Evolution of spacetime arises due to the departure from holographic equipartition in all Lanczos-Lovelock theories of gravity

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In the case of general relativity, one can interpret the Noether charge in any bulk region as the heat content TS of its boundary surface. Furthermore, the time evolution of the spacetime metric in Einstein's theory arises due to the difference $(N_{\text{sur}} - N_{\text{bulk}})$ of suitably defined surface and bulk degrees of freedom. We show that this thermodynamic interpretation generalizes in a natural fashion to all Lanczos-Lovelock models of gravity. The Noether charge, related to the time evolution vector field, in a bulk region of space is equal to the heat content TS of the boundary surface with the temperature T defined by using local Rindler observers and S being the Wald entropy. Using the Wald entropy to define the surface degrees of freedom N_{sur} and the Komar energy density to define the bulk degrees of freedom N_{bulk}, we can also show that the time evolution of the geometry is sourced by $(N_{\text{sur}} - N_{\text{bulk}})$. When it is possible to choose the foliation of spacetime such that the metric is independent of time, the above dynamical equation yields the holographic equipartition for Lanczos-Lovelock gravity with $N_{\text{sur}} = N_{\text{bulk}}$. The implications are discussed.

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I. INTRODUCTION

A surprising connection between gravity and thermodynamics was first demonstrated in the context of black hole mechanics by the fact that one can associate an entropy [1,2] and temperature [3,4] with black holes. It was soon realized that a similar connection exists in the case of several other horizons [5,6] and that the ideas have a far greater domain of applicability [7,8]. Further work in the past decade suggests that these results could be just the tip of the iceberg [9–12]. Recent studies have revealed several curious connections between gravitational dynamics and horizon thermodynamics such as the following:

(i) The gravitational field equations reduce to thermodynamic identities on horizons for a wide class of gravity theories more general than Einstein gravity [13–18].

(ii) The action describing gravity can be separated into a bulk term and a surface term with a specific ("holographic") relation between them, not only in Einstein gravity but also in a more general class of theories [19–22]. In fact, the action functional in all Lanczos-Lovelock gravity can be given a thermodynamic interpretation [22–25].

(iii) Gravitational field equations in all Lanczos-Lovelock models can be obtained from thermodynamic extremum principles [26,27] involving the heat density of null surfaces in the spacetime.

(iv) Gravitational field equations reduce to Navier-Stokes equations of fluid dynamics in arbitrary spacetime projected on a null surface generalizing previous results on black hole spacetime [28–30].

More recently [31], these ideas have been taken significantly further in the context of general relativity. One of us (T.P.) demonstrated that, in the context of general relativity, the following results hold: (a) The total Noether charge in a 3-volume $\mathcal{R}$, related to the time evolution vector field, can be interpreted as the heat content of the boundary $\partial\mathcal{R}$ of the volume. This provides yet another holographic result connecting the bulk and boundary variables. (b) The time evolution of the spacetime itself can be described in an elegant manner by the equation

$$\int_{\mathcal{R}} \frac{d^3x}{8\pi} h_{ab} \pounds_{\xi} p^{ab} = \epsilon \frac{1}{2} k_B T_{\text{avg}} (N_{\text{bulk}} - N_{\text{sur}}), \qquad (1)$$

where h_{ab} is the induced metric on the $t = \text{const}$ surface, the p^{ab} is its conjugate momentum, and $\xi^a = N u^a$ is the time evolution vector corresponding to observers with four-velocity $u_a = -N \nabla_a t$ that is the normal to the $t = \text{const}$ surface. N_{sur} and N_{bulk} are the degrees of freedom in the surface and bulk, respectively, of a three-dimensional region $\mathcal{R}$, and T_{avg} is the average Davies-Unruh temperature of the boundary. (The parameter $\epsilon = \pm 1$ ensures that N_{bulk} is positive even when the Komar energy turns negative.) This equation shows that the rate of change of gravitational momentum is driven by the departure from holographic equipartition, measured by $(N_{\text{bulk}} - N_{\text{sur}})$. The metric will be time independent in the chosen foliation if $N_{\text{sur}} = N_{\text{bulk}}$, which can happen for all static geometries. The validity of Eq. (1) for all observers (i.e., foliations) implies the validity of Einstein's equations. In short, *deviation from holographic equipartition leads to the time evolution of the metric*.

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In the past, virtually every result indicating the emergent nature of gravity in the context of general relativity could be generalized to all Lanczos-Lovelock models of gravity. It is therefore worth investigating whether the above description can be generalized to Lanczos-Lovelock models. This is very important, because the expression for horizon entropy in general relativity is rather trivial and is just a quarter of the horizon area. In Lanczos-Lovelock models, the corresponding expression is much more complex which, in turn, modifies the expression for N_{sur}. It is, therefore, not clear *a priori* whether our results—interpretation of the Noether charge and Eq. (1)—will generalize to Lanczos-Lovelock models. We will show here that these results indeed possess a natural generalization to Lanczos-Lovelock gravity as well.

Note that, throughout the paper, the term holographic has been used in the primitive sense of that term, i.e., to represent a correspondence between bulk and boundary properties of the *same* theory. We also emphasize that the results derived in this work are mathematically rigorous, self-contained, and well within the domain of classical gravity with a single quantum input being the Davies-Unruh temperature. Thus, these results do *not* rely on any speculative models of microscopic physics or quantum gravity. We believe such a deeper connection possibly exists (as discussed in several previous publications), but the results of *this* paper do *not* depend on any such connection.

The rest of the paper is organized as follows: In Sec. II, we review the known results for Einstein gravity and clarify some technical points. (In particular, in Sec. II C, we give some explicit examples to illustrate what happens when the same spacetime admits both static and nonstatic foliations.) In Sec. III, we generalize all these results to Lanczos-Lovelock models of gravity. Section III A provides a brief introduction to Lanczos-Lovelock models and sets up the notation, etc. In Sec. III B, we relate the Noether charge to the surface heat content in the Lanczos-Lovelock models, and in Sec III C, we derive the evolution equation in terms of surface and bulk degrees of freedom. The last section summarizes the conclusions. We work with a mostly positive signature in D-dimensional spacetime and use units with $G = \hbar = c = 1$.

II. WARMUP: REVIEW OF THE RESULTS FOR EINSTEIN GRAVITY

A. The foliation of spacetime

We start with a spacetime foliated by a series of spacelike hypersurfaces each being determined by the constant value of a scalar field $t(x)$, such that on each hypersurface $t(x) = \mathrm{const}$. The unit normal to the constant $t(x)$ hypersurface is $u_a = -N\nabla_a t$, which reduces to $-N\delta_a^0$ when t is considered as one of the coordinates in this spacetime. For this spacetime foliation, we have $g^{00} = -1/N^2$, and $u^a u_a = -1$. Given such a foliation, we can introduce a time evolution vector ζ^a by the condition $\zeta^a \nabla_a t = 1$, which in the coordinate system with t as a coordinate becomes $\zeta^a = \delta_0^a$. In general, we can readily obtain the following decomposition: $\zeta^a = -(\zeta^b u_b)u^a + N^a$, with the property $N^a u_a = 0$ and $N^a = h_b^a \zeta^b$, where $h_b^a = \delta_b^a + u^a u_b$ is the projection tensor. This decomposition also introduces another vector

$$\xi_a = N u_a \to -N^2 \delta_a^0, \tag{2}$$

where the last result holds in the preferred foliation. If we impose the coordinate condition that t becomes one of the spacetime coordinates and $g_{0\alpha} = 0$, this vector reduces to ζ^a. Furthermore, in static spacetimes ξ^a turns out to be the timelike Killing vector. It was shown in Ref. [31] that this vector plays a crucial role in the thermodynamic interpretation and has a rich structure as far as the Noether current and spacetime dynamics are concerned.

