PHYSICAL REVIEW D **90,** 123542 (2014)

Baryon asymmetries in a natural inflation model

Nan Li* and Ding-fang Zeng†

Theoretical Physics Division, College of Applied Sciences, Beijing University of Technology, Beijing 100124, China

(Received 4 August 2014; published 30 December 2014)

A variation of the Affleck-Dine mechanism was proposed to generate the observed baryon asymmetry by Hertzberg and Karouby [Phys. Rev. D 89, 006523 (2014); Phys. Lett. B 737, 34 (2014)], in which the inflaton was assumed to be a complex scalar field with a weakly broken $U(1)$ symmetry, and the baryon asymmetry generation was easily unified with the stage of inflation and reheating. We adapt this mechanism to a general natural inflation scenario and compare the results with those in chaotic inflation models. We compute the net particle number obtained at the end of inflation and transform it into a net baryon number after reheatings. We observed that in our natural inflation model, the desired baryon-to-photon ratio can be achieved equally well as in chaotic models.

DOI: 10.1103/PhysRevD.90.123542 PACS numbers: 98.80.Ft, 98.80.Bp, 98.80.Cq

I. INTRODUCTION

The past 35 years may be the most rapidly developing 35 years for cosmologists. With the help of quickly increasing data from observations, people now have established the so-called standard model of the Universe. According to this model, the early universe experiences a very short time of inflation [1,2], after which matter and antimatter begin to form simultaneously through reheatings. If nothing special happens, the amount of matter and antimatter should be equal. But observations indicate that there is more matter than antimatter in the Universe, the so-called baryon asymmetry. Quantitatively, this is parametrized by the baryon-to-photon ratio η, whose observation value reads

$$\eta_{\rm obs} \approx 6 \times 10^{-10}. \tag{1}$$

It is unreasonable to explain this asymmetry as the initial condition of universe evolution because after inflations any preinflation particle's number density would be diluted to zero so any asymmetries between matter and antimatter would be wiped out totally. So, to implement the observed the baryon asymmetry, one must invoke some mechanism to generate a net baryon number after the inflation.

In 1967, Sakharov [3] came up with three conditions that processes which can produce the baryon asymmetry should satisfy:

(i) The process violates baryon charge conservation.
(ii) The process violates C and CP invariance.
(iii) The process should take place in a nonequilibrium thermodynamic state.

The first condition comes directly, and the second is for the decay of the particles and antiparticles to produce different numbers of baryons and antibaryons. The third is mainly to prevent the inverse process from annihilating the baryon asymmetry.

Among the large number of theories trying to describe the baryon asymmetry, the most interesting one may be the Affleck-Dine mechanism [4], which uses scalar field dynamics to get a net baryon number. Their basic idea is, in a matter or a radiation dominated universe, introducing a complex scalar field with $U(1)$-symmetry broken self-interaction and letting the evolution of the scalar field produce the desired baryon asymmetry. In Ref. [5], by associating with the inflation scenario, Linde give a more physical realization for this mechanism. For more interesting and important discussions about the Affleck-Dine mechanism and baryogenesis, see Refs. [6–18].

In both Affleck-Dine's original work and Linde's improvements, the complex scalar field is identified with the classical squark-slepton scalar field or their avatar, neither of which has direct relevance with inflatons. But in recent works [19,20], Hertzberg and Karouby proposed the idea that the complex scalar field should just be the inflaton field ϕ. So the nonzero net ϕ-particle number is generated just during the latter stage of inflation. While at the end of inflation, by reheating process, the net ϕ particles decay into baryons, thus achieving the desired baryon asymmetry.

Hertzberg and Karouby illustrated their idea with the chaotic inflation scenario [21]. However, ignited by the recent BICEP2 observation [22], more focus of the community is attracted to the "natural inflation" [23] scenario. The natural inflation model is favored by its "naturalness" in physical realizations. As is well known [24], to solve the horizon-, flatness-, and other related questions, any successful slow-roll single-field inflation model must satisfy

$$\chi \equiv \Delta V/(\Delta\phi)^4 \le \mathcal{O}(10^{-6} \sim 10^{-8}), \tag{2}$$

*linan11122@tom.com
†dfzeng@bjut.edu.cn

where ΔV and $\Delta\phi$ are the changes of potential and field, respectively, during the inflation era. This small ratio of mass scales required is known as the fine-tuning problem in inflation. It quantifies how flat the inflaton potential should be. In the natural inflation, the flatness of the potential is easily achieved by a shift symmetry under which $\phi \to \phi + \text{const}$. Our purpose in this paper is just to adapt Hertzberg and Karouby's idea to the natural inflation model.

This paper is organized as follows: In Sec. II we introduce the natural inflation model with complex scalar fields and illustrate the process of inflaton asymmetries generation. In Sec. III we let the inflaton decay into baryons and give our final results on baryon asymmetries. Section IV is a summary of our work and some discussions.

