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Extending the standard model effective field theory with the complete set of dimension-7 operators

Landon Lehman*

Department of Physics, Notre Dame, Indiana 46556, USA

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We present a complete list of the independent dimension-7 operators that are constructed using the standard model degrees of freedom and are invariant under the standard model gauge group. This list contains only 20 independent operators, far fewer than the 63 operators available at dimension 6. All of these dimension-7 operators contain fermions and violate lepton number, and 7 of the 20 violate baryon number as well. This result extends the standard model effective field theory and allows a more detailed exploration of the structure and properties of possible deformations from the standard model Lagrangian.

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I. INTRODUCTION

The discovery of the Higgs boson at the Large Hadron Collider (LHC) provided the latest of many examples of the explanatory power of the standard model. However, because of the hierarchy problem and the accompanying angst regarding naturalness, it is widely believed that beyond-the-standard-model (BSM) physics is necessary at scales not much beyond the TeV scale. As of yet there are no clear experimental signatures of new physics at these scales; in particular, theoretically compelling extensions of the standard model such as supersymmetry have yet to be experimentally verified.

While searches for BSM particles at the LHC driven by explicit theories such as supersymmetry or extra dimensions are useful and ongoing, it may be beneficial to take an alternative and complementary approach in the quest for BSM physics. This avenue is the method of effective field theory, which parametrizes all possible deviations from the standard model that any particular UV theory might explore. Effective field theory can be viewed as a universal bottom-up approach to BSM physics, as opposed to the top-down approach of particular UV theories.

Assuming no undiscovered light ($\lesssim$TeV) particles, integrating out the heavy degrees of freedom in a BSM model will produce effective operators that are invariant under the standard model gauge group. By constructing every possible operator using the standard model degrees of freedom and using standard model gauge group invariance as a constraint, we can remain agnostic as to the specific BSM theory that is producing these operators. Of course, a given UV theory may have symmetries that forbid some of the operators, or various operators may be loop suppressed, but the point of effective field theory is to take a general approach and cast a wide net. The operators constructed in this manner collectively constitute the standard model effective field theory (SMEFT).

Since operators with dimension greater than 4 are suppressed by powers of an energy scale equal to the scale at which the new physics is integrated out, the construction of effective operators from standard model fields can be organized by an expansion in the canonical dimension of the operators. The canonical dimension is the total dimension of the fields making up an operator; dimensionful couplings are not included. This expansion can be treated as an expansion in powers of the dimensionless parameter $\varepsilon = m_{\mathrm{SM}}/\Lambda$, where Λ is the scale of the new physics and standard model mass scales such as the Higgs mass or the top quark mass are represented by m_{SM}.

This constructive program is trivially implemented at dimension 5, or equivalently ε^1, since there is only one possible dimension-5 gauge invariant operator—the Weinberg neutrino mass operator [1]. At the dimension-6 level (ε^2), Buchmüller and Wyler counted 80 operators in 1986 [2]. Some of these 80 operators were redundant and able be interrelated by using the standard model equations of motion to make field redefinitions. An updated classification containing 59 independent dimension-6 operators was published in 2010 [3]. Some dimension-7 and dimension-8 operators have been studied in the literature [4–14], but no complete operator basis has previously been published for any dimension greater than 6.

Table I lists the number of operators in the SMEFT for each order in the operator dimension expansion up to dimension 7. This operator count does not include flavor index permutations or Hermitian conjugates and ignores operators that only contribute to topological quantum effects, such as the QCD theta term $\theta g_3^2 G^A_{\mu\nu}\tilde{G}^{A\mu\nu}/(32\pi^2)$. The Higgs mass term $(H^\dagger H)$ is the solitary dimension-2 operator.

The formal properties of the dimension-6 operators of the SMEFT have been extensively studied. A series of papers calculated the full 59×59 anomalous dimension matrix for the operators that preserve baryon number [15–21], and the anomalous dimension matrix for

*llehman@nd.edu

TABLE I. The number of operators invariant under the standard model gauge group $SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$, organized by canonical operator dimension. This enumeration only includes operators with nontopological effects and does not consider flavor permutations or Hermitian conjugates. Note that four of the dimension-6 operators violate baryon number conservation, leaving 59 that conserve baryon number.

Dimension	Number of operators
2	1
4	13
5	1
6	63
7	20

the baryon-number violating operators followed soon after [22].[1] The dimension-6 operators were also recently noted to have intriguing "holomorphic" properties [24]. Similar examinations of the formal properties of the dimension-7 operators can now be undertaken utilizing the list presented in this paper.

The SMEFT up to dimension 6 has also been widely used in phenomenological studies. The LHC phenomenology of a specific toy model leading to some dimension-6 operators was examined in Ref. [25]. The effects of the dimension-6 SMEFT on lepton flavor violation [26] and top quark processes [27,28] have also been explored. Many papers address the effects of dimension-6 operators on the Higgs sector; for a representative sample, see Refs. [29–36] and references therein. The implementation of operators involving the Higgs into FeynRules has aided such phenomenological efforts, allowing dimension-6 operators to be used in Monte Carlo generators such as MadGraph [37]. The dimension-7 operators will also be useful for phenomenological studies of possible signals of new physics in various channels and processes. In particular, given the lepton and baryon-number violating properties of the dimension-7 operators, they can be used to explore baryogenesis and leptogenesis, giving possible methods of generating the matter-antimatter asymmetry of the Universe.

The organization of this paper is as follows. Section II establishes the relevant symbols and conventions, and the complete list of the independent dimension-7 operators is presented in Sec. III. Some necessary facts regarding fermions and hypercharge are reviewed in Sec. IV. In Sec. V we work through the details of the operator classification used to obtain the list in Sec. III, and we conclude in Sec. VI.

II. NOTATION AND CONVENTIONS

The standard model degrees of freedom are listed for convenience in Tables II and III. The $SU(2)_W$ generators will be represented by τ^I, with $I = 1, 2, 3$ the adjoint representation indices. The indices for the fundamental representation of $SU(2)_W$ will be $\{i, j, k, m, n\} \in \{1, 2\}$. The field-strength tensors will be denoted by $X_{\mu\nu} \in \{G^A_{\mu\nu}, W^I_{\mu\nu}, B_{\mu\nu}\}$, all with dimension 2. Dual tensors are defined as $\tilde{X}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\rho\sigma}X^{\rho\sigma}$. The indices $\{p, r, s, t\}$ will be used to denote fermion flavors (generations), and the chirality indices $\{L, R\}$ will generally be suppressed. To satisfy $SU(2)_W$ invariance, the complex conjugate of the Higgs appears in the construction $\tilde{H}^i \equiv \epsilon_{ij}(H^j)^*$. The symbol C will denote the Dirac charge conjugation matrix, which links together same-chirality fermion fields in a scalar current.

