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Four-quark structure of the excited states of heavy mesons

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We propose a four-quark structure for some of the excited states of heavy mesons containing a single charm or bottom quark. The four-quark wave functions are constructed based on a diquark-antidiquark form under the constraint that they form an antitriplet $\bar{\mathbf{3}}_f$ in $\mathrm{SU}(3)_f$, which seems to be realized in some of the excited states listed by the Particle Data Group. Depending on the structure of the antidiquark, we construct two possible models for its wave functions: Model I, where the antidiquark is symmetric in flavor ($\bar{\mathbf{6}}_f$) and antisymmetric in color ($\mathbf{3}_c$), and Model II, where the antidiquark is antisymmetric in flavor ($\mathbf{3}_f$) and symmetric in color ($\bar{\mathbf{6}}_c$). To test the phenomenological relevance of these wave functions, we calculate the mass differences among the excited states of spin $J = 0$, 1, 2 using color-spin interactions. The four-quark wave functions based on Model I are found to reproduce the observed mass of the excited states of heavy mesons. Also, our four-quark model provides an interesting phenomenology related to the decay widths of the excited states. To further pursue the possibility of the four-quark structure, we make a few predictions for open-charm and open-bottom states that may be discovered in future experiments. Most of these are expected to have broad widths, which would make them difficult to be identified experimentally. However, one resonance with $J = 1$ containing bottom and strange quarks is expected to appear as a sharp peak with a mass around $B_{1N}^{\bar{s}} \sim 5753$ MeV. The confirmation of the existence of such states in future experiments will shed light on our understanding of the structure of heavy meson excited states.

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I. INTRODUCTION

Multiquark states, which refer to hadrons composed of four or more quarks, are very interesting subjects in hadron physics. Although the ground states of hadrons can be well described by the conventional picture of quark-antiquark systems for mesons and three-quark systems for baryons, there has been a controversy over the existence of exotic states including multiquarks and/or glueballs in hadron spectroscopy. This is because the conventional quark models taking into account color and flavor degrees of freedom do not rule out the possible existence of multiquark states. Indeed, there have been various experiments reporting the candidates of exotic states, which include $X(3872)$ [1], $Y(4260)$ [2], and $Z(4430)$ [3]. For these mesons (among various interpretations), the four-quark scenarios containing two heavy and two light quarks are quite promising [4–6]. Also, pentaquark states triggered by the experiments of the LEPS Collaboration at SPring-8 [7] are still under debate both theoretically and experimentally. The existence of hybrid mesons with gluonic excitations will also be investigated by the Hall-D experiments at Thomas Jefferson National Accelerator Facility [8].

The pure exotic states can be distinguished by their unique quantum numbers, but the existence of cryptoexotic states is hard to identify as their quantum numbers can also be produced by the conventional pictures of hadrons. Therefore, some cryptoexotic multiquark states (other than the newly discovered exotic-state candidates) may have already been observed and listed in the current edition of the Particle Data Group (PDG) [9], especially in hadron excited states. The pioneering work in this direction may be the diquark-antidiquark model advocated by Jaffe in the 1970s [10,11], who proposed the four-quark structure for the scalar meson nonet, $a_0(980)$, $f_0(980)$, $\sigma(600)$, and $\kappa(800)$. (For a review, see Ref. [12].) In this model, diquarks, belonging to a color antitriplet and flavor antitriplet having spin 0, are claimed to be tightly bound and they combine with antidiquarks to form four-quark states. Thus, the four-quark states constructed in this way are (if there are no orbital excitations) restricted to have spin 0. Although this model was confronted with different suggestions based on two-quark pictures, such as the P-wave $\bar{q}q$ [13] or a mixture of various configurations [14], there are other calculations that favor the four-quark picture [15,16].

The lesson from the light-quark system certainly provides theoretical motivations for the possibility of a four-quark structure in the excited states of heavy mesons containing a c or b quark. Experimentally, the excited states of heavy mesons—which were scarcely explored in the past—have

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become much richer thanks to recent experimental investigations, and during the last decade or so the excited states in the open-charm and open-bottom sectors listed in the PDG keep accumulating with various decaying properties. This can provide a nice environment for investigating the structure of heavy meson excited states.

Indeed, there have been various theoretical investigations of the four-quark structure in the excited states of open-charm mesons. These include the phenomenological model studies based on the relativistic quark model [17], the Glozman-Riska hyperfine interaction [18], the 't Hooft interaction [19], QCD sum rules [20,21], etc. Even though there are other suggestions based on the two-quark picture [22] or mixing configurations between two-quark and four-quark states [23], it is still worthwhile to pursue additional signatures for four-quark structure in the excited states of heavy meson systems, and this is the main motivation of the present investigation.

Our approach for four-quark states is quite phenomenological rather than dynamical. By closely examining the current data of heavy meson spectroscopy, we will postulate a plausible flavor structure for the excited states of heavy mesons. Then possible four-quark wave functions will be constructed accordingly based on a diquark-antidiquark picture. Here the diquark is composed of one heavy and one light quark, and the antidiquark is a system of two light antiquarks.

In the present study, we do not restrict our consideration for the antidiquark state to the scalar type which belongs to a color triplet and flavor triplet with zero spin. Instead, we extend our consideration to a more general case by allowing for various possible antidiquark states to see their role in heavy meson excited states. Based on the observation that the excited heavy meson states listed in the PDG have spin 0, 1, or 2, we allow other antidiquark structures other than the scalar state and look for plausible scenarios which can accommodate all of these spin states within one framework. To test the phenomenological relevance of various four-quark models generated from this approach, the mass differences among heavy mesons will be calculated using color-spin interactions and compared with the experimental data.

The paper is organized as follows. In Sec. II, we examine the excited states of heavy mesons in the PDG and motivate the four-quark picture. The four-quark wave functions constructed accordingly will be presented in Sec. III. After a brief introduction of color-spin interactions in Sec. IV, we present our calculations of the hyperfine masses from the four-quark wave functions in Sec. V. Results and discussions are given in Sec. VI, and we summarize in Sec. VII.

II. HEAVY MESON SPECTROSCOPY

We start by examining D- and B-meson spectroscopy compiled by the Particle Data Group, which motivates the possible four-quark structure for the excited states of heavy mesons. In Tables I and II we list the open-charm and open-bottom mesons that can be found in the compilation of the PDG [9]. The lowest-lying states listed in Table I are found to have negative parity. Their isospins are either $I = 1/2$ or $I = 0$, and their spins are 0 or 1. There are four (two) mesons in the D (D_s) family, and three (two) mesons in the B (B_s) family. The excited states, which refer to the resonances with higher masses, are listed in Table II. There are seven (four) mesons in the D (D_s) family, and three (two) in the B (B_s) family.[1] The excited states listed in Table II have some notable features. Their parity is positive, which is opposite that in the lowest-lying case; isospins of all the resonances are either $I = 1/2$ or $I = 0$, as in the lowest-lying states; and their spins are $J = 0$, 1, 2. Within each family, there is a hierarchy in the mass spectrum, i.e., the mass increases with spin J, namely, $m_{J=0} < m_{J=1} < m_{J=2}$.

As anticipated, the spectrum of the lowest-lying states is consistent with the conventional $Q\bar{q}$ picture. They form an antitriplet in $\mathrm{SU}(3)_f$ as one can see from Table III, where the mesons are regrouped according to their spin and parity J^P. In most cases, there are three mesons for each J^P, composed of two members in an isodoublet ($I = 1/2$) and one member in an isosinglet ($I = 0$). The mass splitting Δm between $I = 1/2$ and $I = 0$ members is about 90–100 MeV, which, though somewhat smaller than the quark mass difference $m_s - m_u$, still supports the formation of $\bar{\mathbf{3}}_f$. The only exception is the B mesons in the $J^P = 1^-$ channel, where one member in an isodoublet ($I = 1/2$) is missing. But the mass splitting between $B^*(5325)$ and $B_s^*(5415)$ is again 90 MeV, which is similar in magnitude to that of other $\bar{\mathbf{3}}_f$ multiplets. Even though one more member is anticipated in this channel, we expect that it will be discovered soon

TABLE I. The lowest-lying resonances with $J^P = 0^-, 1^-$ in the D, D_s, B, B_s families listed in the PDG [9].

	Lowest-lying states			
Family	Meson	$I(J^P)$	Mass (MeV)	Γ (MeV)
D	D^0	$\frac{1}{2}(0^-)$	1864.86	-
	$D^\pm$	$\frac{1}{2}(0^-)$	1869.62	-
	D^{*0}	$\frac{1}{2}(1^-)$	2006.99	< 2.1
	$D^{*\pm}$	$\frac{1}{2}(1^-)$	2010.29	0.096
D_s	$D_s^\pm$	$0(0^-)$	1968.50	-
	$D_s^{*\pm}$	$0(1^-)$	2112.3	< 1.9
B	$B^\pm$	$\frac{1}{2}(0^-)$	5279.25	-
	B^0	$\frac{1}{2}(0^-)$	5279.58	-
	B^*	$\frac{1}{2}(1^-)$	5325.2	-
B_s	B_s^0	$0(0^-)$	5366.77	-
	B_s^*	$0(1^-)$	5415.4	-

[1]Some mesons are not included in this list because their quantum numbers are unknown and their masses are higher than the states that we are considering in this work.

TABLE II. The low-lying excited states with $J^P = 0^+, 1^+, 2^+$ in the D, D_s, B, B_s families collected from the PDG. According to the PDG, the quantum numbers (I, J, P) of most of the excited mesons have yet to be confirmed. The $D_1^{\pm}(2423)$, whose J^P is unknown, is assigned to have $J^P = 1^+$ in our analysis because its mass is similar to D_1^0.

	Excited states			
Family	Meson	$I(J^P)$	Mass (MeV)	Γ (MeV)
D	D_0^{*0}	$\frac{1}{2}(0^+)$	2318.29	267
	$D_0^{*\pm}$	$\frac{1}{2}(0^+)$	2403	283
	D_1^0	$\frac{1}{2}(1^+)$?	2421.4	27.4
	$D_1^{\pm}$	$\frac{1}{2}(1^+)$	2423.2	25
	D_1^0	$\frac{1}{2}(1^+)$	2427	384
	D_2^{*0}	$\frac{1}{2}(2^+)$	2462.6	49
	$D_2^{*\pm}$	$\frac{1}{2}(2^+)$	2464.3	37
D_s	$D_{s0}^{*\pm}$	$0(0^+)$	2317.8	< 3.8
	$D_{s1}^{\pm}$	$0(1^+)$	2459.6	< 3.5
	$D_{s1}^{\pm}$	$0(1^+)$	2535.12	0.92
	$D_{s2}^{*\pm}$	$0(2^+)$	2571.9	17
B	B_1^0	$\frac{1}{2}(1^+)$	5723.5	-
	B_2^{*0}	$\frac{1}{2}(2^+)$	5743	23
	B_J^*	?(?$^?$)	5698	128
B_s	B_{s1}^0	$0(1^+)$	5828.7	-
	B_{s2}^{*0}	$0(2^+)$	5839.96	1.56

at current experimental facilities and one can safely claim that the B mesons of $J^P = 1^-$ also form $\bar{\mathbf{3}}_f$. This antitriplet structure is consistent with the two-quark systems having a charm (or a bottom) and a light antiquark, namely $c\bar{q}(q = u, d, s)$ (or $b\bar{q}$) in relative S-wave state. The negative parity appears naturally with this quark composition.