B. Noether charge and evolution equation in general relativity

We begin by calculating the Noether charge for the vector field ξ^a. The Noether current in general relativity can be written in an elegant manner by using a new set of variables (f^{ab}, N^c_{ab}) in terms of which several expressions in general relativity become simpler. These variables, defined as

$$f^{ab} = \sqrt{-g} g^{ab}, \qquad N^c_{ab} = Q^{cd}_{ae}\Gamma^e_{bd} + Q^{cd}_{be}\Gamma^e_{ad}, \tag{3}$$

where $2Q^{ab}_{cd} = (\delta^a_c \delta^b_d - \delta^a_d \delta^b_c)$, were earlier used in Refs. [32,33] and their thermodynamic interpretation was provided in Ref. [34]. The variation of the Einstein-Hilbert action in terms of these conjugate variables results into

$$\begin{aligned} \delta(\sqrt{-g}R) &= R_{ab}\delta f^{ab} - \partial_c (f^{ab}\delta N^c_{ab}) \\ &= \sqrt{-g}[G_{ab}\delta g^{ab} - \nabla_c (g^{ik}\delta N^c_{ik})]. \end{aligned} \tag{4}$$

If the above variation results from a Lie variation with respect to some vector field v_a, then from the above expression a conserved current J^a emerges with the property $\nabla_a J^a = 0$. This conserved current is the Noether current and has the following expression:

$$16\pi J^a(v) = 2R^{ab} v_b + g^{ij} \pounds_v N^a_{ij}. \tag{5}$$

(The factor 16π is conventional when we use units with $G = 1$; obviously, any multiple of J^a is conserved.) Given the fact that $\nabla_a J^a = 0$, we can write the Noether current in terms of an antisymmetric second-rank tensor J^{ab}, the Noether potential as $J^a = \nabla_b J^{ab}$. This in the case of general relativity becomes

$$16\pi J^{ab}(v) = \nabla^a v^b - \nabla^b v^a. \tag{6}$$

Though in the above discussion the Noether current has been derived by using the Lie variation, it should be stressed that the same result can be obtained by using differential geometry *without ever using diffeomorphism invariance of the action principle for gravity.* This has been shown explicitly in Ref. [31], and hence we will not repeat the arguments here.

Next, we will calculate the Noether current for the time evolution vector ξ^a. For the evaluation we shall use a relation between the Noether current of two vector fields q^a and v^a such that $v^a = f(x)q^a$, for arbitrary function $f(x)$. In part 2 of the Appendix [see Eq. (A6)], it is shown that

$$16\pi\{q_a J^a(v) - f(x) q_a J^a(q)\} = \nabla_b(\{q^a q^b - g^{ab} q^2\}\nabla_a f). \tag{7}$$

The usefulness of this relation can be realized by noting that for $q_a = \nabla_a \phi$ for some scalar ϕ the Noether current vanishes. Thus, applying the above result for u_a and then for ξ_a, one can arrive at the following simple relation for the Noether current of ξ_a [see part 2 of the Appendix; Eq. (A9)]:

$$16\pi u_a J^a(\xi) = 2D_\alpha(N a^\alpha), \tag{8}$$

where $a^i = u^j \nabla_j u^i$ represents the four acceleration which satisfies the relation $D_i a^i = \nabla_i a^i - a^2$, with D_i representing the surface covariant derivative for the $t = \text{const}$ surface. Then we can integrate Eq. (8) over the $t =$ const hypersurface with $\sqrt{h} d^3x$ being the integration measure and bounded by the $N = \text{const}$ surface leading to the total Noether charge contained in the 3-volume. Then, dividing both sides of Eq. (8) by 16π, we arrive at

$$\int_{\mathcal{V}} d^3x \sqrt{h} u_a J^a(\xi) = \int_{\mathcal{V}} \frac{d^3x\sqrt{h}}{8\pi} D_\alpha(N a^\alpha) = \int_{\partial\mathcal{V}} \frac{\sqrt{\sigma} d^2x}{8\pi} N r_\alpha a^\alpha, \tag{9}$$

which holds for any arbitrary region $\mathcal{V}$ of the spacetime, with the bounding region being the $N(t, \mathbf{x}) = \text{const}$ surface within the $t = \text{const}$ hypersurface. This allows us to identify the vector r_a to be normal to this $N(t, \mathbf{x}) =$ const hypersurface as $r_a = \epsilon D_a N (D_b N D^b N)^{-1/2} = \epsilon h^i_a \nabla_i N / a$, where the ϵ factor is introduced to ensure that r_a is always the *outward*-pointing normal. (When the acceleration a_i is outward pointing, $\epsilon = 1$; otherwise, $\epsilon = -1$.) Here $a = \sqrt{a_i a^i}$ is the magnitude of the acceleration. So we can also write the normal r_α as $r_\alpha = \epsilon a_\alpha / a$, with a representing the magnitude of the acceleration. Then we obtain

$$N r_\alpha a^\alpha = N\epsilon \frac{a_\alpha}{a} a^\alpha = \epsilon N a. \tag{10}$$

The Tolman redshifted Davies-Unruh temperature on the boundary surface $N = \text{const}$ is $T_{\text{loc}} = Na/2\pi$ for observers with four velocity $u_a = -N\delta^0_a$. Locally free-falling observers will observe these observers moving normal to the $t =$ const hypersurface with an acceleration a, and, as a consequence, the local vacuum will appear as a thermal state with temperature T_{loc} to these observers. By using all these results, Eq. (9) can be written as

$$2\int_{\mathcal{V}} d^3x \sqrt{h} u_a J^a(\xi) = \epsilon \int_{\partial\mathcal{V}} \frac{\sqrt{\sigma} d^2x}{2}\left(\frac{Na}{2\pi}\right) = \epsilon \int_{\partial\mathcal{V}} \sqrt{\sigma} d^2x \left(\frac{1}{2} T_{\text{loc}}\right). \tag{11}$$

The above result can be interpreted as *twice the Noether charge contained in the* $N = \text{const}$ *surface is equal to the equipartition energy of the surface*. With the interpretation of $\sqrt{\sigma}/4$ as entropy density, the above result also gives

$$\int_{\mathcal{V}} d^3x \sqrt{h} u_a J^a(\xi) = \epsilon \int_{\partial\mathcal{V}} \frac{\sqrt{\sigma} d^2x}{4} T_{\text{loc}} = \epsilon \int_{\partial\mathcal{V}} d^2x T_{\text{loc}} s, \tag{12}$$

which is the heat density of the bounding surface. The interpretation of $\sqrt{\sigma}/4$ as the entropy density comes naturally when the boundary surface becomes a horizon. Thus, even in the most general (nonstatic) context, the Noether charge of the time development vector in the bulk spacetime region has a simple interpretation as the surface heat content.

We will next obtain the dynamics of gravity in terms of bulk and surface degrees of freedom using the Noether current formalism. For this, we again start with Eq. (8) and use Eq. (5) leading to

$$u_a g^{ij} \pounds_\xi N^a_{ij} = D_\alpha(2Na^\alpha) - 2N R_{ab} u^a u^b. \tag{13}$$

Then we integrate the above expression as in the earlier situation over the three-dimensional region $\mathcal{R}$ with the boundary surface being $N = \text{const}$ within the $t = \text{const}$ surface leading to

$$\int_{\mathcal{R}} d^3x \sqrt{h} u_a g^{ij} \pounds_\xi N^a_{ij} = \int_{\partial\mathcal{R}} d^2x \sqrt{\sigma} r_\alpha (2Na^\alpha) - \int_{\mathcal{R}} d^3x \sqrt{h} 2N R_{ab} u^a u^b, \tag{14}$$

where we have used $d^3x\sqrt{h}$ as the integration measure. Introducing the dynamics through Einstein's equation $R_{ab} = 8\pi(T_{ab} - (1/2)g_{ab}T) = 8\pi\bar{T}_{ab}$ and dividing the whole expression by 8π gives

$$\int_{\mathcal{R}} \frac{d^3x\sqrt{h}}{8\pi} u_a g^{ij} \pounds_\xi N^a_{ij} = \int_{\partial\mathcal{R}} d^2x\sqrt{\sigma} r_\alpha \left(\frac{N a^\alpha}{4\pi}\right) - \int_{\mathcal{R}} d^3x\sqrt{h} 2N\bar{T}_{ab}u^a u^b. \quad (15)$$