II. NET ϕ-PARTICLE GENERATIONS IN NATURAL INFLATION MODELS

Let us begin our investigation from the simplest complex scalar field inflaton models, whose action has the form

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}\mathcal{R} + \frac{1}{2}|\partial\phi|^2 - V(\phi,\phi^*)\right], \quad (3)$$

where g is the determinant of the metric, $\mathcal{R}$ is the Ricci scalar, and $V(\phi,\phi^*)$ is the effective potential of inflaton field ϕ. In the current paper we will use the normal flat Friedmann-Robertson-Walker (FRW) metric with signatures $(+---)$ and natural units $\hbar = c = 1$. Differences among various inflation models root in their potential function V. In the usual natural inflation model, the potential or the lowest order approximation of the potential is generally of the form

$$V(\phi) = \Lambda^4(1 \pm \cos(N\phi/f)), \quad (4)$$

where ϕ is the real scalar field; f is the characteristic scale of global symmetry spontaneously breaking; Λ is a lower scale associating with some explicit soft symmetry breaking; the choice of the sign does not affect the physical results, and we will choose the negative sign in this paper; and the coefficient N is always assumed to be equal to 1.

For our purpose in this paper, we will take ϕ as a complex scalar field whose self-interaction potential has the form

$$V(\phi,\phi^*) = V_s(\phi) + V_b(\phi,\phi^*) \quad (5)$$

with

$$V_s(\phi) = \Lambda^4\left[1 - \cos\left(\frac{|\phi|}{f}\right)\right]. \quad (6)$$

$$V_b(\phi,\phi^*) = \lambda(\phi^n + \phi^{*n}), \quad (7)$$

where the integer $n \geq 3$ and λ is a symmetry breaking parameter. Obviously, V_s preserves the global $U(1)$ symmetry and V_b breaks it down. Although the cross terms like $\phi^{n-m}\phi^{*m} + \phi^{*n-m}\phi^m$ also break the $U(1)$ symmetry, we will not consider them for simplicity.

Similar to the usual model (4) where ϕ is just a real field, our potential model (6) with ϕ being a complex field can also be implemented through the global symmetry breaking and some nonperturbative effects [25]. For example, consider the following global symmetry breaking model:

$$\mathcal{L} = K[\phi,\psi] - \left[(\phi^a\phi^a)^2 - \frac{f^2}{2}\right]^2 + \bar{\psi}_i^\ell t_{ij}^a\psi_j^r\phi^a, \quad (8)$$

where ϕ, ψ, $K[\phi,\psi]$, and t_{ij}^as denote the adjoint boson, fundamental fermion, the corresponding kinetic energy, and fundamental generators of the group, respectively. Physically, this symmetry could be the SU(2) symmetry of 2-flavor QCD or the SU(N) symmetry of some grand unification theory. Taking SU(2) as examples, for arbitrary real function $\Theta(\phi^x,\phi^y)$, as $\langle\phi^a t^a\rangle = \phi_0^z t^z e^{i\Theta(\phi^x,\phi^y)}$, the symmetry of the system will be broken to $U(1)$. If further nonperturbative effects lead that $\langle\bar{\psi}_i^\ell t_{ij}^a\psi_j^r\rangle = \kappa^3$, we will get the goldstone boson interacting potential $V[\phi^x,\phi^y] = \kappa^3\phi_0^z e^{i\Theta(\phi^x,\phi^y)}$. When $\Theta = |\phi|$, we implement the potential (6). Obviously, the usual natural model (4) is covered by (6) as a special example. However, only in a complex field model do we have global $U(1)$ symmetry and the relevant conserving current whose zero component corresponds to the net ϕ-particle number $N_\phi - N_{\bar{\phi}}$.

Returning to the discussion of our model (5), the smallness of λ is natural in physics by 't Hooft's criteria [26]: a small parameter in a theory is natural if, when the limit is set to zero, the symmetry of the system increases. Obviously, when $\lambda = 0$, the $U(1)$ symmetry is recovered, and the symmetry of the system increases. We also need the smallness of λ to preserve the shape of the potential for inflaton; otherwise, the character of the natural inflation would be destructed. From the observation aspect, a small value of λ is also favored by small baryon-to-photon ratios, because λ is just the measure of the $U(1)$-symmetry breaking degree that is responsible for the net particle number's generation.

It is worth mentioning that despite the $U(1)$ symmetry being broken by V_b, the charge conjugation symmetry $\phi \leftrightarrow \phi^*$ is still respected. We assume that this symmetry is broken in the following process, or the Sakharov's conditions would be violated. In the original Affleck-Dine mechanism, it is spontaneously broken by the interaction with some other light fields. However, the detailed mechanism is not important to us, and since it does not affect our results, we will not discuss it in this paper.

A. Net ϕ particles from ϕ and $\bar{\phi}$

First, noting that the function V_s is a periodic function of period $2\pi f$, we restrict the value of $|\phi| \in [0, \pi f]$. Second, using the fact that V_s takes the minimum at $|\phi| = 0$, we make the Taylor expansion of it at this point as $V_s \approx \frac{1}{2}\Lambda^4 \left(\frac{|\phi|}{f}\right)^2$ when $|\phi|$ is small. Third, since $n \geq 3$ in V_b, at the late time of inflation during which $|\phi|$ is small, V_b decreases faster than V_s so it soon becomes negligible. As a result, the effective potential of the inflaton at later times conserves the global $U(1)$ symmetry. According to Noether's theorem, we can derive the conserving charge as the net particle number,

$$\Delta N_\phi = N_\phi - N_{\bar{\phi}} = i \int d^3x \sqrt{g_s} (\phi^* \dot{\phi} - \dot{\phi}^* \phi), \tag{9}$$

where $d^3x\sqrt{g_s}$ is the spatial volume measure, and N_ϕ and $N_{\bar{\phi}}$ are the number of ϕ and $\bar{\phi}$ particles. As the roughest approximation, we take ϕ as spatial homogeneous. Substituting the FRW metric into this definition, we can work out the integral and get

$$\Delta N_\phi = N_\phi - N_{\bar{\phi}} = i V_{\mathrm{com}} a(t)^3 (\phi^* \dot{\phi} - \dot{\phi}^* \phi), \tag{10}$$

where V_{com} is the comoving volume and $a(t)$ is the scale factor.

