

Exact wormhole solutions with nonminimal kinetic couplingR. V. Korolev^{*} and S. V. Sushkov[†]*Institute of Physics, Kazan Federal University, Kremlevskaya 18, Kazan 420008, Russia*

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We consider static spherically symmetric solutions in the scalar-tensor theory of gravity with a scalar field possessing the nonminimal kinetic coupling to the curvature. The Lagrangian of the theory contains the term $(\epsilon g^{\mu\nu} + \eta G^{\mu\nu})\phi_{,\mu}\phi_{,\nu}$ and represents a particular case of the general Horndeski Lagrangian, which leads to second-order equations of motion. We use the Rinaldi approach to construct analytical solutions describing wormholes with nonminimal kinetic coupling. It is shown that wormholes exist only if $\epsilon = -1$ (phantom case) and $\eta > 0$. The wormhole throat connects two anti-de Sitter spacetimes. The wormhole metric has a coordinate singularity at the throat. However, since all curvature invariants are regular, there is no curvature singularity there.

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I. INTRODUCTION

Wormholes are topological handles in spacetime linking widely separated regions of a single universe or “bridges” joining two different universes. Interest in these configurations dates back at least as far as 1916 [1] with punctuated revivals of activity following both the classic work of Einstein and Rosen in 1935 [2] and the later series of works initiated by Wheeler in 1955 [3]. More recently, a fresh interest in the topic has been rekindled by the work of Morris and Thorne [4], leading to a flurry of activity branching off into diverse directions [5–7].

A fundamental property of wormhole physics is that the existence of traversable Lorentzian wormholes as solutions to the equations of general relativity requires exotic matter, which possesses a negative pressure and violates the null energy condition [4,8]. The known classical forms of matter are believed to obey the usual energy conditions; hence, wormholes, if they exist, should belong to the realm of “unusual” physics. In the literature, there are several basic approaches to construct realistic physical models of wormholes. First, as is known, all energy conditions are violated by certain quantum effects, such as the Casimir effect and Hawking evaporation (see relevant comments in [4,9]), and so wormhole solutions can be found in semi-classical gravity [10]. Another way is to consider hypothetical forms of matter possessing exotic properties. Various models of such kind have been considered in the literature, among them, wormholes supported by phantom energy [11,12], the generalized Chaplygin gas [13], tachyon matter [14], etc. It is worth noticing that due to the problematic nature of Lorentzian wormholes, it is useful to minimize the usage of exotic matter, and indeed a wide variety of wormhole solutions have been analyzed in the literature to this effect, ranging from thin-shell

wormholes [15] to rotating [16] and dynamic wormhole geometries [17].

An alternative approach in wormhole physics is based on the fact that generalized energy conditions could be violated in various modified theories of gravity [18]. Therefore, wormhole geometries could be theoretically constructed even without the presence of exotic matter but would be sustained in the context of modified gravity. Examples of wormhole solutions in modified theories of gravity are manifold and, in particular, include Brans-Dicke wormholes [19], in $f(R)$ gravity [20], wormholes in braneworlds [21], and in the curvature-matter coupled generalization of $f(R)$ gravity [22], wormholes in conformal Weyl gravity [23], etc. (see, also, the review [7] and references therein).

A natural way to modify general relativity consists in taking into account possible nonminimal coupling between matter fields and the curvature, and, more specifically, nonminimal coupling which includes derivatives of dynamic quantities of matter fields. The most general scalar-tensor theory of such type was suggested in the 1970s in the Horndeski work [24]. Horndeski developed his theory on the basis of mathematical facts, but later, the same results were obtained on the basis of a more intuitive approach from Galileon’s research [25]. The simplest Lagrangian in the Horndeski theory contains a term $G^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$ providing nonminimal kinetic coupling of a scalar field to the curvature. Cosmological applications of such theory have been intensively investigated (see Ref. [26] and references therein). Less studied is a problem of black hole existence in the theory of gravity with nonminimal kinetic coupling. A while ago, Rinaldi [27] found a class of exact solutions with characteristic features of black holes, particularly, with event horizon. Afterwards, the Rinaldi method was applied in [28–31] to find new particular solutions with event horizons.