Color $SU(3)_C$ indices will always be suppressed, with the convention that an operator with two quarks with color indices $\{\alpha, \beta\}$ will always be contracted as $\delta_{\alpha\beta}q^\alpha q^\beta$, and an operator with three quarks will have the color indices contracted in the totally antisymmetric manner $\epsilon_{\alpha\beta\gamma}q^\alpha q^\beta q^\gamma$. There are no dimension-7 operators with more than three quarks.

The SMEFT Lagrangian can contain the matter fields shown in Table II, the gauge field-strength tensors $X_{\mu\nu}$, and covariant derivatives D_μ. The SMEFT Lagrangian at zeroth order, otherwise known as the standard model Lagrangian, is

TABLE II. The standard model matter degrees of freedom, along with their dimensions and gauge and Lorentz group representations.

Field	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$	Dimension	$SL(2,\mathbb{C})$
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	*3*	*2*	1/6	3/2	$(\frac{1}{2}, 0)$
u_R	*3*	*1*	2/3	3/2	$(0, \frac{1}{2})$
d_R	*3*	*1*	−1/3	3/2	$(0, \frac{1}{2})$
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	*1*	*2*	1/2	1	(0,0)
$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	*1*	*2*	−1/2	3/2	$(\frac{1}{2}, 0)$
e_R	*1*	*1*	−1	3/2	$(0, \frac{1}{2})$

TABLE III. The standard model gauge degrees of freedom and their gauge group representations. All have dimension 1 and transform in the vector representation of the Lorentz group. $A = 1, \ldots, 8$ and $I = 1, \ldots, 3$.

Field	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$
G^A_μ	*8*	*1*	0
W^I_μ	*1*	*3*	0
B_μ	*1*	*1*	0

[1]This reference also noted that only four of the five baryon-number violating operators listed in Ref. [3] are independent, a fact previously realized in Ref. [23].

$$\mathcal{L}_{\rm SM} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu H)^\dagger(D^\mu H) + m^2 H^\dagger H - \frac{1}{2}\lambda(H^\dagger H)^2 + i(\bar{L}\not{D}L + \bar{e}\not{D}e + \bar{Q}\not{D}Q + \bar{u}\not{D}u + \bar{d}\not{D}d) - (\bar{L}Y_e eH + \bar{Q}Y_u u\tilde{H} + \bar{Q}Y_d dH + \text{H.c.}). \quad (1)$$

As mentioned before, we ignore topological terms—i.e. terms that are total spacetime derivatives. The Lagrangian and all operators apply above the electroweak symmetry breaking scale.

The dimension-7 operators presented in Sec. III generically violate lepton and/or baryon number. In fact, all SMEFT operators of odd dimension must violate either lepton number or baryon number, and perhaps both [38–40]. Baryon number is defined as

$$B = \frac{1}{3}(n_q - n_{q^c}), \quad (2)$$

where n_q is the number of quarks and n_{q^c} is the number of antiquarks. Lepton number is

$$L = (n_l - n_{l^c}), \quad (3)$$

with n_l the number of leptons and n_{l^c} the number of antileptons.

III. COMPLETE LIST OF DIMENSION-7 OPERATORS

The complete list of independent dimension-7 operators is presented in Table IV. It will be shown in Sec. IV that all of the possible dimension-7 operators contain fermions, so every operator in Table IV has suppressed flavor indices. This can be contrasted with the dimension-6 case, where four of the eight operator classes are fermion free and thus do not need flavor indices. Some operators which might at first glance seem to be missing from the list can be formed from those included by adding or subtracting combinations with different permutations of flavor indices. For example,

$$\epsilon_{in}\epsilon_{jm}(\bar{d}L^i_p)(Q^jL^m_q)H^n = \mathcal{O}^{(2)pq}_{LLQ\bar{d}H} - \mathcal{O}^{(1)pq}_{LLQ\bar{d}H}, \quad (4)$$

where the Schouten identity $\epsilon_{in}\epsilon_{jm} = \epsilon_{im}\epsilon_{jn} - \epsilon_{ij}\epsilon_{mn}$ was used. Writing out the lepton doublet flavor indices explicitly, we have

$$\mathcal{O}^{(1)pq}_{LLQ\bar{d}H} = \epsilon_{ij}\epsilon_{mn}(\bar{d}L^i_p)(Q^jCL^m_q)H^n \quad \text{and}$$
$$\mathcal{O}^{(2)pq}_{LLQ\bar{d}H} = \epsilon_{im}\epsilon_{jn}(\bar{d}L^i_p)(Q^jCL^m_q)H^n. \quad (5)$$

Another more complicated example using the same two operators is

$$\epsilon_{in}\epsilon_{jm}(\bar{d}Q^j)(L^i_pL^m_q)H^n = -(\mathcal{O}^{(2)pq}_{LLQ\bar{d}H} + \mathcal{O}^{(2)qp}_{LLQ\bar{d}H}) + (\mathcal{O}^{(1)pq}_{LLQ\bar{d}H} + \mathcal{O}^{(1)qp}_{LLQ\bar{d}H}), \quad (6)$$

where Fierz identities were used along with the Schouten identity mentioned above. As a third example, Ref. [5] lists two dimension-7 operators with the field content $\{\bar{L}, Q, Q, d, H\}$, which after passing to the notation used here are

$$\mathcal{O}^{pq}_1 = \epsilon_{ij}\epsilon_{mn}(Q^i_pCQ^j_q)(\bar{L}^m d)\tilde{H}^n \quad \text{and}$$
$$\mathcal{O}^{pq}_2 = \epsilon_{jm}\epsilon_{in}(Q^i_pCQ^j_q)(\bar{L}^m d)\tilde{H}^n. \quad (7)$$

In terms of the single operator $\mathcal{O}^{pq}_{\bar{L}QQdH}$ given in Table IV, these can be written

$$\mathcal{O}^{pq}_2 = \mathcal{O}^{qp}_{\bar{L}QQdH},$$
$$\mathcal{O}^{pq}_1 = \mathcal{O}^{pq}_{\bar{L}QQdH} - \mathcal{O}^{qp}_{\bar{L}QQdH}, \quad (8)$$

with the lepton flavor index assignment

$$\mathcal{O}^{pq}_{\bar{L}QQdH} = \epsilon_{ij}(Q^m_pCQ^i_q)(\bar{L}_m d)\tilde{H}^j. \quad (9)$$

