^{(\mathrm{AN})}_{\rho\mu} - \partial_\rho \delta g^{(\mathrm{AN})}_{\mu\nu}) \tag{21}$$

and

$$\Theta^{\alpha}_{\mu\nu} \equiv \frac{1}{2} \delta g^{\alpha\rho(\mathrm{AN})}(\partial_\mu g^{(\mathrm{LTB})}_{\nu\rho} + \partial_\nu g^{(\mathrm{LTB})}_{\rho\mu} - \partial_\rho g^{(\mathrm{LTB})}_{\mu\nu}), \tag{22}$$

it is possible to write the Christoffel connection at the first order as

$$\Gamma^{\alpha}_{\mu\nu} = \Gamma^{\alpha(\mathrm{LTB})}_{\mu\nu} + \Sigma^{\alpha}_{\mu\nu} + \Theta^{\alpha}_{\mu\nu}. \tag{23}$$

Let us calculate the Ricci tensor

$$R_{\nu\alpha} = \partial_\mu \Gamma^{\mu}_{\nu\alpha} - \partial_\nu \Gamma^{\mu}_{\mu\alpha} + \Gamma^{\mu}_{\mu\rho}\Gamma^{\rho}_{\nu\alpha} - \Gamma^{\mu}_{\nu\rho}\Gamma^{\rho}_{\mu\alpha} \tag{24}$$

with $\Gamma^{\mu}_{\nu\alpha}$ given by Eq. (23). Therefore, we have

$$\begin{aligned}R_{\nu\alpha} &= \partial_\mu(\Gamma^{\mu(\mathrm{LTB})}_{\nu\alpha} + \Sigma^{\mu}_{\nu\alpha} + \Theta^{\mu}_{\nu\alpha}) \\ &\quad - \partial_\nu(\Gamma^{\mu(\mathrm{LTB})}_{\mu\alpha} + \Sigma^{\mu}_{\mu\alpha} + \Theta^{\mu}_{\mu\alpha}) \\ &\quad + (\Gamma^{\mu(\mathrm{LTB})}_{\mu\rho} + \Sigma^{\mu}_{\mu\rho} + \Theta^{\mu}_{\mu\rho})(\Gamma^{\rho(\mathrm{LTB})}_{\nu\alpha} + \Sigma^{\rho}_{\nu\alpha} + \Theta^{\rho}_{\nu\alpha}) \\ &\quad - (\Gamma^{\mu(\mathrm{LTB})}_{\nu\rho} + \Sigma^{\mu}_{\nu\rho} + \Theta^{\mu}_{\nu\rho})(\Gamma^{\rho(\mathrm{LTB})}_{\mu\alpha} + \Sigma^{\rho}_{\mu\alpha} + \Theta^{\rho}_{\mu\alpha}).\end{aligned} \tag{25}$$

When we multiply in Eq. (25), neglecting the second order terms $\Sigma\Sigma$, $\Theta\Theta$, $\Sigma\Theta$, and $\Theta\Sigma$, and putting

$$\begin{aligned}R^{(\mathrm{LTB})}_{\nu\alpha} &= \partial_\mu \Gamma^{\mu(\mathrm{LTB})}_{\nu\alpha} - \partial_\nu \Gamma^{\mu(\mathrm{LTB})}_{\mu\alpha} \\ &\quad + \Gamma^{\mu(\mathrm{LTB})}_{\mu\rho}\Gamma^{\rho(\mathrm{LTB})}_{\nu\alpha} - \Gamma^{\mu(\mathrm{LTB})}_{\nu\rho}\Gamma^{\rho(\mathrm{LTB})}_{\mu\alpha},\end{aligned} \tag{26}$$

$$\begin{aligned}R^{(\Sigma)}_{\nu\alpha} &\equiv \partial_\mu \Sigma^{\mu}_{\nu\alpha} - \partial_\nu \Sigma^{\mu}_{\mu\alpha} + \Sigma^{\mu}_{\mu\rho}\Gamma^{\rho(\mathrm{LTB})}_{\nu\alpha} + \Gamma^{\mu(\mathrm{LTB})}_{\mu\rho}\Sigma^{\rho}_{\nu\alpha} \\ &\quad - \Gamma^{\mu(\mathrm{LTB})}_{\nu\rho}\Sigma^{\rho}_{\mu\alpha} - \Sigma^{\mu}_{\nu\rho}\Gamma^{\rho(\mathrm{LTB})}_{\mu\alpha},\end{aligned} \tag{27}$$

$$\begin{aligned}R^{(\Theta)}_{\nu\alpha} &\equiv \partial_\mu \Theta^{\mu}_{\nu\alpha} - \partial_\nu \Theta^{\mu}_{\mu\alpha} + \Theta^{\mu}_{\mu\rho}\Gamma^{\rho(\mathrm{LTB})}_{\nu\alpha} + \Gamma^{\mu(\mathrm{LTB})}_{\mu\rho}\Theta^{\rho}_{\nu\alpha} \\ &\quad - \Gamma^{\mu(\mathrm{LTB})}_{\nu\rho}\Theta^{\rho}_{\mu\alpha} - \Theta^{\mu}_{\nu\rho}\Gamma^{\rho(\mathrm{LTB})}_{\mu\alpha},\end{aligned} \tag{28}$$

the Ricci tensor to the first order in δ can be written as

$$R_{\nu\alpha} = R^{(\mathrm{LTB})}_{\nu\alpha} + R^{(\Sigma)}_{\nu\alpha} + R^{(\Theta)}_{\nu\alpha}. \tag{29}$$