In Ref. [32], we have studied wormholes in the theory of gravity with nonminimal kinetic coupling. Solutions describing asymptotically flat traversable wormholes have

^{*}korolyovrv@gmail.com
[†]sergey_sushkov@mail.ru

been only obtained by means of numerical methods. The aim of this work is to construct exact analytical wormhole solutions with nonminimal kinetic coupling by using the Rinaldi method.

II. ACTION AND FIELD EQUATIONS

Let us consider a gravitational theory with nonminimal derivative coupling given by the action [33]

$$S = \int dx^4 \sqrt{-g} \left\{ \frac{R}{8\pi} - [\varepsilon g_{\mu\nu} + \eta G_{\mu\nu}] \phi^{;\mu} \phi^{;\nu} \right\}, \quad (1)$$

where $g_{\mu\nu}$ is a metric, $g = \det(g_{\mu\nu})$, R is the scalar curvature, $G_{\mu\nu}$ is the Einstein tensor, ϕ is a real massless scalar field, and η is a parameter of nonminimal kinetic coupling with the dimension of length squared. The ε parameter equals ± 1 . In the case $\varepsilon = 1$, we have a canonical scalar field with a positive kinetic term, and the case $\varepsilon = -1$ describes a phantom scalar field with a negative kinetic term.

Variation of the action (1) with respect to the metric $g_{\mu\nu}$ and scalar field ϕ provides the following field equations [26]:

$$G_{\mu\nu} = 8\pi[\varepsilon T_{\mu\nu} + \eta \Theta_{\mu\nu}], \quad (2a)$$

$$[\varepsilon g^{\mu\nu} + \eta G^{\mu\nu}] \nabla_\mu \nabla_\nu \phi = 0, \quad (2b)$$

where

$$T_{\mu\nu} = \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} (\nabla\phi)^2, \quad (3)$$

$$\begin{aligned} \Theta_{\mu\nu} = & -\frac{1}{2} \nabla_\mu \phi \nabla_\nu \phi R + 2 \nabla_\alpha \phi \nabla_{(\mu} \phi R_{\nu)}^\alpha \\ & + \nabla^\alpha \phi \nabla^\beta \phi R_{\mu\alpha\nu\beta} + \nabla_\mu \nabla^\alpha \phi \nabla_\nu \nabla_\alpha \phi \\ & - \nabla_\mu \nabla_\nu \phi \square \phi - \frac{1}{2} (\nabla\phi)^2 G_{\mu\nu} \\ & + g_{\mu\nu} \left[-\frac{1}{2} \nabla^\alpha \nabla^\beta \phi \nabla_\alpha \nabla_\beta \phi + \frac{1}{2} (\square\phi)^2 - \nabla_\alpha \phi \nabla_\beta \phi R^{\alpha\beta} \right]. \end{aligned} \quad (4)$$

Because of the Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$, Eq. (2a) leads to a differential consequence,

$$\nabla^\mu [\varepsilon T_{\mu\nu} + \eta \Theta_{\mu\nu}] = 0. \quad (5)$$

One can check straightforwardly that the substitution of expressions (3) and (4) into (5) yields Eq. (2b). In other words, the equation of motion of scalar field (2b) is the differential consequence of the system (2a).

III. STATIC SPHERICALLY SYMMETRIC SOLUTIONS

Let us find static spherically symmetric solutions of the field equations (2). Under the assumption of spherical symmetry, the scalar field is a function of the radial coordinate r , i.e., $\phi = \phi(r)$, and the spacetime metric can be taken as follows:

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + \rho^2(r)d\Omega^2, \quad (6)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$. Note that a freedom in choosing the radial coordinate allows us to fix the form of one of the metric functions $f(r)$, $g(r)$, or $\rho(r)$, but at this stage we will not do it for the sake of generality. Now, using the above-mentioned metric and scalar field ansatz, we can represent the field equations (2) in the following form:

$$\frac{\sqrt{fg}}{g} \psi \left[\varepsilon \rho^2 + \eta \left(\frac{\rho \rho' f'}{fg} + \frac{\rho'^2}{g} - 1 \right) \right] = C_0, \quad (7a)$$

$$\rho \rho' \frac{f'}{f} = \frac{g(g - \rho'^2) - 4\pi\eta\psi^2(g - 3\rho'^2) + 4\pi\varepsilon\rho^2\psi^2g}{g - 12\pi\eta\psi^2}, \quad (7b)$$

$$\frac{\rho \rho'}{2} \left(\frac{f'}{f} - \frac{g'}{g} \right) = \frac{g(g - \rho'^2 - \rho \rho'') + 4\pi\eta\psi^2(2\rho'^2 + \rho \rho'') + 4\pi\eta\rho\rho'(\psi^2)'}{g - 12\pi\eta\psi^2}, \quad (7c)$$

where C_0 is an integration constant, and $\psi \equiv \phi'$. It is worth noting that Eq. (7a) is a first integral of the equation of motion (2b).