Using Eq. (10) and introducing the Komar energy density by the definition $\rho_{\rm Komar} = 2N\bar{T}_{ab}u^a u^b$, we obtain

$$\frac{1}{8\pi}\int_{\mathcal{R}} d^3x\sqrt{h} u_a g^{ij} \pounds_\xi N^a_{ij} = \epsilon \int_{\partial\mathcal{R}} d^2x\sqrt{\sigma}\left(\frac{1}{2}T_{\rm loc}\right) - \int_{\mathcal{R}} d^3x\sqrt{h}\rho_{\rm Komar}. \quad (16)$$

We define the surface degrees of freedom by

$$N_{\rm sur} \equiv A = \int_{\partial\mathcal{R}} \sqrt{\sigma} d^2x, \quad (17)$$

which is always positive. We can define an average temperature over the surface such that

$$T_{\rm avg} \equiv \frac{1}{A}\int_{\partial\mathcal{R}} \sqrt{\sigma} d^2x T_{\rm loc}. \quad (18)$$

Finally, we introduce the bulk degrees of freedom by the definition

$$N_{\rm bulk} = \frac{\epsilon}{(1/2)T_{\rm avg}}\int d^3x\sqrt{h}\rho_{\rm Komar}. \quad (19)$$

When the bulk region is in equipartition at the temperature $T_{\rm avg}$, then $N_{\rm bulk}$ represents the correct number of bulk degrees of freedom. Here also we need the factor ϵ to ensure that $N_{\rm bulk}$ is positive definite. We choose $\epsilon = +1$ if the total Komar energy within the volume is positive and $\epsilon = -1$ if the total Komar energy in the volume is negative so as to keep $N_{\rm bulk}$ always positive. With all these definitions, Eq. (16) can be written in the following manner (this corrects a minor typo in Ref. [31]):

$$\frac{1}{8\pi}\int_{\mathcal{R}} d^3x\sqrt{h} u_a g^{ij} \pounds_\xi N^a_{ij} = \frac{\epsilon}{2} T_{\rm avg}(N_{\rm sur} - N_{\rm bulk}). \quad (20)$$

Thus, for comoving observers in static spacetime, we have the holographic equipartition $N_{\rm sur} = N_{\rm bulk}$. When the difference $(N_{\rm sur} - N_{\rm bulk})$ is nonzero for a given foliation, we have a departure from holographic equipartition, and this leads to the time evolution of the metric, as is evident from the left-hand side of Eq. (20). The implications of this result has been discussed extensively in Ref. [31].

C. Aside: Some illustrative examples

An important aspect of the dynamical evolution equation is the following: The structure of Eq. (20) shows that, while it is covariant, it is foliation dependent through the normal u_i. For example, even in a static spacetime (which possesses a timelike Killing vector field), the *nonstatic* observers will perceive a time dependence of the metric and hence a departure from holographic equipartition [so that both sides of Eq. (20) are nonzero], while static observers (with velocities along the Killing direction) will perceive a time-independent metric and holographic equipartition [with both sides of Eq. (20) being zero]. This contrast is most striking when we study two natural classes of observers in a static spacetime. The first set are observers with four-velocities along the timelike Killing vector who have a nonzero acceleration. In this foliation the metric components are independent of time, and the left-hand side of Eq. (20) vanishes, leading to holographic equipartition $N_{\rm sur} = N_{\rm bulk}$. But we know that *any* spacetime metric can be expressed in the synchronous frame coordinates with the line element:

$$ds^2 = -d\tau^2 + g_{\alpha\beta}dx^\alpha dx^\beta. \quad (21)$$

In the synchronous frame, the observers at $x^\alpha = \text{const}$ are comoving with four velocity: $u_a = (-1, 0, 0, 0)$. Obviously, the comoving observer is not accelerating (i.e., the curves $x^\alpha = \text{const}$ are geodesics), and hence the local Davies-Unruh temperature for these observers will vanish. We want to consider Eq. (20) in two such coordinate systems to clarify some of the issues.

Let us begin with the synchronous frame in which $T_{\rm avg} \to 0$, $T_{\rm avg}N_{\rm sur} \to 0$ with $T_{\rm avg}N_{\rm bulk}$ remaining finite, so that Eq. (20) reduces to the following form:

$$\frac{1}{8\pi}\int_{\mathcal{R}} d^3x\sqrt{h} u_a g^{ij} \pounds_\xi N^a_{ij} = -\frac{\epsilon}{2}T_{\rm avg}N_{\rm bulk} = -\int_{\mathcal{R}} d^3x\sqrt{h}\rho_{\rm Komar}. \quad (22)$$

The quantity $u_a g^{ij} \pounds_\xi N^a_{ij}$ in an arbitrary synchronous frame is given by

$$\sqrt{h} u_a g^{ij} \pounds_\xi N^a_{ij} = 2\sqrt{h}(K_{ab}K^{ab} - u^a\nabla_a K) = \sqrt{h}\left(g^{\alpha\beta}\partial_\tau^2 g_{\alpha\beta} + \frac{1}{2}\partial_\tau g^{\alpha\beta}\partial_\tau g_{\alpha\beta}\right), \quad (23)$$

where we have used Eq. (A11). It can be shown that equating this expression to $-16\pi\bar{T}_{ab}u^a u^b$ correctly reproduces the standard time-time component of Einstein's equation in the synchronous frame. So, our Eq. (20) gives the correct result, as it should.

As an explicit example, consider the Friedmann universe for which $g_{\alpha\beta} = a^2(t)\delta_{\alpha\beta}$ leading to the following expressions:

$$\partial_\tau g_{\alpha\beta} = 2a\dot{a}\delta_{\alpha\beta}; \qquad \partial_\tau^2 g_{\alpha\beta} = (2\dot{a}^2 + 2a\ddot{a})\delta_{\alpha\beta};$$
$$\partial_\tau g^{\alpha\beta} = -2\frac{\dot{a}}{a^3}\delta^{\alpha\beta}, \tag{24}$$

and $\bar{T}_{ab}u^a u^b = (1/2)(\rho + 3p)$. On substitution of Eq. (24), in Eq. (23) we arrive at the following expression for the time evolution of the scale factor:

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p). \tag{25}$$

The above equation supplemented by the equation of state leads to the standard results. Thus in the Friedmann universe the dynamical evolution of spacetime leads to the dynamical evolution equation of the scale factor sourced by the Komar energy density. Before proceeding further, it is worthwhile to clarify the following point: In the case of the Friedmann universe, one can *also* obtain [35] the following result:

$$\frac{dV}{dt} = N_{\rm sur} - \sum \epsilon N_{\rm bulk}, \tag{26}$$

where $V = (4\pi/3)H^{-3}$ is the areal volume of the Hubble radius sphere if we define the degrees of freedom using the temperature $T \equiv H/2\pi$. (The ϵ factor has to be chosen for each bulk component appropriately in order to keep all $N_{\rm bulk}$ positive as indicated by the summation; see [35] for a detailed discussion.) Though this is also equivalent to Einstein's equation, it is structurally quite different from the evolution equation in Eq. (20) (and should not be confused with it) for the following reasons: (a) The left-hand sides of Eqs. (20) and (26) are different. (b) The placement of ϵ's are different in the right-hand sides of Eqs. (20) and (26). (c) One uses the Friedmann time coordinate in the left-hand side of Eq. (26) but still attributes a temperature $T \equiv H/2\pi$ to define the degrees of freedom. (d) Most importantly, Eq. (26) holds only for the Friedmann universe, while Eq. (20) is completely general.