To get the equation of motion for ϕ, we vary the total action of the system with respect to ϕ^* and get

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda^4}{f} \sqrt{\frac{\phi}{\phi^*}} \sin\left(\frac{|\phi|}{f}\right) + 2\lambda n \phi^{*n-1} = 0, \tag{11}$$

where $H = \dot{a}/a$ is the Hubble parameter. Taking the time derivative of ΔN_ϕ,

$$\frac{\partial}{\partial t} \Delta N_\phi = i V_{\mathrm{com}} a^3 (3H(\phi^* \dot{\phi} - \dot{\phi}^* \phi) + \phi^* \ddot{\phi} - \ddot{\phi}^* \phi), \tag{12}$$

and substituting the results into an appropriate combination of (11) with its complex conjugate, we will get

$$\Delta N_\phi(t_f) = \Delta N_\phi(t_i) + 2i\lambda V_{\mathrm{com}} n \int_{t_i}^{t_f} dt a(t)^3 (\phi(t)^n - \phi^*(t)^n), \tag{13}$$

where $\Delta N_\phi(t_i)$ is the initial net particle number at time t_i, while t_f denotes the final time. From this equation, we easily see that when the $U(1)$ symmetry is unbroken, i.e., $\lambda \to 0$, the net particle number will indeed be conserved. Since any initial particle number would be diluted by inflation, and the process we are interested in happens at the late time of inflation, we will set $\Delta N_\phi(t_i) = 0$ from now on.

For convenience in the latter derivations, we express the scalar field ϕ in the polar coordinate as

$$\phi(t) = \Phi(t) e^{i\theta(t)} \tag{14}$$

and rewrite the net particle number ΔN_ϕ in the form

$$\Delta N_\phi(t_f) = -4\lambda V_{\mathrm{com}} n \int_{t_i}^{t_f} dt a(t)^3 \Phi(t)^n \sin(n\theta(t)). \tag{15}$$

Like the fields ϕ and ϕ^*, the polar field Φ and angular field θ also satisfy differential equations similar to (11), which can be solved order by order in λ. By Eq. (15), ΔN_ϕ is proportional to λ. So if we need to calculate ΔN_ϕ only to first order approximation, which is reasonable when λ is small, then we need to calculate the integral only to the zeroth order in λ. It can be proved that the evolution of θ is determined by the symmetry breaking term. So when we neglect the effect of λ, θ does not evolve at all, i.e., $\dot{\theta} = 0$. For this reason, the factor $\sin(n\theta(t))$ in (14) can be extracted out of the integrations. Using $\Phi_0(t)$ and $a_0(t)$ to denote $\Phi(t)$ and $a(t)$ when we neglect the effect of λ in the equations of motion, we can write ΔN_ϕ as the form

$$\Delta N_\phi(t_f) = -4\lambda V_{\mathrm{com}} n \sin(n\theta_i) \int_{t_i}^{t_f} dt a_0(t)^3 \Phi_0(t)^n, \tag{16}$$

where θ_i is the initial value of θ.

The equation of motion for Φ_0 is easy to derive,

$$\ddot{\Phi}_0 + 3H_0 \dot{\Phi}_0 + \frac{\Lambda^4}{f} \sin\left(\frac{\Phi_0}{f}\right) = 0, \tag{17}$$

while the corresponding Friedmann equation for H_0 reads

$$H_0^2 = \frac{8\pi}{3m_{Pl}^2} \left(\frac{1}{2}\dot{\Phi}_0^2 + \Lambda^4 \left(1 - \cos\left(\frac{\Phi_0}{f}\right)\right)\right), \tag{18}$$

where $m_{Pl} \equiv 1/\sqrt{G} = 1.22 \times 10^{19}$ GeV is the Plank mass. By these two equations of motion, supplemented with appropriate initial conditions, we will be able to calculate the integral in Eq. (15) very fluently.

B. The value of Λ and f from observations

According to Ref. [27], to be consistent with known cosmic-microwave background observations such as WMAP [28], Planck [29,30], and BICEP2 [22], the Λ and f parameters in the natural inflation should satisfy that $f \gtrsim m_{Pl}$ and $\Lambda \sim m_{\mathrm{GUT}} \sim 10^{16}$ GeV. Although the natural inflation in this paper is implemented with complex scalar fields, the parameter determination logic could be adapted from [27] routinely.

(i) Constraints from the density perturbation spectrum index n_s. In both real and complex scalar fields, we can derive that

$$n_s = 1 - \frac{m_{Pl}^2}{8\pi f^2}. \tag{19}$$

According to Ref. [30], the observation value of $n_s \approx 0.96$. This means that in the complex scalar field natural inflation model, $f \approx m_{Pl}$.