We now speculate on the structure of the excited states listed in Table II. Since these states have positive parity, one can think of two possible ways to construct such states. The first way is based on the two-quark picture. Here, the states with positive parity can be constructed by orbitally exciting the lowest-lying states ($\ell = 1$). By combining these with the spin of the two-quark $j = (0, 1)$, one can generate the total spin $J = 0, 1, 2$ for positive-parity states. Then the mass splitting among the excited states can be generated by spin-orbit forces. In particular, the mass splitting between $J^P = 1^+$ and $J^P = 0^+$ members is expected to be about half of the one between $J^P = 2^+$ and $J^P = 1^+$ members [24]. We see from Table II that this expectation works well for $D_0^{*\pm}(2403)$, $D_1^{*\pm}(2423)$, and $D_2^{\pm}(2464)$, but it fails for $D_0^*(2318)$, $D_1^0(2421)$, and $D_2(2463)$.

Another way to construct the positive-parity excited states, which we want to pursue in the present work, is to make the product of the $\mathrm{SU}(3)_f$ singlet of $\bar{q}q$ of negative parity and the ground states of $c\bar{q}$ (or $b\bar{q}$). The resulting states contain four quarks and they obviously form a $\bar{\mathbf{3}}_f$ in $\mathrm{SU}(3)_f$. Of course, the states constructed in this way are close to the two-meson molecular states. Motivated by this observation, however, in this work we want to investigate the general features of four-quark resonance states in the heavy-quark sector. Thus, a similar approach like the diquarkonia model [10,11,25] will be adopted for quantitative estimates.

The present investigation is also motivated by the $\bar{\mathbf{3}}_f$ structure observed explicitly in the excited states of Table III. In the $J^P = 2^+$ channel of the "D or D_s" family, there are three members, namely, $D_2^{\pm}(2464)$, $D_2^{*0}(2463)$, and $D_{s2}^{*\pm}(2572)$ with the isospins expected from the $\bar{\mathbf{3}}_f$ multiplet. The mass splitting between $I = 1/2$ and $I = 0$ members is about 108 MeV, which is similar to the splitting in the lowest-lying mesons. Thus, the three resonances in $J^P = 2^+$ seem to form a $\bar{\mathbf{3}}_f$.

In the $J^P = 1^+$ channel of the "D or D_s" family, $D_1^{\pm}(2423)$, $D_1^0(2421)$, and $D_{s1}^{*\pm}(2535)$ seem to form a $\bar{\mathbf{3}}_f$ with the mass splitting Δm of 113 MeV. However, there is another state, $D_{s1}^{\pm}(2460)$ of $I = 0$, which is hard to classify as a member of $\bar{\mathbf{3}}_f$. Later, we will discuss the importance implied by the existence of this state. We will find that, in the four-quark picture with $\bar{\mathbf{3}}_f$, there are two possible ways to make the spin-1 states, and, after taking care of the mixing between the two, $D_{s1}^{\pm}(2460)$ fits nicely with the member in the spin-1 channel.

In the $J^P = 1^+$ channel from the "B or B_s" family, there are three resonances. Here, the $B_J^*(5698)$ may not be a member of an isodoublet with $B_1^0(5724)$ because of their large mass difference of 26 MeV. But its existence as well as its quantum number is not well established yet. The other two, $B_1^0(5724)$ and $B_{s1}^0(5829)$, have a mass splitting around 106 MeV, similar to the mass splitting expected from the structure of $\bar{\mathbf{3}}_f$. Also, in the $J^P = 2^+$ channel from the "B or B_s" family there are only two resonances with the mass splitting 97 MeV, which again is a similar magnitude as that expected from $\bar{\mathbf{3}}_f$. So even though one member in the isodoublet is missing, the two resonances seem to be members of $\bar{\mathbf{3}}_f$.

A somewhat puzzling situation can be seen in the $J^P = 0^+$ channel. In the charm sector, even though we have three resonances, the mass of $D_{s0}^{\pm}(2318)$ is almost similar to that of $D_0^{*0}(2318)$. This shows that the $D_{s0}^{\pm}$ cannot be a member of $\bar{\mathbf{3}}_f$ and it may not be described by our four-quark model with $\bar{\mathbf{3}}_f$. This $D_{s0}^{\pm}(2318)$ resonance may be a chiral partner of the $D_s^{\pm}(1969)$ [26,27] in a two-quark picture, or it could be a DK molecule in a multiquark picture [28]. Also, the $D_0^{*\pm}(2403)$ (because of its large mass) may not form an isodoublet with the $D_0^{*0}(2318)$. In the bottom sector, there are no resonances reported from the "B or B_s" family in the $J^P = 0^+$ channel. As we will see later, the resonances belonging to $J^P = 0^+$—if they are constructed with our four-quark picture—are found to have strong components in the pseudoscalar-pseudoscalar decay

TABLE III. D, D_s and B, B_s families compiled by the quantum numbers J^P. Δm is the mass difference between the $I = 1/2$ and $I = 0$ members, which shows that most low-lying resonances in each spin channel form $\bar{\mathbf{3}}_f$ with a mass splitting around 100 MeV. For the excited states, since the mass difference between the $I = 1/2$ states is not small, the mass splitting Δm is calculated using the underlined members in $I = 1/2$ as the reference point. We put the question mark for the B_J^* meson in the $J^P = 1^+$ channel as its quantum numbers are unknown. The other question marks represent the undiscovered states.

	Family	J^P	I	Meson	Δm (MeV)
Lowest-lying states	D or D_s	0^-	$\frac{1}{2}$	$D^\pm(1870)$, $D^0(1865)$	
			0	$D_s^\pm(1968)$	101
		1^-	$\frac{1}{2}$	$D^{*\pm}(2010)$, $D^{*0}(2007)$	
			0	$D_s^{*\pm}(2112)$	104
	B or B_s	0^-	$\frac{1}{2}$	$B^\pm(5279)$, $B^0(5280)$	
			0	$B_s^0(5367)$	87
		1^-	$\frac{1}{2}$	$B^*(5325)$, ?	
			0	$B_s^*(5415)$	90
Excited states	D or D_s	0^+	$\frac{1}{2}$	$D_0^{*\pm}(2403)$, $\underline{D_0^{*0}(2318)}$	
			0	$D_{s0}^{*\pm}(2318)$	−0.2
		1^+	$\frac{1}{2}$	$\underline{D_1^\pm(2423)}$, $D_1^0(2427)$, $\underline{D_1^0(2421)}$	
			0	$D_{s1}^\pm(2460)$	37.3
			0	$D_{s1}^\pm(2535)$	112.8
		2^+	$\frac{1}{2}$	$\underline{D_2^{*\pm}(2464), D_2^{*0}(2463)}$	
			0	$D_{s2}^{*\pm}(2572)$	108.4
	B or B_s	0^+	$\frac{1}{2}$	?, ?	
			0	?	?
		1^+	$\frac{1}{2}$	$\underline{B_1^0(5724)}$, $B_J^*(5698, ?)$	
			0	$B_{s1}^0(5829)$	105.9
		2^+	$\frac{1}{2}$	$\underline{B_2^{*0}(5743)}$, ?	
			0	$B_{s2}^{*0}(5840)$	97

channels with low invariant masses. Because of this, they can have large decay widths, making experimental observation difficult. Indeed, we note that $D_0^{*0}(2318)$ has a broad width of 267 MeV which was only recently reported by the PDG.[2]

In this section, we have examined the excited states of positive parity listed in the PDG, which shows that there are several reasons to believe that most excited states form $\bar{\mathbf{3}}_f$ in flavor space. Though some resonances are still missing in the PDG, this examination motivates us to pursue a possible four-quark structure based on $\bar{\mathbf{3}}_f$ for the study of the excited states of heavy mesons containing a charm or a bottom quark.

III. FOUR-QUARK WAVE FUNCTIONS

In this section, we construct four-quark wave functions for the excited mesons in the D and D_s families. As we have discussed in the previous section, most excited states of heavy mesons listed in the PDG have positive parity with $I = (0, 1/2)$ and $J = (0, 1, 2)$. In addition, they seem to have the flavor structure of $\bar{\mathbf{3}}_f$. From the phenomenological point of view, these properties can be generated by multiplying an SU(3) singlet $\bar{q}^i q_i$ by the two-quark systems, $Q\bar{q}^i$ ($q_i = u, d, s$), where Q stands for a heavy quark. Therefore, $Q = c$ for the D and D_s families and $Q = b$ for the B and B_s families. To construct four-quark resonance states instead of molecular states, we follow the diquark-antidiquark approach [10,11] and impose the phenomenological aspect of the $\bar{\mathbf{3}}_f$ structure mentioned above. Such four-quark states can be schematically expressed as $Qq_i\bar{q}^j\bar{q}^i$. Therefore, to construct the tetraquark structure the possible flavor, color, and spin configurations of each diquark should be determined.

As far as flavor is concerned, one can separate the antidiquark into two terms, namely, symmetric ($\bar{\mathbf{6}}_f$) and antisymmetric ($\mathbf{3}_f$) combinations as

$$\begin{aligned} \bar{q}^j\bar{q}^i &= \frac{1}{2}(\bar{q}^j\bar{q}^i + \bar{q}^i\bar{q}^j) + \frac{1}{2}(\bar{q}^j\bar{q}^i - \bar{q}^i\bar{q}^j) \\ &\equiv (\bar{q}^j\bar{q}^i)_+ + (\bar{q}^j\bar{q}^i)_-. \end{aligned} \quad (1)$$

[2]This resonance was not listed in the PDG before 2010.

Since these two combinations are orthogonal to each other, we have two possible flavor wave functions for four-quark states:

$$\text{Case 1:}\ D^{\bar{q}^j}|_{\text{flavor}} = \frac{1}{\sqrt{2}} \sum_{q_i=u,d,s} Qq_i(\bar{q}^j\bar{q}^i)_+ = \frac{1}{\sqrt{2}}[Qu(\bar{q}^j\bar{u})_+ + Qd(\bar{q}^j\bar{d})_+ + Qs(\bar{q}^j\bar{s})_+], \tag{2}$$

$$\text{Case 2:}\ D^{\bar{q}^j}|_{\text{flavor}} = \sum_{q_i=u,d,s} Qq_i(\bar{q}^j\bar{q}^i)_- = Qu(\bar{q}^j\bar{u})_- + Qd(\bar{q}^j\bar{d})_- + Qs(\bar{q}^j\bar{s})_-. \tag{3}$$

Here $Q = c$ so that these wave functions denote the excited states of D mesons. When $\bar{q}^j = \bar{u}$ or $\bar{d}$ these four-quark wave functions may represent the excited states in the D family, and when $\bar{q}^j = \bar{s}$ they may be the excited states in the D_s family. From this equation, we can clearly see that the $D^{\bar{q}^j}$ in either case form $\bar{\mathbf{3}}_f$ separately in flavor space.

In color space, the diqaurk belongs to either $\bar{\mathbf{3}}_c$ or $\mathbf{6}_c$ and the antidiquark to $\mathbf{3}_c$ or $\bar{\mathbf{6}}_c$. Thus, to make colorless four-quark states, the diquark and antidiquark should be in either $(\bar{\mathbf{3}}_c, \mathbf{3}_c)$ or $(\mathbf{6}_c, \bar{\mathbf{6}}_c)$. The possible spins of the diquark and antidiquark, represented by J_{12} and J_{34}, respectively, are 0 and 1. By combining these spins, one can generate the total spin states for the four-quark states as $J = 0$, 1, 2 since $\boldsymbol{J} = \boldsymbol{J}_{12} + \boldsymbol{J}_{34}$. Depending on the specific flavor combination we choose, we can determine the possible color and spin configurations.