All of the dimension-7 operators violate lepton number, and 7 of the 20 operators violate baryon number as well. In fact, all of the operators that do not violate baryon number do violate lepton number by two units, i.e. $L = +2$. The baryon-number violating operators all have $L = -1$, so that $B - L = 2$; therefore, all of the baryon-number violating operators violate $B - L$ as well. If the operators violating $B - L$ lead to proton decay, they will be suppressed by a scale $\Lambda \gtrsim 10^{10}$ GeV, since the proton lifetime is generically experimentally constrained to be $\gtrsim 10^{32}$ years [41–44]. However, the flavor structure of the $B - L$ violating operators could be such that they do not in fact lead to proton decay within experimentally constrained time scales. This could happen for example for an operator that did not contain first-generation quarks, and such an operator might be suppressed by some scale lower than 10^{10} GeV.[2] The operators that do not violate baryon number lead to neutrino mass generation, since they have $L = +2$. Therefore, these operators are also suppressed by a high scale, namely $\gtrsim 10^4$ TeV [13].

As mentioned in the Introduction, some of these dimension-7 operators have previously been examined in the literature. For example, Ref. [8] lists nine dimension-7 operators with $B = +1$ and ten operators with $L = +2$ in the context of $SO(10)$ grand unified theories and nucleon decay. The nine $B = +1$ operators are captured by $\mathcal{O}^{(1)}_{\bar{L}QddD}$, $\mathcal{O}^{(2)}_{\bar{L}QddD}$, $\mathcal{O}_{ddd\bar{e}D}$, $\mathcal{O}_{\bar{L}QQdH}$, $\mathcal{O}_{\bar{e}QddH}$, $\mathcal{O}_{\bar{L}dddH}$, and $\mathcal{O}_{\bar{L}uddH}$, and the ten $L = +2$ operators correspond to the remaining

[2] I thank an anonymous reviewer for this insight.

TABLE IV. The dimension-7 operators. Color and flavor indices are left implicit, and $SU(2)_W$ indices are left implicit when the contractions are obvious. The symbol C represents the Dirac charge conjugation matrix, as explained in Sec. IV. The six classes of operators shown group the operators by the degrees of freedom H, X, D, and ψ.

1: $\psi^2H^4 + \text{H.c.}$		2: $\psi^2H^2D^2 + \text{H.c.}$	
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^iCL^m)H^jH^n(H^\dagger H)$	$\mathcal{O}_{LHD}^{(1)}$	$\epsilon_{ij}\epsilon_{mn}L^iC(D^\mu L^j)H^m(D_\mu H^n)$
		$\mathcal{O}_{LHD}^{(2)}$	$\epsilon_{im}\epsilon_{jn}L^iC(D^\mu L^j)H^m(D_\mu H^n)$
3: $\psi^2H^3D + \text{H.c.}$		**4: $\psi^2H^2X + \text{H.c.}$**	
$\mathcal{O}_{LHDe}$	$\epsilon_{ij}\epsilon_{mn}(L^iC\gamma_\mu e)H^jH^mD^\mu H^n$	$\mathcal{O}_{LHB}$	$\epsilon_{ij}\epsilon_{mn}(L^iC\sigma_{\mu\nu}L^m)H^jH^nB^{\mu\nu}$
		$\mathcal{O}_{LHW}$	$\epsilon_{ij}(\tau^I\epsilon)_{mn}(L^iC\sigma_{\mu\nu}L^m)H^jH^nW^{I\mu\nu}$
5: $\psi^4D + \text{H.c.}$		**6: $\psi^4H + \text{H.c.}$**	
$\mathcal{O}_{LL\bar{d}uD}^{(1)}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^iCD^\mu L^j)$	$\mathcal{O}_{LLL\bar{e}H}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^jCL^m)H^n$
$\mathcal{O}_{LL\bar{d}uD}^{(2)}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^iC\sigma^{\mu\nu}D_\nu L^j)$	$\mathcal{O}_{LLQ\bar{d}H}^{(1)}$	$\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^jCL^m)H^n$
$\mathcal{O}_{\bar{L}QddD}^{(1)}$	$(QC\gamma_\mu d)(\bar{L}D^\mu d)$	$\mathcal{O}_{LLQ\bar{d}H}^{(2)}$	$\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^jCL^m)H^n$
$\mathcal{O}_{\bar{L}QddD}^{(2)}$	$(\bar{L}\gamma_\mu Q)(dCD^\mu d)$	$\mathcal{O}_{LL\bar{Q}uH}$	$\epsilon_{ij}(\bar{Q}_m u)(L^mCL^i)H^j$
$\mathcal{O}_{ddd\bar{e}D}$	$(\bar{e}\gamma_\mu d)(dCD^\mu d)$	$\mathcal{O}_{\bar{L}QQdH}$	$\epsilon_{ij}(\bar{L}_m d)(Q^mCQ^i)\tilde{H}^j$
		$\mathcal{O}_{\bar{L}dddH}$	$(dCd)(\bar{L}d)H$
		$\mathcal{O}_{\bar{L}uddH}$	$(\bar{L}d)(uCd)\tilde{H}$
		$\mathcal{O}_{Leu\bar{d}H}$	$\epsilon_{ij}(L^iC\gamma_\mu e)(\bar{d}\gamma^\mu u)H^j$
		$\mathcal{O}_{\bar{e}QddH}$	$\epsilon_{ij}(\bar{e}Q^i)(dCd)\tilde{H}^j$

five operators in class 6, along with $\mathcal{O}_{LL\bar{d}uD}^{(1)}$ (there is not an operator corresponding to $\mathcal{O}_{LL\bar{d}uD}^{(2)}$). Another reference listing several of the operators is Ref. [11]; this deals with $L = +2$ operators that could produce Majorana masses for neutrinos and lists the same five $L = +2$ operators from class 6 in Table IV. Mention is also made here of $\mathcal{O}_{LHB}$, $\mathcal{O}_{LHW}$, $\mathcal{O}_{LHDe}$, and the class of operators comprising $\mathcal{O}_{LHD}^{(1)}$ and $\mathcal{O}_{LHD}^{(2)}$, but a reduction to the minimal set of operators is not carried out.