(ii) Constraints from the tensor-to-scalar (perturbation amplitudes) ratio. Theoretical considerations [27] require

$$V_H = (2.2 \times 10^{16}\ \text{GeV})^4 \frac{r}{0.2}, \tag{20}$$

for natural inflation models $V_H = 2\Lambda^4$. According to the observation of BICEP2, $r = 0.20^{+0.07}_{-0.05}$. So $\Lambda \approx 10^{16}$ GeV are very normal choices.

According to Ref. [25], the number of inflation e-foldings to solve the flatness and horizon problem of noninflation cosmologies also implies constraints on the choice of model parameters Λ and f. Its basic logic is as follows.

First, according to the slow-roll scenario, the number of inflation e-foldings reads

$$\begin{aligned} N_e = \ln\left(\frac{a_2}{a_1}\right) &= \int_{t_1}^{t_2} H dt = \frac{8\pi}{m_{Pl}^2} \int_{\phi_2}^{\phi_1} \frac{V(\phi)}{V'(\phi)} d\phi \\ &= \frac{8\pi f^2}{m_{Pl}^2} \ln\left[\frac{1+\cos(|\phi_2|/f)}{1+\cos(|\phi_1|/f)}\right], \end{aligned} \tag{21}$$

where a_1 and ϕ_1 are initial values when the inflation begins, a_2 and ϕ_2 are the values at the end of inflation, and V' denotes $dV/d\phi$.

Second, to associate N_e with f, define a "possibility" $P(f)$ to quantify whether a given f value is likely to generate sufficient inflation that $N_e \approx 60$,

$$P(f) = \frac{\pi f - \phi_{\min}(f)}{\pi f}, \tag{22}$$

where $\phi_{\min}(f)$ is the minimal value of $|\phi_1|$ that gets $N_e \geq 60$ for a given f. Obviously, as long as $\phi_{\min}$ can drive sufficient inflation, all values of $|\phi_1| \in (\phi_{\min}, \pi f)$ will yield $N_e > 60$, as shown in Fig 1.

Third, using the slow-roll parameter definition

$$\epsilon = \frac{m_{Pl}^2 V'(\phi)^2}{16\pi V(\phi)^2} = \frac{m_{Pl}^2}{16\pi f^2}\left[\frac{\sin(\phi/f)}{1-\cos(\phi/f)}\right]^2, \tag{23}$$

and the inflation ending condition $\epsilon \approx 1$, we can get the ϕ field value at the inflation ending,

$$\phi_2(f) = f \arccos\left[\frac{16\pi f^2 - m_{Pl}^2}{16\pi f^2 + m_{Pl}^2}\right]. \tag{24}$$

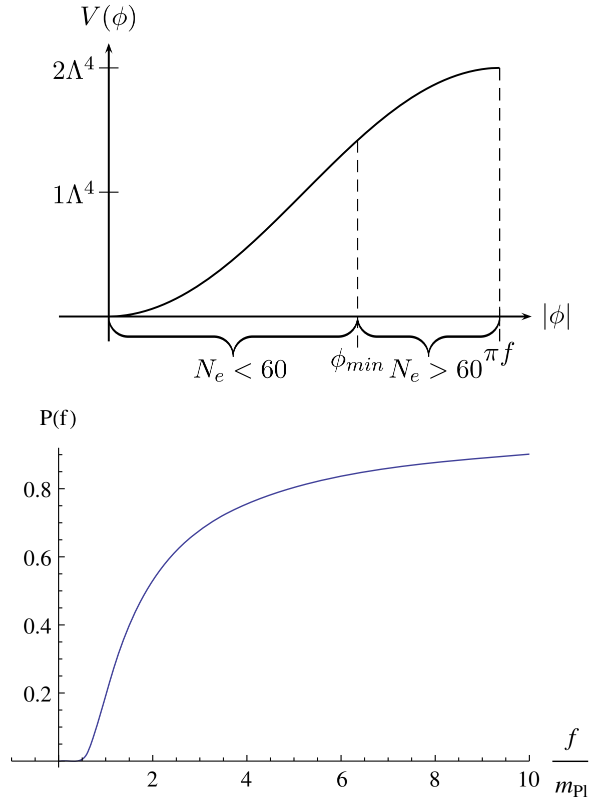

FIG. 1 (color online). The upper figure illuminates the shape of $V(\phi)$. It is clear that a higher potential generates a larger e-folding number, so $|\phi_1| \in (\phi_{\min}, \pi f)$ would get $N_e > 60$. The lower figure shows the numeric feature of $P(f)$, from which we easily see that the larger f is, the closer $P(f) \to 1$.

Finally, combining Eqs. (20), (21), and (23) we can exactly work out $P(f)$, and the result is shown in Fig. 1. While from the definition of $P(f)$, we know that to get enough inflation e-foldings, $P(f)$ should be as close as possible to 1. For $f = m_{Pl}$, we get $P(f) = 0.194$, which is grudgingly in the desired range. More large values of f will generate sufficient inflation.

In the following calculations, we will set $f = m_{Pl}$ and $\Lambda = 10^{16}$ GeV when necessary.

