PHYSICAL REVIEW D **90,** 125017 (2014)

Quantum origin of suppression for vacuum fluctuations of energy

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(Received 18 August 2014; published 18 December 2014)

By using a model with a spatially global scalar field, we show that the energy density of zero-point modes is exponentially suppressed by an average number of field quanta in a finite volume with respect to the energy density in the stationary state of minimal energy. We describe cosmological implications of the mechanism.

DOI: 10.1103/PhysRevD.90.125017 PACS numbers: 03.70.+k, 98.80.Cq, 04.50.Kd

I. INTRODUCTION AND RECAPITULATION OF THE COSMOLOGICAL CONSTANT PROBLEM

After primary speculations presented in [1], we find that the cosmological constant [2] is associated with a vacuum energy density generated by zero-point modes of quantum fields [3,4]. In the framework of quantum field theory, these quantum fluctuations are ordinarily divergent and they should be renormalized. In this respect, the relevant renormalization is related to actual thresholds of energies, at which particles and forces contribute significantly. Then, one usually supposes that some combinations of Planckian scale and particle masses generate the energy density of vacuum ρ_{vac}, so that the maximal estimate corresponds to the greater scale known in real physics and it yields $\rho_{\mathrm{vac}} \sim m_{\mathrm{Pl}}^4$, where the reduced Planck mass $m_{\mathrm{Pl}} \approx 2.4 \times 10^{18}$ GeV is given by the Newton constant G as $8\pi G m_{\mathrm{Pl}}^2 = 1$. Such types of estimates are in direct conflict with the value extracted from the cosmological data [5] giving $\rho_{\mathrm{vac}} \mapsto \rho_\Lambda = \Lambda^4$ at $\Lambda \sim 10^{-3}$ eV, which is 30 orders of magnitude less than the Planck mass.

However, nobody can guarantee that the Planck scale defining the strength of the gravitational interaction has to establish a fundamental mass scale or energy threshold relevant to the cosmological constant. In this respect, one could follow a more realistic way by using the generally accepted description of particle physics in the Standard Model. So the direct observation of the Higgs boson allows us to evaluate the energy density of the electroweak vacuum from the effective potential of the Higgs boson in terms of the masses of the Higgs boson and W boson, which give $\rho_\Lambda^{\mathrm{EW}} \sim m_H^2 m_W^2 \sim (10^2\ \mathrm{GeV})^4$, exceeding the contribution due to the additional condensates in the quantum chromodynamics by at least 8 orders of magnitude. Then, the magnitude of mismatching the scale of the cosmological constant would be significantly relaxed from 30 orders to 14 orders, which is still significant. Notice that in this approach to the estimate of the cosmological constant scale, one ignores an arbitrary constant shift of the Higgs potential. This shift can originate from physics of other fields. In addition, at the observed mass value the Higgs potential can lose its stability at very large fields below the Planckian range due to effects of the renormalization group, which could produce the tunnel decay of the present Universe to another universe with a different vacuum.

Let us show that such a suppression can be explained due to a finite volume effect for quantum fluctuations in an excited nonstationary state. For the sake of clarity we exhibit the mechanism by considering a time-dependent scalar field $\phi(t)$, which is spatially global and free. The action in a finite physical volume V_R is given by the expression

$$S = V_R \int dt \frac{1}{2}(\dot{\phi}^2 - m^2\phi^2), \tag{1}$$

where $\dot{\phi} = d\phi/dt$ and m is the field mass. The specific reference frame of space-time suggestively should be associated with the reference frame of the homogeneous component of cosmic microwave background radiation in the Universe, so that time t could correspond to the cosmic time of Friedmann-Robertson-Walker-Lemaitre metrics. The meaning of V_R is the volume wherein the fluctuations of the field are causal, so that it can be considered as spatially global, while an inhomogeneity is evaluated by $|\nabla\phi| \sim \delta\phi/\lambda_c$, where λ_c is the Compton length, $\lambda_c = 1/m$, and $\delta\phi$ denotes the field fluctuation. The basic motivation and consideration are further considered in the field model of (1), so we ignore the influence of curved space-time on the main result for the moment. However, we return to the discussion of this issue in Sec. III.