Coming back to the consequences of Eq. (20), since this result is true for any Friedmann universe, it is also true for the de Sitter spacetime written in synchronous (Friedmann) coordinates. The de Sitter metric, as seen by comoving observers, has an explicit time dependence $a(t) \propto \exp(Ht)$, and for these observers the perceived Davies-Unruh temperature vanishes. Nevertheless, Eq. (20) will of course give the correct evolution equation. On the other hand, de Sitter spacetime can also be expressed in static coordinates with the line element:

$$ds^2 = -\left(1 - \frac{r^2}{l^2}\right)dt^2 + \frac{dr^2}{(1 - \frac{r^2}{l^2})} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{27}$$

The observers with $x^\alpha = $ const in this coordinate system are not geodesic observers. They have the following four velocity and four acceleration, respectively:

$$u_a = \sqrt{\left(1 - \frac{r^2}{l^2}\right)}(-1, 0, 0, 0), \tag{28}$$

$$a^i = (0, -(r/l^2), 0, 0). \tag{29}$$

Let us see what happens when we use this foliation.

In this case, the acceleration a^i and the normal r_i are directed opposite to each other as r_i is the outward-directed normal. (Note that in the de Sitter spacetime the free-falling observers are moving outwards, and *with respect to them* the static observers are moving *inwards* opposite to the outward-pointing normal.) Hence, in this situation we have $\epsilon = -1$. The magnitude of the acceleration is

$$a = \frac{r}{l^2}\frac{1}{\sqrt{(1 - \frac{r^2}{l^2})}}, \tag{30}$$

which is obtained from Eq. (29). Thus, the local Davies-Unruh temperature turns out to be

$$T_{\rm loc} = \frac{Na}{2\pi} = \frac{r}{2\pi l^2} = T_{\rm avg}. \tag{31}$$

Since the spacetime is static, ξ_i becomes a timelike Killing vector and the Lie derivative of the connection present in Eq. (20) vanishes. Therefore, in this foliation, holographic equipartition should hold. To verify this explicitly, we start by calculating surface degrees of freedom. From Eq. (17), the surface degrees of freedom turn out to be

$$N_{\rm sur} \equiv A = \int_{\partial\mathcal{R}} \sqrt{\sigma} d^2x = 4\pi r^2. \tag{32}$$

Again, the bulk degree of freedom can be obtained from Eq. (19) as

$$N_{\rm bulk} = 4\pi\frac{\frac{8\pi}{3}\rho r^3}{rl^{-2}}. \tag{33}$$

Note that the ϵ factor in the definition of the bulk degrees of freedom keeps it positive, even though the Komar energy density is negative. Then in de Sitter spacetime we have $8\pi\rho = (3/l^2)$, from which we readily observe that

$$N_{\rm bulk} = (8\pi\rho)(l^2/3)4\pi r^2 = 4\pi r^2 = N_{\rm sur}. \tag{34}$$

Hence, for de Sitter spacetime in static coordinates, holographic equipartition does hold, as it should. [Alternatively, setting $N_{\rm bulk} = N_{\rm sur}$ will lead to the correct identification of l in the metric with a source by $8\pi\rho = (3/l^2)$.]

One can easily verify, by explicit computation, how these results generalize to any static spherically symmetric one, with the line element:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \tag{35}$$

which covers several interesting metrics with horizons. In these static coordinates, the holographic equipartition holds, as can be easily checked. A more interesting situation is in the case of geodesic observers in a synchronous frame. To check this, we start with a coordinate transformation: $(t, r, \theta, \phi) \to (\tau, R, \theta, \phi)$ in which the variables are related by the following equations:

$$dt = dR - \frac{1}{\sqrt{1-f(r)}}\frac{dr}{f(r)}, \tag{36}$$

$$dR = d\tau + \frac{dr}{\sqrt{1-f(r)}}. \tag{37}$$

In terms of these newly defined variables, the line element reduces to the synchronous form:

$$ds^2 = -d\tau^2 + [1-f(r)]dR^2 + r^2 d\Omega^2. \tag{38}$$

The comoving observers, having four velocities $u_a = (-1, 0, 0, 0)$, are geodesic observers with zero acceleration, and thus the local Davies-Unruh temperature also becomes zero. We can use Eqs. (22) and (23) to describe the evolution. The relevant derivatives are

$$\begin{aligned}
&\partial_\tau g_{RR} = -f'(r)\dot{r}; \qquad \partial_\tau^2 g_{RR} = -f'(r)\ddot{r} - f''(r)\dot{r}^2;\\
&\partial_\tau g^{RR} = \frac{f'(r)\dot{r}}{[1-f(r)]^2};\\
&\partial_\tau g_{\theta\theta} = 2r\dot{r}; \qquad \partial_\tau^2 g_{\theta\theta} = 2r\ddot{r} + 2\dot{r}^2; \qquad \partial_\tau g^{\theta\theta} = -\frac{2\dot{r}}{r^3};\\
&\partial_\tau g_{\phi\phi} = 2r\dot{r}\sin^2\theta; \qquad \partial_\tau^2 g_{\theta\theta} = (2r\ddot{r} + 2\dot{r}^2)\sin^2\theta;\\
&\partial_\tau g^{\theta\theta} = -\frac{2\dot{r}}{r^3}\frac{1}{\sin^2\theta}.
\end{aligned} \tag{39}$$

On substitution of these in Eq. (23), we obtain the following differential equation satisfied by the unknown function $f(r)$:

$$f''(r) + \frac{2f'(r)}{r} = 16\pi\bar{T}_{\tau\tau} = -16\pi\bar{T}^0_0. \tag{40}$$

It can be easily verified that this is the correct field equation in this case (see, e.g., p. 302 of Ref. [19]). For example, if we consider the metric of a charged particle with $\bar{T}_{\tau\tau} = Q^2/8\pi r^4$, the above equation can be solved to give $f(r) = 1 - (2M/r) + (Q^2/r^2)$, which, of course, is the Reissner-Nordström metric. The description being covariant but foliation dependent is actually a very desirable and inevitable feature from the thermodynamical point of view [36,37].

III. GENERALIZATION TO LANCZOS-LOVELOCK GRAVITY

In the previous section, we have reviewed, in the context of Einstein-Hilbert action, how the departure from holographic equipartition leads to the dynamics of the spacetime and have also shown that in static spacetime the surface degrees of freedom equal the bulk degrees of freedom. We will now generalize the above description to Lanczos-Lovelock gravity.

A. A brief introduction to Lanczos-Lovelock gravity

Consider, in a D-dimensional spacetime, an action functional which is made from the metric and the curvature tensor but does not contain any derivatives of curvature tensor, such that

$$A = \int_{\mathcal{V}} d^D x \sqrt{-g} L(g^{ab}, R^a{}_{bcd}). \tag{41}$$

Let us define

$$P^{abcd} = \left(\frac{\partial L}{\partial R_{abcd}}\right)_{g_{ij}}, \tag{42}$$

which has all the algebraic properties of the curvature tensor. We next define another tensor (which is a generalization of Ricci tensor in general relativity) by

$$\mathcal{R}^{ab} \equiv P^{aijk} R^b{}_{ijk}. \tag{43}$$

This tensor is actually symmetric, though the result is nontrivial to prove (for this result and more properties of these tensors, see [38]). The variation of the action functional leads to

$$\begin{aligned}
\delta A &= \delta \int_{\mathcal{V}} d^D x \sqrt{-g} L\\
&= \int_{\mathcal{V}} d^D x \sqrt{-g} E_{ab}\delta g^{ab} + \int_{\mathcal{V}} d^D x \sqrt{-g}\nabla_j \delta v^j,
\end{aligned} \tag{44}$$

where we have the following expression for equation of motion term E_{ab} and the boundary term δv^a:

$$\begin{aligned}
E_{ab} &\equiv \frac{1}{\sqrt{-g}}\left(\frac{\partial\sqrt{-g}L}{\partial g^{ab}}\right)_{R_{abcd}} - 2\nabla^m\nabla^n P_{amnb}\\
&= \mathcal{R}_{ab} - \frac{1}{2}g_{ab}L - 2\nabla^m\nabla^n P_{amnb},
\end{aligned} \tag{45}$$

$$\delta v^j = 2P^{ibjd}\nabla_b \delta g_{di} - 2\delta g_{di}\nabla_c P^{ijcd}. \tag{46}$$

