Bounds on QCD axion mass and primordial magnetic field from cosmic microwave background μ distortion

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The oscillation of the cosmic microwave background (CMB) photons into axions can cause CMB spectral distortion in the presence of a large-scale magnetic field. With the COBE collaboration limit on the μ parameter and a homogeneous magnetic field with strength $B \lesssim 3.2$ nG at the horizon scale, a stronger lower limit on the axion mass in comparison with the limit of the ADMX experiment is found to be 4.8×10^{-5} eV $\lesssim m_a$ for the Kim-Shifman-Vainshtein-Zakharov (KSVZ) axion model. On the other hand, using the experimental limit on the axion mass 3.5×10^{-6} eV $\lesssim m_a$ from the ADMX experiment together with the COBE bound on μ, $B \lesssim 53$ nG is found for the KSVZ axion model and $B \lesssim 141$ nG for the Dine-Fischler-Sdrenicki-Zhitnitsky (DFSZ) axion model, for a homogeneous magnetic field with coherence length at the present epoch $\lambda_B \sim 1.3$ Mpc. Limits on B and m_a for PIXIE/PRISM expected sensitivity on μ are derived. If CMB μ distortion would be detected by the future space missions PIXIE/PRISM and assuming that the strength of the large-scale magnetic field is close to its canonical value, $B \sim$ 1–3 nG, axions in the mass range 2–3 μeV would be potential candidates of CMB μ distortion.

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The cosmic microwave background (CMB) presents small temperature anisotropy of the order of $\delta T/T \sim 10^{-5}$ on a small angular scale, and its spectrum is supposed to be slightly distorted [1] due to various mechanisms that might have operated in the early Universe. In general, these distortions are described in the terms of the so-called μ, i, and y parameters of which their values quantify the type of each distortion [2]. The COBE [3] space mission obtained stringent limits on $|\mu| < 9 \times 10^{-5}$ and $|y| < 1.5 \times 10^{-5}$ parameters, thus implying that there might be a very narrow window in which to look for the process leading to spectral distortion. Other planned space missions include PIXIE [4] and PRISM [5], which expect to reach better sensitivity on μ and y with respect to COBE of the order of $\mu \simeq 5 \times 10^{-8}$ and $y \simeq 10^{-8}$.

Generally speaking, the most popular proposed mechanisms that can create spectral distortion can be classified as "secondary" mechanisms in the sense that the original CMB spectrum is affected indirectly. Indeed, in these models, the energy and photon number are injected into the medium from external sources such as decaying dark matter particles [6], sound waves [7], etc. On the other hand, CMB can also have "primary" spectral distortions that can be disentangled from the secondary ones. An interesting mechanism that can be classified as primary is oscillation of the CMB photons into light bosons such as axions, axionlike particles (ALPs), and gravitons. These processes, in the cosmological context, are possible in the presence of an external magnetic field in which the photon has a vertex coupling with them. In the case of axions, the relevant term that describes the coupling of photons with axions is given by the interaction Lagrangian density

$$\mathcal{L}_{a\gamma} = -\frac{g_{a\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}, \tag{1}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor, $\tilde{F}^{\mu\nu}$ is its dual, and a is the axion field. In general, the coupling constant of axions can be written as

$$g_{a\gamma} = \frac{\alpha_s}{2\pi f_a}\left(\frac{E}{N} - \frac{2}{3}\frac{4+w}{1+w}\right), \tag{2}$$

where α_s is the fine structure constant, f_a is the axion decay constant, E is the electromagnetic anomaly associated with the axial current, and N is the color anomaly. Among all of the axion models, two of them, namely, the KSVZ [8] and DFSZ [9] axion models, have been extensively studied in the literature. For the KSVZ model, we have $E/N = 8/3$ and $E/N = 0$ for the DFSZ model. In both models, the coupling constant of axions to photons $g_{a\gamma}$ is proportionally related to axion mass m_a. The latter is related with quark masses up (u) and down (d), and the relation between axion mass m_a and axion decay constant f_a is given by

$$m_a = \frac{m_\pi f_\pi}{f_a}\frac{w^{1/2}}{1+w}, \tag{3}$$

where $m_\pi = 135$ MeV is the pion mass, $f_\pi \simeq 92$ MeV is the pion decay constant, and $w = m_u/m_d$ with m_u, m_d being respectively the up- and down-quark masses. The