In order to consider the perturbations of the energy-momentum tensor, we consider a general anisotropic density energy given by

$$\begin{aligned}\rho_{\mathrm{mat}}(r,t,\theta) &\equiv \rho_{\|\mathrm{mat}}(r,t)\sin^2\theta + \rho_{\perp\mathrm{mat}}(r,t)\cos^2\theta \\ &= \rho_{\|\mathrm{mat}}(r,t) + \delta_{\mathrm{mat}}(r,t)\cos^2\theta,\end{aligned} \tag{30}$$

where $\delta_{\mathrm{mat}} \equiv \rho_{\perp\mathrm{mat}} - \rho_{\|\mathrm{mat}}$. The density that we choose has a planar symmetry because of the consistency with the metric that we are working with. This choice allows us to rewrite the energy-momentum tensor and its trace as

$$T^{\nu}_{\mu} \equiv T^{\nu(\mathrm{LTB})}_{\mu} + \Delta T^{\nu}_{\mu}, \tag{31}$$

$$T \equiv T^{(\mathrm{LTB})} + \Delta T, \tag{32}$$

where, in general,

$$T^{\nu(\mathrm{LTB})}_{\mu} = \mathrm{diag}[\rho(r,t), -p(r,t), -p(r,t), -p(r,t)]. \tag{33}$$

However, all these definitions must be viewed as a first order correction to the usual energy-momentum tensor in the LTB case. This point of view becomes clear if we look at the usual perturbation theory provided by Ref. [43]. In fact, our particular definition is fully consistent with Mukhanov's one whether we fix $\delta p = V = \sigma = 0$ in Eq. (5.1) of Ref. [43]: this means that we are only considering the perturbation in the energy density, and we neglect the effect of a different pressure along two directions (in particular, we continue using a pressureless matter fluid everywhere). For sure, what we did is a strong constraint. By the way, the particular choice of the perturbation is not relevant for the purposes of this paper.

In order to be explicit, we write

$$T_\mu^\nu = T_\mu^{\nu(\mathrm{LTB})} + \delta T_\mu^{\nu(\mathrm{AN})} \tag{34}$$

and

$$\begin{aligned} T_{\nu\alpha} &= (g_{\nu\mu}^{(\mathrm{LTB})} + \delta g_{\nu\mu}^{(\mathrm{AN})})(T_\alpha^{\mu(\mathrm{LTB})} + \delta T_\alpha^{\mu(\mathrm{AN})}) \\ &\simeq T_{\nu\alpha}^{(\mathrm{LTB})} + \delta g_{\nu\mu}^{(\mathrm{AN})} T_\alpha^{\mu(\mathrm{LTB})} + g_{\nu\mu}^{(\mathrm{LTB})} \delta T_\alpha^{\mu(\mathrm{AN})}. \end{aligned} \tag{35}$$

As regards the energy conditions, we consider the general energy-momentum tensor:

$$T_{\mu\nu} = \rho u_\mu u_\nu + p(-g_{\mu\nu} + u_\mu u_\nu) \tag{36}$$

with $u_\mu u^\mu = 1$. Hence, energy conditions state the following:

(i) the weak energy condition is $T_{\mu\nu} u^\mu u^\nu \geq 0$,
(ii) the dominant energy condition can be shown by defining $W^\mu = T^{\mu\nu} u_\nu$, $W^\mu W_\mu \geq 0$,
(iii) the strong energy condition is $T_{\mu\nu} u^\mu u^\nu \geq \frac{1}{2} T u^\mu u_\mu$, and
(iv) the null energy condition is $T_{\mu\nu} k^\mu k^\nu \geq 0$, where k^μ is a lightlike vector.

As is well known, the hierarchy among these conditions is the following: strong implies null, dominant implies weak, and weak implies null. In this way, by providing that the dominant condition holds, weak and null are also satisfied as well. In particular, for energy-momentum (36), with $p = 0$, they become

(i) weak: $\rho_{\mathrm{mat}} \geq 0$,
(ii) dominant: $\rho_{\mathrm{mat}}^2 \geq 0$,
(iii) strong: $\rho_{\mathrm{mat}} \geq 0$, and
(iv) null: $\rho_{\mathrm{mat}} \geq 0$.

The Einstein's equations in this universe are

$$\begin{aligned} &R_\mu^{\nu(\mathrm{LTB})} + R_{\mu\nu}^{(\mathrm{LTB})} \delta g^{\alpha\nu} + (R_{\mu\alpha}^{(\Sigma)} + R_{\mu\alpha}^{\Theta}) g^{\nu\alpha(\mathrm{LTB})} \\ &= 8\pi G[T_\mu^{\nu(\mathrm{LTB})} + \delta T_\mu^{\nu(\mathrm{AN})} + \frac{1}{2}\delta_\mu^\nu (T^{(\mathrm{LTB})} + \delta T^{(\mathrm{AN})})]. \end{aligned} \tag{37}$$

IV. LUMINOSITY DISTANCE

The concept of distance depends on the assumed model of the Universe and on the matter distribution in it. The measured distances are influenced by inhomogeneities and anisotropy of the Universe; see, for example, Refs. [44,45].