C. Dimensionless representation

Since ΔN_ϕ is proportional to the size of the expanding universe, it is not a good quantity for numerics, even though it is dimensionless. The more appropriate quantity measuring the baryon asymmetry is

$$\alpha \equiv \frac{\Delta N_\phi}{N_{\text{tot}}} = \frac{\Delta n_\phi}{n_\phi + n_{\bar\phi}}, \tag{25}$$

where N_{tot} is the total number of ϕ and $\bar\phi$ particles and $n = N/V_{\text{com}} a^3$ stands for particle number densities. After the inflation finishes, but before the decay of ϕ particles into baryons, all energies that fill the Universe are stored in the nonrelativistic ϕ particles. So we have

$$m_\phi(n_\phi + n_{\bar\phi}) = \varepsilon_0 = \frac{1}{2}\dot{\Phi}_0^2 + \Lambda^4\left(1 - \cos\left(\frac{\Phi_0}{f}\right)\right), \tag{26}$$

where $m_\phi = \frac{\Lambda^2}{f}$ is the mass of the ϕ particle and ε_0 is the energy density of the Universe. From the above two equations, we can derive

$$\alpha = \frac{m_\phi \Delta n_\phi}{\varepsilon_0}. \tag{27}$$

To get further dimensionless representation for α, we introduce the following dimensionless quantities:

$$\tau \equiv m_\phi t = \frac{\Lambda^2 t}{f}, \qquad \tilde{\Phi} \equiv \frac{\Phi_0}{f}, \qquad \tilde{H} \equiv \frac{H_0}{m_\phi} = \frac{f H_0}{\Lambda^2}, \tag{28}$$

and write

$$\alpha = -\frac{\lambda f^n}{\Lambda^4}\sin(n\theta_i) A_n(\tau_i, \tau_f), \tag{29}$$

where τ and $\tilde{\Phi}$ are dimensionless time and field variables, respectively, while

$$A_n(\tau_i, \tau_f) = \frac{4n \int_{\tau_i}^{\tau_f} d\tau a_0(\tau)^3 \tilde{\Phi}(\tau)^n}{a_0(\tau_f)^3\left(\frac{1}{2}\dot{\tilde{\Phi}}(\tau_f)^2 + 1 - \cos\left(\tilde{\Phi}(\tau_f)\right)\right)}. \tag{30}$$

By numerically solving the dimensionless version of Eqs. (16) and (17), we will obtain the time dependence of $\tilde{\Phi}_0(\tau)$, $a_0(\tau)$ very easily; see Fig. 2 for references.

From Fig. 2, we can easily see that at the beginning of inflation, $a_0(\tau_i)$ is very small [relative to $a_0(t_f)$]. As a result, in the integration (29) contributions from the early time are negligible. While at the matter dominating era marked by τ_f, $\tilde{\Phi}$ is evolving to almost zero. So integrations from that period also contribute little to A_n; see Fig. 3 for quantitative references. From the figure, it is easy to see that A_n is totally determined by the "middle" area of the integrand. Under the limit that $\tau_i \to 0$ and $\tau_f \to \infty$, A_n can be looked at as a constant that depends only on the lower index n;

$$A_n(\tau_i \to 0, \tau_f \to \infty) \equiv c_n. \tag{31}$$

As an example, we numerically compute this parameter when $n = 3, 4, \ldots, 10$, and the result is as follows:

$$c_3 \approx 8.0, \qquad c_4 \approx 3.7, \qquad c_5 \approx 1.3, \qquad c_6 \approx 0.56$$
$$c_7 \approx 0.25, \qquad c_8 \approx 0.12, \qquad c_9 \approx 0.060, \qquad c_{10} \approx 0.031. \tag{32}$$

Substituting this results into Eqs. (29) and (30), we will get the final expression for the dimensionless baryon asymmetry parameter as follows:

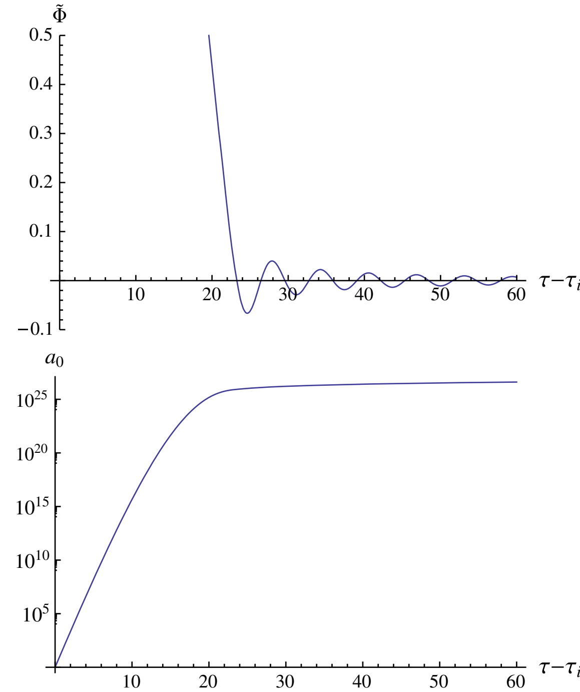

FIG. 2 (color online). The evolution of the dimensionless field variable $\tilde{\Phi}$ and scale factor a_0. In this plot, the initial conditions were set to $\tilde{\Phi} = 2.5$, $f = m_{Pl}$, and $\Lambda = 10^{16}$ GeV to implement the e-folding number $\sim$60. Without loss of generality, we set $\dot{\tilde{\Phi}} = 0$ and $a_i = 1$. The lower figure shows that indeed about 60 e-folding numbers are generated.