Then, following Rinaldi [27], we will search for analytical solutions of the system (7) in the particular case supposing that

$$C_0 = 0. \quad (8)$$

In this case, Eq. (7a) yields

$$\rho \rho' \frac{f'}{f} = g - \rho'^2 - \frac{\varepsilon \rho^2 g}{\eta}. \quad (9)$$

This gives the following expression for the function $f(r)$:

$$f(r) = \frac{C_1}{\rho} \exp \left(- \int \frac{(\varepsilon \rho^2 - \eta)g}{\eta \rho \rho'} dr \right), \quad (10)$$

where C_1 is an integration constant. From Eq. (7b), using the relation (9), one can derive ψ^2 :

$$\psi^2(r) = \frac{\varepsilon \rho^2 g}{8\pi\eta(\varepsilon\rho^2 - \eta)}. \quad (11)$$

The scalar curvature for the metric (6) takes the following form:

$$R = \frac{2}{\rho^2} - \frac{2\rho'f'}{\rho g f} + \frac{2\rho'g'}{\rho g^2} - \frac{2\rho'^2}{\rho^2 g} - \frac{4\rho''}{\rho g} + \frac{f'^2}{2g f^2} - \frac{f''}{g f} + \frac{g'f'}{2g^2 f}. \quad (12)$$

By using the relation (9), one can eliminate the function $f(r)$ from the expression for R . As a result, we find

$$R = \frac{2(\varepsilon\rho^2 + \eta)}{\eta\rho^2} - \frac{3\rho'}{2\rho g} \left(\frac{\rho'}{\rho} - \frac{g'}{g} \right) - \frac{3\rho''}{\rho g} - \frac{g}{2\rho^2 \rho'^2} - \frac{\varepsilon g(\varepsilon\rho^2 - 2\eta)}{2\eta^2 \rho'^2} + \frac{g'(\varepsilon\rho^2 - \eta)}{2\eta\rho\rho'g} - \frac{\rho''(\varepsilon\rho^2 - \eta)}{\eta\rho\rho'^2}. \quad (13)$$

Formulas (10), (11), and (13) state the functions $f(r)$, $\psi(r)$, and $R(r)$ in terms of $g(r)$. The equation for $g(r)$ can be obtained by eliminating $f(r)$ and $\psi(r)$ from Eq. (7c) by using the relations (9) and (11):

$$\rho\rho'(\varepsilon\rho^2 - 2\eta)\frac{g'}{g} - \left(\frac{1}{\eta}\rho^4 - 3\varepsilon\rho^2 + 2\eta \right)g + \rho'^2(3\varepsilon\rho^2 + 2\eta) - 2\rho\rho''(\varepsilon\rho^2 - 2\eta) - \frac{4\rho^4\rho'^2}{\varepsilon\rho^2 - \eta} = 0. \quad (14)$$

It is worth noting that a general solution of Eq. (14) could be obtained analytically for an arbitrary function $\rho(r)$. Depending on the sign of $\varepsilon\eta$, the solution takes the following forms.

A. $\varepsilon\eta > 0$:

$$g(r) = \frac{\rho'^2(\rho^2 - 2l_\eta^2)^2}{(\rho^2 - l_\eta^2)^2 F(r)}, \quad (15)$$

$$F(r) = 3 - \frac{8m}{\rho} - \frac{\rho^2}{3l_\eta^2} + \frac{l_\eta}{\rho} \operatorname{arctanh} \frac{\rho}{l_\eta}. \quad (16)$$

B. $\varepsilon\eta < 0$:

$$g(r) = \frac{\rho'^2(\rho^2 + 2l_\eta^2)^2}{(\rho^2 + l_\eta^2)^2 F(r)}, \quad (17)$$

$$F(r) = 3 - \frac{8m}{\rho} + \frac{\rho^2}{3l_\eta^2} + \frac{l_\eta}{\rho} \operatorname{arctan} \frac{\rho}{l_\eta}. \quad (18)$$

Here, m is an integration constant, and $l_\eta = |\varepsilon\eta|^{1/2}$ is a characteristic scale of nonminimal kinetic coupling.

