Degeneracy between CCDM and ΛCDM cosmologies

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The creation of cold dark matter cosmology model is studied beyond the linear perturbation level. The skewness is explicitly computed, and the results are compared to those from the ΛCDM model. It is explicitly shown that both models have the same signature for the skewness and cannot be distinguished by using this observable.

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I. INTRODUCTION

In a recent work [1], we have investigated the creation of cold dark matter (CCDM) cosmology as an alternative to explain cosmic acceleration. The CCDM cosmology [2] is a phenomenological scenario in which it is assumed that the gravitational field induces particle creation such that a special choice of the particle production rate produces a cosmology that, at the background level, is indistinguishable from the standard ΛCDM model. There have been some recent papers that try to give a more than phenomenological basis for CCDM particle creation, e.g., Ref. [3], while a more detailed study and review of the thermodynamics of cosmologies of particle creation, including CCDM, has been discussed in Refs. [4,5]. By assuming zero effective sound speed, in Ref. [1], we have compared CCDM with ΛCDM showing that these models are observationally degenerated not only at background but also at the first-order perturbation level.

More recently, some of our results have been criticized in Ref. [6]. The authors of this paper claim that, since in CCDM dark matter particles are being continuously created while baryons are conserved, the ratio between dark matter and baryons energy densities is not constant and would change with redshift in these models. According to them, the existence of such variation would be detectable by current observations breaking the above-mentioned degeneracy. In fact, they use this argument to rule out CCDM. The key point here is that, to obtain their result, these authors have considered in their analysis all the amount of dark matter (clustered or not) in estimating the baryon-dark matter energy densities ratio. However, what can be measured in gravitational experiments with clusters of galaxies is only clustered matter, of course. Smoothly distributed matter or energy at scales of 20–30h^{-1} Mpc, like, for instance, that associated with Λ, is undetectable in cosmological tests on these scales. Since in both CCDM and ΛCDM clustered matter (baryons and dark matter) redshifts as $(1+z)^3$, the ratio between dark matter and baryons energy densities is expected to be constant in both models. Therefore, contrary to the arguments used in Ref. [6], these models cannot be observationally distinguished by using, for instance, the gas mass fraction test [7].

In this paper, we extend the results presented in Ref. [1] and consider the nonlinear dynamics in the CCDM. Here, we explicitly show that the degeneracy between the CCDM and the ΛCDM remains at any order in perturbartion theory. It is also shown that both models have the same signature for the skewness and cannot be distinguished by using this observable.

This paper is organized as follows. In Sec. II, we briefly present the CCDM scenario, emphasizing the reason it is not possible to distinguish CCDM from ΛCDM with gravitational experiments like the gas mass fraction test. In Sec. III, we extend our previous results, obtained in Ref. [1], and consider nonlinear dynamics in the CCDM models. Finally, our conclusions and final remarks are given in Sec. IV.

II. CCDM MODEL: CONSERVED, CREATED, AND CLUSTERED MATTER

Throughout this work, a flat Friedmann–Robertson–Walker metric is assumed. Since we are mainly interested in processes that occurred after radiation domination, we neglect radiation, and, for the sake of simplicity, unless explicitly stated, we also neglect baryons considering only the presence (and creation) of pressureless ($p=0$) dark matter particles.

The relevant cosmological equations for the CCDM model are ($c=1$)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho, \tag{2.1}$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3p_c), \tag{2.2}$$