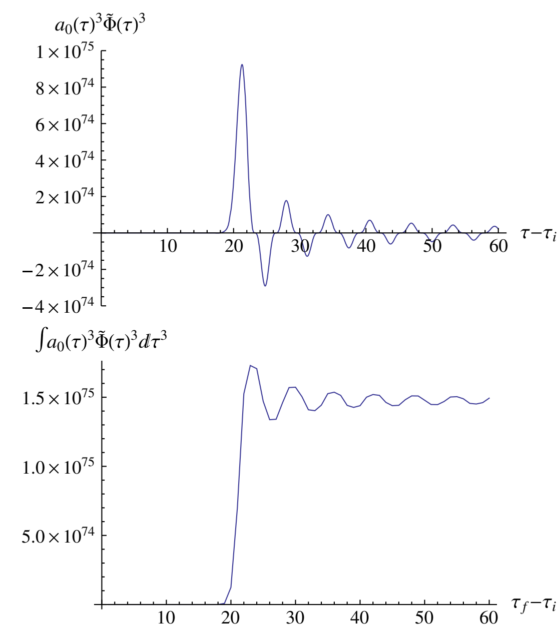

FIG. 3 (color online). The upper graph plots the integrand in Eq. (29), which is proportional to the production rate of the net particle number. The lower graph plots the integrated value and expresses the whole net particle number produced till τ_f. We can easily see that the early and late time contributions do not significantly affect the final net particle number. Almost all of the net particles are produced during a very short time around $\tau = 20$.

$$\alpha = -c_n \frac{\lambda f^n}{\Lambda^4} \sin(n\theta_i). \quad (33)$$

Obviously, for some special values of the initial angle θ_i, for instance $\theta_i = \pi$, the factor $\sin(n\theta_i)$ vanishes. In such cases, no baryon asymmetry is generated. Such special values can generate large isocurvature fluctuations [19], but that is not our main goal. We will set θ_i to be general values so that $|\sin(n\theta_i)| \approx 1$.

Now we have implemented the goal of generating net inflaton ϕ particles from symmetric initial conditions in natural inflations. Our next goal is transferring the ϕ particles into baryons and associating α with the observable baryon-to-photon ratio η in the next section.

III. ϕ PARTICLES DECAY INTO BARYONS

According to inflationary theory, at the late time of inflation, the inflaton field oscillates near the minimum of its effective potential and gradually decays into standard model particles [31]. This stage of the early universe is called "reheating." Almost all elementary particles populating the Universe are created during reheating, and these particles interact with each other and finally come to a state of thermal equilibrium at a temperature T_r, which is called the reheating temperature.

Now we assume that each ϕ particle carries a baryon number B, and it will decay into baryons through a process that conserves the baryon number during the stage of reheating. We assume that all the subsequent interactions also conserve the baryon number, so we have

$$(N_b - N_{\bar{b}})_f = B(N_\phi - N_{\bar{\phi}})_i, \quad (34)$$

where the lower index f means a final time on which reheating finishes, and i stands for an initial time in which all the energy stored in the inflaton field is translated into ϕ particles, but ϕ's decay does not begin. By these symbols, we can write down η in the following form:

$$\eta = \frac{(N_b - N_{\bar{b}})_f}{(N_\gamma)_f} = B\frac{(N_\phi - N_{\bar{\phi}})_i}{(N_\gamma)_f} = \alpha B \frac{(N_\phi + N_{\bar{\phi}})_i}{(N_\gamma)_f}. \quad (35)$$

Obviously, to calculate η, we need to work out the initial total number of ϕ particles and the photon number at late times.

At initial times, all the energy congesting the Universe is provided by ϕ particles, so

$$m_\phi (N_\phi + N_{\bar{\phi}})_i = \frac{\Lambda^2}{f}(N_\phi + N_{\bar{\phi}})_i = V_{\mathrm{com}}(a^3\varepsilon)_i. \quad (36)$$

Using the Friedmann equation, we can relate the energy density to the Hubble parameter as

$$(\varepsilon)_i = \frac{3m_{Pl}^2}{8\pi}(H^2)_i. \quad (37)$$

While on the number of photons at late times, we can relate it with the temperature,

$$(N_\gamma)_f = V_{\mathrm{com}}(a^3 n_\gamma)_f = V_{\mathrm{com}}\frac{2\zeta(3)}{\pi^2}(a^3T^3)_f, \quad (38)$$

where $\zeta(3) \approx 1.202$ is the so-called Apéry's constant. Using these two results, we can rewrite the ratio η in Eq. (35) as follows:

$$\eta = \frac{3\pi B\alpha}{16\zeta(3)} \frac{m_{Pl}^2 f}{\Lambda^2} \frac{(a^3H^2)_i}{(a^3T^3)_f}. \quad (39)$$