The luminosity distance is one of the most important quantities for understanding the presence of dark energy in the Universe, considering the photon coming from type Ia supernovae. In this section, we want to calculate the luminosity distance for our metric Eq. (9). The reciprocity theorem by Etherington [46] and popularized by Ellis [47] connects the angular diameter distance d_A and the luminosity distance d_L by

$$d_L = (1+z)^2 d_A, \tag{38}$$

where

$$d(\ln d_A) = \frac{1}{2}\nabla_\alpha p^\alpha d\tau \tag{39}$$

with τ the temporal affine parameter and $p^\alpha = dx^\alpha/d\tau$ the quadrimomentum of a generic signal that is started from the supernova and reaches us. To our end, it is necessary to calculate $\nabla_\alpha p^\alpha \equiv \partial_\alpha p^\alpha + \Gamma_{\alpha\mu}{}^\alpha p^\mu$:

$$\begin{aligned} \nabla_\alpha p^\alpha &= \partial_\alpha p^\alpha + \Gamma_{\alpha\mu}{}^\alpha p^\mu = \partial_\alpha p^\alpha + \frac{\partial_\mu \sqrt{-g}}{\sqrt{-g}} p^\mu \\ &= \partial_0 p^0 + \partial_1 p^1 + \frac{\partial_0\sqrt{-g}}{\sqrt{-g}} p^0 + \frac{\partial_1\sqrt{-g}}{\sqrt{-g}} p^1 \end{aligned} \tag{40}$$

with

$$-g = A_\parallel^2 (g_{11}g_{22} - g_{12}^2)\sin^2\theta \equiv A_\parallel^2 B^2(t,r,\theta)\sin^2\theta, \tag{41}$$

where we define

$$B(t,r,\theta) \equiv \sqrt{g_{11}(t,r,\theta)g_{22}(t,r,\theta) - g_{12}^2(t,r,\theta)}. \tag{42}$$

In this way, we obtain

$$\frac{\partial_0\sqrt{-g}}{\sqrt{-g}} = \frac{(\partial_0 A_\parallel B + A_\parallel \partial_0 B)\sin\theta}{A_\parallel B\sin\theta} = \frac{\partial_0 A_\parallel}{A_\parallel} + \frac{\partial_0 B}{B}, \tag{43a}$$

$$\frac{\partial_1\sqrt{-g}}{\sqrt{-g}} = \frac{(\partial_1 A_\parallel B + A_\parallel \partial_1 B)\sin\theta}{A_\parallel B\sin\theta} = \frac{\partial_1 A_\parallel}{A_\parallel} + \frac{\partial_1 B}{B}. \tag{43b}$$

This permits to write Eq. (40) as

$$\begin{aligned} &\partial_0 p^0 + \partial_1 p^1 + \left(\frac{\partial_0 A_\parallel}{A_\parallel} + \frac{\partial_0 B}{B}\right)p^0 + \left(\frac{\partial_1 A_\parallel}{A_\parallel} + \frac{\partial_1 B}{B}\right)p^1 \\ &= \partial_0 p^0 + \partial_1 p^1 + \frac{1}{A_\parallel}\frac{dA_\parallel}{d\tau} + \left(\frac{\partial_0 B}{B}p^0 + \frac{\partial_1 B}{B}p^1\right). \end{aligned} \tag{44}$$

In the last equation, we have considered the general relation between partial derivatives in the coordinates x^α and total derivatives in the affine time τ. In fact, if Φ is a generic function that depends from the coordinates, it is possible to write

$$\frac{d\Phi(x^\alpha(\tau))}{d\tau} = \frac{\partial\Phi(x^\alpha)}{\partial x^\beta}\frac{dx^\beta}{d\tau} \equiv \partial_\beta\Phi p^\beta. \tag{45}$$

On the other hand, as regards B, we must write $\frac{dB}{d\tau} = \partial_0 B p^0 + \partial_1 B p^1 + \partial_2 B p^2$, but we are considering the radial signal; therefore, $p^2 \equiv d\theta/d\tau$—that is to say, $\theta(\tau) = \mathrm{cost}$. In this way, we can consider θ as a parameter that is able to locate the trajectory of propagation of light. This employment permits us to write

$$B(r,t,\theta) \approx B(r(\tau),t(\tau),\theta) \Rightarrow \frac{dB}{d\tau} \approx \partial_0 B p^0 + \partial_1 B p^1. \tag{46}$$

Therefore, Eq. (40) becomes

$$\nabla_\alpha p^\alpha = \partial_0 p^0 + \partial_1 p^1 + \frac{1}{A_\parallel}\frac{dA_\parallel}{d\tau} + \frac{1}{B}\frac{dB}{d\tau}. \tag{47}$$