Formally, action (1) corresponds to the harmonic oscillator of "frequency" m and "inertial mass" V_R. Then, the dimensionless operators

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$$\hat{Q} = \frac{\phi}{\phi_0}, \qquad \hat{P} = \frac{\dot{\phi}}{\dot{\phi}_0},$$

at

$$\phi_0^2 = \frac{1}{V_R m}, \qquad \dot{\phi}_0^2 = \frac{m}{V_R},$$

define the operators of annihilation and creation for the spatially global field quanta

$$\hat{a} = \frac{1}{\sqrt{2}}(\hat{Q} + i\hat{P}), \qquad \hat{a}^\dagger = \frac{1}{\sqrt{2}}(\hat{Q} - i\hat{P}),$$

with the standard commutator

$$[\hat{a}, \hat{a}^\dagger] = 1.$$

The Hamiltonian takes the form

$$\hat{H} = \frac{1}{2}V_R(\dot{\phi}^2 + m^2\phi^2) = \frac{1}{2}m(\hat{P}^2 + \hat{Q}^2) = m\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right). \tag{2}$$

If the field is nonstationary excited, its quantum state can be considered as a superposition of oscillatory coherent states, which minimize uncertainties in the field ϕ and its rate $\dot{\phi}$. Let us consider the coherent state $|\alpha\rangle$ with the average number of quanta n in the volume V_R:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle, \qquad \alpha^*\alpha = n, \qquad \alpha = \frac{1}{\sqrt{2}}(Q_0 + iP_0).$$

The averaged energy reads as

$$\langle E\rangle = \langle\alpha|\hat{H}|\alpha\rangle = m\left(n + \frac{1}{2}\right).$$

Usually, the minimal energy shift of this state from the minimum of the potential is referred to as the energy level of zero-point mode (ZPM),

$$\delta_{\min}E = \frac{1}{2}m. \tag{3}$$

So, it defines the quantity that we call the bare cosmological constant,

$$\rho_\Lambda^{\text{bare}} = \frac{m}{2V_R} = \langle\text{vac}|\rho|\text{vac}\rangle = m^2\langle\text{vac}|\phi^2|\text{vac}\rangle = \langle\text{vac}|\dot{\phi}^2|\text{vac}\rangle. \tag{4}$$

This shows that the finite volume sets nonzero fluctuations of the spatially global field.

If the fluctuations of the field match the Planckian scale, $\langle\text{vac}|\phi^2|\text{vac}\rangle \sim m_{\text{Pl}}^2$, then the bare cosmological constant $\rho_\Lambda^{\text{bare}}$ takes a huge value, which constitutes the cosmological constant problem. On the other hand, if we restrict ourselves to the range of the Standard Model, then we expect that the mass of the scalar field is given by the mass of the Higgs boson M, while the fluctuations of the field square are of the order of the natural scale in the electroweak physics, i.e., M^2. So one can arrive at an estimate that is consistent with the expectations of the Standard Model, but it still would have a huge value.

However, in the next section we show that if the number of quanta for $\phi(t)$ is not equal to zero, $n \neq 0$, then only a fraction of an energy shift from the potential minimum refers to the ZPM and the fractional part of the energy does originate from the suppressed vacuum fluctuations; hence, the suppressed fraction indeed corresponds to the vacuum energy.

II. SUPPRESSION MECHANISM IN THE ACTION

Let us find the fraction of the ZPM in the energy shift from the minimum of the potential for the coherent state. So, decomposing the state into the sum of the vacuum $|\text{vac}\rangle$ and the state $|\text{quanta}\rangle$ with nonzero numbers of stationary field quanta,

$$|\alpha\rangle = \mathcal{A}_{\text{vac}}|\text{vac}\rangle + \mathcal{A}_q|\text{quanta}\rangle,$$

we evaluate the average density of energy,

$$\langle\alpha|\rho|\alpha\rangle = |\mathcal{A}_{\text{vac}}|^2\langle\text{vac}|\rho|\text{vac}\rangle + |\mathcal{A}_q|^2\langle\text{quanta}|\rho|\text{quanta}\rangle, \tag{5}$$

where the probability to find k quanta in the coherent state is given by the Poisson distribution,

$$|\mathcal{A}_k|^2 = \frac{n^k}{k!}\mathrm{e}^{-n},$$

while the free Hamiltonian does not mix the stationary states with different numbers of quanta.