For the specified function $\rho(r)$, formulas (15)–(18) together with (10) and (11) give a solution to the problem of $g(r)$, $f(r)$, and $\psi(r)$ construction. Below we consider two special examples of the function $\rho(r)$.

IV. SCHWARZSCHILD COORDINATES: $\rho(r) = r$

As was mentioned above, the freedom in choosing the radial coordinate r allows us to specify, additionally, the form of one of the metric functions. Let us make a choice:

$$\rho(r) = r. \quad (19)$$

This case corresponds to Schwarzschild coordinates, so that r is the curvature radius of coordinate sphere $r = \text{const} > 0$.

Substituting $\rho(r) = r$ into the formulas (10), (11), and (15)–(18) and calculating the integral in (10), we derive the solutions for $g(r)$, $f(r)$, $\psi(r)$. For the first time, the solutions in the case $\rho(r) = r$ were obtained by Rinaldi [27]. Below we briefly discuss these solutions separately, depending on a sign of the product $\varepsilon\eta$.

A. $\varepsilon\eta > 0$. In this case, the solution reads

$$f(r) = C_1 F(r), \quad (20)$$

$$g(r) = \frac{(r^2 - 2l_\eta^2)^2}{(r^2 - l_\eta^2)^2 F(r)}, \quad (21)$$

$$\psi^2(r) = \frac{\varepsilon}{8\pi l_\eta^2} \frac{r^2(r^2 - 2l_\eta^2)^2}{(r^2 - l_\eta^2)^3 F(r)}, \quad (22)$$

where C_1 is an integration constant and

$$F(r) = 3 - \frac{r^2}{3l_\eta^2} - \frac{8m}{r} + \frac{l_\eta}{r} \operatorname{arctanh} \frac{r}{l_\eta}. \quad (23)$$

In the limit $r \rightarrow 0$, the solution (20) for the function $f(r)$ takes the asymptotical form

$$f(r) \approx 4C_1 \left(1 - \frac{2m}{r} \right).$$

To compare the derived asymptotic with the Schwarzschild solution, it is convenient to set $C_1 = \frac{1}{4}$.

Note that the expressions (20)–(22) contain the function $(l_\eta/r)\operatorname{arctanh}r/l_\eta$, which is defined in the domain $r \in (0, l_\eta)$. To continue the solution into the interval $r \in (l_\eta, \infty)$, one should make use of identity

$$\frac{l_\eta}{r} \operatorname{arctanh} \frac{r}{l_\eta} = \frac{l_\eta}{2r} \ln \frac{l_\eta + r}{l_\eta - r},$$

and then turn to the function $\frac{l_\eta}{2r} \ln \left| \frac{l_\eta + r}{l_\eta - r} \right|$. At $r \rightarrow \infty$, the asymptotic of the function $f(r)$ with the domain extended to the interval (l_η, ∞) has a form of de Sitter solution:

$$f(r) \approx \frac{3}{4} - \frac{r^2}{12l_\eta^2}.$$

Also note that at $r = l_\eta$, the function $\frac{l_\eta}{2r} \ln \left| \frac{l_\eta+r}{l_\eta-r} \right|$ has logarithmic singularity, and that is why its domain consists of two disconnected parts $\mathcal{R}_1: 0 < r < l_\eta$ and $\mathcal{R}_2: l_\eta < r < \infty$. This implies that we have two different classes of solutions of the form (20)–(22) which are defined independently within separate domains \mathcal{R}_1 and \mathcal{R}_2 .

Further, let us take into account the fact that we consider the real scalar field, so the value ψ^2 should be nonnegative, i.e., $\psi^2 \geq 0$. In view of this requirement, the formula (22) imposes additional restrictions on the r domain. In particular, it should be noted that in each of the intervals \mathcal{R}_1 and \mathcal{R}_2 at fixed ε , a sign of the function $\psi^2(r)$ is defined by a sign of $F(r)$ and reverses where the function $F(r)$ reverses its sign. Hence, we can conclude that the solution (20)–(22) cannot be considered as a solution which describes a black hole in the theory of gravity with nonminimal kinetic coupling.