As regards the partial derivative of p^α, remembering that we are considering the radial propagation of signals, the relevant components are

$$dp^0 + \Gamma_{00}{}^0 dx^0 p^0 + \Gamma_{10}{}^0(dx^1 p^0 + dx^0 p^1) + \Gamma_{11}{}^0 dx^1 p^1 = 0, \tag{48a}$$

$$dp^1 + \Gamma_{00}{}^1 dx^0 p^0 + \Gamma_{10}{}^1(dx^1 p^0 + dx^0 p^1) + \Gamma_{11}{}^1 dx^1 p^1 = 0, \tag{48b}$$

from which we have

$$\partial_0 p^0 = -(\Gamma_{00}{}^0 p^0 + \Gamma_{10}{}^0 p^1), \tag{49a}$$

$$\partial_1 p^1 = -(\Gamma_{10}{}^1 p^0 + \Gamma_{11}{}^1 p^1). \tag{49b}$$

In order to complete the analysis, observe that $\Gamma_{00}{}^0 = \Gamma_{10}{}^0 = 0$ and

$$\begin{aligned}\Gamma_{10}{}^1 &= \frac{1}{2}g^{11}(\partial_1 g_{01} + \partial_0 g_{11} - \partial_1 g_{10}) \\ &\quad + \frac{1}{2}g^{12}(\partial_1 g_{02} + \partial_0 g_{21} - \partial_2 g_{10}) \\ &= -(g^{11}X\partial_0 X + g^{12}F\partial_0 F),\end{aligned} \tag{50}$$

$$\begin{aligned}\Gamma_{11}{}^1 &= \frac{1}{2}g^{11}(\partial_1 g_{11} + \partial_1 g_{11} - \partial_1 g_{11}) \\ &\quad + \frac{1}{2}g^{12}(\partial_1 g_{12} + \partial_1 g_{21} - \partial_2 g_{11}) \\ &= -(g^{11}X\partial_1 X + 2g^{12}F\partial_1 F - X\partial_2 X),\end{aligned} \tag{51}$$

where we have put

$$g_{11} \equiv X^2, \qquad g_{22} \equiv Y^2, \qquad g_{12} \equiv F^2. \tag{52}$$

Now, we work in a small approximation of anisotropy, in order to use Eq. (23) to the lower order; in this way, it is possible to write

$$\Gamma_{10}{}^1 \to \Gamma_{10}^{1(\mathrm{LTB})} = \frac{\partial_0\partial_1 A_\parallel}{\partial_1 A_\parallel}, \tag{53a}$$

$$\Gamma_{11}{}^1 \to \Gamma_{11}^{1(\mathrm{LTB})} = \frac{\partial_1^2 A_\parallel}{\partial_1 A_\parallel}, \tag{53b}$$

where $\partial_1^2 \equiv \frac{\partial^2}{\partial r^2}$. Therefore, Eq. (47) is

$$\nabla_\alpha p^\alpha \approx -\left(\frac{\partial_0\partial_1 A_\parallel}{\partial_1 A_\parallel}p^0 + \frac{\partial_1^2 A_\parallel}{\partial_1 A_\parallel}p^1\right) + \frac{1}{A_\parallel}\frac{dA_\parallel}{d\tau} + \frac{1}{B}\frac{dB}{d\tau}. \tag{54}$$

Taking into account Eq. (45), it is possible to write the first two terms in Eq. (54) as $\frac{1}{\partial_1 A_\parallel}\frac{dA_\parallel}{d\tau}$.

Inserting Eq. (54) into Eq. (39), it is possible to obtain d_A; in fact, we have

$$\frac{dd_A}{d_A} \approx \frac{1}{2}\left(\frac{1}{A_\parallel}\frac{dA_\parallel}{d\tau} + \frac{1}{B}\frac{dB}{d\tau} - \frac{1}{\partial_1 A_\parallel}\frac{d\partial_1 A_\parallel}{d\tau}\right)d\tau, \tag{55}$$

and integrating in τ, we obtain

$$d_A(r,t,\theta) = \sqrt{\frac{A_\parallel(r,t)B(r,t,\theta)}{\partial_1 A_\parallel(r,t)}}. \tag{56}$$

This expression of the luminosity distance reduces to the isotropic limit of the LTB metric; in fact, we have

$$\begin{aligned}&X(r,t,\theta) \to \partial_1 A_\parallel(r,t), \\ &A(r,t,\theta) \to A_\parallel(r,t), \\ &F(r,t,\theta) \to 0;\end{aligned} \tag{57}$$

therefore, we obtain the limit

$$\begin{aligned}B(r,t,\theta) \to A_\parallel(r,t)\partial_1 A_\parallel(r,t) &\Rightarrow d_A(r,t,\theta) \\ &\to d_A^{(\mathrm{LTB})}(r,t) = A_\parallel(r,t).\end{aligned} \tag{58}$$