IV. FERMIONS AND HYPERCHARGE

It is not possible to construct any dimension-7 operators without using fermion fields. To see this, recall that in the absence of fermions the available field content consists only of objects of dimension 1 (the Higgs doublet H or covariant derivatives D_μ) and objects of dimension 2 (the field strength tensors $X_{\mu\nu}$). Therefore, an odd number of dimension-1 objects must be included in order to obtain a total dimension of 7. However, since the Higgs has hypercharge $1/2$ and $X_{\mu\nu}$ and D_μ each have zero hypercharge, each fermion-free operator must contain an even number of Higgs fields in order to remain $U(1)_Y$ invariant. Similarly, since the total number of Lorentz indices in each operator must be even, each fermion-free operator must contain an even number of derivatives (since there are no fermions, there are no γ_μ's to provide another source of Lorentz indices). The dimensional constraint requires an odd number of dimension-1 objects, and Lorentz plus hypercharge constraints require an even number of dimension-1 objects, so we can conclude that there are no possible fermion-free operators of dimension 7, or more generally of any odd dimension.

Furthermore, consider the possibility of dimension-7 operators with multiple fermion currents. These operators must always have an even number of fermions since fermion fields have fractional dimensionality. The maximum possible number of fermion currents is therefore 2. Since two fermion currents have total dimension 6, we must add a dimension-1 object in order to get a dimension-7 operator, giving the two classes ψ^4D and ψ^4H discussed in Sec. V.

Because of the ubiquity of fermions in dimension-7 operators, hypercharge constraints will play a crucial role in the operator classification in Sec. V. For this reason, the remainder of this section reviews the basics of fermion currents and hypercharges in the standard model and establishes two facts for easy reference when carrying out the classification.

First, we do not need to use γ_5 in fermion currents (such as in the pseudovector matrix $\gamma^\mu\gamma^5$) because all fermions under discussion are chiral and thus eigenstates of γ_5. So the only fermion currents to consider are the scalar, vector, and tensor currents. Second, recall that there are two ways to write down a scalar fermion current—one connecting left-handed fields with right-handed fields and the other connecting fields of the same chirality with an insertion of charge conjugation,

$$\psi_{1_{L(R)}} C \psi_{2_{L(R)}} \quad \text{and} \quad \bar{\psi}_{1_{L(R)}} \psi_{2_{R(L)}}, \tag{10}$$

where C is the Dirac charge conjugation matrix. The operator C can be explicitly written as $i\gamma_2\gamma_0$ and causes a Lorentz spinor to transform as its conjugate spinor. These two possibilities can also be written in two-component fermion notation; for a review see Ref. [45]. In this case it is easiest to define all fields as left handed, so the standard model fermions are Q, L, u^c, d^c, and e^c, all transforming under the $(\frac{1}{2}, 0)$ representation of the Lorentz group (a bar is often used instead of the superscript c in the field names, but this would further confuse the notation). Then the four-component currents (LCL) and (dCd) become

$$L^\alpha L_\alpha = \epsilon_{\alpha\beta} L^\alpha L^\beta \quad \text{and} \quad d^{c\dagger}_{\dot{\alpha}} d^{c\dagger\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}} d^{c\dagger}_{\dot{\alpha}} d^{c\dagger}_{\dot{\beta}} \tag{11}$$

in two-component notation, with α and β labelling spinor indices. Similarly, the four-component current $(\bar{L}d)$ is written in two-component notation as

$$L^\dagger_{\dot{\alpha}} d^{c\dagger\dot{\alpha}} = (L_\alpha)^\dagger d^{c\dagger\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}} (L^\beta)^\dagger d^{c\dagger\dot{\alpha}}. \tag{12}$$

Four-component notation is used in the operator list, and classification for continuity with the notation used in Ref. [3], but it is useful to realize the two-component counterparts, especially when applying Fierz identities. Appendix A reproduces the list of dimension-7 operators from Table IV in two-component notation.

Having established some conventions regarding fermions, we now move to the hypercharge constraints. Forming all possible scalar fermion currents using the two methods in Eq. (10) and the fields from Table II shows that there are no scalar fermion currents that have zero hypercharge. Hereafter this statement will be referred to as rule 1. The situation for tensor currents mirrors that of scalar currents so far as the chirality of the two fields is concerned, so rule 1 also applies to tensor currents.

Similarly, there are two ways to construct a vector fermion current:

$$\bar{\psi}_{1_{L(R)}} \gamma_\mu \psi_{2_{L(R)}} \quad \text{and} \quad \psi_{1_{L(R)}} C \gamma_\mu \psi_{2_{R(L)}}. \tag{13}$$

Translating to two-component notation gives for example (suppressing the spinor indices)

$$L^\dagger \bar{\sigma}^\mu Q \quad \text{and} \quad Q \sigma^\mu d^{c\dagger} \tag{14}$$

for the four-component currents $(\bar{L}\gamma^\mu Q)$ and $(QC\gamma_\mu d)$. Again taking account of all of the possibilities using the fields in Table II shows that there are no vector currents with hypercharge $\pm 1/2$. This will be called rule 2.

Summarizing:

Rule 1. *There are no scalar or tensor fermion currents with zero hypercharge.*

Rule 2. *There are no vector fermion currents with hypercharge* $\pm 1/2$.

We will make extensive use of these rules in the following section.

V. CLASSIFICATION OF OPERATORS

As was done in the previous dimension-6 classifications, the dimension-7 classification will use the field equations of motion (EOMs) in order to make field redefinitions and thus allow some classes of operators to be subsumed into other classes. For dimension-7 operators, the EOMs are needed at $O(1/\Lambda^3)$, and we will neglect $O(1/\Lambda^4)$ effects. Therefore, the EOMs can be calculated using only the original standard model Lagrangian $\mathcal{L}_{\rm SM}$ as given in Eq. (1). It is important to note that this use of just the classical EOMs is an approximation that may not always be justified. In particular, it assumes that all of the operators at a given dimension have the same cutoff scale Λ. This may not always be the case, as the particles that are integrated out to give an operator $\mathcal{O}_a$ may be much lighter than the particles integrated out to give operator $\mathcal{O}_b$, leaving $\mathcal{O}_a$ with a lower cutoff than $\mathcal{O}_b$. Hence, the cutoff scale Λ should technically have an index i for each independent operator: $\Lambda = \Lambda_i$. As an example, the dimension-5 Weinberg operator has a cutoff scale $\Lambda = \Lambda_{\rm dim5}$ that is experimentally constrained by the light neutrino masses to be much higher than many of the experimentally allowed values of $\Lambda = \Lambda_{i,\rm dim6}$ for dimension-6 operators.