B. $\varepsilon\eta < 0$. In this case, the solution reads as follows:

$$f(x) = \frac{1}{4}F(r), \quad (24)$$

$$g(x) = \frac{(r^2 + 2l_\eta^2)^2}{(r^2 + l_\eta^2)^2 F(r)}, \quad (25)$$

$$\psi^2(r) = -\frac{\varepsilon}{8\pi l_\eta^2} \frac{r^2(r^2 + 2l_\eta^2)^2}{(r^2 + l_\eta^2)^3 F(r)}, \quad (26)$$

where

$$F(r) = 3 + \frac{r^2}{3l_\eta^2} - \frac{8m}{r} + \frac{l_\eta}{r} \arctan \frac{r}{l_\eta}. \quad (27)$$

Now the solution contains a function $\frac{l_\eta}{r} \arctan \frac{r}{l_\eta}$ and has a domain $r \in (0, \infty)$. In the limit $r \rightarrow 0$, the function $f(r)$ yields the Schwarzschild asymptotics $f(r) \approx 1 - \frac{2m}{r}$, and in the limit $r \rightarrow \infty$, the anti-de Sitter one $f(r) \approx \frac{3}{4} + \frac{r^2}{12l_\eta^2}$. However, the obtained solution cannot be considered as an analogue of the Schwarzschild–anti-de Sitter solution, as in the case of $m > 0$, the function $F(r)$ reverses sign at a point r_h inside the interval $r \in (0, \infty)$, and, hence, the value of ψ^2 becomes negative in one of the intervals $(0, r_h)$ or (r_h, ∞) according to the sign of ε .

From a physical point of view, the case $m = 0$ may be of some interest. In this case, the function $F(r)$ is everywhere positive and regular. About a zero point, the metric functions have asymptotics $f(r) = 1 + O(r^2)$ and $g(r) = 1 + O(r^2)$, as well as $\psi^2(r) = \frac{\varepsilon}{8\pi l_\eta^2} \frac{r^2}{l_\eta^2} (1 + O(r^2))$. Thus, at $\varepsilon = +1$, we obtain a static spherically symmetric

configuration with a regular center and the anti-de Sitter structure at infinity.

V. WORMHOLE CONFIGURATION:

$$\rho(r) = \sqrt{r^2 + a^2}$$

In this section, we consider static spherically symmetric configurations with the metric function $\rho(r)$ of the following form:

$$\rho(r) = \sqrt{r^2 + a^2}, \quad (28)$$

where $a > 0$ is a parameter. Then, the metric (6) reads

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + (r^2 + a^2)d\Omega^2. \quad (29)$$

If $f(r)$ and $g(r)$ are everywhere positive and regular functions with a domain $r \in (-\infty, \infty)$, then the metric (29) describes a wormhole configuration with a throat at $r = 0$; parameter a is a throat radius.

Substituting $\rho(r) = \sqrt{r^2 + a^2}$ into the formulas (11) and (15)–(18), we derive the solutions for $g(r)$ and $\psi^2(r)$ in an explicit form. The solution (10) for $f(r)$ contains the indefinite integral, which in this case cannot be expressed in terms of elementary functions. Below we consider these solutions for each sign of the product $\varepsilon\eta$.

A. $\varepsilon\eta > 0$. In this case, the solution for $g(r)$ is given by the formulas (15) and (16). The solution contains the function $(l_\eta/\sqrt{r^2 + a^2})\operatorname{arctanh}(\sqrt{r^2 + a^2}/l_\eta)$ with the domain that could be found from condition $\sqrt{r^2 + a^2}/l_\eta < 1$, i.e., $|r| < r_1 \equiv (l_\eta^2 - a^2)^{1/2}$. At the points $|r| = r_1$, the function $(l_\eta/\sqrt{r^2 + a^2})\operatorname{arctanh}(\sqrt{r^2 + a^2}/l_\eta)$ logarithmically diverges. Consequently, the metric function $g(r)$ guides the singular behavior near $|r| = r_1$, which makes this solution inconsiderable from a physical point of view.