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range of the parameter w is between $0.35 \leq w \leq 0.6$ [10], where in general its standard value is taken as $w = 0.56$. For recent reviews on axions and ALPs, see Ref. [11], and for earlier works on axions in cosmology, see Ref. [12].

The origin of the large-scale magnetic field (which makes possible the transition of photons into axions) is interesting by itself since its presence would have an enormous impact in several situations in cosmology (such as bing bang nucleosynthesis, CMB temperature anisotropy, etc.) and in astrophysics (such as cosmic rays deflection, etc). Thus, its strength B_e and its direction are of fundamental importance. The most common ways to constrain large-scale magnetic field strength have been essentially from CMB temperature anisotropy and Faraday rotation of the CMB [13]. In the former case, it is supposed that the external magnetic field would contribute to the total energy density of the Universe, and therefore it would be possible that this additional energy density could cause CMB temperature anisotropy [14]. In the latter case, the presence of the magnetic field would cause polarization of the CMB, through the so-called Faraday effect, namely, the rotation of the polarization plane of the CMB. It has also been shown that the Faraday effect can be induced by a coupling of a quintessential background field with pseudoscalar coupling to the CMB; see Ref. [15] (for a link between the Faraday effect and CMB B-mode polarization, see Ref. [16]). For a review on large-scale magnetic fields, see Ref. [17].

In a previous work [18], we obtained tight limits on the ALP parameter space by using coupling of CMB photons with ALPs in a primordial magnetic field. In this work, we study the oscillation of CMB photons into axions in the presence of a large magnetic field and derive new limits on axion mass and magnetic field strength. Photon-axion mixing is phenomenologically different from oscillation into ALPs, since in the axion case the two quantities that characterize axions, its mass m_a and coupling constant to photons $g_{a\gamma}$, are directly proportional with each other. Consequently, in the case of photon-axion mixing, the number of independent parameters is reduced to only B_e and m_a or $g_{a\gamma}$ with respect to the photon-ALP mixing. Therefore, based on phenomenological or experimental results, it would be possible that, knowing one of the parameters B_e or m_a, we can constrain the remaining one.

First, knowing the upper bounds on the magnetic field strength at the present time, we can find limits for the axion mass. In this case, the field strength and coherence length are fixed *a priori*. Second, if we know the experimental limits on the axion mass, we can bound the magnetic field strength and discuss its coherence length *a posteriori*. In this work, we consider only a uniform (homogeneous) magnetic field. The effect on the CMB oscillation due to a nonhomogeneous (stochastic) magnetic field will not be considered. In connection with the first case, we use limits on the magnetic field from the CMB temperature anisotropy and Faraday rotation, where the field coherence length is greater than or comparable to the horizon scale. For a magnetic field with coherence length comparable to the horizon scale, CMB temperature anisotropy gives $B \lesssim 4$ nG [19], and Faraday rotation of the Lyman-α forest gives [20] $B \lesssim 1$ nG. As far as the second case, we consider existing limits on the axion mass to constrain the strength of the homogeneous magnetic field with coherence length at least comparable to the horizon scale during the μ epoch. In the formalism of the density matrix that we use below, the magnetic field is assumed to be homogeneous at given coherence length λ_B, where the field strength changes only due to the expansion of the Universe. Here, we adopt the rationalized Lorentz–Heaviside natural units, $c = \hbar = k_B = \epsilon_0 = \mu_0 = 1$.