Therefore, the average density of energy observed in gravity, is decomposed as

$$\langle\rho\rangle = |\mathcal{A}_{\text{vac}}|^2\rho_\Lambda^{\text{bare}} + \rho_q, \tag{6}$$

at

$$|\mathcal{A}_{\text{vac}}|^2 = \mathrm{e}^{-n}, \tag{7}$$

with ρ_q being the energy density of nonzero-point modes that, for the coherent state, equals

$$\rho_q = \frac{m}{V_R}\left(n + \frac{1}{2} - \frac{1}{2}\mathrm{e}^{-n}\right) = \rho_\Lambda^{\text{bare}}(2n + 1 - \mathrm{e}^{-n}).$$

Thus, the true energy density generated by ZPMs in the coherent state is suppressed and is given by

$$\rho_\Lambda = |\mathcal{A}_{\rm vac}|^2 \rho_\Lambda^{\rm bare} \mapsto {\rm e}^{-n} \rho_\Lambda^{\rm bare}. \quad (8)$$

Relation (8) remains valid generically by the order of magnitude not only for the coherent state, but also for the most ordinary, nonexotic states yielding $|\mathcal{A}_{\rm vac}|^2 \sim {\rm e}^{-n}$. We can hold the relation for the probability of the ZPM in the quantum state as the definition of the effective number of quanta in this state. Moreover, if the fluctuations of field quanta are *statistically occasional*, then the probability with respect to the number of quanta has to fit the *Poisson distribution*, and hence, the quantum state should be the coherent state.

Let us stress that the described effect of vacuum fluctuation suppression in the excited nonstationary state necessarily involves the finite volume, but it has no connection to the well-known Casimir effect, which is also related to a restricted volume. Indeed, the Casimir effect takes place due to the relevant modification of ZPMs in the state of *minimal* energy for the system of restricted volume, while setting the state of minimal energy in our consideration will mean $n \to 0$ and the suppression factor will disappear, which results in the ordinary situation for the Casimir effect. In other words, the suppression factor becomes essential if the system is excited to the nonstationary state, when the Casimir effect is irrelevant, and vice versa, the Casimir effect takes place when the suppression we are studying is inactive. In this respect, the problem of the cosmological constant comes back when we deal with the stationary state of minimal energy, while near this state the Casimir effect represents the evidence for the reality of vacuum energy.

The meaning of the bare cosmological constant is the following: if the system is in the vacuum state, the bare cosmological constant would be the energy density in the system, i.e., in the empty vacuum without any fields, particles, or quanta, only vacuum fluctuations. If the system is not empty and it is occasionally excited, the actual vacuum fluctuations are suppressed, which means the suppression of the observed cosmological constant. In this way, we assume in our model that there is not any different contribution to the cosmological constant, say, like some induced terms of various nature, since those additional terms cannot be suppressed in the same manner.

The decomposition of (6)–(8) would remain formal, if the field is free and it does not interact. Then $\rho_\Lambda^{\rm bare}$ would set the minimal density of energy, of course, while we would observe the total density of field energy without the possibility to extract the energy density of suppressed zero-point fluctuations. However, if the field interacts, then it goes through a nontrivial evolution: the quanta can mix and transform into quanta of matter fields, while the vacuum transfers itself into itself, i.e., into the *vacuum*, since it is *stable*; hence, the contribution of ZPMs to the energy density remains constant with the evolution, and this contribution is much less than the bare term because of the excited state of the field, which makes the decomposition in (6)–(8) observable for the interacting field.[1] The suppressed contribution of ZPMs is observed as the cosmological constant in the presence of matter quanta.

In other words, the decomposition of (6)–(8) is based on quantum mechanics, and it is detectable. The detection suggests the interaction of field system. Roughly speaking, the field quanta decay to visible particles of matter, while the suppressed vacuum fluctuations form the observable cosmological constant.