Taking into account the fact that there are no fermion-free operators, the following 11 distinct classes can be formed from the dimension-7 combinations of the degrees of freedom $\{X, D, \psi, H\}$:

$$\psi^2 X^2, \psi^2 H^4, \psi^2 H^2 D^2, \psi^2 H^3 D, \psi^2 H D^3, \psi^2 D^4, \\ \psi^2 H^2 X, \psi^2 D^2 X, \psi^2 H D X, \psi^4 D, \psi^4 H. \tag{15}$$

These classes are individually examined in the following subsections. Five classes are completely ruled out by rule 1 and rule 2, leaving the following six classes that will require a closer look and that do end up containing operators: 1) $\psi^2 H^4$, 2) $\psi^2 H^2 D^2$, 3) $\psi^2 H^3 D$, 4) $\psi^2 H^2 X$, 5) $\psi^4 D$, and 6) $\psi^4 H$.

Out of these six classes of operators, the EOMs only need to be used in two classes: $\psi^2 H^2 D^2$ and $\psi^4 D$. In each of these classes, the use of the EOMs only reduces some subset of the class of operators to other classes. This can be contrasted with the situation for the dimension-6 operators, where the EOMs are instrumental in removing entire classes from consideration [3]. Hypercharge constraints are much more useful for the dimension-7 operators, a fact that can be traced back to the absence of fermion-free operators of dimension 7.

A. $\psi^2 X^2$

The total number of Lorentz indices must be even, so the fermion current has to be a scalar or a tensor. But since the field-strength tensors do not carry hypercharge, the fermion current must have zero hypercharge, and rule 1 rules this out.

B. $\psi^2 H^4$

The fermion current must be a scalar and have hypercharge 0, ± 1, or ± 2. Rule 1 eliminates the zero hypercharge case. Examining the other scalar fermion current possibilities using the fields in Table II shows that only the current with two lepton doublets has hypercharge ± 1, and only the current with two right-handed electrons has hypercharge ± 2. The lepton doublet current leads to the only operator in this class:

$$\mathcal{O}_{LH} = \epsilon_{ij}\epsilon_{mn}(L^i C L^m) H^j H^n (H^\dagger H). \tag{16}$$

The other ways of contracting the $SU(2)_W$ indices are either equivalent or identically zero. Forming $SU(2)_W$ triplets instead of singlets does not produce anything new, because of the group identity

$$\tau^I_{jk}\tau^I_{mn} = 2\delta_{jn}\delta_{mk} - \delta_{jk}\delta_{mn}. \tag{17}$$

For the right-handed electron current, there is no way to contract the $SU(2)_W$ indices in a way that is not identically zero,

$$(eCe)(H^\dagger_j \tilde{H}^j)^2 = (eCe)\epsilon_{ij}\epsilon_{mn} H^*_i H^*_j H^*_m H^*_n = 0. \tag{18}$$

C. $\psi^2 H^2 D^2$

To form a Lorentz scalar, the fermion current can be either a tensor or scalar, and it must have hypercharge 0 or ± 1. Rule 1 eliminates the zero hypercharge case, and the only scalar or tensor current that has hypercharge ± 1 is the one with two lepton doublets. The remaining analysis in this section closely follows the calculations done in Ref. [3] for the dimension-6 operator class $\psi^2 H D^2$.

Consider first the case with a scalar fermion current. Then there are five possibilities for where the derivatives act: 1) both on a single Higgs, 2) both on a single fermion, 3) one on each fermion, 4) one on each Higgs, and 5) one on a fermion and the other on a Higgs. In the work that follows, boxes signify generic classes of operators. If both derivatives act on a single Higgs, the operator can be reduced using the Higgs equation of motion

$$\psi^2 H(D_\mu D^\mu H) = m^2 \boxed{\psi^2 H^2} + \boxed{\psi^2 H^4} + \boxed{\psi^4 H}. \tag{19}$$

If both derivatives act on a single fermion, the operator can also be reduced with EOMs

$$\begin{aligned}\psi H^2(D_\mu D^\mu \psi) &= \psi H^2(\eta^{\mu\nu} D_\mu D_\nu \psi) \\ &= \psi H^2 \not{D}\not{D}\psi + \boxed{\psi^2 H^2 X} \\ &= \boxed{\psi^2 H^3 D} + \boxed{\psi^2 H^2 X}.\end{aligned} \tag{20}$$

The second equality in Eq. (20) follows upon using $[D_\mu, D_\nu] \sim X_{\mu\nu}$ and the identity

$$\gamma^\mu\gamma^\nu = \eta^{\mu\nu} - i\sigma^{\mu\nu}, \tag{21}$$

and the third follows from the fermion EOMs. If the derivatives act one on each fermion, a combination of integration by parts and the two previous reductions can be used to change the operator to possibility number 5,

$$\begin{aligned}H^2(D_\mu\psi)(D^\mu\psi) &= -2H\psi(D_\mu H)(D^\mu\psi) \\ &\quad - \psi H^2(D_\mu D^\mu \psi) + \boxed{T} \\ &= -2H\psi(D_\mu H)(D^\mu \psi) + \boxed{\psi^2 H^3 D} \\ &\quad + \boxed{\psi^2 H^2 X} + \boxed{T},\end{aligned} \tag{22}$$

where $\boxed{T}$ represents total derivatives and the second equality follows from Eq. (20). Similarly, if the derivatives act one on each Higgs,

$$\begin{aligned}\psi^2(D_\mu H)(D^\mu H) &= -2H\psi(D_\mu\psi)(D^\mu H) \\ &\quad - \psi^2 H(D_\mu D^\mu H) + \boxed{T} \\ &= -2H\psi(D_\mu\psi)(D^\mu H) + m^2\boxed{\psi^2 H^2} \\ &\quad + \boxed{\psi^2 H^4} + \boxed{\psi^4 H} + \boxed{T},\end{aligned} \tag{23}$$

where the second equality follows from Eq. (19). So possibility 4 can also be reduced to possibility 5. Now consider the final possibility, number 5, where one derivative acts on a fermion and the other acts on a Higgs. Integrating by parts in this case just gives back the same structure, so we need to check if it can be otherwise reduced through gamma matrix algebra followed by integration by parts,