A. Antidiquark: flavor-symmetric case $(\bar{q}^j\bar{q}^i)_+$

We first discuss the case when the antidiquark is symmetric in flavor, i.e., $(\bar{q}^j\bar{q}^i)_+$. Since the antidiquark should be totally antisymmetric when spin, flavor, and color are considered all together, it can be either $\mathbf{3}_c$ or $\bar{\mathbf{6}}_c$ in color space. When it is in $\mathbf{3}_c$, since this is antisymmetric in color indices, the antidiquark spin is restricted to $J_{34} = 1$ in order to make totally antisymmetric $(\bar{q}^j\bar{q}^i)_+$ systems. On the other hand, the Qq diquark that contains a heavy quark is not constrained by the Pauli principle. Thus, if the four-quark state (namely diquark-antidiquark system) has spin 0, the possible spin configurations for the Qq diquark and the $\bar{q}\,\bar{q}$ antidiquark are $J_{12} = 1$, and $J_{34} = 1$, respectively, which we denote as $|J, J_{12}, J_{34}\rangle = |011\rangle$. For the spin-1 case, we have two spin configurations: i) $|J, J_{12}, J_{34}\rangle = |101\rangle$ and ii) $|J, J_{12}, J_{34}\rangle = |111\rangle$. If this situation is realized in meson spectroscopy, the physical states should be mixing states of these two states in the $J = 1$ channel. For $J = 2$, the only possible spin configuration is $|J, J_{12}, J_{34}\rangle = |211\rangle$. Thus, if the four-quark states are constructed under the assumption that the antidiquark is in a flavor-symmetric and color-antisymmetric state $(\mathbf{3}_c)$, there is one state with $J = 0$, two states with $J = 1$, and one state with $J = 2$. These states are seemingly consistent with the experimental spectra observed for the D and D_s family, as one can see from Table III, suggesting that this model is promising for the excited states of open-charm mesons.

Given the flavor part of the four-quark wave function in Eq. (2), it is straightforward to incorporate the color part. Since the diquark (antidiquark) belongs to $\bar{\mathbf{3}}_c$ $(\mathbf{3}_c)$ in color, we obtain the four-quark wave function as

$$D_J^{\bar{q}^j}[(\bar{q}^j\bar{q}^i)_+ \in \mathbf{3}_c] = \frac{1}{\sqrt{24}} \sum_{q_i=u,d,s} \left\{ \sum_{a,b,d,e,f} \varepsilon_{abd}\varepsilon^{aef}[(Q)^b(q_i)^d]_{J_{12}=0,1}[((\bar{q}^j)_e(\bar{q}^i)_f)_+]_{J_{34}=1} \right\}, \tag{4}$$

where a, b, d, e, and f are color indices. The numerical factor $1/\sqrt{24}$ in Eq. (4) includes the color normalization $1/\sqrt{12}$ as well as the flavor normalization $1/\sqrt{2}$ from Eq. (2). We have also indicated that the Qq diquark can have spin 0 or 1, but the $\bar{q}\,\bar{q}$ antidiquark in the present configuration can only have spin 1.

When the antidiquark is in a color-symmetric state of $\bar{\mathbf{6}}_c$, its spin is restricted to an antisymmetric state, i.e., $J_{34} = 0$. Then the possible spin configurations are $|J, J_{12}, J_{34}\rangle = |000\rangle$ for $J = 0$ and $|J, J_{12}, J_{34}\rangle = |110\rangle$ for $J = 1$. This model with $\bar{\mathbf{6}}_c$ cannot generate a $J = 2$ state and thus this scenario alone cannot explain the observed excited states whose spins range from 0 to 2. If one wants to describe all the states with spin 0, 1, or 2 within the same framework, one should construct a model allowing for both color configurations ($\mathbf{3}_c$ and $\bar{\mathbf{6}}_c$) for the antidiquark, since the two configurations can mix with each other. This is the only way that the $\bar{\mathbf{6}}_c$ configuration can enter into the framework. However, with this mixing scheme—even though we can generate all the spin states—the number of states generated from this scenario seems to be too many. There should be two states in spin 0, three states in spin 1, and one state in spin 2, which is not consistent with the observed excited states. For example, in Table III, if one counts the number of mesons in the D family with charge zero, there is one meson in spin 0, two mesons in spin 1, and one meson in spin 2. For charged mesons in the D family, there is one meson in spin 0, one in spin 1, and one in spin 2. Therefore, the mixing scheme requires the discovery of additional two or three mesons of similar masses in the D family, which seems to be inconsistent with present observations. Thus, the mixing scheme—which allows for both color states ($\mathbf{3}_c$ and $\bar{\mathbf{6}}_c$) for the antidiquark—may be implausible for the excited states. In the present work, when the antidiquark is flavor symmetric $(\bar{q}^j\bar{q}^i)_+$, we consider the antidiquark with the color state $\mathbf{3}_c$ only. This model will be referred to as Model I. Our discussion on colors and possible spin configurations for the diquark and antidiquark (when the antidiquark is in a flavor-symmetric state) is summarized in Table IV.

TABLE IV. Possible spins and colors of the Qq diquark, the $\bar{q}\,\bar{q}$ antidiquark, and four-quark states when the $\bar{q}\,\bar{q}$ antidiquark is symmetric in flavor, $(\bar{q}^j\bar{q}^i)_+$. The case with the antidiquark in the color state $\mathbf{3}_c$ is referred to as Model I.

Qq_i		$(\bar{q}^j\bar{q}^i)_+$		$Qq_i(\bar{q}^j\bar{q}^i)_+$
Spin $(=J_{12})$	Color	Spin $(=J_{34})$	Color	$\vert J,J_{12},J_{34}\rangle$
0	$\bar{\mathbf{3}}_c$	1	$\mathbf{3}_c$	$\vert 101\rangle$
1	$\bar{\mathbf{3}}_c$	1	$\mathbf{3}_c$	$\vert 011\rangle$, $\vert 111\rangle$, $\vert 211\rangle$
0	$\mathbf{6}_c$	0	$\bar{\mathbf{6}}_c$	$\vert 000\rangle$
1	$\mathbf{6}_c$	0	$\bar{\mathbf{6}}_c$	$\vert 110\rangle$

B. Antidiquark: flavor-antisymmetric case $(\bar{q}^j\bar{q}^i)_-$

The other flavor configuration of the $\bar{q}\,\bar{q}$ antidiquark is an antisymmetric combination, $(\bar{q}^j\bar{q}^i)_-$. Again, the Pauli principle requires that the antidiquark is antisymmetric when spin, flavor, and color degrees of freedom are considered all together. We begin with the color-symmetric state $\bar{\mathbf{6}}_c$. Since the antidiquark is flavor antisymmetric, its spin state is restricted to the symmetric state of $J_{34}=1$ in order to make a totally antisymmetric $(\bar{q}^j\bar{q}^i)_-$ system. Since the spin of the Qq diquark can be $J_{12}=0$, 1, the spin of the four-quark states can be $J=0,1,2$. If the four-quark state (namely, the diquark-antidiquark system) has spin 0, the possible spins for the diquark and antidiquark are $J_{12}=1$, $J_{34}=1$, so that the spin configuration of the four-quark system is $|J,J_{12},J_{34}\rangle=|011\rangle$. When $J=1$, however, we again have two spin configurations: i) $|J,J_{12},J_{34}\rangle=|101\rangle$ and ii) $|J,J_{12},J_{34}\rangle=|111\rangle$. When $J=2$, the only possible spin configuration is $|J,J_{12},J_{34}\rangle=|211\rangle$. Thus, in this scenario, one can construct one state in spin 0, two states in spin 1, and one state in spin 2, again seemingly agreeing with the excited meson spectra in the charm sector.

The four-quark wave function can be constructed straightforwardly. Incorporating the color part into Eq. (3), we obtain the four-quark wave functions as

$$D_J^{\bar{q}^j}[(\bar{q}^j\bar{q}^i)_-\in\bar{\mathbf{6}}_c]=\frac{1}{\sqrt{24}}\sum_{q_i=u,d,s}\sum_{a,b}\{[(Q)_a(q_i)_b]_{J_{12}=0,1}[((\bar{q}^j)^a(\bar{q}^i)^b)_-]_{J_{34}=1}+[(Q)_a(q_i)_b]_{J_{12}=0,1}[((\bar{q}^j)^b(\bar{q}^i)^a)_-]_{J_{34}=1}\}, \quad (5)$$

where the possible spins for the diquark J_{12} and antidiquark J_{34} are indicated explicitly. Here, the factor $1/\sqrt{24}$ comes from the color part.

When the antidiquark is in the color-antisymmetric state $\mathbf{3}_c$ (since we are considering a flavor-antisymmetric wave function for the antidiquark), its spin is restricted to an antisymmetric state, i.e., $J_{34}=0$. With this constraint, the possible spin configurations are $|J,J_{12},J_{34}\rangle=|000\rangle$ for $J=0$ and $|J,J_{12},J_{34}\rangle=|110\rangle$ for $J=1$. We cannot generate the spin-2 state in this configuration. Here we have a similar situation as that discussed in the last part of the previous subsection. With a similar argument, this scheme with $\mathbf{3}_c$—even if we allow the mixing among the $\mathbf{3}_c$ and $\bar{\mathbf{6}}_c$ cases—may not be relevant for the excited states. In this work, when the antidiquark is flavor antisymmetric $(\bar{q}^j\bar{q}^i)_-$, we consider the color state with $\bar{\mathbf{6}}_c$ only. This model is referred to as Model II from now on. Our discussion on colors and possible spin configurations for the Qq diquark and $\bar{q}\,\bar{q}$ antidiquark (when the antidiquark is in a flavor-antisymmetric state) is summarized in Table V.

TABLE V. Possible spins (and colors) of the Qq diquark, the $\bar{q}\,\bar{q}$ antidiquark, and four-quark states when the antidiquark is antisymmetric in flavor, $(\bar{q}^j\bar{q}^i)_-$. The case with the antidiquark in the color state $\bar{\mathbf{6}}_c$ is referred to as Model II.

Qq_i		$(\bar{q}^j\bar{q}^i)_-$		$Qq_i(\bar{q}^j\bar{q}^i)_-$
Spin $(=J_{12})$	Color	Spin $(=J_{34})$	Color	$\vert J,J_{12},J_{34}\rangle$
0	$\bar{\mathbf{3}}_c$	0	$\mathbf{3}_c$	$\vert 000\rangle$
1	$\bar{\mathbf{3}}_c$	0	$\mathbf{3}_c$	$\vert 110\rangle$
0	$\mathbf{6}_c$	1	$\bar{\mathbf{6}}_c$	$\vert 101\rangle$
1	$\mathbf{6}_c$	1	$\bar{\mathbf{6}}_c$	$\vert 011\rangle$, $\vert 111\rangle$, $\vert 211\rangle$

IV. COLOR-SPIN INTERACTIONS

To test the four-quark wave functions constructed in the previous section, we now use the color-spin interaction to estimate the mass splittings among the heavy mesons of our concern. The color-spin interaction takes the following simple form [29–33]:

$$V=\sum_{i<j}v_0\lambda_i\cdot\lambda_j\frac{J_i\cdot J_j}{m_im_j}, \quad (6)$$

where the spatial dependence is integrated out. Here λ_i denotes the Gell-Mann matrix, J_i is the spin, and m_i is the constituent mass of the ith quark. The overall strength of the color-spin interaction is controlled by the parameter v_0, which needs to be determined from the experimental data. This interaction is basically a generalization of the dipole-dipole electromagnetic interaction to effectively take into account the gluon exchange among constituent quarks.