Since we are not going to compute the result by using detailed decaying processes, we need to assume the decay of ϕ particles and the subsequent process of thermalization occur very fast. So we can set both $(a^3H^2)_i$ and $(a^3T^3)_f$ to be values around the end of reheating and get an approximation for the final result. We insert an $\mathcal{O}(1)$ factor β to account for the deviation caused by this assumption,

$$\eta = \frac{3\beta\pi B\alpha}{16\zeta(3)} \frac{m_{Pl}^2 f}{\Lambda^2} \frac{H_r^2}{T_r^3}. \quad (40)$$

The stage of reheating ends when the Hubble parameter becomes smaller than the decay rate of ϕ, $H \lesssim \Gamma_\phi$ [31]. And the reheating temperature can be estimated by [32]: $T_r \approx 0.2\sqrt{\Gamma_\phi m_{Pl}}$. Substituting these two relations into the approximate expression of η, we obtain

$$\eta = \frac{3\pi\beta B\alpha}{0.2^3 \times 16\zeta(3)} \frac{m_{Pl}^{1/2}\Gamma_\phi^{1/2} f}{\Lambda^2}. \quad (41)$$

Inserting the α expression (33) obtained in the previous section into this equation, we will get our result for η,

$$\eta = -c_n \frac{3\pi\beta B\lambda}{0.2^3 \times 16\zeta(3)} \frac{m_{Pl}^{1/2}\Gamma_\phi^{1/2} f^{n+1}}{\Lambda^6} \sin(n\theta_i). \quad (42)$$

Before further discussion of physical features of this expression for η, we should first determine the range of the symmetry breaking parameter λ. Since we assumed that the symmetry breaking term in the potential of ϕ is subdominant during the inflation to ensure the feature of the natural inflation, we have to impose constraint

$$\lambda(\phi_i^n + \phi_i^{*n}) = \lambda\Phi_i^n \cos(n\theta_i) \ll \Lambda^4\left(1 - \cos\left(\frac{\Phi_i}{f}\right)\right). \quad (43)$$

While to assure this constraint holds for all the possible values of θ_i, the value of λ has to be limited from the upper bound,

$$\lambda \ll \lambda_{\max} = \frac{\Lambda^4(1-\cos(\Phi_i/f))}{\Phi_i^n}. \quad (44)$$

With this constraint, we can test whether our result is physically acceptable.

Now we use the boundary value of λ to work out the required Γ_ϕ for generating the observed $\eta \approx 6\times 10^{-10}$. The expression of $\Gamma_{\phi,\mathrm{req}}$ can be derived from Eq. (41),

$$\Gamma_{\phi,\mathrm{req}} \approx c_n^{-2}\left(\frac{\lambda_{\max}}{\lambda}\right)^2 \frac{\eta^2(0.2^3\times 16\zeta(3)^2)}{(3\pi)^2} \times \frac{\Lambda^4 f^{-2n-2}\Phi_i^{2n}}{m_{Pl}(1-\cos(\frac{\Phi_i}{f}))^2}(\beta B|\sin(n\theta_i)|)^{-2}. \quad (45)$$

In the previous sections, we have set $f = m_{Pl}$, $\Lambda = 10^{16}$ GeV, and $\Phi_i = 2.5f$, so the expression can be simplified to

$$\Gamma_{\phi,\mathrm{req}} \approx 1.6\times 10^{-7}\ \mathrm{eV} \times 2.5^{2n} c_n^{-2}\left(\frac{\lambda_{\max}}{\lambda}\right)^2 (\beta B|\sin(n\theta_i)|)^{-2}. \quad (46)$$

If we set an appropriate value for λ, for example, $\lambda = \frac{1}{10}\lambda_{\max}$, and assume that $\beta B|\sin(n\theta_i)| \approx 1$, we will get the required decay rate for different ns. Our results, and those from [19] with a magnitude correction for comparisons, are shown as follows:

n	$\Gamma_{\phi,\mathrm{req}}$in natural inflation	$\Gamma_{\phi,\mathrm{req}}$in chaotic inflation
$n=3$	6.1×10^{-5} eV	4×10^{-3} eV
$n=4$	1.8×10^{-3} eV	2×10^{-1} eV
$n=5$	9.0×10^{-2} eV	10^1 eV
$n=6$	3.0 eV	6×10^2 eV
$n=7$	9.5×10 eV	2×10^4 eV
$n=8$	2.6×10^3 eV	9×10^5 eV
$n=9$	6.5×10^4 eV	3×10^7 eV
$n=10$	1.5×10^6 eV	10^9 eV

Obviously, a larger power n of symmetry breaking interaction requires a larger decay width to give the desired photon-to-baryon ratio, while from derivations (42)–(46), we know that $\Gamma_\phi \propto \Lambda^4$; that is, the higher energy scale of inflation requires a larger width of inflaton decays, or otherwise, the theoretical photon-to-baryon ratio will deviate remarkably from expectations.

With these results of $\Gamma_{\phi,\mathrm{req}}$, we can work out the corresponding reheating temperature using the relation $T_r \approx 0.2\sqrt{\Gamma_\phi m_{Pl}}$. An important condition is that the reheating temperature must be higher than the typical temperature of big bang nucleosynthesis ~MeV. For the lowest value of $\Gamma_{\phi,\mathrm{req}}$ in natural inflation, when $n=3$, the reheating temperature $T_r \approx 173$ GeV, and for $n=10$, the corresponding $T_r \approx 2.7\times 10^7$ GeV. All the T_r's in our model are much higher than MeV, this is obviously consistent with the big bang nucleosynthesis, and thus is physically acceptable.