B. $\varepsilon\eta < 0$. In this case, by substituting $\rho(r) = \sqrt{r^2 + a^2}$ into the formulas (17), (18), and (10), we obtain the following solutions for the metric functions $g(r)$ and $f(r)$ and the function $\psi^2(r)$:

$$g(r) = \frac{r^2(r^2 + a^2 + 2l_\eta^2)^2}{(r^2 + a^2)(r^2 + a^2 + l_\eta^2)^2 F(r)}, \quad (30)$$

$$f(r) = \frac{a}{\sqrt{r^2 + a^2}} \times \exp \left[\int_0^r \frac{r(r^2 + a^2 + 2l_\eta^2)^2}{l_\eta^2(r^2 + a^2)(r^2 + a^2 + l_\eta^2)F(r)} dr \right], \quad (31)$$

$$\psi^2(r) = -\frac{\varepsilon}{8\pi l_\eta^2} \frac{r^2(r^2 + a^2 + 2l_\eta^2)^2}{(r^2 + a^2)(r^2 + a^2 + l_\eta^2)^3 F(r)}, \quad (32)$$

where

$$F(r) = 3 - \frac{8m}{\sqrt{r^2 + a^2}} + \frac{r^2 + a^2}{3l_\eta^2} + \frac{l_\eta}{\sqrt{r^2 + a^2}} \arctan\left(\frac{\sqrt{r^2 + a^2}}{l_\eta}\right), \quad (33)$$

and the integration constant $C_1 = a$ in the expression for $f(r)$ is chosen so that $f(0) = 1$. The function $F(r)$ has a minimum at $r = 0$; thus, to make it everywhere positive, one should demand $F(0) > 0$. Hence, one can derive the limitation on the upper value of the parameter m :

$$2m < a \left(\frac{3}{4} + \frac{\alpha^2}{12} + \frac{1}{4\alpha} \arctan \alpha \right), \quad (34)$$

where $\alpha \equiv a/l_\eta$ is the dimensionless parameter which defines the ratio of two characteristic sizes: the wormhole throat radius a and the scale of nonminimal kinetic coupling l_η . In the particular case of $a \ll l_\eta$, we get $2m < a$. Further, we assume that the value of m satisfies the condition (34), and, therefore, the function $F(r)$ is positively definite, i.e., $F(r) > 0$.

Let us consider asymptotical properties of the obtained solution. Far from the throat in the limit $|r| \rightarrow \infty$, the metric functions $g(r)$ and $f(r)$ have the following asymptotics:

$$\begin{aligned} g(r) &= 3 \frac{l_\eta^2}{r^2} + O\left(\frac{1}{r^4}\right), \\ f(r) &= A \frac{r^2}{l_\eta^2} + O(r^0), \end{aligned} \quad (35)$$

where the constant A depends on the parameters a , l_η , and m and can be calculated only numerically. Let us note that the derived asymptotics correspond to geometry of anti-de Sitter space with the constant negative curvature.

In the neighborhood of the throat $r = 0$, we obtain

$$g(r) = B \frac{r^2}{l_\eta^2} + O(r^4), \quad f(r) = 1 + O(r^2), \quad (36)$$

where

$$B = \frac{(\alpha^2 + 2)^2}{\alpha^2 (\alpha^2 + 1)^2 \left(3 + \frac{1}{3} \alpha^2 - \frac{8m}{a} + \frac{1}{\alpha} \arctan \alpha \right)}.$$

It is worth noticing that at the throat $r = 0$, the metric function $g(r)$ vanishes, i.e., $g(0) = 0$. This implies that there is a coordinate singularity at $r = 0$. To answer the question whether there is a geometric singularity at this point, one should compute the curvature invariants for the metric (6). In this paper, we confine ourselves to a discussion of the scalar curvature (13). By substituting the solution (30) into (13), one can check that the curvature near the throat is regular: $R(r) = R_0 + O(r^2)$, where the value $R_0 = R(0)$ is clumsily expressed in terms of the parameters a , $l_\eta m$. Far from the throat in the limit $|r| \rightarrow \infty$, the scalar curvature tends asymptotically to a constant negative value, i.e., $R \rightarrow R_\infty$, where

$$R_\infty = -\frac{5 + 3l_\eta^2}{2l_\eta^2}. \quad (37)$$

It is worthwhile to note that the asymptotical value R_∞ is determined only by the characteristic scale of nonminimal kinetic coupling l_η and does not depend on a and m . We also note that $R_\infty \rightarrow -\infty$ in the limit $l_\eta \rightarrow 0$.