This is fairly general, but we impose the condition that the field equation should be second order in the metric. Since the quantity P^{abcd} involves the second derivative of the metric, the term $\nabla^m \nabla^n P_{amnb}$ in E_{ab} contains the fourth-order derivative of the metric. We can get a second-order field equation by imposing an extra condition on P^{abcd} such that

$$\nabla_a P^{abcd} = 0. \tag{47}$$

Thus, finding an action functional which would lead to equations of motion which are second order in the metric reduces to finding scalars such that Eq. (47) is satisfied. Such an action functional is unique and coincides with the Lanczos-Lovelock Lagrangian in D dimensions given by [39–42]

$$L = \sum_m c_m L^{(m)} = \sum_m c_m (\delta^{aba_2 b_2 \ldots a_m b_m}_{cdc_2 d_2 \ldots c_m d_m} R^{c_2 d_2}_{a_2 b_2} \ldots R^{c_m d_m}_{a_m b_m}) R^{cd}_{ab}. \tag{48}$$

Because of the complete antisymmetry in the indices of the determinant tensor, we have in a D-dimensional spacetime the following restriction: $2m \leq D$. (Otherwise, the determinant tensor would vanish identically.) In four dimensions, this property uniquely fixes the result to be the Einstein-Hilbert action for $m = 1$. The nature of Lanczos-Lovelock models at $D = 2m$ is of quiet importance, as these are known as critical dimensions for a given Lanczos-Lovelock term. In these situations, the variation of the action functional reduces to a pure surface term [43].

To proceed further, we need the expression for the Noether current in Lanczos-Lovelock gravity. Recall that the standard result for the Noether current, for diffeomorphism invariance of a Lagrangian $L(g^{ab}, R^a{}_{bcd})$, is given by [9]

$$16\pi J^a = 2E^a_b \xi^b + L\xi^a + \delta_\xi v^a, \tag{49}$$

where E_{ab} is defined in Eq. (45) and $\delta_\xi v^a$ represents the surface term in the Lagrangian variation. The following three relations can be used:

$$2E^a_b \xi^b + L\xi^a = 2\mathcal{R}^a_b \xi^b, \tag{50}$$

$$\delta_\xi v^a = -\pounds_\xi v^a = -2\mathcal{R}^a_b \xi^b + 2P^{abdi} \nabla_b \nabla_d \xi_i, \tag{51}$$

$$\delta_\xi v^i = 2P_a{}^{bci} \pounds_\xi \Gamma^a_{bc} \tag{52}$$

to express the Noether current in two different, useful, forms as follows:

$$16\pi J^a = 2\mathcal{R}^a_b \xi^b + \delta_\xi v^a = 2P^{abcd} \nabla_b \nabla_c \xi_d \tag{53}$$

$$= 2\mathcal{R}^a_b \xi^b + 2P_i{}^{jka} \pounds_\xi \Gamma^i_{jk}. \tag{54}$$

The corresponding expression for the Noether potential in Lanczos-Lovelock gravity is given by [9]

$$16\pi J^{ab}(\xi) = 2P^{abcd} \nabla_c \xi_d. \tag{55}$$

We can obtain the entropy of horizons from the relevant Noether charge. In Lanczos-Lovelock gravity, the entropy is defined in terms of the tensor P^{abcd} and is known as Wald entropy with the expression [44–51]

$$S = -\frac{1}{8} \int d^{D-2}x \sqrt{\sigma} P^{abcd} \mu_{ab} \mu_{cd} \equiv \int d^{D-2}x s, \tag{56}$$

where σ is the metric determinant over the $(D-2)$-dimensional hypersurface and μ_{ab} is the binormal to the hypersurface. The last equation defines the entropy density s which will be used frequently in our later discussion.

B. Heat content of spacetime in Lanczos-Lovelock gravity

We will work with the same spacetime foliations defined in Eq. (2) throughout and thus will use the vectors u^a, ξ^a. We begin by performing the same calculation as before, viz. connecting the Noether charge in a volume to the heat content of the boundary. To do this, we will start by relating the Noether current for a vector q_a to that of another vector $f(x) q_a = v_a$ for any arbitrary function $f(x)$. From part 2 of the Appendix, using Eq. (A16) we obtain the desired relation as

$$16\pi \{ q_a J^a(fq) - f q_a J^a(q) \} = \nabla_b (2P^{abcd} q_a q_d \nabla_c f). \tag{57}$$

The usefulness of the above equation again originates from the fact that, if $q_a = \nabla_a \phi$, then its Noether current vanishes, and thus the Noether current for $v_a = f(x) q_a$ acquires a particularly simple form. Applying the above result for the two natural vector fields u^a and ξ^a from Eq. (A24), we obtain the simple relation

$$16\pi u_a J^a(\xi) = 2D_\alpha (N \chi^\alpha), \tag{58}$$

where we have introduced a new vector field χ^a given by [see Eq. (A18)]

$$\chi^a = -2P^{abcd} u_b u_d a_c, \tag{59}$$

which satisfies the condition $u_a \chi^a = 0$ (so that it is a spatial vector) and also has the property $D_i \chi^i = \nabla_i \chi^i - a_i \chi^i$. We can integrate Eq. (58) over $(D-1)$-dimensional volume bounded by the $N = \text{const}$ surface within the $t = \text{const}$ hypersurface leading to

$$\int_{\mathcal{V}} d^{D-1}x \sqrt{h} u^a J_a(\xi) = \int_{\partial \mathcal{V}} \frac{d^{D-2}x \sqrt{\sigma}}{8\pi} N r_\alpha \chi^\alpha. \tag{60}$$

As in general relativity, here also the vector r_α is the unit normal to the $N = \text{const}$ hypersurface. This vector is either parallel or antiparallel to the acceleration four vector such that $r_\alpha = \epsilon a_\alpha/a$, where $\epsilon = +1$ implies parallel to acceleration and vice versa. With this notion, we obtain the following result from the vector field χ_α:

$$\sqrt{\sigma}\frac{N r_\alpha \chi^\alpha}{8\pi} = \epsilon\left(\frac{Na}{2\pi}\right)\left(\frac{1}{2}\sqrt{\sigma}P^{\alpha b d\beta} r_\alpha u_b u_d r_\beta\right). \quad (61)$$

The term in brackets is closely related to the entropy density of the surface in Lanczos-Lovelock gravity, defined in Eq. (56) as [9,45]

$$s = -\frac{1}{8}\sqrt{\sigma}P^{abcd}\mu_{ab}\mu_{cd} = \frac{1}{2}\sqrt{\sigma}P^{\alpha b d\beta} r_\alpha u_b u_d r_\beta. \quad (62)$$

Using this expression for entropy density in Eq. (61), we obtain

$$\sqrt{\sigma}\frac{N r_\alpha \chi^\alpha}{8\pi} = \epsilon T_{\text{loc}} s, \quad (63)$$

where $T_{\text{loc}} = Na/2\pi$ is the redshifted local Unruh-Davies temperature as measured by the observers moving normal to the $t = \text{const}$ surface, with respect to the local vacuum of freely falling observers. We thus see that the results in general relativity have a natural generalization to Lanczos-Lovelock models. With all these results, Eq. (60) reduces to

$$\int_{\mathcal{V}} d^{D-1}x\sqrt{h}u^a J_a(\xi) = \epsilon\int_{\partial\mathcal{V}} d^{D-2}x T_{\text{loc}} s. \quad (64)$$

Thus *in Lanczos-Lovelock gravity as well the Noether charge in a bulk region is equal to the surface heat content of the boundary.* The similar result derived for general relativity can be thought of as a special case of Lanczos-Lovelock gravity; the connection between the bulk Noether charge and the surface heat content goes way beyond general relativity. This result *is nontrivial, because the expression for entropy density in the general Lanczos-Lovelock models is nontrivial* in contrast with general relativity in which it is just one-quarter per unit area.