$$\begin{aligned}2\psi H(D^\mu\psi)(D_\mu H) &= 2\psi H(\eta^{\mu\nu}D_\nu\psi)(D_\mu H)\\ &= \psi H((\gamma^\mu \not{D} + \not{D}\gamma^\mu)\psi)(D_\mu H)\\ &= \boxed{\psi^2H^3D} - (D_\nu\psi)(\gamma^\nu\gamma^\mu\psi)H(D_\mu H) - \psi(\gamma^\nu\gamma^\mu\psi)(D_\nu H)(D_\mu H) - \psi(\gamma^\nu\gamma^\mu\psi)H(D_\nu D_\mu H) + \boxed{T}\\ &= \boxed{\psi^2H^3D} - \psi^2(D^\mu H)(D_\mu H) + i\psi\sigma^{\mu\nu}\psi(D_\nu H)(D_\mu H) - \psi^2 H(D^\mu D_\mu H) + i\psi\sigma^{\mu\nu}\psi H(D_\nu D_\mu H) + \boxed{T}\\ &= \boxed{\psi^2H^3D} + \boxed{\psi^2H^4} + \boxed{\psi^4H} + m^2\boxed{\psi^2H^2} + \boxed{T} + 2\psi H(D^\mu\psi)(D_\mu H) + i\psi\sigma^{\mu\nu}\psi(D_\nu H)(D_\mu H)\\ &\quad + i\psi\sigma^{\mu\nu}\psi(D_\nu D_\mu H),\end{aligned} \tag{24}$$

where the penultimate equality follows from the fermion EOMs and Eq. (21) and the final equality follows from Eqs. (19) and (23). Since we get the same structure back along with some tensor current operators, this is an independent contribution and must be included in the operator list. There are two independent ways to contract the $SU(2)_W$ indices, since there are four distinct fundamental representations of $SU(2)_W$ in the tensor product. These give the following two operators:

$$\mathcal{O}^{(1)}_{LHD} = \epsilon_{ij}\epsilon_{mn}L^iC(D^\mu L^j)H^m(D_\mu H^n), \tag{25}$$

$$\mathcal{O}^{(2)}_{LHD} = \epsilon_{im}\epsilon_{jn}L^iC(D^\mu L^j)H^m(D_\mu H^n). \tag{26}$$

The other contraction with $\epsilon_{in}\epsilon_{jm}$ is not independent, because of the Schouten identity $\epsilon_{in}\epsilon_{jm} = \epsilon_{im}\epsilon_{jn} - \epsilon_{ij}\epsilon_{mn}$. Note that we can always choose which fermion the derivative acts on, since the other case is equivalent up to an integration by parts after using the above results.

Next consider the case with a tensor fermion current. In this case, if both derivatives act on a single object, since $\sigma_{\mu\nu}$ is antisymmetric we are led to $[D_\mu, D_\nu]$ and thus to the class ψ^2XH^2, since $[D_\mu, D_\nu] \sim X_{\mu\nu}$. If one derivative acts on a fermion and one on a Higgs, by using $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ and the fermion EOMs we have

$$\begin{aligned}-(2i)\psi\sigma_{\mu\nu}(D^\mu\psi)H(D^\nu H) &= \psi(\not{D}\gamma_\nu - \gamma_\nu\not{D})\psi H(D^\nu H)\\ &= 2\psi(D_\nu\psi)H(D^\nu H)\\ &\quad - 2\psi\gamma_\nu(\not{D}\psi)H(D^\nu H)\\ &= \boxed{\psi^2H^3D} + 2\psi H(D_\nu\psi)(D^\nu H),\end{aligned} \tag{27}$$

so this possibility can be eliminated in favor of the two operators already constructed. If the derivatives act one on each fermion, integrating by parts gives

$$\begin{aligned}(D^\mu\psi)\sigma_{\mu\nu}(D^\nu\psi)H^2 &= -\psi\sigma_{\mu\nu}H^2(D^\mu D^\nu\psi)\\ &\quad - 2\psi\sigma_{\mu\nu}H(D^\mu\psi)(D^\nu H) + \boxed{T}\\ &= \boxed{\psi^2XH^2} + \boxed{\psi^2H^3D}\\ &\quad - 2i\psi H(D_\nu\psi)(D^\nu H) + \boxed{T},\end{aligned} \tag{28}$$

where the second equality follows from Eq. (27). So this possibility can also be reduced to the operators in Eqs. (25) and (26). Similarly, the case with the derivatives acting one on each Higgs reduces to classes ψ^2H^3D, ψ^2XH^2, and the previous operators after integration by parts. Therefore, there are no tensor current operators in this class.

D. ψ^2H^3D

The fermion current must be a Lorentz vector with hypercharge $\pm1/2$ or $\pm3/2$. Rule 2 eliminates the hypercharge $\pm1/2$ case, and the only remaining vector current that will work is the one connecting the lepton doublet with the right-handed electron field.

If the derivative acts on either of the fermion fields, two identical Higgs doublets must be contracted in order to satisfy $SU(2)_W$ invariance, giving a result that is identically zero. If the derivative does not act on either fermion field, we only have to consider the derivative acting on a single Higgs doublet, since all of the remaining cases reduce to this. Then there is only one way to contract the $SU(2)_W$ indices that is not identically zero, giving the operator

$$\mathcal{O}_{LHDe} = \epsilon_{ij}\epsilon_{mn}(L^iC\gamma_\mu e)H^jH^mD^\mu H^n. \tag{29}$$

E. ψ^2HD^3

The fermion current must be a Lorentz vector with hypercharge $\pm1/2$. But this possibility is removed by rule 2.

F. ψ^2D^4

The fermion current must be a scalar or tensor with hypercharge zero, and therefore rule 1 eliminates this class.

G. ψ^2H^2X

The fermion current must be a tensor with hypercharge 0 or ±1, since the field-strength tensors are traceless. The only current that works is the one with two lepton doublets. For $B_{\mu\nu}$ there is only one independent way to contract the $SU(2)_W$ indices, giving the following operator:

$$\mathcal{O}_{LHB} = \epsilon_{ij}\epsilon_{mn}(L^iC\sigma_{\mu\nu}L^m)H^jH^nB^{\mu\nu}. \tag{30}$$

For $W^I_{\mu\nu}$, we need to include the triplet τ^I, and there are generally six $SU(2)$ singlets in the product of four fundamentals and two triplets. However, some cases vanish because of the two identical H fields, leaving four independent singlets. Then allowing family index transpositions between the two leptons (as in the examples in Sec. III) and accounting for the fact that the labels on the Higgs fields are interchangeable leaves only one independent operator:

$$\mathcal{O}_{LHW} = \epsilon_{ij}(\tau^I\epsilon)_{mn}(L^iC\sigma_{\mu\nu}L^m)H^jH^nW^{I\mu\nu}. \tag{31}$$

The gluon field-strength tensor $G^A_{\mu\nu}$ cannot be used since there are no other objects available with a nontrivial $SU(3)_C$ transformation. The possibility of using the dual tensors $\tilde{B}^{\mu\nu}$ or $\tilde{W}^{I\mu\nu}$ does not give any new operators, because of the identities

$$\epsilon_{\alpha\beta\mu\nu}\sigma^{\mu\nu} = 2i\sigma_{\alpha\beta}\gamma_5 \tag{32}$$

and

$$\gamma_5\psi_{L,R} = \mp\psi_{L,R}. \tag{33}$$

H. ψ^2D^2X

The fermion current must be a scalar or tensor with zero hypercharge, so this class is eliminated by rule 1.