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where the creation pressure $p_c = -\rho\Gamma/(3H)$ and Γ is the particle production rate. The fluid equation for ρ is

$$\dot{\rho} + 3H\rho = \rho\Gamma. \tag{2.3}$$

In the CCDM cosmological model, Γ is given by [1,2]

$$\Gamma = \frac{3\beta H_0^2}{H} = 3\beta\left(\frac{\rho_{c0}}{\rho}\right)H, \tag{2.4}$$

where β is a $\mathcal{O}(1)$ dimensionless constant, H_0 is the current value of the Hubble parameter, and $\rho_{c0} \equiv 3H_0^2/(8\pi G)$ is the critical density at the present time, which, in our flat-space and simple-fluid approximation, is equal to the value of dark matter energy density at the present time. With the above choice for the particle production rate, Eq. (2.3) can be easily integrated obtaining

$$\rho = \rho_{c0}[(1-\beta)(1+z)^3 + \beta]. \tag{2.5}$$

By substituting Eq. (2.5) in Eq. (2.1), we obtain

$$\frac{H^2}{H_0^2} = (1-\beta)(1+z)^3 + \beta. \tag{2.6}$$

The two terms in the right-hand side of Eq. (2.6) have a clear meaning: The first term redshifts exactly as matter, while the second term, the constant β, plays the role of the cosmological constant density parameter at the present time, $\Omega_{\Lambda 0}$. The CCDM model is then able to mimic exactly the ΛCDM background expansion history.

In our previous work, Ref. [1], we have split the total dark matter energy density ρ in a conserved and created part, $\rho = \rho_{\rm conserved} + \rho_{\rm created}$, and assumed that the conserved part of the dark matter energy density is given by $\rho_{\rm conserved} = \rho_{c0}(1-\beta)(1+z)^3$, while the created one is $\rho_{\rm created} = \rho_{c0}\beta$. Although, from the physical point of view, there is nothing wrong with this choice, it should be remarked that it is not mandatory since there are other possibilities. To better understand this statement, notice that from the integration of Eq. (2.3) we get that the conserved energy density part can be written as $\rho_{\rm conserved} = A/a^3$, while the created part becomes $\rho_{\rm created} = B/a^3 + \beta\rho_{c0}$, where A and B are integration constants. In the special case in which $\beta = 0$, from Eq. (2.4), we have $\Gamma = 0$, and, since there is no matter creation, it follows from the expression for $\rho_{\rm created}$ that we should also have $B = 0$. Thus, we can think of B as an arbitrary function of β such that it vanishes when $\beta = 0$. For the sake of simplicity, let us assume a linear function, $B = \rho_{c0}\alpha\beta$, where α is a constant. In this case, it is straightforward to show from the above equations that

$$A = \rho_{c0}[1-(1+\alpha)\beta]. \tag{2.7}$$

In Ref. [1], we have assumed $\alpha = 0$. This corresponds to the special case in which all the energy density of the created part is constant and all the clustered dark matter (the part that redshifts as matter as expected) is conserved. As remarked above, this choice in not mandatory. Indeed, since the energy density of both created and conserved parts should be positive definite, we have the constraint $0 \le \alpha \le 1/\beta - 1$. In the case in which $\alpha = 1/\beta - 1$, i.e., for the upper limit for α, there is no conserved dark matter. In other words, in this special case, all the dark matter (clustered and unclustered) is created during the Universe evolution. Notice that, since current observations indicate $\beta \sim 0.7$ (recalling that β in CCDM plays the role of $\Omega_{\Lambda 0}$ in ΛCDM), the choice $\alpha = 1$, made by the authors of Ref. [6], is unphysical since, in this case, the energy density of the conserved part would be negative. Thus, it is important to emphasize that in the CCDM scenario we do not know *a priori* which part of the total dark matter particles has been created and which one is conserved. Only by fixing a value for α is this choice specified. Although we do not know *a priori* which part is created (or conserved), we do know which one clusters and which one does not. In other words, independent of the value of α (the way we split the created and conserved parts for ρ), it is clear from Eq. (2.6) that the unclustered part of the energy density is constant, while the clustered one redshifts as a^{-3}.