Using the color-spin interaction, the hadron mass can be calculated by

$$M_H \sim \sum_i m_i + \langle V \rangle, \tag{7}$$

where the hyperfine mass $\langle V \rangle$ is obtained by using an appropriate hadron wave function. A nice aspect of this approach is that, even though Eq. (7) is not precise enough to reproduce the experimental masses, the mass differences among hadrons are successfully explained by the differences in the hyperfine masses,

$$\Delta M_H \sim \Delta \langle V \rangle. \tag{8}$$

To illustrate this feature, the computed mass differences among several baryons are presented in Table VI with the experimental mass splittings. In the baryon sector, the overall strength v_0 of the color-spin interaction is fitted from the measured $\Delta - N$ mass splitting, which leads to $v_0 \sim (-199.6)^3$ MeV^3. We use this value to calculate the hyperfine masses of other baryons. For the constituent quark masses, we take the conventional values: $m_u = m_d = 330$ MeV, $m_s = 500$ MeV, $m_c = 1500$ MeV, and $m_b = 4700$ MeV. As one can see from Table VI, the splittings from the hyperfine masses are quite consistent with the experimental mass splittings. The largest error is found in the mass difference of $\Sigma_b - \Lambda_b$. But, even in this case, the experimental mass gap is only 13 MeV higher than the calculated hyperfine mass gap. Therefore, Table VI shows that the hyperfine mass splittings are useful in calculating the mass splittings between baryons with different spins and different spin configurations but with the same flavor.[3]

TABLE VI. The hyperfine mass splittings, given in MeV, are compared with the experimental mass differences of baryons. The coupling strength in the color-spin interaction, v_0, is fitted from the $\Delta - N$ mass difference and is used to determine the mass splittings of other resonances.

	Δm from data	Δm from $\langle V \rangle$
$\Delta - N$	292	292 (fit)
$\Sigma - \Lambda$	77	66.2
$\Sigma^* - \Sigma$	192	192.7
$\Xi^* - \Xi$	211	192.7
$\Sigma_c - \Lambda_c$	167	151.8
$\Sigma_c^* - \Sigma_c$	65	64.5
$\Sigma_b - \Lambda_b$	194	181
$\Sigma_b^* - \Sigma_b$	19	20.5

TABLE VII. The hyperfine mass splittings, given in MeV, are compared with the experimental mass differences of mesons. The coupling strength v_0 (fixed from the $\rho - \pi$ mass difference) is used to determine the mass splittings of other resonances.

	Δm from data	Δm from $\langle V \rangle$
$\rho - \pi$	635	635 (fit)
$K^* - K$	396	419.1
$D^* - D$	140	139.7
$D_s^* - D_s$	144	92.2
$B^* - B$	45.8	44.6
$B_s^* - B_s$	48.6	29.4

Similar calculations can be performed for the meson sector, and the results are given in Table VII. In this case, we fit v_0 from the observed $\rho - \pi$ mass splitting, which leads to $v_0 \sim (-235)^3$ MeV^3. This strength is somewhat different from the one fixed in the baryon sector. There could be various reasons for this difference. In particular, it is often believed that the pseudoscalar mesons involved in the analysis acquire contributions from the instanton-induced interactions. Moreover, the pion mass calculated from Eq. (7) involves the hyperfine mass of about 480 MeV, which is comparable in magnitude with the leading quark mass contribution. This situation is rather different from the baryon case where the hyperfine masses are much smaller than the quark mass contribution. Nevertheless, if we use this value to calculate the hyperfine masses of the other mesons, then the mass differences among them seem to be comparable to the experimental ones. As one can see from Table VII, the hyperfine masses generate the experimental mass splittings of $K^* - K$, $D^* - D$, and $B^* - B$ very well, although the agreement is not as good for $D_s^* - D_s$ and $B_s^* - B_s$.

V. HYPERFINE MASSES FROM FOUR-QUARK SYSTEMS

In Sec. III we constructed the four-quark wave functions, which are relevant for our study of heavy meson excited states. Depending on the symmetric aspect of the antidiquark, we come up with the following two plausible models for the four-quark wave functions:
Model I: The antidiquark is symmetric in flavor ($\bar{\mathbf{6}}_f$) and belongs to the color state $\mathbf{3}_c$. In this model, the four-quark wave functions are given by Eq. (4).

[3]Note that the Λ baryon contains a spin-0 diquark while Σ has a spin-1 diquark. Thus Λ and Σ have different spin configurations, although they both have spin-1/2 [24].

Model II: The antidiquark is antisymmetric in flavor ($\mathbf{3}_c$) and belongs to the color state $\bar{\mathbf{6}}_c$. In this model, the four-quark wave functions are given by Eq. (5).

The hyperfine masses of the four-quark systems are matrix elements of the hyperfine potential V between these four-quark wave functions. To explain our calculation in detail, we write the color-spin interaction for the four-quark systems as

$$V = v_0\left[\lambda_1\cdot\lambda_2\frac{J_1\cdot J_2}{m_1m_2} + \lambda_3\cdot\lambda_4\frac{J_3\cdot J_4}{m_3m_4} + \lambda_1\cdot\lambda_3\frac{J_1\cdot J_3}{m_1m_3} + \lambda_1\cdot\lambda_4\frac{J_1\cdot J_4}{m_1m_4} + \lambda_2\cdot\lambda_3\frac{J_2\cdot J_3}{m_2m_3} + \lambda_2\cdot\lambda_4\frac{J_2\cdot J_4}{m_2m_4}\right], \tag{9}$$

where the indices 1, 2, 3, 4 refer to Q, q_i, $\bar{q}^j$, and $\bar{q}^i$ in Eqs. (4) and (5). Thus, 1, 2 quarks form the diquark (Q, q_i) and 3, 4 quarks form the antidiquark $(\bar{q}^j, \bar{q}^i)$. The corresponding quark masses are denoted by m_1, m_2, m_3, and m_4, respectively. Given one specific flavor combination, one can calculate the color part and spin part separately.

A. Color part

Here we calculate the color part $\lambda_i\cdot\lambda_j$ in the potential V. In the case of Model I, where the wave function is given by Eq. (4), the antidiquark (namely [3, 4] quarks) is in the color triplet state $\mathbf{3}_c$, which restricts the diquark (namely [1, 2] quarks) to be in $\bar{\mathbf{3}}_c$ in order to make colorless four-quark states. Thus, the expectation values of $\lambda_1\cdot\lambda_2$ and $\lambda_3\cdot\lambda_4$ can be calculated as

$$\langle\lambda_1\cdot\lambda_2\rangle_{\bar{\mathbf{3}}_c,\mathbf{3}_c} = \langle\lambda_3\cdot\lambda_4\rangle_{\bar{\mathbf{3}}_c,\mathbf{3}_c} = -\frac{8}{3}. \tag{10}$$

In the case of Model II, where the wave function is given by Eq. (5), the antidiquark is in $\bar{\mathbf{6}}_c$, which restricts the diquark to be in the color state $\mathbf{6}_c$. The expectation values of $\lambda_1\cdot\lambda_2$ and $\lambda_3\cdot\lambda_4$ can be calculated in the [1, 2] [3, 4] basis as

$$\langle\lambda_1\cdot\lambda_2\rangle_{\mathbf{6}_c,\bar{\mathbf{6}}_c} = \langle\lambda_3\cdot\lambda_4\rangle_{\mathbf{6}_c,\bar{\mathbf{6}}_c} = \frac{4}{3}. \tag{11}$$

To calculate the expectation values of other operators like $\lambda_1\cdot\lambda_3$, $\lambda_2\cdot\lambda_3$, etc., it is necessary to rearrange the wave function of definite color states in the diquark-antidiquark ([1, 2] [3, 4]) basis into the [1, 3] [2, 4] basis or the [1, 4] [2, 3] basis. This can be done by using the following decomposition:

$$q_a\bar{q}^b = \underbrace{q_a\bar{q}^b - \frac{1}{3}\delta_a^b q_d\bar{q}^d} + \underbrace{\frac{1}{3}\delta_a^b q_d\bar{q}^d} = \mathbf{8}_a^b + \delta_a^b\mathbf{1}, \tag{12}$$

which expresses a quark-antiquark pair in terms of an octet and a singlet in color space.

When the diquark and the antidiquark are in $(\bar{\mathbf{3}}_c, \mathbf{3}_c)$ as in Eq. (4), we find

$$\langle\lambda_1\cdot\lambda_3\rangle_{\bar{\mathbf{3}}_c,\mathbf{3}_c} = \langle\lambda_2\cdot\lambda_4\rangle_{\bar{\mathbf{3}}_c,\mathbf{3}_c} = \langle\lambda_2\cdot\lambda_3\rangle_{\bar{\mathbf{3}}_c,\mathbf{3}_c} = \langle\lambda_1\cdot\lambda_4\rangle_{\bar{\mathbf{3}}_c,\mathbf{3}_c} = -\frac{4}{3}. \tag{13}$$

Inserting all the factors into Eq. (9) leads to

$$\langle V\rangle_{\bar{\mathbf{3}}_c,\mathbf{3}_c} = -\frac{8}{3}v_0\left[\frac{J_1\cdot J_2}{m_1m_2} + \frac{J_3\cdot J_4}{m_3m_4} + \frac{J_1\cdot J_3}{2m_1m_3} + \frac{J_1\cdot J_4}{2m_1m_4} + \frac{J_2\cdot J_3}{2m_2m_3} + \frac{J_2\cdot J_4}{2m_2m_4}\right]. \tag{14}$$

When the diquark and the antidiquark are in $(\mathbf{6}_c, \bar{\mathbf{6}}_c)$, the expectation values are obtained as

$$\langle\lambda_1\cdot\lambda_3\rangle_{\mathbf{6}_c,\bar{\mathbf{6}}_c} = \langle\lambda_2\cdot\lambda_4\rangle_{\mathbf{6}_c,\bar{\mathbf{6}}_c} = \langle\lambda_2\cdot\lambda_3\rangle_{\mathbf{6}_c,\bar{\mathbf{6}}_c} = \langle\lambda_1\cdot\lambda_4\rangle_{\mathbf{6}_c,\bar{\mathbf{6}}_c} = -\frac{10}{3}, \tag{15}$$

which leads to

$$\langle V\rangle_{\bar{\mathbf{6}}_c,\mathbf{6}_c} = \frac{4}{3}v_0\left[\frac{J_1\cdot J_2}{m_1m_2} + \frac{J_3\cdot J_4}{m_3m_4} - \frac{5}{2}\left(\frac{J_1\cdot J_3}{m_1m_3} + \frac{J_1\cdot J_4}{m_1m_4} + \frac{J_2\cdot J_3}{m_2m_3} + \frac{J_2\cdot J_4}{m_2m_4}\right)\right]. \tag{16}$$

B. Spin part

The spin parts can be calculated in a similar way. For an illustration, we take the four-quark wave function of spin 0, which has the spin configuration $|J, J_{12}, J_{34}\rangle = |011\rangle$ in the [1, 2] [3, 4] basis. The calculation for the other spin configurations can be done similarly. The spin interactions $J_1\cdot J_2$ and $J_3\cdot J_4$ can be calculated directly on $|011\rangle$. For instance, since the diquark [1, 2] is in the spin-1 state, $J_1\cdot J_2$ acting on $|011\rangle$ is

$$J_1 \cdot J_2|011\rangle = \frac{1}{2}(J_{12}^2 - J_1^2 - J_2^2)|011\rangle = \frac{1}{4}|011\rangle. \quad (17)$$

Similarly, $J_3 \cdot J_4|011\rangle = \frac{1}{4}|011\rangle$ since the antidiquark [3, 4] is also in the spin-1 state.