C. Evolution equation of spacetime in Lanczos-Lovelock gravity

Let us next generalize our result presented in Eq. (1) for Lanczos-Lovelock models obtaining the dynamical evolution as due to deviation from holographic equipartition. We will start by substituting the Noether current expression for ξ^a as presented in Eq. (54) to Eq. (58) which leads to the following result:

$$2u_a P_i^{\ jka}\pounds_\xi \Gamma^i_{jk} = D_\alpha(2N\chi^\alpha) - 2N\mathcal{R}_{ab}u^a u^b. \quad (65)$$

Let us first consider the pure Lanczos-Lovelock theory with the mth-order Lanczos-Lovelock Lagrangian. (We shall consider the generalization to Lanczos-Lovelock models with a sum of Lagrangians, at the end.) Contracting the field equation $\mathcal{R}_{ab} - (1/2)g_{ab}L = 8\pi T_{ab}$ in Lanczos-Lovelock gravity with g^{ab}, we get $L = [8\pi]/[m-(D/2)]T$, where D is spacetime dimension. Therefore, the field equation can also be rewritten as

$$\mathcal{R}_{ab} = 8\pi\bar{T}_{ab} = 8\pi\left(T_{ab} - \frac{1}{2}\frac{1}{(D/2)-m}g_{ab}T\right) \equiv 8\pi\bar{T}_{ab}. \quad (66)$$

Using this and integrating Eq. (65) over $(D-1)$-dimensional volume, we arrive at

$$\int_{\mathcal{R}}\frac{d^{D-1}x\sqrt{h}}{8\pi}2u_a P_i^{\ jka}\pounds_\xi\Gamma^i_{jk} = \int_{\partial\mathcal{R}}\frac{d^{D-2}x\sqrt{\sigma}}{4\pi}N\chi^\alpha r_\alpha - \int_{\mathcal{R}} d^{D-1}x\sqrt{h}2N\bar{T}_{ab}u^a u^b. \quad (67)$$

As before, the r_α is the normal to the $N = \text{const}$ surface within the $t = \text{const}$ surface and is either parallel or antiparallel to the four acceleration. The energy-momentum term can be written in an identical fashion by using the Komar energy density, defined as $\rho_{\text{Komar}} = 2N\bar{T}_{ab}u^a u^b$. We can proceed by using Eq. (61), which on substitution into Eq. (67) leads to

$$\int_{\mathcal{R}}\frac{d^{D-1}x\sqrt{h}}{8\pi}2u_a P_i^{\ jka}\pounds_\xi\Gamma^i_{jk} = -2\epsilon\int_{\partial\mathcal{R}} d^{D-2}x\sqrt{\sigma}P^{\alpha b\beta d} r_\alpha u_b r_\beta u_d\left(\frac{1}{2}T_{\text{loc}}\right) - \int_{\mathcal{R}} d^{D-1}x\sqrt{h}\rho_{\text{Komar}}. \quad (68)$$

The rest of the analysis requires proper definition of N_{sur}, N_{bulk}, etc., which we do in analogy with the case of general relativity. The number of surface degrees of freedom is defined as 4 times the entropy as in the case of general relativity:

$$N_{\text{sur}} \equiv 4S = 2\int_{\partial\mathcal{R}} d^{D-2}x\sqrt{\sigma}P^{\alpha b d\beta} r_\alpha u_b u_d r_\beta. \quad (69)$$

The average temperature is properly defined by using the surface degrees of freedom as the local weights leading to ensure that the total heat content is reproduced:

$$\frac{1}{2}N_{\text{sur}}k_B T_{\text{avg}} = \frac{1}{2}\int dN_{\text{sur}}k_B T_{\text{loc}}; \qquad T_{\text{avg}}S = \int T_{\text{loc}}dS. \quad (70)$$

This result can be written more explicitly as

$$T_{\rm avg} = \frac{\int_{\partial\mathcal{R}} d^{D-2}x\sqrt{\sigma}P^{\alpha b\beta d}r_\alpha u_b r_\beta u_d T_{\rm loc}}{\int_{\partial\mathcal{R}} d^{D-2}x\sqrt{\sigma}P^{\alpha b\beta d}r_\alpha u_b r_\beta u_d} = \frac{1}{S}\int dS T_{\rm loc} = \frac{1}{N_{\rm sur}}\int dN_{\rm sur}T_{\rm loc}. \tag{71}$$

Once $T_{\rm avg}$ is defined, the number of bulk degrees of freedom is given by the equipartition value:

$$N_{\rm bulk} = \frac{\epsilon}{(1/2)T_{\rm avg}}\int_{\mathcal{R}} d^{D-1}x\sqrt{h}\rho_{\rm Komar}, \tag{72}$$

with ϵ included (as in general relativity), to ensure that $N_{\rm bulk}$ is always positive. Inserting Eqs. (69), (71), and (72) in Eq. (68), we find that the dynamical evolution of the spacetime in Lanczos-Lovelock gravity is determined by the following relation:

$$\int_{\mathcal{R}} \frac{d^{D-1}x\sqrt{h}}{8\pi} 2u_a P_i^{\ jka} \pounds_\xi \Gamma^i_{jk} = \epsilon\left(\frac{1}{2}T_{\rm avg}\right)(N_{\rm sur} - N_{\rm bulk}), \tag{73}$$

which is a direct generalization of the corresponding result for general relativity.

For a static spacetime, the Lie variation of connection vanishes as ξ^a becomes a timelike Killing vector. Hence, in that situation we have, even in Lanczos-Lovelock gravity, the holographic equipartition given by

$$N_{\rm sur} = N_{\rm bulk}. \tag{74}$$

(This result has been obtained earlier in terms of equipartition energies in Ref. [52].) When the foliation leads to a time-dependent metric, the departure from holographic equipartition drives dynamical evolution of the metric through the Lie derivative term on the left-hand side of Eq. (73).

The above result was derived for the mth-order Lanczos-Lovelock Lagrangian. The definition of $\bar{T}_{ab}$, $\rho_{\rm Komar}$, and $N_{\rm bulk}$ introduces the m dependence though the expression for $\mathcal{R}_{ab}$ in Eq. (66). If, instead, we consider a Lanczos-Lovelock Lagrangian made of a sum of Lagrangians with different m, then the equation of motion $\mathcal{R}_{ab} - (1/2)g_{ab}L = 8\pi T_{ab}$ on contraction with g^{ab} leads to the result

$$\sum_m c_m[m - (D/2)]L_{(m)} = 8\pi T, \tag{75}$$

which cannot be solved in closed form for L in terms of T. However, one can take care of this issue by redefining $\rho_{\rm Komar}$ and $N_{\rm bulk}$ formally in terms of $\mathcal{R}_{ab}$. That is, we define the Komar energy density as $\rho = 2N(\mathcal{R}_{ab}/8\pi)u^a u^b$, and then the bulk degrees of freedom reduce to the following form:

$$N_{\rm bulk} = \frac{\epsilon}{(1/2)T_{\rm avg}}\int_{\mathcal{R}} d^{D-1}x\sqrt{h}\rho. \tag{76}$$

Then we again obtain the same result:

$$\int_{\mathcal{R}} \frac{d^{D-1}x\sqrt{h}}{8\pi} 2u_a P_i^{\ jka} \pounds_\xi \Gamma^i_{jk} = \epsilon\left(\frac{1}{2}T_{\rm avg}\right)(N_{\rm sur} - N_{\rm bulk}) \tag{77}$$

with the understanding that, for a given model, one should reexpress the variables in terms of T_{ab}.