I. ψ^2HDX

The fermion current must be a vector with hypercharge $\pm 1/2$, so rule 2 removes this class.

J. ψ^4D

The fermions must have zero total hypercharge, and one of the two currents must be a vector current. If the derivative acts on either of the two fermions in the vector current, the operator can be reduced by using the fermion EOMs to the class ψ^4H. If the derivative acts on the scalar or tensor current, the operator cannot be reduced. There are only five current combinations that have a single vector current and zero total hypercharge: $(\bar{d}L)(LC\gamma_\mu u)$, $(\bar{L}d)(QC\gamma_\mu d)$, $(\bar{L}\gamma_\mu Q)(dCd)$, $(\bar{d}\gamma_\mu u)(LCL)$, and $(dCd)(\bar{e}\gamma_\mu d)$. Using integration by parts along with the fermion EOMs allows the derivative to be switched back and forth between the two fermions in the scalar or tensor current

$$\begin{aligned}(\psi_1\gamma_\mu\psi_2)((D^\mu\psi_3)\psi_4) &= (\psi_1\gamma_\mu\psi_2)(\psi_3D^\mu\psi_4) \\ &\quad + (D^\mu\psi_1\gamma_\mu\psi_2)(\psi_3\psi_4) \\ &\quad + (\psi_1\gamma_\mu D^\mu\psi_2)(\psi_3\psi_4) + \boxed{T} \\ &= (\psi_1\gamma_\mu\psi_2)(\psi_3D^\mu\psi_4) + \boxed{\psi^4H} + \boxed{T},\end{aligned} \tag{34}$$

so we can arbitrarily pick one of the two fermions for the derivative to act on. Doing this for each of the five current combinations gives the following five operators:

$$\mathcal{O}^{(1)}_{LL\bar{d}uD} = \epsilon_{ij}(\bar{d}\gamma_\mu u)(L^iCD^\mu L^j), \tag{35}$$

$$\mathcal{O}^{(2)}_{LL\bar{d}uD} = \epsilon_{ij}(\bar{d}\gamma_\mu u)(L^iC\sigma^{\mu\nu}D_\nu L^j), \tag{36}$$

$$\mathcal{O}^{(1)}_{\bar{L}QddD} = (QC\gamma_\mu d)(\bar{L}D^\mu d), \tag{37}$$

$$\mathcal{O}^{(2)}_{\bar{L}QddD} = (\bar{L}\gamma_\mu Q)(dCD^\mu d), \tag{38}$$

$$\mathcal{O}_{ddd\bar{e}D} = (\bar{e}\gamma_\mu d)(dCD^\mu d). \tag{39}$$

The tensor current was chosen for $\mathcal{O}^{(2)}_{LL\bar{d}uD}$ to avoid crossing color indices between currents. The scalar current not included can then be formed by using Fierz identities and flavor index transpositions of $\mathcal{O}^{(1)}_{LL\bar{d}uD}$ and $\mathcal{O}^{(2)}_{LL\bar{d}uD}$. See Ref. [45] or Ref. [46] for the relevant Fierz identities.

K. ψ^4H

The fermions must have total hypercharge $\pm 1/2$. The operator must be constructed from two scalar currents, two vector currents, or two tensor currents in order to preserve Lorentz invariance. This places constraints on the hypercharge combinations that actually work.

For a given field content, we can calculate the number of $SL(2,\mathbb{C})$ singlets in the tensor product, thus allowing a direct statement of the number of independent operators with that field content without an explicit calculation of all the possible Fierz transformations. For example, the field content $\{L, L, \bar{Q}, u\}$ can be written as the product of two

TABLE V. The eight sets of four fermion fields that can be joined into two scalar or two vector currents, have a total hypercharge $\pm 1/2$, and have an odd number of $SU(2)_W$ doublets. The entries in the first column allow a single $SL(2,\mathbb{C})$ singlet, and the entries in the second column give two $SL(2,\mathbb{C})$ singlets.

Fields	
$\{L, L, \bar{Q}, u\}$	$\{\bar{L}, u, d, d\}$
$\{\bar{L}, Q, Q, d\}$	$\{\bar{L}, d, d, d\}$
$\{\bar{e}, Q, d, d\}$	$\{L, L, Q, \bar{d}\}$
$\{L, e, u, \bar{d}\}$	$\{L, L, L, \bar{e}\}$

TABLE VI. The dimension-7 operators in two-component fermion notation. All of the fermions are defined to be fundamentally left handed, so the fermion fields are $Q, L, \bar{d}, \bar{u}$, and $\bar{e}$, all transforming under the $(\frac{1}{2}, 0)$ representation of the Lorentz group. The bar here is part of the field name and in particular does *not* mean the Dirac bar used in four-component notation. Spinor indices are contracted within parentheses. Color and flavor indices are left implicit, and $SU(2)_W$ indices are left implicit when the contractions are obvious. The symbol C represents the Dirac charge conjugation matrix, as explained in Sec. IV.