The study of the oscillation of the CMB photons into axions with an essential loss of coherence is best formulated in terms of the density operator of the system $\hat{\rho}$ (in our case, the system is composed of axions and photons). To the linear order of approximation, it satisfies the quantum kinetic equation [21]

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] - \{\hat{\Gamma}, (\hat{\rho} - \hat{\rho}_{\rm eq})\}, \tag{4}$$

where $\hat{H}$ is the Hamiltonian of the photon-axion system including a refraction index (first-order effects), $\hat{\Gamma}$ is the coherence breaking operator of photons and axions with the background medium, and $\hat{\rho}_{\rm eq}$ is the equilibrium density operator. Since the magnetic field mixes only the $(\times)$ photon state (see below) with the axion, the matrix elements of the operators $\hat{\rho}$, $\hat{\Gamma}$, and $\hat{H}$ in the basis spanned by the two component fields $\Psi^T = (A_\times, a)$ are, respectively, given by

$$\rho = \begin{pmatrix} n_\gamma & \rho_{\gamma a} \\ \rho_{a\gamma} & n_a \end{pmatrix}, \quad \Gamma = \begin{pmatrix} \Gamma_\gamma & 0 \\ 0 & \Gamma_a \end{pmatrix}, \quad H = \begin{pmatrix} M_\times & M_{a\gamma} \\ M_{a\gamma} & M_a \end{pmatrix}, \tag{5}$$

where $\rho_\gamma = n_\gamma$ and $\rho_a = n_a$ are, respectively, the photon and axion occupation numbers, $\rho_{\gamma a} = \rho^*_{a\gamma} = R + iI$ with R and I being, respectively, the real and the imaginary parts of $\rho_{\gamma a}$. The matrix elements of the equilibrium density operator in the flavor space are given by the equilibrium occupation number $n_{\rm eq} = 1/(e^x - 1)$ times the identity matrix $\mathbf{I}$, $\rho_{\rm eq} = n_{\rm eq}\mathbf{I}$, where $x = \omega/T$ with T being the photon temperature. The coherence breaking matrix (Γ) is diagonal in the flavor space, and its entries are given by the *sum* of the scattering and the annihilation/absorption rates of photons (Γ_γ) and axions (Γ_a). Matrix elements that enter the interaction Hamiltonian are [22], $M_\times = \omega(n-1)_\times$, $M_{a\gamma} = g_{a\gamma}B_T/2$, and $M_a = -m_a^2/2\omega$. Here, B_T is the strength of the external magnetic field $\mathbf{B}_e$, which is transverse to the direction $\mathbf{x}$, of the photon/axion propagation.

$A_{+,\times}$ are the photon polarization states with $+,\times$ being the polarization indexes (helicity) of the photon. The helicity state $(+)$ corresponds to the polarization perpendicular to the external magnetic field, and $(\times)$ describes the polarization parallel to the external field. For the purpose of this work and the cosmological epoch in which we are interested, the total refraction index is given by the sum of two main components: the refraction index due to electronic plasma $n_{\rm pla}$ and the refraction index due to vacuum polarization $n_{\rm QED}$. The refraction index due to electronic plasma is given by $(n_{\rm pla}-1)_{\times,+} = -\omega_{\rm pla}^2/2\omega^2$, where $\omega_{\rm pla}^2 = 4\pi n_e/m_e$ with n_e being the number density of free electrons in the plasma. The refraction index due to QED effects, for $\omega \ll (2m_e/3)\times(B_c/B)$, is given by Ref. [23] $(n-1)_{\times,+} = (\alpha/4\pi)(B_T/B_c)^2[(14/45)_\times, (8/45)_+]$, where $B_c = m_e^2/e = 4.41\times 10^{13}$ G is the critical magnetic field.