V. RELATION BETWEEN COORDINATES AND REDSHIFT

In this section, we want to calculate the luminosity distance in order to obtain an operative expression and therefore to apply it to experimental data. Equation (56) gives us the luminosity distance

$$d_L = (1+z)^2\sqrt{\frac{A_\parallel(r,t)B(r,t,\theta)}{\partial_1 A_\parallel(r,t)}}, \tag{59}$$

but this expression is not directly applicable because it depends on (r,t) coordinates and on the redshift. Therefore, it is necessary to find the relations $r(z)$ and $t(z)$. To this end, let us consider the definition of redshift and let us use the static observator classes that are also geodetic (${\Gamma_{00}}^\mu = 0$). We consider light signal, that is to say, $p^0 \propto 1/\delta t$; in this way, we write

$$1+z \equiv \frac{(g_{\mu\nu}u^\mu p^\nu)_{\rm em}}{(g_{\mu\nu}u^\mu p^\nu)_{\rm oss}} = \frac{p^0_{\rm em}}{p^0_{\rm oss}} = \frac{\delta t_{\rm oss}}{\delta t_{\rm em}} \Rightarrow 1+z(\tau) = \frac{\delta t_{\rm oss}}{\delta t(\tau)}, \tag{60}$$

where $u^\mu = (1,0,0,0)$ and $g_{00} = 1$. Now, we derive with respect to τ and we have

$$\frac{dz(\tau)}{d\tau} = -\frac{\delta t_{\rm oss}}{\delta t(\tau)^2}\frac{d\delta t(\tau)}{d\tau} \equiv -\frac{1+z(\tau)}{\delta t(\tau)}\frac{d\delta t(\tau)}{d\tau} \Rightarrow \frac{d\delta t}{d\tau} = -\frac{\delta t}{1+z}\frac{dz}{d\tau}. \tag{61}$$

On the other hand, for geodetic radial signals, we have $ds^2 = 0$ and $d\theta = d\phi = 0$ that give

$$dt^2 - X(r,t,\theta)^2 dr^2 = 0 \Rightarrow dt = \pm X(r,t,\theta)dr. \tag{62}$$

As regards the ambiguity of the sign, we must consider the minus sign; because of increasing the distance ($dr > 0$), we have a more ancient signal ($dt < 0$). When we consider the signals that, respectively, start at time t and $t + \delta t$, Eq. (62) must be valid; therefore, we have

$$\frac{dt}{d\tau} = -X(r,t,\theta)\frac{dr}{d\tau}, \tag{63a}$$

$$\frac{d(t+\delta t)}{d\tau} = -X(r,t+\delta t,\theta)\frac{dr}{d\tau}. \tag{63b}$$

Equation (63b) can be written as

$$\frac{dt}{d\tau} + \frac{d\delta t}{d\tau} \approx -[X(r,t,\theta) + \delta t\partial_0 X(r,t,\theta)]\frac{dr}{d\tau} \tag{64}$$

that, taking into account Eq. (63a), can be written as

$$\frac{d\delta t}{d\tau} \approx -\delta t\partial_0 X(r,t,\theta)\frac{dr}{d\tau} = -\delta t\partial_0 X(r,t,\theta)\frac{dr}{dz}\frac{dz}{d\tau}. \tag{65}$$

Now, Eqs. (61) and (65) are equal; therefore, we have

$$\frac{dr}{dz} = \frac{1}{1+z}\frac{1}{\partial_0 X(r,t,\theta)}. \tag{66}$$

As regards $t(z)$, it is important to remember that

$$\frac{dt}{d\tau} = \frac{dt}{dz}\frac{dz}{d\tau} \quad \text{and} \quad \frac{dr}{d\tau} = \frac{dr}{dz}\frac{dz}{d\tau}; \tag{67}$$

in this way, taking into account Eq. (63b), we obtain the relation

$$\frac{dt}{dz} = -\frac{1}{1+z}\frac{X(r,t,\theta)}{\partial_0 X(r,t,\theta)}. \tag{68}$$

Putting it all together, we are able to write the luminosity distance as a function of the redshift z and the angle θ:

$$d_L(z,\theta) = (1+z)^2\left[\frac{A_\parallel(r_\theta(z),t_\theta(z))}{\partial_1 A_\parallel(r_\theta(z),t_\theta(z))}B(r_\theta(z),t_\theta(z),\theta)^{\frac{1}{2}}\right]^{\frac{1}{2}}, \tag{69a}$$

$$\frac{dr_\theta(z)}{dz} = \frac{1}{1+z}\frac{1}{\partial_0 X(r_\theta(z),t_\theta(z),\theta)}, \tag{69b}$$

$$\frac{dt_\theta(z)}{dz} = -\frac{1}{1+z}\frac{X(r_\theta(z),t_\theta(z),\theta)}{\partial_0 X(r_\theta(z),t_\theta(z),\theta)}. \tag{69c}$$

It is important to observe that the subscript θ reminds us that the functions r and t are determined by the resolution of the system given by Eqs. (69b) and (69c), where the angle θ is fixed and considered as a constant parameter during the propagation of light.

VI. COMPARISON WITH EXPERIMENTAL DATA

The accelerating expansion of the Universe is driven by mysterious energy with negative pressure known as dark energy. In spite of all the observational evidence, the nature of dark energy is still a challenging problem in theoretical physics; therefore, there has been a new interest in studying alternative cosmological models [14].

In the context of FLRW models, the acceleration of the Universe requires the presence of a cosmological constant. But, it does not appear to be natural to introduce the presence of a cosmological constant and does not appear to be natural to introduce the dark energy.