The above results provide a direct connection between the evolution of spacetime and departure from holographic equipartition. The results also encode the holographic behavior of gravity by introducing naturally defined bulk and surface degrees of freedom. The difference between the description of evolution along these lines and that of standard field equation $\mathcal{R}_{ab} - (1/2)g_{ab}L = 8\pi T_{ab}$ is the following: For the standard gravitational field equations, the left-hand side *does not* have a clear physical meaning. There is also no distinction between static and dynamic spacetime, and hence the standard treatment cannot answer the question: what drives the time dependence of the metric? The answer is obviously not T_{ab}, since we can obtain time-dependent solutions even when $T_{ab} = 0$ and static solutions with $T_{ab} \neq 0$. In contrast, the evolution depicted in Eq. (77) addresses all these issues, and we have a natural separation between static and evolving metrics via holographic equipartition. When the surface and bulk degrees of freedom are unequal, resulting in departure from holographic equipartition, it drives the time dependence of the metric. Thus, the driving force behind the dynamical evolution of spacetime is the departure from holographic equipartition, providing a physically transparent statement about spacetime dynamics.

IV. DISCUSSION

Our aim in this work was to consider the relationship between the Noether current and gravitational dynamics in a useful manner. Noether currents can be thought of as originating from mathematical identities in differential geometry, with *no connection to the diffeomorphism invariance of gravitational action* [31]. This result holds not only in general relativity but also in Lanczos-Lovelock gravity (see part 1 of the Appendix).

Even though such conserved currents can be associated with any vector field, the time development vectors are always special. This is the motivation for introducing the vector ξ^a in the spacetime through Eq. (2). The vector ξ^a is parallel to velocity vector u^a for fundamental observers and represents proper time flow normal to the $t = \text{const}$ surface. As we saw, its Noether charge and current associated with this vector have an elegant and physically interesting thermodynamic interpretation. We showed that, for the vector field ξ^a in Lanczos-Lovelock gravity in arbitrary spacetime dimension, the *total Noether charge* in any bulk volume $\mathcal{V}$, bounded by a constant lapse surface, equals *the heat content of the boundary surface*. Also, the equipartition energy of the surface equals twice the Noether charge. While defining the heat content, we have used the local Unruh-Davies temperature and Wald entropy. This result holds for Lanczos-Lovelock gravity of all orders and does not rely on static spacetime or the existence of Killing vectorlike criteria.

The above identification allows us to study holographic equipartition for static spacetime and relate the time evolution of the metric as due to departure from holographic equipartition. With a suitable and natural definition for the degrees of freedom in the surface and in the bulk, we find that for static spacetimes (described in the natural foliation) the surface and the bulk degrees of freedom are equal in number, yielding holographic equipartition. It is the departure from this holographic equipartition that drives spacetime evolution. This result holds not only in general relativity but also in Lanczos-Lovelock gravity.

All the results derived above are generally covariant, but they do depend on the foliation. This implies that these results depend on observers and their acceleration, which is inevitable, since the Davies-Unruh temperature is intrinsically observer dependent. Since the dynamical evolution is connected to thermodynamic concepts in this approach, different observers *must* perceive the dynamical evolution differently. For example, the de Sitter spacetime is time dependent when written in the synchronous frame and becomes time independent in static spherically symmetric coordinates. Our description adapts naturally to the two different situations.

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APPENDIX: CALCULATIONAL DETAILS

Some calculations are not presented in an explicit format in the main text, which would affect the flow of ideas in the paper. Most of these relations exist in the literature; however, we collect the derivations together here with the hope that they will be useful to the reader.

1. Derivation of Noether current from differential identities in Lanczos-Lovelock gravity

In this section, the Noether current for Lanczos-Lovelock gravity will be derived by starting from identities in differential geometry without using any diffeomorphism invariance of action principles. The conceptual importance of this approach has already been emphasized in Ref. [31], in the context of Einstein gravity, and we shall generalize the result for Lanczos-Lovelock models. We start with the fact that the covariant derivative of any vector field can be decomposed into a symmetric and an antisymmetric part. From the antisymmetric part, we can define another antisymmetric tensor field as

$$16\pi J^{aj} = 2P^{ajki}\nabla_k v_i = P^{ajki}(\nabla_k v_i - \nabla_i v_k). \tag{A1}$$

It is evident from the antisymmetry of P^{abcd} that a conserved current exists such that $J^a = \nabla_j J^{aj}$. We recall the identities

$$(\nabla_j\nabla_k - \nabla_k\nabla_j)v^i = R^i{}_{cjk}v^c \tag{A2}$$

and

$$\mathcal{L}_v\Gamma^i_{jk} = \nabla_j\nabla_k v^i - R^i{}_{kjm}v^m \tag{A3}$$

and use them in the definition in Eq. (43) to get

$$\begin{aligned}\mathcal{R}^{ab}v_b &= P^{aijk}R^b{}_{ijk}v_b = -P^{aijk}(\nabla_j\nabla_k - \nabla_k\nabla_j)v_i\\ &= P^{aijk}\nabla_k\nabla_j v_i + (P^{akij} + P^{ajki})\nabla_j\nabla_k v_i\\ &= P^{aijk}\nabla_k\nabla_j v_i + P^{akij}\nabla_j\nabla_k v_i + \nabla_j(P^{ajki}\nabla_k v_i),\end{aligned} \tag{A4}$$

where in the second line we have used the identity $P^{a(bcd)} = 0$. Then from Eq. (A1) we obtain

$$\begin{aligned}16\pi J^a &= 2\mathcal{R}^{ab}v_b - 2P^{aijk}\nabla_k\nabla_j v_i - 2P^{akij}\nabla_j\nabla_k v_i\\ &= 2\mathcal{R}^{ab}v_b + 2P_i{}^{ajk}\nabla_k\nabla_j v^i - 2P_i{}^{jak}\nabla_j\nabla_k v^i\\ &= 2\mathcal{R}^{ab}v_b + 2P_i{}^{ajk}(\mathcal{L}_v\Gamma^i_{kj} + R^i{}_{jkm}v^m)\\ &\quad - 2P_i{}^{jak}(\mathcal{L}_v\Gamma^i_{jk} + R^i{}_{kjm}v^m)\\ &= 2\mathcal{R}^{ab}v_b + 2P_i{}^{jka}\mathcal{L}_v\Gamma^i_{jk},\end{aligned} \tag{A5}$$

while arriving at the third line we have used Eq. (A3) and for the last line we have used the fact that $P^{ijak}R_{ikjm} = P^{akij}R_{ikjm} = -P^{kaij}R_{ikjm} = P^{kaij}R_{kijm}$. Thus, Eq. (54) can be derived without any reference to the diffeomorphism invariance of the gravitational action, by using only the identities in differential geometry and various symmetry properties.

2. Identities regarding Noether current in Lanczos-Lovelock action

The Noether potential J^{ab} is antisymmetric in (a,b), and from its expression given by Eq. (6) it is evident that $J^{ab}(q)$ would identically vanish for $q_a = \nabla_a \phi$. We will use the above fact in order to obtain a relation between the Noether current for two vector fields q_a and v_a connected by $v_a = f(x)q_a$. This result in the case of general relativity is detailed in Ref. [31]. Expanding the expression for Noether current for $v_a = fq_a$ and taking the dot product with q_a along with subtracting the Noether current for q_a, one can show that

$$16\pi\{q_a J^a(fq) - fq_a J^a(q)\} = \nabla_b[(q^a q^b - q^2 g^{ab})\nabla_a f]. \tag{A6}$$

This is the result used in the main text. By using this result, it is easy to determine the Noether currents for $u_a = -N\nabla_a t$ and $\xi_a = Nu_a$. Using Eq. (A6) with $q_a = -u_a/N$ and $f = -N$, we obtain

$$16\pi u_a J^a(u) = \nabla_i a^i - a^2 = D_\alpha a^\alpha, \tag{A7}$$

where the acceleration is defined as

$$a_j = u^i \nabla_i u_j = (u^i \nabla_i N)\frac{u_j}{N} + Nu^i \nabla_j \left(\frac{u_i}{N}\right) = h^i_j \frac{\nabla_i N}{N}. \tag{A8}$$

Next, in order to obtain the Noether current for ξ^a we use Eq. (A6) with $q_a = u_a$ and $f = N$ leading to

$$\begin{aligned} 16\pi u_a J^a(\xi) &= Nu_a J^a(u) + \nabla_j(Na^j) = 2N\nabla_j a^j \\ &= D_\alpha(2Na^\alpha), \end{aligned} \tag{A9}$$

which is the desired relation in Eq. (8).

