

Evidence for an energy-invariant “edge” in proton-proton scattering at very high energies

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Accurate fits to pp and $\bar{p}p$ cross section, data up to Tevatron energies, incorporating the constraints imposed by analyticity and unitarity, successfully predict the results of recent LHC and cosmic ray measurements and suggest that the cross sections approach a black disc limit asymptotically. The approach to the limit is, however, very slow. We present a simple geometric picture which explains these features in a natural way. A “black disc” of logarithmically growing radius is supplemented by a soft “edge” whose properties are invariant with energy. The constancy of the edge results in the prediction that the quantity $(\sigma^{\text{TOT}} - 2\sigma^{\text{El}})/\sqrt{\sigma^{\text{TOT}}}$ approaches a constant at high energy. Using the existing fits, this prediction appears to be verified. The value of the limiting constant allows an estimate of the thickness of the edge, which turns out to be on the order of 1 fm. One thus arrives at a picture where the proton-proton scattering at lower energies is dominated by what becomes the edge, while at higher energies it is dominated by the disc. The crossover between the two regimes is only at $\sqrt{s} \geq 10$ TeV, accounting for the slow approach to asymptotic behavior. Some questions as to the nature of the edge are discussed.

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I. INTRODUCTION

Since the earliest days of high energy physics, questions and speculations have been raised about the ultimate nature and geometric form of very high energy elementary particle interactions. A Yukawa-like mass-energy density around the proton plus the assumption of a strongly energy-dependent interaction strength leads to an effective radius of interaction $R \sim \ln s$ and thus a cross section, $\sim R^2 \sim \ln^2 s$ [1]. A model with diffusion in the transverse dimensions leads to a similar conclusion, with additionally a relation between the cross section, and the multiplicity which appears to be satisfied [2]. Analyticity arguments lead to the conclusion that a $\ln^2 s$ growth of the cross section is in fact the most rapid allowable behavior [3,4].

Indeed, the very high energy pp and $\bar{p}p$ cross sections approach a $\sigma \sim \ln^2 s$ form at the highest energies as shown

by the fits in [5]. These included data up to Tevatron energies of ~ 1.8 TeV, and *accurately predicted* both the LHC and cosmic ray experimental results [6,7]. It is found that both the total cross section, and the elastic cross section, have a leading $\ln^2 s$ behavior. Furthermore the ratio of the coefficients of these terms are closely 2:1, as would be expected in a black-disc picture. However, in these fits the $\ln^2 s$ terms are not totally dominant, even at LHC energies, indicating a very slow approach to “asymptopia.”

Here we present a picture of the scattering in which these features arise in a natural way, and a test which seems to confirm the model and to determine some of its parameters.

II. DISC PLUS EDGE MODEL

A simple “black disc” scattering amplitude with a sharp, step-function edge is physically implausible, even if the black disc picture is basically correct. It seems more plausible to us to assume a fixed, soft edge on the disc, where the edge has *fixed, energy-independent* properties.

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This edge will then gradually become relatively less important as the disc grows in size.

A. Test for an energy-independent edge

We test the picture of a finite, energy-independent edge in the pp scattering amplitude as follows.

We assume initially that at high energies, the elastic scattering amplitude is purely imaginary. We can then write the total and elastic cross sections in two-dimensional impact parameter space b [8] as

$$\begin{aligned}\sigma^{\text{TOT}} &= 8\pi \int_0^\infty \frac{1}{2}(1-\eta)bdb, \\ \sigma^{\text{EL}} &= 8\pi \int_0^\infty \left(\frac{1}{2}(1-\eta)\right)^2 bdb.\end{aligned}\quad (1)$$

The quantity $\eta(b, s)$ is the “transparency” at impact parameter b . In an eikonal picture, it is given by the eikonal function $\chi(b, s)$ as $\eta(b) = \exp(-\chi)$ [8]. It has the general form indicated in Fig. 1, starting from approximately zero at $b = 0$ and rising rapidly to 1 in the vicinity of the black disc radius $b = R$. If the jump to 1 at $b = R$ were simply a step function, one would have the ideal black disc with radius R and a sharp edge. However with a smooth rise as we have indicated in the figure, there is an edge, whose properties we wish to study.