For the other spin interactions ($J_1 \cdot J_3$, $J_2 \cdot J_4$, etc.), it is necessary to write the spin state $|011\rangle$ in the [1, 3] [2, 4] basis using Racah coefficients. To do this, we first write $|011\rangle$ in terms of the diquark spin and its projection $|J_{12}M_{12}\rangle$, and the antidiquark part $|J_{34}M_{34}\rangle$, with appropriate Clebsch-Gordan coefficients, namely,

$$|011\rangle_{[12][34]} = \frac{1}{\sqrt{3}}[|11\rangle_{12}|1-1\rangle_{34} - |10\rangle_{12}|10\rangle_{34} + |1-1\rangle_{12}|11\rangle_{34}]. \quad (18)$$

Here the subscripts in the kets indicate the quarks or antiquarks that make the designated spin state. Then, after writing down each spin state in terms of spinors of participating quarks, we reorganize the $|011\rangle$ state with respect to $|J_{13}, M_{13}\rangle$ and $|J_{24}, M_{24}\rangle$. This procedure applied to Eq. (18) yields the spin wave functions,

$$|011\rangle_{[13][24]} = \frac{\sqrt{3}}{6}[|10\rangle_{13}|10\rangle_{24} + 3|00\rangle_{13}|00\rangle_{24} - |11\rangle_{13}|1-1\rangle_{24} - |1-1\rangle_{13}|11\rangle_{24}] \quad (19)$$

in the [1, 3] [2, 4] basis. Of course, this state is not an eigenstate of J_{13}, as expected. Similarly, one can write Eq. (18) in the [1, 4] [2, 3] spin basis, $|J_{14}, M_{14}\rangle$ and $|J_{23}, M_{23}\rangle$, which gives

$$|011\rangle_{[14][23]} = \frac{\sqrt{3}}{6}[|10\rangle_{14}|10\rangle_{23} + 3|00\rangle_{14}|00\rangle_{23} - |11\rangle_{14}|1-1\rangle_{23} - |1-1\rangle_{14}|11\rangle_{23}] \quad (20)$$

in the [1, 4] [2, 3] basis. Using these expressi

$$\langle 011|J_1 \cdot J_4|011\rangle = \langle 011|J_2 \cdot J_3|011\rangle = \langle 011|J_2 \cdot J_4|011\rangle = -\frac{1}{2}.$$

ons, it is now straightforward to calculate the expectation values of the spin operators of concern in this particular four-quark state ($\langle 011|J_2 \cdot J_4|011\rangle$, etc.). They are obtained as

$$\langle 011|J_1 \cdot J_4|011\rangle = \langle 011|J_1 \cdot J_3|011\rangle = \langle 011|J_2 \cdot J_3|011\rangle = \langle 011|J_2 \cdot J_4|011\rangle = -\frac{1}{2}. \quad (21)$$

One interesting remark is that, under the change of basis, one can identify the decay channels of the relevant four-quark state. For example, in Eq. (20), the [1, 4] indices correspond to $Q\bar{q}(q = u, d, s)$ and the [2, 3] indices correspond to $q\bar{q}$. The spin state $|00\rangle_{14}|00\rangle_{23}$ in Eq. (20) contains a Fock space of pseudoscalar-pseudoscalar particles, which can decay, for instance, to πD for $Q = c$ if the decay occurs through a "fall-apart" mechanism. The colors of course should be combined into a singlet separately in [1, 4] and [2, 3] for such a decay to happen. The other spin states in Eq. (20) correspond to a vector-vector channel like the ρD^* channel. From this change of spin basis, we see that the state $|011\rangle$ consists of pseudoscalar-pseudoscalar and vector-vector components with the probability ratio of 3:1. Thus, this four-quark state in the spin-0 channel has a large component in the pseudoscalar-pseudoscalar channel like πD. Usually the invariant mass of this decay channel is expected to be quite lower than the possible four-quark mass. This means that the four-quark state with $|011\rangle$ may have a large decay width, making experimental observation difficult. Indeed, as we mentioned in Sec. II, $D_0^{*0}(2318)$ (which is one candidate for the four-quark states) has a broad width of 267 MeV. Also, by applying the same argument to the bottom sector, we expect B-meson excited states with spin 0 to be broad. Currently, B mesons with spin 0 are missing in the PDG (see Table II), which might be due to experimental difficulties coming from their broad widths.

Our prescription for evaluating the spin part can be similarly applied to the other spin states, which include the spin-1 state with two possible configurations, $|101\rangle$ and $|111\rangle$, and the spin-2 state with the configuration $|211\rangle$. The two configurations in $J = 1$, $|101\rangle$ and $|111\rangle$, can mix because of the nonzero mixing term $\langle 101|V|111\rangle$. Therefore, one needs to diagonalize the 2×2 matrix in order to calculate the physical hyperfine masses in the spin-1 channel.

C. Flavor part

The hyperfine masses for a general flavor combination, $q_1 q_2 \bar{q}_3 \bar{q}_4$, are presented in Table VIII, where the corresponding spin configurations as well as the color structure of the antidiquark are given. Using these formulas, one can calculate $\langle V_D \rangle_J^{\bar{u}}$, $\langle V_D \rangle_J^{\bar{d}}$, and $\langle V_D \rangle_J^{\bar{s}}$. The final hyperfine masses corresponding to the states $D_J^{\bar{u}}$, $D_J^{\bar{d}}$, and $D_J^{\bar{s}}$ can be obtained by summing over all the flavor combinations according to Eq. (2) for Model I and Eq. (3) for Model II. To be specific, in the case of Model I, the hyperfine masses $\langle V_D \rangle_J^{\bar{u}}$, $\langle V_D \rangle_J^{\bar{d}}$, and $\langle V_D \rangle_J^{\bar{s}}$ are calculated schematically as follows:

TABLE VIII. The hyperfine mass $\langle V\rangle$ for a given spin configuration of the four-quark states and the color states of the antidiquark. The hyperfine masses presented here are for a general flavor combination, $q_1q_2\bar{q}_3\bar{q}_4$, without the flavor normalization. Thus, to obtain the final hyperfine masses, one needs to combine all the flavor combinations as well as the normalization according to Eqs. (2) and (3).

$\vert J,J_{12},J_{34}\rangle$	Color of $\bar{q}_3\bar{q}_4$	Hyperfine mass $\langle V\rangle_{q_1q_2\bar{q}_3\bar{q}_4}$
$\vert 011\rangle$		$-\frac{2}{3}v_0[\frac{1}{m_1m_2}+\frac{1}{m_3m_4}-\frac{1}{m_1m_3}-\frac{1}{m_1m_4}-\frac{1}{m_2m_3}-\frac{1}{m_2m_4}]$
$\vert 101\rangle$		$-\frac{2}{3}v_0[-\frac{3}{m_1m_2}+\frac{1}{m_3m_4}]$
$\vert 111\rangle$	$\mathbf{3}_c$	$-\frac{2}{3}v_0[\frac{1}{m_1m_2}+\frac{1}{m_3m_4}-\frac{1}{2m_1m_3}-\frac{1}{2m_1m_4}-\frac{1}{2m_2m_3}-\frac{1}{2m_2m_4}]$
$\vert 211\rangle$	(Model I)	$-\frac{2}{3}v_0[\frac{1}{m_1m_2}+\frac{1}{m_3m_4}+\frac{1}{2m_1m_3}+\frac{1}{2m_1m_4}+\frac{1}{2m_2m_3}+\frac{1}{2m_2m_4}]$
Mixing ($\vert 101\rangle,\vert 111\rangle$)		$-\frac{\sqrt{2}}{3}v_0[-\frac{1}{m_1m_3}-\frac{1}{m_1m_4}+\frac{1}{m_2m_3}+\frac{1}{m_2m_4}]$
$\vert 011\rangle$		$\frac{v_0}{3}[\frac{1}{m_1m_2}+\frac{1}{m_3m_4}+\frac{5}{m_1m_3}+\frac{5}{m_1m_4}+\frac{5}{m_2m_3}+\frac{5}{m_2m_4}]$
$\vert 101\rangle$		$\frac{v_0}{3}[-\frac{3}{m_1m_2}+\frac{1}{m_3m_4}]$
$\vert 111\rangle$	$\bar{\mathbf{6}}_c$	$\frac{v_0}{6}[\frac{2}{m_1m_2}+\frac{2}{m_3m_4}+\frac{5}{m_1m_3}+\frac{5}{m_1m_4}+\frac{5}{m_2m_3}+\frac{5}{m_2m_4}]$
$\vert 211\rangle$	(Model II)	$\frac{v_0}{6}[\frac{2}{m_1m_2}+\frac{2}{m_3m_4}-\frac{5}{m_1m_3}-\frac{5}{m_1m_4}-\frac{5}{m_2m_3}-\frac{5}{m_2m_4}]$
Mixing ($\vert 101\rangle,\vert 111\rangle$)		$\frac{5\sqrt{2}}{6}v_0[\frac{1}{m_1m_3}+\frac{1}{m_1m_4}-\frac{1}{m_2m_3}-\frac{1}{m_2m_4}]$

Model I

$$\begin{aligned}
\langle V_D\rangle_J^{\bar{u}} &= \frac{1}{8}[4\langle V_D\rangle_{Qu\bar{u}\bar{u}}+\langle V_D\rangle_{Qd\bar{u}\bar{d}}+\langle V_D\rangle_{Qd\bar{d}\bar{u}}+\langle V_D\rangle_{Qs\bar{u}\bar{s}}+\langle V_D\rangle_{Qs\bar{s}\bar{u}}],\\
\langle V_D\rangle_J^{\bar{d}} &= \frac{1}{8}[\langle V_D\rangle_{Qu\bar{d}\bar{u}}+\langle V_D\rangle_{Qu\bar{u}\bar{d}}+4\langle V_D\rangle_{Qd\bar{d}\bar{d}}+\langle V_D\rangle_{Qs\bar{d}\bar{s}}+\langle V_D\rangle_{Qs\bar{s}\bar{d}}],\\
\langle V_D\rangle_J^{\bar{s}} &= \frac{1}{8}[\langle V_D\rangle_{Qu\bar{s}\bar{u}}+\langle V_D\rangle_{Qu\bar{u}\bar{s}}+\langle V_D\rangle_{Qd\bar{s}\bar{d}}+\langle V_D\rangle_{Qd\bar{d}\bar{s}}+4\langle V_D\rangle_{Qs\bar{s}\bar{s}}].
\end{aligned} \tag{22}$$

The specified flavor combination and the associated numerical factors follow from Eq. (2). Here each term with specified flavors—for example, the term like $\langle V_D\rangle_{Qu\bar{u}\bar{u}}$—can be obtained from the general formulas given in Table VIII with Model I. The isospin symmetry requires $\langle V_D\rangle_J^{\bar{u}} = \langle V_D\rangle_J^{\bar{d}}$.

In the case of Model II, the hyperfine masses can be calculated schematically as follows:

Model II

$$\begin{aligned}
\langle V_D\rangle_J^{\bar{u}} &= \frac{1}{4}[\langle V_D\rangle_{Qd\bar{u}\bar{d}}+\langle V_D\rangle_{Qd\bar{d}\bar{u}}+\langle V_D\rangle_{Qs\bar{u}\bar{s}}+\langle V_D\rangle_{Qs\bar{s}\bar{u}}],\\
\langle V_D\rangle_J^{\bar{d}} &= \frac{1}{4}[\langle V_D\rangle_{Qu\bar{d}\bar{u}}+\langle V_D\rangle_{Qu\bar{u}\bar{d}}+\langle V_D\rangle_{Qs\bar{d}\bar{s}}+\langle V_D\rangle_{Qs\bar{s}\bar{d}}],\\
\langle V_D\rangle_J^{\bar{s}} &= \frac{1}{4}[\langle V_D\rangle_{Qu\bar{s}\bar{u}}+\langle V_D\rangle_{Qu\bar{u}\bar{s}}+\langle V_D\rangle_{Qd\bar{s}\bar{d}}+\langle V_D\rangle_{Qd\bar{d}\bar{s}}],
\end{aligned} \tag{23}$$

where the specified flavor combination and the numerical factors follow from Eq. (3). Here each term with specified flavors is again obtained from the general formulas given in Table VIII with Model II.