	1: $\psi^2 H^4 + \mathrm{H.c.}$		2: $\psi^2 H^2 D^2 + \mathrm{H.c.}$
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^i L^m)H^j H^n (H^\dagger H)$	$\mathcal{O}_{LHD}^{(1)}$	$\epsilon_{ij}\epsilon_{mn}(L^i D^\mu L^j)H^m D_\mu H^n$
		$\mathcal{O}_{LHD}^{(2)}$	$\epsilon_{im}\epsilon_{jn}(L^i D^\mu L^j)H^m D_\mu H^n$
	3: $\psi^2 H^3 D + \mathrm{H.c.}$		4: $\psi^2 H^2 X + \mathrm{H.c.}$
$\mathcal{O}_{LHDe}$	$\epsilon_{ij}\epsilon_{mn}(L^i \sigma_\mu \bar{e}^\dagger)H^j H^m D^\mu H^n$	$\mathcal{O}_{LHB}$	$\epsilon_{ij}\epsilon_{mn}(L^i \sigma_{\mu\nu} L^m)H^j H^n B^{\mu\nu}$
		$\mathcal{O}_{LHW}$	$\epsilon_{ij}(\tau^I \epsilon)_{mn}(L^i \sigma_{\mu\nu} L^m)H^j H^n W^{I\mu\nu}$
	5: $\psi^4 D + \mathrm{H.c.}$		6: $\psi^4 H + \mathrm{H.c.}$
$\mathcal{O}_{LL\bar{d}uD}^{(1)}$	$\epsilon_{ij}(\bar{d}\sigma_\mu \bar{u}^\dagger)(L^i D^\mu L^j)$	$\mathcal{O}_{LLL\bar{e}H}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^j L^m)H^n$
$\mathcal{O}_{LL\bar{d}uD}^{(2)}$	$\epsilon_{ij}(\bar{d}\sigma_\mu \bar{u}^\dagger)(L^i \sigma^{\mu\nu} D_\nu L^j)$	$\mathcal{O}_{LLQ\bar{d}H}^{(1)}$	$\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^j L^m)H^n$
$\mathcal{O}_{\bar{L}QddD}^{(1)}$	$(Q\sigma_\mu \bar{d}^\dagger)(L^\dagger D^\mu \bar{d}^\dagger)$	$\mathcal{O}_{LLQ\bar{d}H}^{(2)}$	$\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^j L^m)H^n$
$\mathcal{O}_{\bar{L}QddD}^{(2)}$	$(Q\sigma_\mu L^\dagger)(\bar{d}^\dagger D^\mu \bar{d}^\dagger)$	$\mathcal{O}_{LL\bar{Q}uH}$	$\epsilon_{ij}(Q_m^\dagger \bar{u}^\dagger)(L^m L^i)H^j$
$\mathcal{O}_{ddd\bar{e}D}$	$(\bar{e}\sigma_\mu \bar{d}^\dagger)(\bar{d}^\dagger D^\mu \bar{d}^\dagger)$	$\mathcal{O}_{\bar{L}QQdH}$	$\epsilon_{ij}(L_m^\dagger \bar{d}^\dagger)(Q^m Q^i)\tilde{H}^j$
		$\mathcal{O}_{\bar{L}dddH}$	$(\bar{d}^\dagger \bar{d}^\dagger)(L^\dagger \bar{d}^\dagger)H$
		$\mathcal{O}_{\bar{L}uddH}$	$(L^\dagger \bar{d}^\dagger)(\bar{u}^\dagger \bar{d}^\dagger)\tilde{H}$
		$\mathcal{O}_{Leu\bar{d}H}$	$\epsilon_{ij}(L^i \sigma_\mu \bar{e}^\dagger)(\bar{d}\sigma^\mu \bar{u}^\dagger)H^j$
		$\mathcal{O}_{\bar{e}QddH}$	$\epsilon_{ij}(\bar{e}Q^i)(\bar{d}^\dagger \bar{d}^\dagger)\tilde{H}^j$

vector currents or as the product of two scalar currents, but there is only one $SL(2,\mathbb{C})$ singlet in the tensor product $(\frac{1}{2},0)\otimes(\frac{1}{2},0)\otimes(0,\frac{1}{2})\otimes(0,\frac{1}{2})$. So we can choose the product of scalar currents as the representative operator (this choice does not cross color indices between currents, so it is aesthetically more pleasing). Now we need to consider the $SU(2)_W$ contraction. There are three ways to contract the $SU(2)_W$ indices, two of which are independent. But since there are two identical fields in the operator, two of the three $SU(2)_W$ contractions are equivalent under a transposition of flavor indices. Therefore, there is only one independent operator for this set of fermion fields:

$$\mathcal{O}_{LL\bar{Q}uH} = \epsilon_{ij}(\bar{Q}_m u)(L^m C L^i)H^j. \qquad (40)$$

The field contents that work for this class can be found by a simple search, and they are listed in Table V. Carrying out the procedure described in the previous paragraph for these eight sets of fermion fields leads to the operators listed for this class in Table IV. Whenever possible the currents are chosen so that color indices are not crossed between currents. Note that for the field content $\{L, L, Q, \bar{d}, H\}$ there are two $SL(2,\mathbb{C})$ singlets and two ways to contract the $SU(2)_W$ indices for each of these singlets, leading to two independent operators after considering possible flavor index permutations. In this case, the color indices must be crossed between currents so that all of the possible $SU(2)_W$ singlets and Lorentz contractions can be formed from the two operators given.

VI. CONCLUSION

The standard model works extremely well at explaining particle physics as we know it, and no clear BSM signals have come into view at the LHC 7 and 8 TeV runs. It is therefore imperative to study in detail all possible deviations from the standard model in order to better understand the specific channels where new physics might materialize. To further this program, we have presented a complete classification of the dimension-7 operators in the standard model effective field theory. There are 20 dimension-7 operators, all lepton-number violating, with seven of them also violating baryon number. All of the operators include fermions, so the use of hypercharge constraints plays a central role in the operator classification. This catalog allows a closer examination of the SMEFT structure and properties and provides a guide for detailed studies utilizing an effective field theory approach to physics beyond the standard model. Even though most of the operators are generically suppressed by a very high scale, it would be interesting to try to find loopholes in this supression, perhaps by utilizing the flavor structure of the $B-L$ operators as mentioned in Sec. III.

Some simple modifications allow the construction of more independent operators, extending the reach of the SMEFT beyond the standard model. For example, adding another distinct Higgs doublet, as is done in supersymmetry, allows the construction of a new operator at the dimension-5 level[3] and would give several new operators at dimensions 6 and 7. Modifying the standard model by including right-handed neutrinos also gives many new operators, some of which were examined at the dimension-6 level in Ref. [22]. The SMEFT could also be extended while remaining strictly within the confines of the standard model by perfoming a classification of the dimension-8 operators. This would certainly be possible but would be tedious considering the large number of operators available at dimension 8 as opposed to dimension 7.

It would also be interesting to calculate the one-loop anomalous dimension matrix for the dimension-7 operators and check what, if any, of the holomorphy properties defined in Ref. [24] are present, but this is beyond the scope of this work. At this point, we simply note that, according to the definition of holomorphy given in Ref. [24], there are 10 holomorphic and antiholomorphic operators and 10 nonholomorphic operators at dimension 7. Since none of the dimension-7 operators is self-conjugate, the Hermitian conjugates of the holomorphic (antiholomorphic) operators will be antiholomorphic (holomorphic), and the Hermitian conjugates of the nonholomorphic operators will also be nonholomorphic. Perhaps some new structure may emerge when the formal properties of the standard model effective field theory are examined in more detail.

[3]This operator is $\epsilon_{ij}\epsilon_{mn}(L_p^i C L_q^j)H_1^m H_2^n$.

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APPENDIX: THE OPERATORS IN 2-COMPONENT NOTATION

See Table VI.

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