If in addition to dark matter we had considered also baryons, assuming $B = \rho_{c0}\alpha\beta$ and that baryons are conserved, instead of Eq. (2.7), we would get $A = \rho_{c0}[1-(1+\alpha)\beta - \Omega_{B0}]$, where Ω_{B0} is the present value of the baryons density parameter. The total (baryons+dark matter) energy density will be $\rho = \rho_{c0}[\Omega_{B0}a^{-3} + (1-\beta-\Omega_{B0})a^{-3} + \beta]$. Therefore, regardless of the value of α again, the ratio between the baryons energy density (ρ_B) and the clustered dark matter energy density ($\rho_{\rm clust}$) is independent of redshift:

$$\frac{\rho_B}{\rho_{\rm clust}} = \frac{\Omega_{B0}}{1-\beta-\Omega_{B0}}. \tag{2.8}$$

As stressed in Ref. [1], it is this ratio, and not ρ_B/ρ, that is estimated, for instance, in x-ray surveys [7]. Thus, since it does not depend on redshift, it will not be possible to distinguish the CCDM scenario from ΛCDM by using measurements of the baryon mass fraction in clusters, as originally suggested in Ref. [8] and considered in Ref. [6].

III. DARK DEGENERACY AND SKEWNESS

In Ref. [1], we have investigated the growth of linear perturbations in CCDM models and compared the neo-Newtonian [9] and the general-relativistic frameworks. We have shown that both approaches are formally identical only when the effective sound speed ($c_{\rm eff}$) vanishes [10]. We also showed, assuming $c_{\rm eff}^2 = 0$, that CCDM and ΛCDM models are degenerate not only at the background level but at the linear perturbation order as well. In this section, we will explicitly show that this result is also valid in the nonlinear regime (nonlinear perturbations have been extensively examined, for example, in Ref. [11]).

Instead of using the neo-Newtonian formulation, as considered in Ref. [1], in this work, we follow a different, but to some extent equivalent, approach and start considering Raychaudhuri's equation for a nonrotating and shearless fluid,

$$\dot{\Theta} + \frac{1}{3}\Theta^2 = R_{\mu\nu}u^\mu u^\nu, \tag{3.1}$$

where $R_{\mu\nu}$ is the Ricci tensor, and in our coordinate system the fluid 4-velocity is given by $u^\mu = (1, \dot{a}\,\vec{x} + \vec{v})$, where $\vec{v}$ is the peculiar velocity. Therefore, $\Theta \equiv u^\mu{}_{;\mu}$ can be written as $\Theta = 3\dot{a}/a + \theta/a$, where $\theta \equiv \nabla\cdot\vec{v}$. By using Einstein equations, we can write Eq. (3.1) as

$$\theta' + \frac{\theta}{a} + \frac{\theta^2}{3H} = -\frac{4\pi G}{H}(\delta\rho + 3\delta P), \tag{3.2}$$

where the prime denotes differentiation with respect to the scale factor a and, as usual, we decompose the dynamical variables into their background and inhomogeneus parts; i.e., we write $\rho = \tilde{\rho} + \delta\rho = \tilde{\rho}(1+\delta)$ and $P = \tilde{P} + \delta P$. Here, the tilde denotes background quantities.

By using the conservation equation $\dot{\rho} + (\rho + P)\Theta = 0$, we obtain that the density contrast δ satisfies the differential equation

$$\delta' + \frac{3}{a}(c_{\rm eff}^2 - w)\delta + [1 + w + \delta(1 + c_{\rm eff}^2)]\frac{\theta}{Ha^2} = 0, \tag{3.3}$$

where $w = \tilde{P}/\tilde{\rho}$ and $c_{\rm eff}^2 = \delta P/\delta\rho$.