VI. RESULTS AND DISCUSSION

We now present and discuss the results obtained from the two models using Eqs. (22) and (23). In our calculations, there are a few parameters that need to be fixed. For the constituent quark masses, we use $m_u = m_d = 330$ MeV, $m_s = 500$ MeV, $m_c = 1500$ MeV, and $m_b = 4700$ MeV as discussed in Sec. IV.

One additional parameter is the strength of the color-spin interaction v_0. Our analyses in Sec. IV show that the fitted parameter v_0 takes different values for the baryon sector and the meson sector. Keeping this limitation in mind, we fix v_0 separately within our four-quark systems. Specifically, we fix this strength from the experimental mass difference between $D_0^{*0}(2318)$ and $D_2^{*0}(2463)$ by identifying $D_0^{\bar{u}}$ with $D_0^{*0}(2318)$ and $D_2^{\bar{u}}$ with $D_2^{*0}(2463)$. Of course, the extracted parameter v_0 depends on the two models presented above. Once v_0 is fixed, one can calculate the hyperfine masses of the other resonances, such as spin-1 mesons without strangeness and spin-0, -1, or -2 mesons with nonzero strangeness. The obtained mass difference will be compared with the measured data to test the idea of the four-quark structure. To check the parameter dependence of our results, we will also show the results using the v_0 value fixed from the $\Delta - N$ mass difference.

The calculations are performed for the charm sector and the bottom sector. For B mesons, we will use a similar nomenclature, i.e., $B_J^{\bar{q}^j}$ represents a state and $\langle V_B\rangle_J^{\bar{q}^j}$ ($\bar{q}^j = \bar{u}, \bar{d}, \bar{s}$) is the corresponding hyperfine mass.

A. Results from Model I

In Model I, the antidiquark is symmetric in flavor space and its color wave function belongs to $\mathbf{3}_c$. The hyperfine masses are calculated using Eq. (22). From the mass splitting between $D_0^{*0}(2318)$ and $D_2^{*0}(2463)$, we have $v_0 \sim (-193)^3$ MeV^3 in Model I, which is somewhat close to the one obtained from the $\Delta - N$ mass difference, $v_0 \sim (-199.6)^3$ MeV^3. Using this parameter, we calculate the hyperfine masses from the four-quark states, $|J, J_{12}, J_{34}\rangle = |011\rangle,\ |101\rangle,\ |111\rangle,\ |211\rangle$ as well as the mixing terms between the two states in the spin-1 channel. The resulting hyperfine masses are presented in Table IX.

We now discuss the results for the mesons without strangeness, $D_J^{\bar{u}}$, using the corresponding hyperfine masses $\langle V_D\rangle_J^{\bar{u}}$. In the spin-1 channel, because of the two spin configurations and the mixing between them, the hyperfine masses form a 2×2 matrix. The physical hyperfine masses can be obtained by diagonaliziation,

$$\begin{array}{c|cc} & |101\rangle & |111\rangle \\ \hline |101\rangle & 13.66 & 42.00 \\ |111\rangle & 42.00 & 0.82 \end{array} \xrightarrow{\text{diagonalization}} \begin{array}{c|cc} & |D_{1P}^{\bar{u}}\rangle & |D_{1N}^{\bar{u}}\rangle \\ \hline |D_{1P}^{\bar{u}}\rangle & 49.73 & 0.00 \\ |D_{1N}^{\bar{u}}\rangle & 0.00 & -35.24 \end{array}. \tag{24}$$

Thus, in the spin-1 channel, the physical hyperfine masses are

$$\langle V_D\rangle_{1P}^{\bar{u}} = 49.73\ \text{MeV}, \qquad \langle V_D\rangle_{1N}^{\bar{u}} = -35.24\ \text{MeV}. \tag{25}$$

Here we have denoted the corresponding eigenstates as $D_{1P}^{\bar{u}}$ and $D_{1N}^{\bar{u}}$, where the subscript P (N) is introduced to indicate a positive (negative) hyperfine mass.

The hyperfine mass difference between $D_{1P}^{\bar{u}}$ and the spin-2 meson is $\langle V_D\rangle_{1P}^{\bar{u}} - \langle V_D\rangle_2^{\bar{u}} = -47.48$ MeV, which means that the $D_{1P}^{\bar{u}}$ mass is lower than the spin-2 meson by about -48 MeV. If we use the experimental mass of the spin-2 meson, i.e., 2462.6 MeV, Model I predicts the mass of $D_{1P}^{\bar{u}}$ to be 2415 MeV, which is very close to the observed mass of $D_1^0(2421)$.

TABLE IX. The hyperfine masses obtained for open-charm $(D_J^{\bar{u}}, D_J^{\bar{s}})$ and open-bottom $(B_J^{\bar{u}}, B_J^{\bar{s}})$ excited mesons in Model I. Here the diquark and the antidiquark belong to the color states $\bar{\mathbf{3}}_c$ and $\mathbf{3}_c$, respectively. The strength of the color-spin interaction v_0 fixed by the mass difference $D_2^{*0}(2463) - D_0^{*0}(2318)$ is $v_0 \sim (-193)^3$ MeV^3. We also indicate the charge of the four-quark states corresponding to the hyperfine masses.

$\lvert J, J_{12}, J_{34}\rangle$	$\langle V_D\rangle_J^{\bar{u}}$	$\langle V_D\rangle_J^{\bar{s}}$	$\langle V_B\rangle_J^{\bar{u}}$	$\langle V_B\rangle_J^{\bar{s}}$
$\lvert 011\rangle$	−47.37	−37.89	−40.80	−33.55
$\lvert 211\rangle$	97.21	67.05	84.90	56.70
$\lvert 101\rangle$	13.66	0.00	31.71	16.38
$\lvert 111\rangle$	0.82	−2.91	1.10	−3.47
mixing ($\lvert 101\rangle, \lvert 111\rangle$)	42.00	29.12	50.90	36.05
Charge	0	+1	−1	0

The other member in the spin-1 channel, $D_{1N}^{\bar{u}}$, has a hyperfine mass of -35.24 MeV. The hyperfine mass difference from the spin-2 meson is then $\langle V_D\rangle_{1N}^{\bar{u}} - \langle V_D\rangle_2^{\bar{u}} = -132.45$ MeV, which indicates that the $D_{1N}^{\bar{u}}$ mass should be around 2330 MeV. The current compilation from the PDG does not list the resonance corresponding to $D_{1N}^{\bar{u}}$ in the spin-1 channel. The listed $D_1^0(2427)$ has a mass that is 100 MeV larger than this estimation. We expect that this state (if it exists) has a large decay width coming from the kinematically favorable πD^* mode, and therefore it may not be easy to identify it in experiments.

This can be explained by writing the two eigenstates in the spin-1 channel with respect to the original spin configurations via

$$|D_{1P}^{\bar{u}}\rangle = \alpha|101\rangle + \beta|111\rangle, \tag{26}$$

$$|D_{1N}^{\bar{u}}\rangle = -\beta|101\rangle + \alpha|111\rangle. \tag{27}$$

The mixing parameters are calculated to be $\alpha = -0.76$ and $\beta = -0.65$. Because of the sign difference between Eqs. (26) and (27), the two spin configurations in the $J=1$ channel either add together or partially cancel when making the eigenstate $D_{1P}^{\bar{u}}$ or $D_{1N}^{\bar{u}}$. If the two spin configurations $|101\rangle$ and $|111\rangle$ are rewritten in terms of the [1, 4] [2, 3] basis [similarly to Eq. (20)], one can see that they contain the spin components ($J_{14} = 1,\ J_{23} = 0$), ($J_{14} = 0,\ J_{23} = 1$), and ($J_{14} = 1,\ J_{23} = 1$). The spin component ($J_{14} = 1,\ J_{23} = 0$) contains the πD^* decay mode in addition to the kinematically forbidden mode KD_s^*. The πD^* decay mode is kinematically favorable because the threshold energy is about 150 MeV lower than the expected mass of $D_{1N}^{\bar{u}}$, which is around 2330 MeV. If we count only the spin part of the wave functions, the spin component containing the πD^* mode constitutes 25% in the configuration $|101\rangle$, while it is 50% in $|111\rangle$ before the mixing. After the mixing through Eqs. (26) and (27), this component is enhanced (∼74%) in $D_{1N}^{\bar{u}}$ but strongly suppressed (∼0.7%) in $D_{1P}^{\bar{u}}$. Because of the strong enhancement of the component containing πD^*, $D_{1N}^{\bar{u}}$ is expected to have a large decay width. On the other hand, $D_{1P}^{\bar{u}}$ contains a small component containing the πD^* mode and is expected to be a sharp resonance. Indeed, the $D_1^0(2421)$—which we identify as $D_{1P}^{\bar{u}}$ in our model—has a decay width of only about 27 MeV.

We now discuss the results for the mesons with nonzero net strangeness, $D_J^{\bar{s}}$. From Table IX, we see that the hyperfine mass difference between the $J=0$ and $J=2$ channels is $\langle V_D\rangle_2^{\bar{s}} - \langle V_D\rangle_0^{\bar{s}} = 104$ MeV. If we identify $D_2^{\bar{s}}$ as $D_{s2}^{*\pm}(2572)$, the spin-0 resonance $D_0^{\bar{s}}$ must have a mass around 2470 MeV, i.e., about 105 MeV lower than $D_{s2}^{*\pm}(2572)$. As we have discussed in Sec. II, the current PDG listing does not have a corresponding spin-0 resonance in this nonzero strangeness channel. $D_{s0}^{*\pm}(2318)$ cannot be a candidate because this resonance does not belong to $\bar{\mathbf{3}}_f$. Again, the absence of this resonance may be due to its large decay width, which makes it difficult to experimentally identify $D_0^{\bar{s}}$. A careful inspection of Eq. (20) (where the spin-0 wave function is written in the [1, 4] [2, 3] basis) shows that $D_0^{\bar{s}}$ contains a large component for the KD decay channel, namely, the $|00\rangle_{14}|00\rangle_{23}$ component. Since the KD threshold energy is 2364 MeV and is less than the expected mass of $D_0^{\bar{s}}$, which is 2470 MeV, the KD decay channel is kinematically favorable, which again leads to a large decay width for $D_0^{\bar{s}}$.