In this section, we consider the comparison between experimental data, in particular, with the UNION 2 data set of type Ia supernovae and our inhomogeneous and anisotropic Universe. Let us suppose a small anisotropy in order to write the functions $A_\parallel$ and $A_\perp$ as solutions of a LTB universe with null curvature and matter dominated. We have

$$A_\parallel(r,t) = r\left(1+\frac{3}{2}H_\parallel(r)t\right)^{\frac{2}{3}}, \tag{70a}$$

$$A_\perp(r,t) = r\left(1+\frac{3}{2}H_\perp(r)t\right)^{\frac{2}{3}}, \tag{70b}$$

where we have considered the following parametrization:

$$H_{\parallel/\perp}(r) = H_{\parallel/\perp} + \Delta H_{\parallel/\perp} \exp\left(-\frac{r}{r_{\parallel/\perp}}\right). \quad (71)$$

In this way, we have the possibility to obtain again the simple model in which $H_\parallel = H_\perp$, $\Delta H_\parallel = \Delta H_\perp$, and $r_\parallel = r_\perp$. Let us consider that today, and in our position in the Universe ($t = 0$ and $r = 0$), the Hubble constant is $67.3 \pm 1.2 \frac{\mathrm{km}}{s}/\mathrm{Mpc}$ [48]. We have the following conditions $H_\parallel + \Delta H_\parallel = H_\perp + \Delta H_\perp = 67.3$; therefore, we have

$$H_\parallel = 67.3 - \Delta H_\parallel, \quad (72a)$$

$$H_\perp = 67.3 - \Delta H_\perp. \quad (72b)$$

In this way, we do not have the six parameters of the model; now, they are four: $\Delta H_\parallel$, $\Delta H_\perp$, $r_\parallel$, and $r_\perp$. At this point, we remember the limits given by (14); therefore, for $\epsilon \sim 0$, we have

$$A_\perp(r,t) \approx A_\parallel(r,t) \Rightarrow H_\parallel(r) \approx H_\perp(r). \quad (73)$$

This condition must be transferred to the four parameters.

The condition Eq. (73) is obtained when $\alpha \sim 1$ and $\omega \sim 1$. On the other hand, it also must be $\epsilon' \sim 0$. Therefore, we have

$$A'_{\parallel/\perp} = \left(1 + \frac{2}{2} H_{\parallel/\perp} t\right)^{\frac{2}{3}} + \frac{r H'_{\parallel/\perp} t}{(1 + \frac{3}{2} H_{\parallel/\perp} t)^{\frac{1}{3}}}$$
$$= \frac{A_{\parallel/\perp}}{r} + \frac{r^{\frac{3}{2}} H'_{\parallel/\perp} t}{A_{\parallel/\perp}}, \quad (74)$$

from which we obtain

$$\alpha \equiv \frac{r_\perp}{r_\parallel}, \quad (75a)$$

$$\omega \equiv \frac{\Delta H_\perp}{\Delta H_\parallel}, \quad (75b)$$

$$A'_\parallel - A'_\perp = \frac{A_\parallel - A_\perp}{r} + r^{\frac{3}{2}} t \left(\frac{H'_\perp}{A_\perp} - \frac{H'_\parallel}{A_\parallel}\right)$$
$$= \frac{\epsilon}{r} + r^{\frac{3}{2}} t \left(-\frac{\Delta H_\perp}{A_\perp r_\perp} + \frac{\Delta H_\parallel}{A_\parallel r_\parallel}\right). \quad (76)$$

In this way, for $\epsilon \sim 0$ and $\epsilon' \sim 0$, we must write

$$\frac{\Delta H_\perp}{\Delta H_\parallel} = \frac{A_\perp}{A_\parallel} \frac{r_\perp}{r_\parallel} \Rightarrow \omega = \frac{A_\perp}{A_\parallel} \alpha. \quad (77)$$

TABLE I. Best-fit values for the parameters.

$\Delta H_\parallel$[km(s Mpc)$^{-1}$]	$r_\parallel$[Gpc]	α
25.4	2.88	1.1

Therefore, $A_\perp / A_\parallel \approx 1$, from which we obtain $\omega \approx \alpha$. In conclusion, we also have three parameters: $\Delta H_\parallel$, $r_\parallel$, and α. The advantage of this parametrization is that we can change $\Delta H_\parallel$ and $r_\parallel$ as we want, taking into account that $\alpha \simeq 1$. In our work, we have changed α in the range [0.9, 1].

In Table I, we have the best-fit values of the parameters for $\tilde{\chi}^2 = 0.95$. In Fig. 1, we have the Hubble diagram for the 557 type Ia supernovae of the UNION 2 catalog. The best-fit curve is in the same diagram. The fit of the cosmological observational data is in very good agreement, without using any dark energy.