Assuming $c_{\rm eff}^2 = 0$, differentiating (3.3) with respect to the scale factor, and using (3.2), after some algebra, we obtain the following differential equation for the density contrast:

$$\begin{aligned} &a^2\delta'' + a\delta'\left(\frac{3-9w}{2} - \frac{aw'}{1+w+\delta}\right) - \frac{4a^2\delta'^2}{3(1+w+\delta)} \\ &\quad + \frac{5a\delta\delta' w}{1+w+\delta} + \frac{3\delta}{2}\left(3w^2 - 2w - 1 - 2aw' + \frac{2aww'}{1+w+\delta}\right) \\ &\quad - 3\delta^2\left(\frac{w^2}{1+w+\delta} + \frac{1}{2}\right) = 0. \end{aligned} \tag{3.4}$$

The same differential equation for the density contrast as above was obtained in Ref. [12] by using the neo-Newtonian formulation (note that in Ref. [12] the derivatives were taken with respect to the conformal time). As discussed in Ref. [1], assuming $c_{\rm eff}^2 \neq 0$ introduces a scale dependence of the perturbations even at linear order. This scale dependence can cause strong oscillations if $c_{\rm eff}^2 > 0$, or exponential growth if $c_{\rm eff}^2 < 0$. Thus, only models with $|c_{\rm eff}^2| \ll 1$ are acceptable at linear scales.

Since we are interested in studying the weakly nonlinear regime of structure formation and to compute higher-order moments of the density distribution, it is useful to expand δ as [13]

$$\delta = \sum_{i=1}^{\infty}\delta_i = \sum_{i=1}^{\infty}\frac{D_i(a)}{i!}\delta_0^i, \tag{3.5}$$

where δ_0 is a small perturbation. For special models, like CCDM, in which the adiabatic sound speed is equal to zero and recalling that $c_s^2 = \tilde{P}'/\tilde{\rho}' = w - aw'/[3(1+w)]$, we obtain that D_1 satisfies

$$D_1'' + \frac{3}{2a}(1-5w)D_1' + \frac{3}{2a^2}(3w^2 - 8w - 1)D_1 = 0. \tag{3.6}$$

By using that in CCDM $w(a) = -\beta/[\beta + (1-\beta)a^{-3}]$, Eq. (3.6) can be integrated, and, in terms of the hypergeometric functions, ${}_2F_1(a,b;c;x)$, the growing mode can be expressed as [1]

$$D_1(a,\beta) = \frac{a}{1 + \frac{a^3\beta}{1-\beta}}\,{}_2F_1\left(\frac{1}{3}, 1; \frac{11}{6}; -\frac{a^3\beta}{1-\beta}\right). \tag{3.7}$$

As remarked in our previous work [1], at first order in CCDM, as we increase β, there is a density contrast suppression as compared to ΛCDM. The suppression factor is given by $1/[1 + \beta a^3/(1-\beta)] = 1 + w(a)$ and, as it will be demonstrated below, remains in any order of perturbation theory. As discussed in Ref. [1], the presence of this suppression is related to the fact that in CCDM dark matter clusters in the same manner as it does in ΛCDM. The suppression appears when the constant and nonclustered part of CCDM energy density starts to become non-negligible, and, as a consequence, the equation of state parameter deviates from zero. However, as we have already pointed out, in CCDM when considering tests that involve the growth factor like, for instance, the redshift-space-distortion $f(z)\sigma_8(z)$ test, what should be considered in the calculation of these quantities is only the clustered part of ρ (we direct the interested reader to Ref. [1], where this issue has been more closely discussed).

Let us now illustrate the validity of the above result also in second order of perturbation theory. The second-order solution is obtained by using in Eq. (3.4) that $\delta = D_1\delta_0 + D_2\delta_0^2/2$ and $1/(1+w+\delta) = 1/(1+w) - 1/(1+w)^2\delta + \mathcal{O}(\delta^2)$. We then find that the second-order factor in the expansion (3.5), when keeping only second-order terms in δ_0, satisfies the differential equation

$$\begin{aligned} &D_2'' + \frac{3D_2'}{2a}(1-5w) + \frac{3D_2}{2a^2}(3w^2 - 8w - 1) - \frac{8D_1'^2}{3(1+w)} \\ &\quad + \frac{16D_1D_1'w}{a(1+w)} - \frac{3D_1^2}{a^2}\left(\frac{8w^2}{1+w} + 1\right) = 0. \end{aligned} \tag{3.8}$$