Let us look at the pressure of ZPMs in order to justify their vacuum status. The bare ZPMs have the energy $E^{\rm bare} = \frac{1}{2} m$ independent of the reference volume. This means that the pressure is given by $p^{\rm bare} = \partial E^{\rm bare}/\partial V_R \equiv 0$, which can also be calculated by means of averaging the spatial components of the energy-momentum tensor,

$$\langle {\rm vac}|T_\alpha^\beta|{\rm vac}\rangle = -\delta_\alpha^\beta p^{\rm bare} = -\delta_\alpha^\beta \frac{1}{2} \langle {\rm vac}|\{\dot{\phi}^2(t) - m^2\phi^2(t)\}|{\rm vac}\rangle = 0.$$

In contrast, the true energy of suppressed vacuum fluctuations in volume V_R,

$$E_{\rm vac} = V_R \rho_\Lambda = \frac{1}{2} m |\mathcal{A}_{\rm vac}|^2,$$

can obtain the correct dependence on the volume if we make the vacuum density of energy constant, which implies $|\mathcal{A}_{\rm vac}|^2/V_R = {\rm const}$ and the pressure obtains the value,

$$p = -\frac{\partial E_{\rm vac}}{\partial V_R} = -\rho_\Lambda,$$

hence,

$$\frac{\partial n}{\partial \ln V_R} = -1.$$

Therefore, the reference volume exponentially declines with the growth of quantum number n, and there is a maximal number corresponding to a minimal volume of Planckian length.

This derivation of the actual value for the parameter of the vacuum state is elementary, but it could be absolutely impossible if we ignored the variation of the suppression

[1] The energy-momentum tensor with interactions acts as the source of matter production; hence, it can cause the creation of quanta from the vacuum. However, this effect cannot influence the contribution of ZPMs themselves into its energy density.

"factor," in contrast to the case of the stationary ground state.

Let us evaluate the relative inhomogeneity of the field with respect to the energy density. Thus,

$$\frac{|\nabla\phi|^2}{m^2\langle\phi\rangle^2} \sim \frac{m^2(\delta\phi)^2}{m^2\langle\phi\rangle^2} \sim \frac{\delta E}{E} \sim \frac{1}{\sqrt{n}} \ll 1.$$

Therefore, the inhomogeneity is negligible if the field is nonstationary excited to a large value of quanta.

III. COSMOLOGY AND MODEL ESTIMATES

As we already emphasized, we ignored effects due to a curved space-time, while we derived our mechanism for the suppressed cosmological constant even though it is relevant to the system with gravity. In this respect, we assume that the relevant quantities can enter as the *initial conditions* for the further evolution of the system by taking into account the gravitational expansion. So the energy density of the vacuum remains constant, while the quanta and their energy densities follow the transformations in accordance with the relative field equations, taking into account the gravity as well.

Nevertheless, we have to mention that in the literature there are computations of energy density for the ZPMs in a curved background, which take into account the dependence on the space-time curvature (see, for instance, the textbook by Birrell and Davies [6]). Modern investigations in [7,8] argue that in the curved space-time, for instance, in the de Sitter space-time close to the space-time of inflation in the early Universe the ZPMs themselves produce the energy density that quadratically evolves with the Hubble rate. We stress that such an effect means that the energy density of ZPM is not the cosmological constant at all, since the emergent equation of state (EOS) deviates from the vacuum equation of state, when the ratio of pressure to the energy density equals -1, and; hence, ZPMs generate the form of dark energy. Thus, one has the opportunity to evaluate the relevant parameter of EOS for such dark energy. The effect found in [7,8] essentially changes the energy density of ZPM if the Hubble rate H exceeds the scale of ZPM energy density in the limit of $H \to 0$, i.e., in the flat space-time. In this respect, we expect that this influence of space-time evolution on the ZPM energy density could be suppressed as the energy density divided by the second degree of Planck mass and the second degree of a huge energy scale in the bare cosmological constant. In addition, the effect found in [7,8] corresponds to the local fields considered in the whole space-time, including Hubble and super-Hubble distances, when, for instance, the dynamics of light scalar fields would be essential [8]. In contrast, in our model we deal with the global field in the limited volume, which is much less than the Hubble volume, when the approximation of the field with the ordinary machinery of particle representation is very close to the exact solutions at such distances deep inside the Hubble horizon. Thus, we hope that the dynamical aspects of ZPM energy density are not crucial for the scheme offered in the present paper.