A quantity which exhibits the nature of the edge is the following:

$$\sigma^{\text{TOT}} - 2\sigma^{\text{EL}} = 4\pi \int_0^\infty \eta(1-\eta)bdb. \quad (2)$$

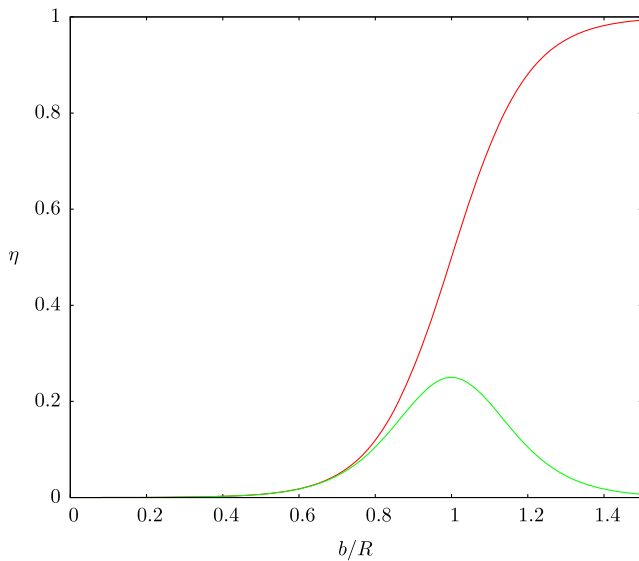


FIG. 1 (color online). Schematic shape of the transparency η as a function of impact parameter b (upper curve, red). With a sharp edge, η would simply jump to 1 at $b/R = 1$. Also shown is the corresponding $\eta(1-\eta)$ (lower curve, green).

We note that $\eta(1-\eta)$ vanishes both at $b = 0$ and $b = \infty$ while peaking near $b = R$ and, therefore, seems to be a suitable quantity for isolating the edge.

If we now assume that the important contribution to the integral in Eq. (2) occurs over a relatively narrow range of b around R , we can set the factor b in the integrand to R and write

$$\sigma^{\text{TOT}} - 2\sigma^{\text{EL}} \approx 4\pi R\mathcal{I}, \quad (3)$$

where \mathcal{I} is the integral $\int_0^\infty \eta(1-\eta)db$. If the edge has energy-independent properties, we expect \mathcal{I} to be constant.

Since the disc scattering dominates the cross section at high energies, we approximate R as $R = \sqrt{\sigma^{\text{TOT}}/2\pi}$. Thus, we expect for a constant edge that

$$\frac{(\sigma^{\text{TOT}} - 2\sigma^{\text{EL}})}{\sqrt{(\pi/2)\sigma^{\text{TOT}}}} \approx 4\mathcal{I} \rightarrow \text{constant} \quad (4)$$

at high energies. Since the maximum value of $\eta(1-\eta)$ is $1/4$, one may think of \mathcal{I} as $\mathcal{I} = \frac{1}{4}t$, where t is an effective “thickness” of the edge. With this definition, the ratio in Eq. (4) is simply t .

Figure 2 shows an evaluation of the left-hand side of Eq. (4) using the even combination of cross sections $\frac{1}{2}(pp + \bar{p}p)$ from the preexisting fit from Ref. [5]. This fit did not include data above ~ 1.8 TeV, but successfully and accurately predicted the cross sections measured at LHC and cosmic ray energies. The dashed (blue) line represents t . For comparison the dashed-dotted line (red) represents the radius inferred from the total cross section, namely $R = \sqrt{\sigma^{\text{TOT}}/2\pi}$.

The thickness t is approximately 1 fermi and is quite constant over an enormous energy region.

In defining R , one could also try using $R = \sqrt{\sigma^{\text{EL}}/\pi}$. This does not make a large quantitative difference and leads to the same asymptotic value for t . However, due to the slow logarithmic behavior of the quantities, the crossover is moved to higher energy, over 100 TeV.