On the other hand, very interesting phenomena can be foreseen in the spin-1 resonance $D_1^{\bar{s}}$. The hyperfine mass matrix for $D_1^{\bar{s}}$ in the basis of spin configurations $|J, J_{12}, J_{34}\rangle = |101\rangle$ and $|111\rangle$ can be read off from Table IX, and its diagonalized form is as follows:

$$\begin{array}{c|cc} & |101\rangle & |111\rangle \\ \hline |101\rangle & 0.00 & 29.12 \\ |111\rangle & 29.12 & -2.91 \end{array} \xrightarrow{\text{diagonalization}} \begin{array}{c|cc} & |D_{1P}^{\bar{s}}\rangle & |D_{1N}^{\bar{s}}\rangle \\ \hline |D_{1P}^{\bar{s}}\rangle & 27.7 & 0.00 \\ |D_{1N}^{\bar{s}}\rangle & 0.00 & -30.61 \end{array}. \tag{28}$$

Thus, the physical hyperfine masses are $\langle V_D\rangle_{1P}^{\bar{s}} = 27.7$ MeV and $\langle V_D\rangle_{1N}^{\bar{s}} = -30.61$ MeV, which correspond to two spin-1 mesons $D_{1P}^{\bar{s}}$ and $D_{1N}^{\bar{s}}$, respectively. The two eigenstates, $D_{1P}^{\bar{s}}$ and $D_{1N}^{\bar{s}}$, are related to the original spin configurations via

$$|D_{1P}^{\bar{s}}\rangle = \alpha|101\rangle + \beta|111\rangle, \tag{29}$$

$$|D_{1N}^{\bar{s}}\rangle = -\beta|101\rangle + \alpha|111\rangle, \tag{30}$$

where the mixing parameters are calculated as $\alpha = -0.725$ and $\beta = -0.689$.

These two states in the spin-1 channel, $D_{1P}^{\bar{s}}$ and $D_{1N}^{\bar{s}}$, seem to fit well with $D_{s1}^{\pm}(2535)$ and $D_{s1}^{\pm}(2460)$ of the PDG. The predicted mass of $D_{1P}^{\bar{s}}$, determined from the hyperfine mass difference, $\langle V_D\rangle_{1P}^{\bar{s}} - \langle V_D\rangle_2^{\bar{s}} = -39$ MeV, is 2530 MeV. This is very close to the observed mass (2535 MeV) of $D_{s1}^{\pm}$. For $D_{1N}^{\bar{s}}$, the predicted mass is about 2475 MeV, which is only 15 MeV larger than the observed mass of $D_{s1}^{\pm}(2460)$. For $D_{s1}^{\pm}(2460)$, there is an alternative explanation based on chiral models [26,27], where the $D_{s1}^{\pm}(2460)$ is the chiral partner of $D_s(2112, J^P = 1^-)$ with the same mass splitting as $D_s(2318, J^P = 0^+)$–$D_s(1969, J^P = 0^-) = 349$ MeV. This explanation is certainly interesting, but it may be challenging to explain the other spin-1 resonance, $D_{s1}^{\pm}(2535)$ MeV. In this sense, our four-quark model can provide an alternative picture for the excited states of heavy mesons, which should be tested in future experiments.

One very interesting feature of this model is that $D_{1N}^{\bar{s}}$ [which we identify as $D_{s1}^{\pm}(2460)$] has a narrow width (see Table II), while the corresponding state in the nonstrange sector ($D_{1N}^{\bar{u}}$, discussed above) has a broad width. The reason for this feature is that the possible decay channel of $D_{1N}^{\bar{s}}$ with the lowest invariant mass is kinematically forbidden. To illustrate this, we again reorganize the spin configurations $|101\rangle$ and $|111\rangle$ in terms of the [1, 4] [2, 3] basis. Because of the nonzero strangeness, one can see that, in the case of $D_{1N}^{\bar{s}}$, the decay channel with the lowest invariant mass is KD^*. This is in contrast to the case of $D_{1N}^{\bar{u}}$ where the lowest decay channel is πD^*. Since the KD^* threshold is ∼2504 MeV and is larger than the predicted mass of $D_{1N}^{\bar{s}}$ (that is, ∼2474 MeV), $D_{1N}^{\bar{s}}$ cannot decay into KD^* even if it acquires a large KD^* component from the mixing. The predicted mass of $D_{1P}^{\bar{s}}$ (2533 MeV) is larger than the KD^* threshold (2504 MeV). But in this case, the KD^* component is strongly suppressed through the mixing, which again leads to a narrow resonance. The agreement with the experimental masses as well as the possible explanation for their decay patterns provide strong support for the four-quark structure of excited heavy mesons.

This model can also be applied to B-meson systems, and the results for the hyperfine masses read

$$J=0:\ \langle V_B\rangle_0^{\bar u}=-40.8\ \mathrm{MeV},\quad \langle V_B\rangle_0^{\bar s}=-33.55\ \mathrm{MeV}, \tag{31}$$

$$J=1:\ \langle V_B\rangle_{1P}^{\bar u}=69.56\ \mathrm{MeV},\quad \langle V_B\rangle_{1P}^{\bar s}=43.84\ \mathrm{MeV}, \tag{32}$$

$$J=1:\ \langle V_B\rangle_{1N}^{\bar u}=-36.75\ \mathrm{MeV},\quad \langle V_B\rangle_{1N}^{\bar s}=-30.94\ \mathrm{MeV}, \tag{33}$$

$$J=2:\ \langle V_B\rangle_2^{\bar u}=84.9\ \mathrm{MeV},\quad \langle V_B\rangle_2^{\bar s}=56.7\ \mathrm{MeV}. \tag{34}$$

We mention that the states with the superscript $\bar u$ have charge -1 and the states with the superscript $\bar s$ have charge 0. Currently, the PDG lists only four resonances in the excited states of B mesons with relatively well-known spin, $B_1^0(5724)(J=1)$, $B_2^{*0}(5743)(J=2)$, $B_{s1}^0(5829)\times(J=1)$, and $B_{s2}^{*0}(5840)(J=2)$, as can be seen in Table II. Certainly, these are not enough to test the four-quark structure. But we can find that the four mesons listed in the PDG seem to fit well with the four-quark states $B_{1P}^{\bar u}$, $B_2^{\bar u}$, $B_{1P}^{\bar s}$, and $B_2^{\bar s}$. The experimental mass splitting between $B_2^{*0}(5743)$ and $B_1^0(5724)$ is about 20 MeV, which is quite close to the corresponding value from the hyperfine mass difference $\langle V_B\rangle_2^{\bar u}-\langle V_B\rangle_{1P}^{\bar u}\simeq$ 15 MeV. In the B_s family, the mass difference between $B_{s2}^{*0}(5840)$ and $B_{s1}^0(5829)$ is about 10 MeV, and this is again not so different from the hyperfine mass difference $\langle V_B\rangle_2^{\bar s}-\langle V_B\rangle_{1P}^{\bar s}\simeq 13$ MeV. Therefore, as far as the mass difference is concerned, the B mesons in the current PDG list fit our four-quark model very well. As information from B-meson spectroscopy continues to accumulate in the PDG's lists, we expect that the predicted B-meson spectrum will be tested in near future.

The hyperfine mass differences obtained in this model are collected in Table X in the column of Model I. Two sets of the results are shown that depend on the value of the color-spin strength v_0. The first set uses the v_0 value fixed from the mass splitting between $D_0^{*0}(2318)$ and $D_2^{*0}(2463)$ and the results are listed in the column labeled "v_0 from four-quark." The other set uses the v_0 value fitted from the $\Delta-N$ mass splitting and the results are given in the column labeled "v_0 from $\Delta-N$." In this calculation, we make use of the following identifications of the four-quark states:

$$\begin{aligned} D_0^{\bar u}&=D_0^{*0}(2318), & D_{1P}^{\bar u}&=D_1^0(2421),\\ D_2^{\bar u}&=D_2^{*0}(2463), & D_{1P}^{\bar s}&=D_{s1}^{\pm}(2535),\\ D_{1N}^{\bar s}&=D_{s1}^{\pm}(2460), & D_2^{\bar s}&=D_{s2}^{*\pm}(2572),\\ B_{1P}^{\bar u}&=B_1^0(5724), & B_2^{\bar u}&=B_2^{*0}(5743),\\ B_{1P}^{\bar s}&=B_{s1}^0(5829), & B_2^{\bar s}&=B_{s2}^{*0}(5840). \end{aligned} \tag{35}$$

Once the model parameter is fixed, we can make predictions on the masses of the unobserved mesons of spin 0 and spin 1, i.e., $B_0^{\bar u}$, $B_0^{\bar s}$, $B_{1N}^{\bar u}$, and $B_{1N}^{\bar s}$. For the $B_0^{\bar u}$ mass, and using the fact that $\langle V_B\rangle_{1P}^{\bar u}-\langle V_B\rangle_0^{\bar u}\simeq 110$ MeV from Eqs. (31) and (32), the $B_0^{\bar u}$ mass should be 110 MeV smaller than the $B_{1P}^{\bar u}$ mass. Since $B_{1P}^{\bar u}$ is identified as $B_1^0(5724)$, the $B_0^{\bar u}$ mass is expected to be around 5613 MeV. One can also estimate the $B_0^{\bar u}$ mass from the spin-2 meson $B_2^{*0}(5743)$, which gives 5617 MeV. Thus, the two methods give a quite consistent prediction.

TABLE X. The mass splittings among the excited heavy mesons in MeV. The results given in the column labeled "v_0 from four-quark" are obtained with the v_0 value fixed from the mass difference of $D_2^{*0}(2463)-D_0^{*0}(2318)$, which gives $v_0=(-192.9)^3$ MeV^3 for Model I and $v_0=(-147.8)^3$ MeV^3 for Model II. The results given in the column labeled "v_0 from $\Delta-N$" are obtained with the v_0 value fixed from the $\Delta-N$ mass difference, which gives $(-199.6)^3$ MeV^3 in both models. The experimental data are from Ref. [9].

		Model I		Model II	
Mass difference	Δm_{expt} [9]	v_0 from four-quark	v_0 from $\Delta-N$	v_0 from four-quark	v_0 from $\Delta-N$
$D_2^{*0}(2463)-D_0^{*0}(2318)$	144.6	144.6 (fit)	160.3	144.6 (fit)	356
$D_1^0(2421)-D_0^{*0}(2318)$	103.3	97.1	107.6	124.6	306.7
$D_2^{*0}(2463)-D_1^0(2421)$	41.3	47.5	52.6	20	49.3
$D_{s2}^{*\pm}(2572)-D_{s1}^{*\pm}(2535)$	36.8	39.4	43.6	16.78	41.3
$D_{s2}^{*\pm}(2572)-D_{s1}^{\pm}(2460)$	112.3	97.7	108.2	124.7	306.9
$D_{s1}^{*\pm}(2535)-D_{s1}^{\pm}(2460)$	75.5	58.3	64.6	107.9	265.6
$B_2^{*0}(5743)-B_1^0(5724)$	19.5	15.3	17	6.98	16.7
$B_{s2}^{*0}(5840)-B_{s1}^0(5829)$	10.3	12.9	14.3	5.7	14.0

We take the average value of the two values as our prediction. In a similar way, we have

$$J=0\colon\ B_0^{\bar{u}}\ \text{mass} \sim 5615\ \text{MeV},$$
$$B_0^{\bar{s}}\ \text{mass} \sim 5751\ \text{MeV}, \tag{36}$$
$$J=1\colon\ B_{1N}^{\bar{u}}\ \text{mass} \sim 5619\ \text{MeV},$$
$$B_{1N}^{\bar{s}}\ \text{mass} \sim 5753\ \text{MeV}. \tag{37}$$

The resonances with the superscript $\bar{u}$ have charge -1 and isospin $1/2$ (isodoublet), so their isospin partners should appear with the same mass. The resonances with the superscript $\bar{s}$ have charge 0 and $I=0$ (isosinglet). We note that the $J=0$ resonances have masses that are quite close to their $J=1$ counterparts, which may cause some difficulties in discovering these new resonances. Additionally—based on a similar discussion as that for D mesons—we expect that the three resonances $B_0^{\bar{u}}$, $B_0^{\bar{s}}$, and $B_{1N}^{\bar{u}}$ will have broad widths, which hampers the discovery of these mesons. However, the resonance with $J=1$ of nonzero strangeness, $B_{1N}^{\bar{s}}$, should appear as a sharp resonance, if it exists. Therefore, the discovery of $B_{1N}^{\bar{s}}$ at a mass of ∼5750 MeV may be a good probe for understanding the structure of excited heavy mesons.