In this context, there are similar and more general arguments in favor of the situation, when the vacuum energy cannot be constant and, hence, it would never represent the cosmological constant since the energy of the vacuum in the curved space-time evolves due to the renormalization group equations with the Hubble rate as the evolution parameter [9–13]. Making use of the conformal anomaly and other constructions in models, such investigations [14,15] argue that the dynamical vacuum energy can be tested by precision data in cosmology. Again, these studies deal with dark energy but not the cosmological constant, which is considered in our paper.

In our treatment of the suppression factor, the calculations concerning ZPMs in the curved space-time would change the exact value of the bare cosmological constant; however, the mechanism itself starts to work if the system is essentially excited to the occasional nonstationary state, which yields the suppression factor of the bare cosmological constant even in the presence of space-time curvature. Thus, in our study we hold the standard point of view on the vacuum energy equivalent to the cosmological constant and do not involve the dynamical treatment of vacuum energy evolving with the Universe expansion. In principle, we see that the mechanism can be implemented in the studies with dynamical vacuum energy, too, simply by the insertion of the suppression factor for the evolving value of the vacuum energy, which could be considered in further developments presented elsewhere.

Another aspect of curved space-time is particle creation, say, during the cosmological evolution [6]. Such creation changes the energy density of matter by an additional term depending on the square of the primary density of the particles or the Hubble rate in the fourth degree, which is typical for the quantum effects in curved space-time. In this respect, the additional term is suppressed as the primary density of energy to the Planck mass in the fourth degree. This means that such a contribution is negligible if the energy density is significantly below the Planckian density, which is assumed for the Universe beyond the region of quantum gravity. Moreover, such gravitational creation of matter does not influence the initial cosmological constant established at the start of evolution. Thus, we expect that our model can be considered in cosmological aspects.

In the form of expression (8), the described mechanism rigorously sets the quantum suppression of the bare cosmological constant. It is relevant to the cosmology because, at first, the spatially global scalar field could be associated with the spatially global part of the inflaton [16–19]; second, a finite volume of causal fluctuations corresponds to a primary volume of the Universe at the start of inflation, wherein the inflaton field can be considered as spatially global.[2] In this way, we can make estimations in a simple manner, say, by setting the primary fluctuations of the field as

$$\langle \mathrm{vac}|\phi^2|\mathrm{vac}\rangle \sim m_{\mathrm{Pl}}^2 \Rightarrow \frac{1}{V_R} \sim m m_{\mathrm{Pl}}^2;$$

hence, the average number of the spatially global field quanta is evaluated by

$$n = \ln \frac{\rho_\Lambda^{\mathrm{bare}}}{\rho_\Lambda} \sim 275 - 2\ln \frac{m_{\mathrm{Pl}}}{m} \gg 1,$$

since the inflaton is quite heavy, i.e., $m \sim 10^{13}$ GeV and $n \sim 250$.

Since the mechanism should be accepted as real, we need to figure out what the number of n is? Our studies show that the answer can be found, for instance, in the framework of the model with the inflaton nonminimally coupled to gravity, i.e., due to the interaction term of the Lagrangian in the form

$$L_{\mathrm{int}} = \frac{1}{2}\xi \phi m_{\mathrm{Pl}} R,$$

where R is the scalar curvature of the metric in the Jordan frame [20–24]. In the Einstein frame, the inflation scale is $\Lambda_{\mathrm{inf}} \sim 10^{16}$ GeV, while the transformed inflaton obtains the mass of the order of $\Lambda_{\mathrm{inf}}/\xi$. The parameters obey the relation for the strong coupling,[3]