B. Real part of the amplitude

In writing the expressions in Eq. (1), we have taken the elastic scattering amplitude to be purely imaginary and neglected its real part. A small real part is known to be present in the forward scattering amplitude $f(s, 0)$ at high energy, both from direct measurements and from dispersion relations [6]. It reaches a peak of about 10% of the imaginary part for energies \sqrt{s} in the range 100–1000 GeV and is smaller at lower and higher energies. Since our relation Eq. (4) deals with cross sections, where the real part enters squared, we may expect a real part correction to be small.

For example, in the eikonal representation of the scattering amplitude [8], where one introduces an imaginary as well as real part to the eikonal function, $\chi = \chi_R + i\chi_I$, one finds that

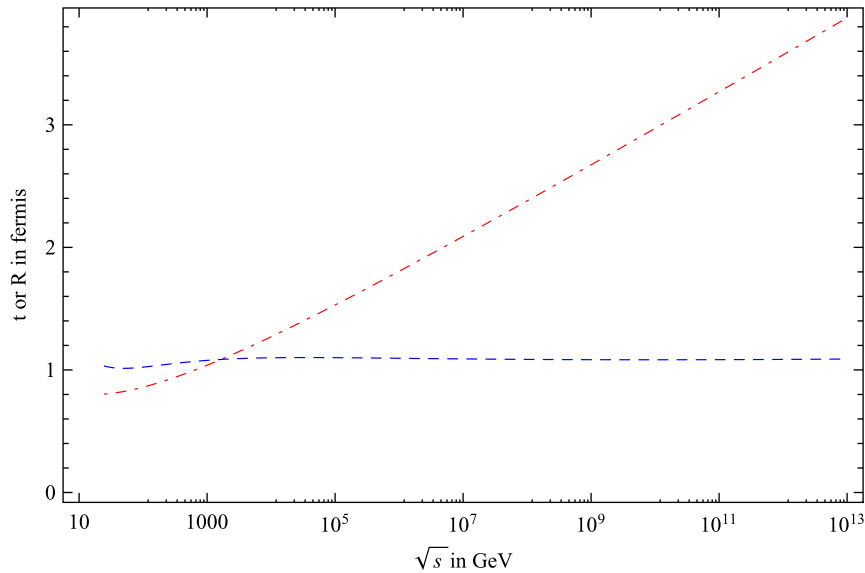


FIG. 2 (color online). Plot of the ratio Eq. (4). The dashed (blue) line represents t , the effective thickness of the edge in fermis, as explained in the text. For comparison the dashed-dotted line (red) represents the black-disc radius R inferred from the total cross section, namely $R = \sqrt{\sigma^{\text{TOT}}/2\pi}$, in Fermis.

$$\begin{aligned} \text{Re}f(s, 0) &= - \int_0^\infty \eta \sin \chi_I b db, \\ \text{Im}f(s, 0) &= \int_0^\infty (1 - \eta \cos \chi_I) b db, \end{aligned} \quad (5)$$

where $\eta = \exp(-\chi_R)$ is the transparency. The corresponding edge integral is

$$\sigma^{\text{TOT}} - 2\sigma^{\text{EL}} = 4\pi \int_0^\infty \eta(\cos \chi_I - \eta) b db. \quad (6)$$

Since $\eta \approx 0$ in the black disc region, and χ is expected to vanish rapidly at large b , we conclude from the observed smallness of the forward real-to-imaginary ratio that χ_I is itself small in the edge region. The factor $(\cos \chi_I - \eta) = (1 - \eta) - \frac{1}{2}\eta\chi_I^2 + \dots$ in Eq. (6) therefore, differs from the factor $(1 - \eta)$ in Eq. (2) only in second order in χ_I . We, therefore, expect corrections of, at most, a few percent to the results discussed in the preceding section. These expectations are confirmed by a numerical calculation in this eikonal model [8], where the change in the integrand of the edge integral Eq. (4) with and without χ_I , is barely perceptible.

III. INTERPRETATION

The flatness of the curve for t in Fig. 2 is impressive. It strongly supports the picture of a proton-proton scattering amplitude consisting of a growing black disc with a constant smooth edge with an energy-independent shape.