Analogously, higher-order terms are obtained recursively by using the solutions of the differential equations for the lower-order ones. The solution for D_2 in CCDM is obtained by numerically integrating Eq. (3.8), using the solution D_1 in CCDM given by Eq. (3.7), and assuming initial conditions such that D_1 and D_2 at high redshift (small values of the scale factor) behave like in an Einstein–de Sitter model ($D_1 \propto a$, $D_2 \propto a^2$ and such that the skewness assumes the value $S_3 = 34/7$ at that time). The solution D_2 for ΛCDM is obtained analogously by numerical integration of the differential equation, equivalent to Eq. (3.8), valid in the ΛCDM case (see also, e.g., Ref. [14]).

In Fig. 1(a), we show the ratio for D_2 between CCDM and ΛCDM. It is clear again the presence of suppression in CCDM as compared to ΛCDM as we increase β. The suppression is found again to be such that $D_2^{\rm CCDM} = [1 + w(a)]D_2^{\Lambda\rm CDM}$. This result holds for any order in perturbation theory, and it can be proved as follows. First, note from Eq. (2.5) that $\rho^{\rm CCDM}(a) = \rho_{c0}[(1-\beta)/a^3 + \beta]$, while $\rho_m^{\Lambda\rm CDM} = \rho_{c0}\Omega_{m0}/a^3$, with $\Omega_{m0} = 1 - \beta$. Thus, at any order of perturbation theory, it holds that $\delta\rho = \delta\rho_m$ for a constant β, and then we also find that $\rho_m^{\Lambda\rm CDM} = [1 + w(a)]\rho^{\rm CCDM}$. Hence, the density contrast in each case is then simply related by $\delta^{\rm CCDM} = [1 + w(a)]\delta^{\Lambda\rm CDM}$. Using this result back in the definition (3.5), it immediately allows us to conclude that at any nth-order in perturbation theory $D_n^{\rm CCDM} = [1 + w(a)]D_n^{\Lambda\rm CDM}$. This concludes our proof.

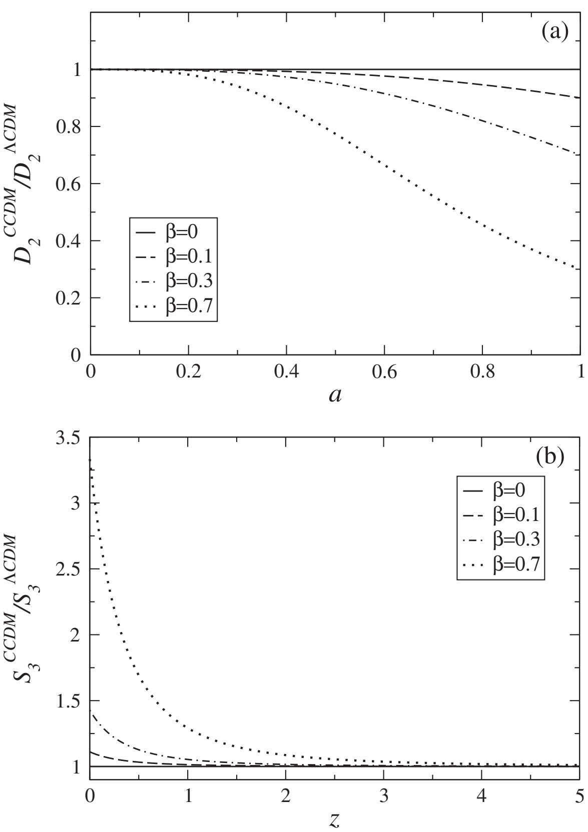

FIG. 1. (a) The ratio of growth function at second order D_2, as a function of the scale factor, and (b) the ratio of skewness S_3, as a function of the redshift, between the CCDM and ΛCDM models and for different values of β. In both cases, the total density ρ is included, Eq. (2.5), for the CCDM. When including only the clustering part (see the text), $w = 0$, $\rho^{\rm CCDM} \to \rho_m^{\Lambda\rm CDM}$, and both CCDM and ΛCDM become fully degenerate.