$$n \sim \xi \sim \frac{m_{\mathrm{Pl}}}{\Lambda_{\mathrm{inf}}}.$$

Note that $\xi \gg 1$ results in a strong suppression of amplitude in a spectrum of relic gravitational waves, if the inflaton potential is exactly quadratic, when it satisfies the form of the cosmic attractor for parameters of inflation [22,24]. A valuable amplitude of relic gravitational waves would point to the fact that the potential should involve some nonquadratic terms breaking the attractor predictions at $\xi \gg 1$. This amplitude of primary gravitational waves could be unambiguously extracted from the detection of B modes of cosmic microwave background radiation if the foreground polarization generated by the dust is suppressed in a region of detection. At present, the enforced amplitude of the B mode of cosmic microwave background radiation detected by BICEP2 [25] corresponds to the secondary foreground produced by the measured dust distribution as the Planck Collaboration has reported in [26].

Thus, the issue of the enigmatic value of n is transformed to fix the hierarchy of $\Lambda_{\mathrm{inf}} \ll m_{\mathrm{Pl}}$. Why does the inflation involve two energy scales? In our opinion, the answer would solve the cosmological constant problem [9,27–29].

IV. DISCUSSION AND GENERALIZATIONS

Let us argue for the relevance of the spatially global scalar field to the cosmological constant. In the framework of quantum field theory, the sum of divergent contributions of any existing fields into the vacuum energy density should be treated as a new independent global dimensional quantity with zero charges of vacuum. Therefore, we can consider this quantity as reducible from the vacuum expectation of the appropriate scalar field ϕ by introducing the contribution to the Lagrangian in the form of $\phi\Lambda_0^3$, which reproduces the cosmological constant at some $\phi \mapsto \langle\phi\rangle = \phi_0$. Without gravity, the value of the cosmological constant is irrelevant to the physics, and it can take any value that corresponds to the global shift symmetry $\phi \mapsto \phi + \phi_c$, while the action can contain any scalar terms dependent on $\partial_\mu\phi$ and the trivial flat potential of ϕ. These properties are characteristic of the inflaton field. This gravity is responsible for a generation of terms breaking the global shift symmetry, particularly, a nonflat potential as well as the kinetic term for ϕ, which makes it the dynamical field of the inflaton.

Finally, we can straightforwardly generalize the mechanism to the calculation of vacuum energy for the nonhomogeneous scalar field. In this case, the bare expression for the average tensor of energy and momentum,

$$\langle \mathrm{vac}|T_\mu^\nu|\mathrm{vac}\rangle = \langle \mathrm{vac}|\left\{\partial_\mu\phi\partial^\nu\phi - \frac{1}{2}\delta_\mu^\nu(\partial\phi)^2 + \frac{1}{2}\delta_\mu^\nu m^2\phi^2\right\}|\mathrm{vac}\rangle,$$

can be written as the integral

$$\langle \mathrm{vac}|T_\mu^\nu|\mathrm{vac}\rangle = \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2 - m^2 + i0}\left\{k_\mu k^\nu - \frac{1}{2}\delta_\mu^\nu(k^2 - m^2)\right\}.$$

After the Wick rotation $k_0 = ik_4$ to the Euclidean space, wherein $k^2 = -k_E^2$, we get

$$\langle \mathrm{vac}|T_\mu^\nu|\mathrm{vac}\rangle = \int \frac{d^4k_E}{(2\pi)^4}\frac{1}{k_E^2 + m^2}\left\{-k_{E\mu}k_E^\nu + \frac{1}{2}\delta_\mu^\nu(k_E^2 + m^2)\right\},$$

and the isotropic integration makes the replacement

$$k_{E\mu}k_E^\nu \mapsto \frac{1}{4}k_E^2\delta_\mu^\nu,$$

[2] The volume of causal fluctuations is not equivalent to the Hubble volume determined by the initial density of energy. The Hubble volume could be greater than the finite volume of causal fluctuations. Therefore, the inflaton can obtain a valuable inhomogeneity in the initial Hubble volume.