It should be kept in mind that the data used for the fit of the pp and $\bar{p}p$ cross sections in Ref. [5] extended only up

to $\sqrt{s} \sim 1.8$ TeV. This fit was used unchanged in making the curves in Ref. [7]; these include the new LHC and cosmic ray data and show the accuracy of the predictions. Since the fit gives a very good representation of the new experimental data up to ~ 80 TeV, our curves up to this energy could just as well have been made directly from the experimental data. However, for energies above 80 TeV, the extrapolation could depend possibly on the functional forms used in making the fit.

The essential feature of the fit which leads to the constancy of the ratio in Eq. (4) is that the leading $\ln^2 s$ terms in σ^{TOT} and σ^{EL} appear with coefficients in the ratio 2:1 as noted in [6] and so cancel in the cross sections. The next-to-leading terms are proportional to $\sim \ln s$, but with different coefficients, so that the difference $(\sigma^{\text{TOT}} - 2\sigma^{\text{EL}})$ is proportional to $\ln s$. This logarithm is effectively canceled by the leading $\ln s$ from the square root of σ^{TOT} in the denominator, leaving a constant difference up to terms of order $1/\ln s$.

It is conceivable that some other next-to-leading parametrization—provided it gives a good lower energy fit and is consistent with the Froissart bound—could give a different asymptotic behavior for Eq. (4). However, it should be noted both that $\ln s$ terms are naturally the leading subdominant terms generated in eikonal models in which χ decreases exponentially at large b and that such terms are also implicit when the leading term is $\sim \ln^2 s$ since s must appear with a scale factor s_0 , and this is interchangeable with a $\ln s$ term.

Thus, in the simplest picture where all cross sections have the same leading black disc behavior, one expects asymptotically

$$\begin{aligned}\sigma_j^{\text{TOT}} &= \beta \ln^2 \sqrt{s/s_0^j} + \dots \\ \sigma_j^{\text{EL}} &= \frac{1}{2} \beta \ln^2 \sqrt{s/s_0^j} + \dots,\end{aligned}\quad (7)$$

where β is universal, according to [7] $\beta = 1.1$ mb, but the s_0 depend on the particular reaction, reflecting differences in the scale where universal behavior sets in. When subtracting two cross sections as in Eq. (4), this necessarily leads to a $\ln s$ term: $\sigma^{\text{TOT}} - 2\sigma^{\text{EL}} = 2\beta \ln \sqrt{s/s_0^j} \ln \sqrt{s_0^j/s_0^j} + \dots$. This said, it would of course be helpful if even higher energy data were somehow to become available in order to extend and confirm the fits [6].

The estimate of the thickness of the edge, about 1 fm, arrived at from the constant on the right-hand side of Eq. (4), is quite reasonable. Since $\pi(1 \text{ fm})^2 \approx 30$ mb is on the order of the low energy pp cross section, ~ 40 mb at $\sqrt{s} = 30$ GeV, one might say that at low energy the pp scattering amplitude is “all edge,” that is, dominated by pp interactions at impact parameters $b \approx 1$ fm. Since the disc-type behavior only sets in around 1 TeV and grows slowly, the relatively large thickness of the “edge” means that the transition to disc domination occurs at high energy, at least around $\sqrt{s} \sim 10$ TeV. This gives a natural explanation of the slow approach to “asymptopia.”

The question of the nature or physical constitution of the edge raises some interesting points. Is it the same for reactions with different particles? If the apparently

universal behavior of cross sections at high energy originates in the gluon field, one would suppose that the disc and its edge are asymptotically the same for all particle species. On the other hand, our observation that at low energy the scattering amplitudes appear to be all edge might suggest that the differences in cross sections at low energy are preserved to high energy through differences in the edge. Since, as we have explained, the edge gives a subdominant contribution to the total cross sections, this would not affect the universality of the cross sections themselves. Unfortunately, data for other particle species are not obtainable as directly as for protons, where one has the LHC and cosmic rays, but it would be of great interest if such information became available.

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