According to our ansatz, it is important to stress that $\rho_{\mathrm{mat}} = \rho_{\parallel\mathrm{mat}} + \delta_{\mathrm{mat}} \cos^2\theta$, so the dominant energy condition up to first order gives

$$\delta_{\mathrm{mat}} \cos^2\theta \geq -\frac{\rho_{\parallel\mathrm{mat}}}{2} \quad (78)$$

while the other ones give

$$\delta_{\mathrm{mat}} \cos^2\theta \geq -\rho_{\parallel\mathrm{mat}}. \quad (79)$$

Furthermore, $\delta_{\mathrm{mat}} = \rho_{\perp\mathrm{mat}} - \rho_{\parallel\mathrm{mat}}$, so, from the Einstein's equation, we have that

$$\rho_{\parallel\mathrm{mat}} = \frac{1}{8\pi G}\left[\left(\frac{\dot{A}_\parallel}{A_\parallel}\right)^2 + 2\frac{\dot{A}_\parallel}{A_\parallel}\frac{\dot{A}'_\parallel}{A'_\parallel}\right],$$
$$\delta_{\mathrm{mat}} = \frac{1}{8\pi G}\left[\left(\frac{\dot{A}_\perp}{A_\perp}\right)^2 + 2\frac{\dot{A}_\perp}{A_\perp}\frac{\dot{A}'_\perp}{A'_\perp} - \left(\frac{\dot{A}_\parallel}{A_\parallel}\right)^2 - 2\frac{\dot{A}_\parallel}{A_\parallel}\frac{\dot{A}'_\parallel}{A'_\parallel}\right]. \quad (80)$$

Hence, we have from solutions (70a) and (70b), with conditions (75a), (75b), and (77), that Eq. (78) gives a

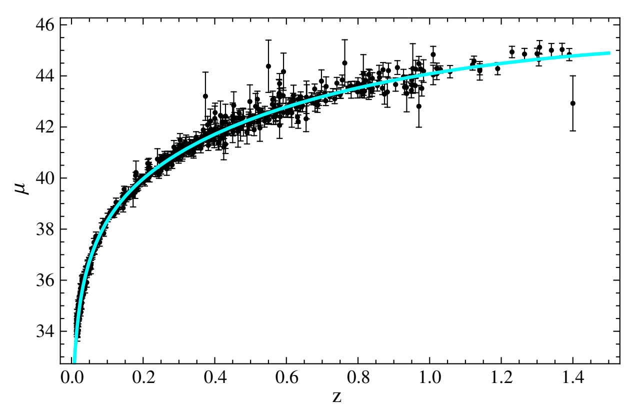

FIG. 1 (color online). Hubble diagram for type Ia supernovae in the UNION 2 catalog. The curve is the best fit.

constraint on parameters which must be satisfied. In particular, by using the best-fit values for $r_{\parallel}$ and $\Delta H_{\parallel}$, the dominant energy condition requires $\alpha < 1.5$, which is in full agreement with our analysis.

VII. CONCLUSION

In the present paper, we have studied the possible effects of anisotropy and inhomogeneity in the expansion of the Universe.

The motivation behind this choice is that singly, inhomogeneous cosmological models and the Bianchi type I cosmological model of the Universe have motivations of truth that must not be left out for one of the two models. Both models may be unified in an anisotropic expansion of the inhomogeneous Universe. The LTB Bianchi type I model possesses important specific properties, and at the same time, this is not too complicated from a physical and mathematical point of view.

In particular, we have connected this model on present-day observations as the luminosity distance of type Ia supernovae. We fit observational data from the UNION 2 catalog of type Ia supernovae with a LTB Bianchi type I model of the Universe. The agreement is good. We have no dark energy in this model.

We are sure that the voids in the Universe dominate, while matter is distributed in a filamentary structure. Therefore, photons must travel through the voids and the presence of inhomogeneities can alter the observable with respect to the corresponding FLRW model of the Universe, which is homogeneous and isotropic.

The key point is that in this model, we have two contributions to the Hubble diagram of type Ia supernovae: inhomogeneity to the large-scale geometry and anisotropy can dynamically generate effects that may remove the need for the postulate of dark energy.

This model must be intended as a first step towards a most general case. The model is oversimplifying for different reasons. First, we have considered only the first order in ϵ. Second, it is necessary to generalize this paper; a very interesting open question that we will study in the future is to obtain how to treat a light-cone average in a more realistic cosmological calculation. Third, we have considered the simple LTB model of the Universe, but it may be very interesting to study a more completed inhomogeneous model such as, for example, the Swiss cheese model. Note, finally, that several possibilities are allowed by our model; it will be interesting to compare this model with other experimental cosmological data.

In the future, we will study the possibility that inhomogeneity and anisotropy can have significant effects on the propagation of light, with potentially very important effects on cosmological observations, and we want to study different observational tests that may confirm this model of the Universe.