For all values of n listed above, $\Gamma_{\phi,\mathrm{req}}$ in natural inflations is smaller than that in chaotic inflations, and the growth of $\Gamma_{\phi,\mathrm{req}}$ with the increasing of n is slower than in chaotic inflation. These differences may be used to distinguish these two models in the future.

IV. SUMMARY AND DISCUSSION

In this paper, we apply a variation of the Affleck-Dine mechanism to a general natural inflation scenario and generate the observed baryon-to-photon ratio. In that mechanism, the process of baryon asymmetry generation is unified with the stage of inflation and reheating. The mechanism is originally set up in chaotic inflation scenarios, and we apply it to a generalized natural inflation. In our natural model, we use a global SU(N) symmetry breaking to implement a complex goldstone field whose self-interaction has the form $\Lambda^4(1-\cos|\phi|/f)$. This symmetry breaking pattern may occur in QCD or some grand unification theories, while the usual $U(1)$-symmetry breaking and real-scalar model could be looked at as a special form of our model.

The baryon asymmetry is first implemented using the inflaton field with a weakly broken global $U(1)$ symmetry. It is in the second stage that the net inflaton ϕ particles decay into standard model particles. By numerical calculations, we work out parameter α describing the asymmetric evolution of $\phi-\bar{\phi}$ particles during the natural inflation era and derive out formulas relating it with the baryon-to-photon ratio η. We calculate the decaying rate of ϕ particles required to generate the observed $\eta \approx 6\times 10^{10}$. It is observed that the reheating temperatures in this inflation model is much higher than the desired temperature of big bang nucleosynthesis. From this aspect, this model is physically acceptable. We also compare our results with those in chaotic inflation models. The differences between the two may be useful for the future distinguishing of them through observations.

As discussions, we note that parameter resonance phenomena [32], superheavy fermions production [33], detailed particle physics model implementation, dark matter particles formation and properties *etc*. in this natural inflation plus reheating mechanism are all interesting future directions.

ACKNOWLEDGMENTS

We thank very much Qian-jun Wang, Meng Sun, Jian-feng Wu, and Professor Yong-chang Huang for meaningful discussions. This work is supported by Beijing Municipal Natural Science Foundation, Grant No. Z2006015201001.

[1] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[2] A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[3] A. D. Sakharov, JETP Lett. **5**, 24 (1967).
[4] I. Affleck and M. Dine, Nucl. Phys. **B249**, 361 (1985).
[5] A. D. Linde, Phys. Lett. **160B**, 243 (1985).
[6] M. Yu. Khlopov, S. G. Rubin, and A. S. Sakharov, Phys. Rev. D **62**, 083505 (2000).
[7] M. Bastero-Gil, A. Berera, R. O. Ramos, and J. G. Rosa, Phys. Lett. B **712**, 425 (2012).
[8] M. Bastero-Gil, A. Berera, R. O. Ramos, and J. G. Rosa, J. Cosmol. Astropart. Phys. 10 (2014) 053.
[9] M. Dine, L. Randall, and S. D. Thomas, Nucl. Phys. **B458**, 291 (1996).
[10] M. Dine, L. Randall, and S. D. Thomas, Phys. Rev. Lett. **75**, 398 (1995).
[11] K. Enqvist and J. McDonald, Phys. Rev. Lett. **83**, 2510 (1999).
[12] K. Koyama and J. Soda, Phys. Rev. Lett. **82**, 2632 (1999).
[13] A. Mazumdar and A. Perez-Lorenzana, Phys. Rev. D **65**, 107301 (2002).
[14] R. Allahverdi, M. Drees, and A. Mazumdar, Phys. Rev. D **65**, 065010 (2002).
[15] O. Seto, Phys. Rev. D **73**, 043509 (2006).
[16] S. Kasuya and M. Kawasaki, Phys. Rev. D **74**, 063507 (2006).
[17] B. Dutta and K. Sinha, Phys. Rev. D **82**, 095003 (2010).
[18] D. Marsh, J. High Energy Phys. 05 (2012) 041.
[19] M. P. Hertzberg and J. Karouby, Phys. Rev. D **89**, 063523 (2014).
[20] M. P. Hertzberg and J. Karouby, Phys. Lett. B **737**, 34 (2014).
[21] A. D. Linde, Phys. Lett. **129B**, 177 (1983).
[22] P. Ade *et al.* (BICEP2 Collaboration), Phys. Rev. Lett. **112**, 241101 (2014).
[23] K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).
[24] F. C. Adams, K. Freese, and A. H. Guth, Phys. Rev. D **43**, 965 (1991).
[25] F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. D **47**, 426 (1993).
[26] G. 't Hooft, in *Recent Developments in Gauge Theories*, edited by G. 't Hooft *et al.* (Plenum Press, New York and London, 1979), p. 135.
[27] K. Freese and W. H. Kinney, arXiv:1403.5277.
[28] G. Hinshaw *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **208**, 19 (2013).
[29] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **571**, A16 (2014).
[30] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **571**, A22 (2014).
[31] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994).
[32] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. D **56**, 3258 (1997).
[33] P. B. Greene and L. Kofman, Phys. Rev. D **62**, 123516 (2000).