B. Results from Model II

Another four-quark wave function that we have constructed in Sec. V is called Model II, where the antidiquark is antisymmetric in flavor space and its color wave function belongs to $\bar{\mathbf{6}}_c$. Within this model, the formulas for the hyperfine masses of one specific flavor combination are given in Table VIII. After putting them into Eq. (23), we then calculate the hyperfine masses in Model II. Again, the strength of the color-spin interaction v_0 is determined by fitting the mass splitting between $D_0^{*0}(2318)$ and $D_2^{*0}(2463)$, which gives $v_0 \sim (-147.8)^3\ \text{MeV}^3$. Using this strength, we calculate the hyperfine masses of the four-quark states $|J, J_{12}, J_{34}\rangle = |011\rangle, |101\rangle, |111\rangle, |211\rangle$, as well as the mixing term between the two spin-1 states. Again, for the spin-1 case it is necessary to diagonalize the hyperfine masses in order to obtain the physical states.

Then we can make predictions on the excited heavy meson spectrum as we did for Model I. The hyperfine masses for the D and D_s families are obtained as

$$J=0\colon\ \langle V_D\rangle_0^{\bar{u}} = -106.43\ \text{MeV},\quad \langle V_D\rangle_0^{\bar{s}} = -108.82\ \text{MeV}, \tag{38}$$

$$J=1\colon\ \langle V_D\rangle_{1P}^{\bar{u}} = 18.18\ \text{MeV},\quad \langle V_D\rangle_{1P}^{\bar{s}} = 24.58\ \text{MeV}, \tag{39}$$

$$J=1\colon\ \langle V_D\rangle_{1N}^{\bar{u}} = -79.2\ \text{MeV},\quad \langle V_D\rangle_{1N}^{\bar{s}} = -83.34\ \text{MeV}, \tag{40}$$

$$J=2\colon\ \langle V_D\rangle_2^{\bar{u}} = 38.2\ \text{MeV},\quad \langle V_D\rangle_2^{\bar{s}} = 41.36\ \text{MeV}. \tag{41}$$

The hyperfine masses for the B and B_s families in Model II read

$$J=0\colon\ \langle V_B\rangle_0^{\bar{u}} = -91.65\ \text{MeV},\quad \langle V_B\rangle_0^{\bar{s}} = -95.05\ \text{MeV}, \tag{42}$$

$$J=1\colon\ \langle V_B\rangle_{1P}^{\bar{u}} = 25.87\ \text{MeV},\quad \langle V_B\rangle_{1P}^{\bar{s}} = 31.0\ \text{MeV}, \tag{43}$$

$$J=1\colon\ \langle V_B\rangle_{1N}^{\bar{u}} = -82.57\ \text{MeV},\quad \langle V_B\rangle_{1N}^{\bar{s}} = -86.58\ \text{MeV}, \tag{44}$$

$$J=2\colon\ \langle V_B\rangle_2^{\bar{u}} = 32.65\ \text{MeV},\quad \langle V_B\rangle_2^{\bar{s}} = 36.69\ \text{MeV}. \tag{45}$$

Alternatively, within Model II we can again calculate the mass differences by using the v_0 value determined by the $\Delta - N$ mass difference. The mass differences in Model II for these two values of v_0 are presented in Table X. These results are compared with the experimental mass splittings as well as the predictions of Model I. As one can see in Table X, the results from Model I have a better agreement with the experimental data than those of Model II. Therefore, we conclude that the four-quark wave functions constructed in Model I are more reliable for the excited heavy meson states as far as the mass differences are concerned.

VII. SUMMARY

In this work, we have constructed four-quark wave functions, which might be relevant for excited states of open-charm and open-bottom mesons. The four-quark wave functions were constructed from a diquark-antidiquark picture under the assumption that they form the $\bar{\mathbf{3}}_f$ multiplet in the SU(3) flavor space. The formation of $\bar{\mathbf{3}}_f$ seems to be realized in some of the observed excited states. Within this approach, we proposed two models for the four-quark wave functions, which we called Model I and Model II. In Model I, the antidiquark is symmetric in flavor ($\bar{\mathbf{6}}_f$) and antisymmetric in color ($\mathbf{3}_c$). On the contrary, in Model II, the antidiquark is antisymmetric in flavor ($\mathbf{3}_f$) and symmetric in color ($\bar{\mathbf{6}}_c$). In both models, the possible spin structures are found to be $|J, J_{12}, J_{34}\rangle = |011\rangle$, $|101\rangle$, $|111\rangle$, and $|211\rangle$, where J is the spin of the four-quark system, J_{12} is the diquark spin, and J_{34} is the antidiquark spin. There exists a mixing between the two spin-1 states, which must be diagonalized in order to find the physical states. To test these four-quark structures, we calculated the hyperfine masses using the color-spin interactions and investigated whether they can reproduce the observed mass splittings among the excited states of the D, D_s, B, and B_s families listed in the PDG.

By comparing our results with the experimental masses, we found that Model I gives a good description of the observed mass splittings (as shown in Table X) while Model II fails. It should be noted that all these results are obtained with only one model parameter v_0, which is fixed by either the mass splitting between $D_0^{*0}(2318)$ and $D_2^{*0}(2463)$ or by the $\Delta - N$ mass splitting. We found that Model I gives a nice description of the mass splittings with these two values of v_0.

Another supporting result of these four-quark structures is the appearance of two spin-1 states. This is indeed consistent with the two experimentally observed resonances, $D_{s1}^{\pm}(2460)$ and $D_{s1}^{*\pm}(2535)$, the masses of which are well explained by our Model I. On the other hand, in the charm sector one of the two spin-1 states fits nicely with the $D_1^0(2421)$ meson, but there is a missing resonance. We have demonstrated that the missing spin-1 state may have a large component of the πD^* decay mode, which is substantially magnified through the mixing. Because of this decay channel, this resonance is expected to be a broad resonance and it may not be easily identified in experiments. However, the two states in the D_s mesons have smaller decay widths. In this case, the decay mode with the lowest invariant mass is KD^*, which is kinematically forbidden in one state and strongly suppressed through the mixing in the other state.

Our Model I can predict some other resonances that are currently missing in the PDG compilation. Motivated by its success in explaining the observed spectroscopy, we make the following predictions on some missing resonances:

$$\begin{aligned}
&J=0:\ D_0^{\bar{s}} \sim 2468\ \text{MeV}, \quad \text{broad resonance;}\\
&J=1:\ D_{1N}^{\bar{u}}, D_{1N}^{\bar{d}} \sim 2330\ \text{MeV}, \quad \text{broad resonances;}\\
&J=0:\ B_0^{\bar{u}}, B_0^{\bar{d}} \sim 5615\ \text{MeV}, \quad \text{broad resonances;}\\
&J=0:\ B_0^{\bar{s}} \sim 5751\ \text{MeV}, \quad \text{broad resonance;}\\
&J=1:\ B_{1N}^{\bar{u}}, B_{1N}^{\bar{d}} \sim 5619\ \text{MeV}, \quad \text{broad resonance;}\\
&J=1:\ B_{1N}^{\bar{s}} \sim 5753\ \text{MeV}, \quad \text{narrow resonance.}
\end{aligned} \tag{46}$$

This shows that most of these resonances are expected to have broad widths due to the decay modes that are kinematically allowed. Therefore, these resonances may not be easily identified in experiments. However, there is one exception: $B_{1N}^{\bar{s}}$ of spin 1 is expected to be a narrow resonance because its possible decay mode KB^* is not kinematically allowed. So the discovery of $B_{1N}^{\bar{s}}(5753)$ in future experiments will shed light on our understanding of the four-quark structure of excited heavy mesons.

Throughout the present work, our discussions were limited to the masses of resonances based on the group structure of four-quark systems. Thus, the next area of study would be the dynamical origin of such a structure, which may also provide a key to understanding the reason why Model I is better than Model II for explaining heavy meson excited states in the four-quark picture. Therefore, it is highly desirable to test the four-quark picture based on the dynamical model approaches to calculate the full mass spectra and the couplings of meson resonances. Such studies should also address the question of whether the real physical states would be mixtures of orbitally excited two-quark states and four-quark states. Testing the four-quark interpolating fields in QCD sum rules may also be an interesting way to compute the physical properties of excited heavy mesons, and it could help us verify which structure has a strong overlap with the physical hadron states.

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[1] S. K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003).
[2] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **95**, 142001 (2005).
[3] S. K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **100**, 142001 (2008).
[4] S. L. Olsen, Nucl. Phys. **A827**, 53c (2009).
[5] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Phys. Rev. D **89**, 114010 (2014).
[6] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **112**, 222002 (2014).
[7] T. Nakano *et al.* (LEPS Collaboration), Phys. Rev. Lett. **91**, 012002 (2003).
[8] E. S. Smith, Heavy Ion Phys. **16**, 187 (2002).
[9] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012); K. A. Olive *et al.* (Particle Data Group), Chin. Phys. C **38**, 090001 (2014).
[10] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977).
[11] R. L. Jaffe, Phys. Rev. D **15**, 281 (1977).
[12] R. L. Jaffe, Phys. Rep. **409**, 1 (2005).
[13] N. A. Törnqvist, Z. Phys. C **68**, 647 (1995).
[14] H.-J. Lee, N. I. Kochelev, and Y. Oh, Phys. Rev. D **87**, 117901 (2013).
[15] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Phys. Rev. Lett. **93**, 212002 (2004).
[16] D. Ebert, R. Faustov, and V. Galkin, Eur. Phys. J. C **60**, 273 (2009).
[17] D. Ebert, R. Faustov, and V. Galkin, Phys. Lett. B **696**, 241 (2011).
[18] V. Borka Jovanovic, Phys. Rev. D **76**, 105011 (2007).
[19] V. Dmitrašinović, Phys. Rev. Lett. **94**, 162002 (2005).
[20] M. E. Bracco, A. Lozea, R. D. Matheus, F. S. Navarra, and M. Nielsen, Phys. Lett. B **624**, 217 (2005).
[21] H. Kim and Y. Oh, Phys. Rev. D **72**, 074012 (2005).
[22] E. van Beveren and G. Rupp, Phys. Rev. Lett. **91**, 012003 (2003).
[23] J. Vijande, F. Fernandez, and A. Valcarce, Phys. Rev. D **73**, 034002 (2006); **74**, 059903(E) (2006).
[24] F. E. Close, *An Introduction to Quarks and Partons* (Academic Press, London, 1979).
[25] M. Anselmino, E. Predazzi, S. Ekelin, S. Fredriksson, and D. B. Lichtenberg, Rev. Mod. Phys. **65**, 1199 (1993).
[26] W. A. Bardeen, E. J. Eichten, and C. T. Hill, Phys. Rev. D **68**, 054024 (2003).
[27] M. A. Nowak, M. Rho, and I. Zahed, Acta Phys. Pol. B **35**, 2377 (2004).
[28] T. Barnes, F. E. Close, and H. J. Lipkin, Phys. Rev. D **68**, 054006 (2003).
[29] S. H. Lee and S. Yasui, Eur. Phys. J. C **64**, 283 (2009).
[30] B. Keren-Zur, Ann. Phys. (N.Y.) **323**, 631 (2008).
[31] M. Oka and S. Takeuchi, Phys. Rev. Lett. **63**, 1780 (1989).
[32] B. Silvestre-Brac, Phys. Rev. D **46**, 2179 (1992).
[33] S. Gasiorowicz and J. L. Rosner, Am. J. Phys. **49**, 954 (1981).