In general relativity the quantity $u_a g^{ij} \pounds_\xi N^a_{ij}$ can be evaluated in terms of the extrinsic curvature [19]. Then from the standard identity

$$\nabla_i a^i - R_{ab}u^a u^b = K_{ij}K^{ij} - u^a \nabla_a K \tag{A10}$$

we obtain

$$\begin{aligned} u_a g^{ij} \pounds_\xi N^a_{ij} &= 2N(\nabla_i a^i - R_{ab}u^a u^b) \\ &= 2N(K_{ij}K^{ij} - u^a \nabla_a K). \end{aligned} \tag{A11}$$

Next, we will generalize the above results to Lanczos-Lovelock gravity. For that purpose, we note that even in Lanczos-Lovelock gravity the Noether potential J^{ab} for a vector field $q_a = \nabla_a f$ vanishes identically. Thus, the Noether current for a vector field $v_a = f(x)q_a$ can be decomposed as

$$\begin{aligned} 16\pi J^{ab}(v) &= 2P^{abcd}\nabla_c(fq_d) \\ &= 2P^{abcd}q_d \nabla_c f + 2fP^{abcd}\nabla_c q_d. \end{aligned} \tag{A12}$$

Then the corresponding Noether current has the following expression:

$$\begin{aligned} 16\pi J^a(v) &= 2P^{abcd}\nabla_b(q_d \nabla_c f) + 2P^{abcd}\nabla_b(f\nabla_c q_d) \\ &= 2P^{abcd}q_d \nabla_b \nabla_c f + 2P^{abcd}\nabla_c f \nabla_b q_d \\ &\quad + 2P^{abcd}\nabla_b f \nabla_c q_d + 2fP^{abcd}\nabla_b \nabla_c q_d. \end{aligned} \tag{A13}$$

From the above equation we readily arrive at

$$\begin{aligned} 16\pi\{J^a(v) - fJ^a(q)\} &= 2P^{abcd}q_d \nabla_b \nabla_c f \\ &\quad + 2P^{abcd}\nabla_c f \nabla_b q_d \\ &\quad + 2P^{abcd}\nabla_b f \nabla_c q_d \\ &= P^{abcd}\nabla_b A_{cd} + 16\pi J^{ab}(q)\nabla_b f, \end{aligned} \tag{A14}$$

where we have defined the antisymmetric tensor A_{cd} as $A_{cd} = q_d \nabla_c f - q_c \nabla_d f$. Now consider the following result: $q_a \nabla_b A_{cd} = \nabla_b(q_a A_{cd}) - A_{cd}\nabla_b q_a$, which leads to

$$\begin{aligned} P^{abcd}q_a \nabla_b A_{cd} &= \nabla_b(P^{abcd}q_a A_{cd}) - 2P^{abcd}q_d \nabla_c f \nabla_b q_a \\ &= \nabla_b(P^{abcd}q_a A_{cd}) - 16\pi q_a J^{ab}(q)\nabla_b f. \end{aligned} \tag{A15}$$

Then Eq. (A14) can be rewritten in the following manner:

$$\begin{aligned} 16\pi\{q_a J^a(fq) - fq_a J^a(q)\} &= 16\pi J^{ab}(q)\nabla_b f q_a \\ &\quad + \nabla_b(P^{abcd}q_a A_{cd}) \\ &\quad - 16\pi q_a J^{ab}(q)\nabla_b f \\ &= \nabla_b(2P^{abcd}q_a q_d \nabla_c f). \end{aligned} \tag{A16}$$

It can be easily verified that in the general relativity limit $P^{abcd} = Q^{abcd} = (1/2)(g^{ac}g^{bd} - g^{ad}g^{bc})$, under which the above equation reduces to Eq. (A6).

Applying the above equation to $u_a = -N\nabla_a t$ with $q_a = \nabla_a t = -u_a/N$ and $f = -N$, we arrive at

$$16\pi u_a J^a(u) = 2N\nabla_b\left(P^{abcd}u_a u_d \frac{\nabla_c N}{N^2}\right). \tag{A17}$$

In order to proceed, we define a new vector field such that

$$\begin{aligned} \chi^a &= -2P^{abcd}u_b u_d \frac{\nabla_c N}{N} \\ &= -2P^{abcd}u_b u_d\left(a_c - \frac{1}{N}u_c u^j \nabla_j N\right) \\ &= -2P^{abcd}u_b a_c u_d. \end{aligned} \tag{A18}$$

Note that in the general relativity limit this vector reduces to the acceleration four vector as follows:

$$\chi^a = -2P^{abcd}u_b a_c u_d = -(g^{ac}g^{bd} - g^{ad}g^{bc})u_b a_c u_d$$
$$= -u^b u_b a^a + u^b a_b u^a = a^a. \quad \text{(A19)}$$

Also just as in the case of acceleration for the vector χ^a as well we have

$$u_a\chi^a = -2aP^{ab\beta d}u_a u_b r_\beta u_d = 0, \quad \text{(A20)}$$

where antisymmetry of P^{abcd} in the first two components has been used. We can also have the following relation for the vector field χ^a:

$$Na_b\chi^b = \chi^b\nabla_b N + \chi^b u_b u^j\nabla_j N = \chi^b\nabla_b N, \quad \text{(A21)}$$

where we have used the relation $u_a\chi^a = 0$ from Eq. (A20). Thus, Eq. (A17) can be written in terms of the newly defined vector field χ^a in the following way:

$$16\pi u_a J^a(u) = N\nabla_b\left(\frac{\chi^b}{N}\right)$$
$$= \nabla_b\chi^b - \frac{\nabla_b N}{N}\chi^b$$
$$= D_\alpha\chi^\alpha. \quad \text{(A22)}$$

The last relation follows from the fact that

$$D_\alpha\chi^\alpha = D_b\chi^b = \nabla_b\chi^b - a_b\chi^b = \nabla_b\chi^b - \frac{\nabla_b N}{N}\chi^b. \quad \text{(A23)}$$

Then it is straightforward to get the Noether current for ξ^a by using $q_a = u_a$ and $f = N$ in Eq. (A16) with Eq. (A22) as

$$16\pi u^a J_a(\xi) = 16\pi N u_a J^a(u) + \nabla_b(N\chi^b)$$
$$= ND_\alpha\chi^\alpha + \nabla_b(N\chi^b)$$
$$= D_\alpha(2N\chi^\alpha). \quad \text{(A24)}$$

Here also we have used the following identity:

$$D_\alpha(N\chi^\alpha) = (g^{ij} + u^i u^j)\nabla_i(N\chi_j)$$
$$= \nabla_i(N\chi^i) + u^i u^j\nabla_i(N\chi_j)$$
$$= N\nabla_i\chi^i + N\chi^i a_i - N\chi^j(u^i\nabla_i u_j)$$
$$= N\nabla_i\chi^i. \quad \text{(A25)}$$

Thus, we have derived the desired relation for the Noether current of the vector field ξ_a, and it turns out to have an identical structure as that of general relativity action with χ^a playing the role of four acceleration.

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