Finally, we discuss briefly the solution (32) for $\psi^2(r)$. Since $F(r) > 0$, hence, the condition $\psi^2(r) \geq 0$ holds only for $\varepsilon = -1$. Now taking into account that the solutions (30), (31), and (32) were obtained in the case of $\varepsilon \eta < 0$, we can conclude that $\eta > 0$.

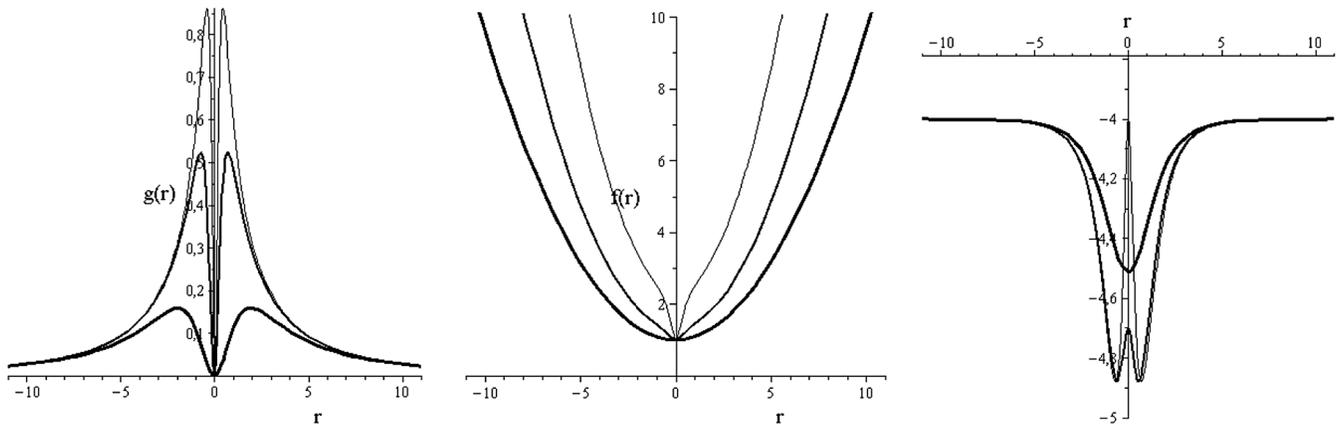


FIG. 1. Graphs for the metric functions $g(r)$, $f(r)$, and the scalar curvature $R(r)$ with $l_\eta = 1$, $m = 0.1$. Curves, from thin to thick, are given for $a = 0.3, 0.5, 1.5$.

To illustrate the performed analysis, we show numerical solutions for $g(r)$, $f(r)$, and the scalar curvature $R(r)$ in Fig. 1.

VI. CONCLUSIONS

In this paper, we have explored static spherically symmetric solutions in the scalar-tensor theory of gravity with a scalar field possessing the nonminimal kinetic coupling to the curvature. The Lagrangian of the theory contains the term $(\epsilon g^{\mu\nu} + \eta G^{\mu\nu})\phi_{,\mu}\phi_{,\nu}$ and represents a particular case of the general Horndeski Lagrangian [24], which leads to second-order equations of motion. Previously, Rinaldi [27] found a class of exact solutions with nonminimal kinetic coupling with characteristic features of black holes, particularly with event horizon. In this work, using the Rinaldi approach, we have found and examined analytical solutions describing wormholes. A detailed analysis revealed a number of characteristic features of the obtained solutions. In particular,

- (i) The wormhole solution exists only if $\epsilon = -1$ (phantom case) and $\eta > 0$.
- (ii) The wormhole metric has a specific coordinate singularity at the wormhole throat. Namely, the metric component g_{rr} is vanished at $r = 0$. However,

there is no curvature singularity at the throat, since all the curvature invariants stay regular.

- (iii) The wormhole throat connects two asymptotical regions with anti-de Sitter geometry of spacetime.

The stability of wormhole configurations is an important test of their possible viability. The stability of wormholes supported by scalar fields was intensively investigated in the literature [34]. To answer the question—are found scalar wormholes with nonminimal derivative coupling stable or not—we need additional investigations, which are in progress.

Also, in the future, we intend to explore more carefully the coordinate singularity of the obtained wormhole solution. In particular, we plan to study the geodesic motion of test massive and massless particles and discuss their behavior near the wormhole's throat.

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