[3] We will present more details elsewhere.

resulting in the expected divergent expression with the vacuum signature of δ_μ^ν in the tensor structure,

$$\langle \mathrm{vac}|T_\mu^\nu|\mathrm{vac}\rangle = \delta_\mu^\nu \frac{1}{4}\int \frac{d^4k_E}{(2\pi)^4}\frac{1}{k_E^2+m^2}\{k_E^2+2m^2\}.$$

At this stage we can use the introduction of the effective number of quanta with the Euclidean four-momentum k_E, $n = n(k_E^2)$ in order to get the true expression for the energy density of suppressed vacuum fluctuations,

$$\rho_\Lambda = \frac{1}{4}\int \frac{d^4k_E}{(2\pi)^4}\frac{\mathrm{e}^{-n(k_E^2)}}{k_E^2+m^2}\{k_E^2+2m^2\}. \tag{9}$$

This value is finite if n increases with k_E^2, say, polynomially. Thus, setting the ansatz of linear dependence

$$n(k_E^2) = \bar{n} + \frac{k_E^2}{\bar{\Lambda}^2},$$

we find

$$\rho_\Lambda = \frac{1}{32\pi^2}\mathrm{e}^{-\bar{n}}\left\{\bar{\Lambda}^4 + m^2\bar{\Lambda}^2 - m^4\left(\ln\frac{\bar{\Lambda}^2}{m^2} - \gamma_E + \mathcal{O}\left(\frac{m^2}{\bar{\Lambda}^2}\ln\frac{\bar{\Lambda}^2}{m^2}\right)\right)\right\},$$

where $\gamma_E \approx 0.5772$ is the Euler gamma. In the limit of $\bar{n} \mapsto n$ given in the case of the spatially global field, the cosmological constant obtains the leading term of $\bar{\Lambda}^4$ by the virtual modes and subleading contribution of $m^2\bar{\Lambda}^2$ analogous to the expression derived for the ZPMs of the global field[4] at $\bar{\Lambda} \mapsto m_{\mathrm{Pl}}$. The suppression factor gets the form of exponentiating the effective number of quanta for the spatially global field. Thus, the quantum description of vacuum energy density in the nonstationary state shows the justified difference from the naive expectations on the cosmological constant formed by fluctuations of the ZPMs, i.e., the bare cosmological constant.

Note that the energy density in the model of growing $n(k_E^2)$ formally becomes infinite, unless we introduce an evident cutoff,

$$V_R\int_0^{\Lambda_{\mathrm{cut}}}\frac{d^3k}{(2\pi)^3}n(k^2)\sqrt{k^2+m^2} = E_{\mathrm{tot}},$$

by setting a finite total energy E_{tot} in the reference volume. Nevertheless, our consideration remains valid in the case of $\Lambda_{\mathrm{cut}} \gg \bar{\Lambda}$.

As is evident, our mechanism does not appear to be straightforwardly effective in the case of any fermionic field, when the occupation number takes only two values: zero and unit. However, the *global* fermionic field is not relevant to the cosmological constant. At this point, it is important to note that we assign the bare cosmological constant to the sum over *all* contributions of physical fields, including fermionic fields and, say, the Higgs boson, quark-gluon condensates and so on. The gravity is the reason why *the bare cosmological constant is transformed into the real dynamical scalar field* with the vacuum quantum numbers of charges, i.e., into the inflaton, which is nonstationary excited from the state of minimal energy in the finite volume. Then, by quantum-mechanical means, the excitation produces the suppression of the bare cosmological constant.

[4] If $\bar{\Lambda} \mapsto \Lambda_{\mathrm{inf}}$, then one can expect that $\bar{\Lambda}^4 \sim m^2 m_{\mathrm{Pl}}^2$ yielding $m \sim \Lambda_{\mathrm{inf}}^2/m_{\mathrm{Pl}} \sim 10^{13}$ GeV.

ACKNOWLEDGMENTS

We address this problem of the total cosmological constant in our new paper [30], wherein we discuss the pseudo-Goldstone nature of the inflaton with respect to the global shift of the energy scale of vacuum energy density. In this mechanism, the primary cosmological constant induced by all of the actual contributions is matched to the bare cosmological constant of inflaton. Our hope is that this matching solves the problem of copious ingredients of the total cosmological constant. This work is supported by the Russian Foundation for Basic Research, Grant No. 14-02-00096.

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