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[1] J. Ehlers, P. Geren, and R. K. Sachs, J. Math. Phys. (N.Y.) **9**, 1344 (1968).
[2] C. A. Clarkson and R. Barrett, Classical Quantum Gravity **16**, 3781 (1999).
[3] C. Clarkson and R. Maartens, Classical Quantum Gravity **27**, 124008 (2010).
[4] W. Percival, Mon. Not. R. Astron. Soc. **381**, 1053 (2010).
[5] H. J. de Vega, M. C. Falvella, and N. G. Sanchez, arXiv:1009.3494.
[6] A. Blanchard, Astron. Astrophys. Rev. **18**, 595 (2010).
[7] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Phys. Rep. **513**, 1 (2012).
[8] E. L. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006).
[9] G. E. Tauber and J. W. Weinberg, Phys. Rev. **122**, 1342 (1961).
[10] F. S. Labini, Classical Quantum Gravity **28**, 164003 (2011).
[11] G. Lemaître, Ann. Soc. Sci. Bruxelles, Ser. 1 **53**, 51 (1933); G. Lemaître, Gen. Relativ. Gravit. **29**, 641 (1997).
[12] R. C. Tolman, Proc. Natl. Acad. Sci. U.S.A. **20**, 169 (1934).
[13] H. Bondi, Mon. Not. R. Astron. Soc. **107**, 410 (1947).
[14] See, for example, A. Krasinsky, *Inhomogeneous Cosmological Models* (Cambridge University Press, Cambridge, England, 2006).
[15] R. Maartens, Phil. Trans. R. Soc. A **369**, 5115 (2011).
[16] E. Russel, C. B. Kilinc, and O. Pashev, Mon. Not. R. Astron. Soc. **442**, 2331 (2014).
[17] V. G. Gurzadyan, A. A. Starobinsky, T. Ghahramanyan, A. L. Kashin, H. G. Khachatryan, H. Kuloghlian, D. Vetrugno, and G. Yegorian, Astron. Astrophys. **490**, 929 (2008).
[18] V. G. Gurzadyan, T. Ghahramanyan, A. L. Kashin, H. G. Khachatryan, A. A. Kocharyan, H. Kuloghlian, D. Vetrugno, and G. Yegorian, Astron. Astrophys. **498**, L1 (2009).

[19] C. L. Bennet, R. S. Hill, G. Hinshaw, M. R. Nolta, N. Odegard, L. Page, D. N. Spergel, and J. L. Weiland, Astrophys. J. Suppl. Ser. **148**, 97 (2003).

[20] K. Land and J. Magueijo, Phys. Rev. Lett. **95**, 071301 (2005).

[21] J. P. Ralston and P. Jain, Int. J. Mod. Phys. D **13**, 1857 (2004).

[22] A. de Oliveira-Costa, M. Tegmark, M. Zaldarriaga, and A. Hamilton, Phys. Rev. D **69**, 063516 (2004).

[23] C. J. Copi, D. Huterer, and G. D. Starkman, Phys. Rev. D **70**, 043515 (2004).

[24] H. K. Eriksen, F. K. Hansen, A. J. Banday, K. M. Gorski, and P. B. Lilje, Astrophys. J. **605**, 14 (2004); **609**, 1198 (2004).

[25] F. K. Hansen, A. J. Banday, and K. M. Gorski, Mon. Not. R. Astron. Soc. **354**, 641 (2004).

[26] P. Vielva, E. Martinez-Gonzalez, R. B. Barreiro, J. L. Sanz, and L. Cayon, Astrophys. J. **609**, 22 (2004).

[27] J. D. Barrow, Phys. Rev. D **55**, 7451 (1997).

[28] A. Berera, R. V. Buniy, and T. W. Kephart, J. Cosmol. Astropart. Phys. 10 (2004) 016.

[29] L. Campanelli, P. Cea, and L. Tedesco, Phys. Rev. Lett. **97**, 131302 (2006); L. Campanelli, P. Cea, and L. Tedesco, Phys. Rev. Lett. **97**, 209903 (2006);

[30] L. Campanelli, P. Cea, and L. Tedesco, Phys. Rev. D **76**, 063007 (2007).

[31] E. Martinez-Gonzalez and J. L. Sanz, Astron. Astrophys. **300**, 346 (1995).

[32] R. Maartens, G. F. R. Ellis, and W. Stoeger, Phys. Rev. D **51**, 1525 (1995).

[33] R. Maartens, G. F. R. Ellis, and W. Stoeger, Phys. Rev. D **51**, 5942 (1995).

[34] D. Shogin and S. Hervik, J. Cosmol. Astropart. Phys. 10 (2013) 005.

[35] Y. B. Zel'dovich, Sov. Astron. **8**, 13 (1964).

[36] B. Bertotti, Proc. R. Soc. A **294**, 195 (1966).

[37] V. M. Dashevskii and V. I. Slysh, Sov. Astron. **9**, 671 (1966).

[38] E. J. Gunn, Astrophys. J. **150**, 737 (1967).

[39] R. Kantowski, Astrophys. J. **155**, 89 (1969).

[40] C. C. Dyer and R. Roeder, Astrophys. J. **174**, L115 (1972).

[41] S. Weinberg, Astrophys. J. **208**, L1 (1976).

[42] G. F. Ellis, B. A. Bassett, and P. K. S. Dunsby, Classical Quantum Gravity **15**, 2345 (1998).

[43] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).

[44] J. Kristian and R. K. Sachs, Astrophys. J. **143**, 379 (1966).

[45] J. Adamek, E. Di Dio, R. Durrer, and M. Kunz, Phys. Rev. D **89**, 063543 (2014).

[46] I. M. H. Etherington, Philos. Mag. **15**, 761 (1933).

[47] G. Ellis, Gen. Relativ. Gravit. **39**, 1047 (2007).

[48] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **571**, A16 (2014).