Returning again to the second-order result, D_2, and assuming Gaussian initial conditions, we can associate D_2 with the emergence of non-Gaussian features in the matter density field. Indeed, D_2 is related to the skewness of the cosmic field [15,16]. The skewness is defined by

$$S_3 = 3D_2/D_1^2. \tag{3.9}$$

We show in Fig. 1(b) the ratio for the skewness between the CCDM the ΛCDM models. The numerical result, in fact, can be expressed analytically as $S_3^{\rm CCDM}(a) = 1/[1+w(a)]S_3^{\Lambda\rm CDM}(a)$. In particular, $S_3(z=0) = 1/(1-\beta) \times S_3^{\Lambda\rm CDM}(z=0)$. For ΛCDM, the skewness is nearly constant, with $S_3^{\Lambda\rm CDM} \approx 4.86$, and is weakly sensitive to Ω_{m0}. One may be tempted to interpret that this difference is an indication that CCDM models with $\beta \simeq 0.7$ ($\Omega_{\rm clust0} \simeq 0.3$) are inconsistent with large-scale skewness measurements [17]. However, care should be taken when analyzing this issue for two reasons. First, in our discussion, baryons were neglected, and measurements of skewness from large-scale galaxy distribution are based on counting luminous objects, not the dark component. Our analysis can easily be generalized to include a small amount of baryons as observed ($\Omega_{b0} \approx 0.04$). It can be shown that in CCDM the baryonic skewness does not change much with redshift;, it is nearly constant such that $S_{3b} \simeq 4.86$ as expected for dark matter and/or baryons in ΛCDM. Therefore, skewness from large-scale galaxy distribution will not be able to break the degeneracy. Second, one may still argue that lensing (convergence) skewness [18] could break the degeneracy between CCDM and ΛCDM. However, lensing skewness is sensitive to clustered matter (baryons and dark matter), and since dark matter in CCDM clusters in the very same manner as it does in ΛCDM, measurements of lensing skewness will also not be able to break the degeneracy between the models.

IV. DISCUSSION AND CONCLUSIONS

A common assumption in cosmology is that we live in a Universe with a dark sector composed by two separately conserved components: clustering dark matter, responsible for large-scale structure formation, and a nonclustering dark energy, responsible for the cosmic acceleration. Cosmologies, like ΛCDM, with the dark sector defined in this manner can fit well the observations. However, this kind of division of the dark sector is not unique and is, in a certain sense, arbitrary. As a matter of fact, we have physically different cosmologies, based on distinct assumptions and/or a separation of the dark sector, that cannot be observationally distinguished from the two dark components cosmologies. Different aspects of this degeneracy in the dark sector, and how cosmological models can be observationally distinguished, have been considered in the literature by several authors (see, for instance, Refs. [19,20]). Dark degeneracy is what this property has been called [19].

Perhaps, the first controversy we find in the literature regarding the above-mentioned difficulty is related to the ΛCDM limit of the so-called generalized Chaplygin gas (GCG) cosmological model [21,22]. Indeed, in Ref. [21] (see also Ref. [23]), it has been shown that gravity alone cannot distinguish the $\alpha = 0$ quartessence GCG model from ΛCDM. The basis of the results we have presented in this paper is the same. In other words, under certain assumptions like, for instance, zero effective sound speed, we cannot distinguish with cosmological observations CCDM from ΛCDM (and, of course, also the $\alpha = 0$ GCG model).

By assuming zero effective sound speed, we have shown in Ref. [1] that CCDM and ΛCDM are observationally degenerated at both the background and at first-order perturbation levels. In this paper, we have extended those results considering nonlinear dynamics in these models. In particular we have shown that they have the same signature for the skewness and explained why they cannot be distinguished by using this observable.

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