When the total interaction rate that enters the problem is much bigger than expansion rate $\Gamma \gg H$ and photon-axion oscillation frequency $\omega_{\rm osc} \gg H$, the equation of motion for the density matrix is given by steady-state approximation; see Ref. [21] for details. In this case, it is possible to express the imaginary part I and real part R through n_γ and n_a; see Ref. [18] for more details. Moreover, if the interaction rate of axions with the medium is small, we can approximate the interaction rate of axions with the medium in Eq. (4), as $\Gamma_a \simeq 0$. Indeed, this is a good approximation for the cosmological epoch in which we are interested and for the axion mass range we are going to consider (see below). Also, assuming that the photon-axion transition is dominated by the resonance, one can find an analytic solution for the production probability of axions at the resonance temperature $\bar{T}$,

$$P_a(\bar{T}) = -\frac{2\pi M_{a\gamma}^2}{kHT}\bigg|_{T=\bar{T}}, \tag{6}$$

where $M_{a\gamma}(\bar{T}) = (g_{a\gamma}B_0/2)(\bar{T}/T_0)^2$ and $k(\bar{T}) = d(\Delta M)/dT|_{T=\bar{T}}$ with $\Delta M(\bar{T}) = M_\times(\bar{T}) - M_a(\bar{T}) = M_{\rm QED}(\bar{T}) - M_{\rm pla}(\bar{T}) - M_a(\bar{T})$. Here, $M_{\rm QED}$ and $M_{\rm pla}$ are, respectively, the QED and plasma contributions to the refraction index in ΔM. The field strength of the transverse part of magnetic field, B_T, scales with temperature as $B_T \sim B = B_0(\bar{T}/T_0)^2$ (magnetic flux conservation) with B_0 being the strength of the magnetic field at present epoch. The term $H(\bar{T})\bar{T}$ can be written as $H(\bar{T})\bar{T} = H_0 T_0\sqrt{\Omega_R}(\bar{T}/T_0)^3$, where $\Omega_R = 9.21\times 10^{-5}$ is the present-day density parameter of relativistic particles (photons and nearly massless neutrinos). During the μ epoch, the Universe is radiation dominated where ionization fraction of free electrons is unity, $X_e = 1$. In this case, we can expand $k(T)$ up to first order in power series and write $k(\bar{T}) = (3/\bar{T})[M_{\rm QED}(\bar{T}) - M_{\rm pla}(\bar{T})]$. Inserting all necessary terms into Eq. (6), we get the expression for P_a at the resonance temperature $\bar{T}$,

$$P_a(\bar{T}) = -\frac{2\pi}{3H(\bar{T})}\frac{M_{a\gamma}^2(\bar{T})}{M_{\rm QED}(\bar{T}) + M_a(\bar{T})}, \tag{7}$$

where in deriving Eq. (7) we have used the fact that for $T = \bar{T}$ we have $\Delta M(\bar{T}) = 0$. We may note that, in the case $M_{\rm QED}(\bar{T}) = -M_a(\bar{T})$, the denominator of Eq. (7) is zero, and the probability goes to infinity. In such a case, one must consider the expansion of $\Delta M(T)$ up to the second order in T around the resonance temperature $\bar{T}$. However, for our purpose, we do not need it here.

To confront Eq. (7) with the numerical results and because it is easier to calculate, let us consider the case in which $M_{\rm QED} \ll M_a$. In the redshift of interest for μ distortion and the photon energy considered here, the QED term in $M_\times$ is small with respect to the plasma term, and therefore from the resonance condition $\Delta M = M_\times - M_a = 0$, we get

$$\left(\frac{\bar{T}}{T_0}\right) = 9\times 10^6 n_e^{-1/3}\bar{m}_a^{2/3}\ \mathrm{cm}^{-1}, \tag{8}$$

where $\bar{m}_a = m_a/\mathrm{eV}$, $n_e \simeq 0.88 n_B(T_0)$ is the number density of the free electrons at the present epoch and $n_B(T_0) = 2.47\times 10^{-7}\ \mathrm{cm}^{-3}$ is the number density of baryons. Equation (8) is a constraint relation for the axion mass in the resonant case. Inserting all necessary quantities into Eq. (7), we get the expression for P_a,

$$P_a(\bar{T}) = 5.75\times 10^{-27} x C_{a\gamma}^2 B_{\rm nG}^2\left(\frac{\bar{T}}{T_0}\right)^3, \tag{9}$$

where $B_{\rm nG} = (B_0/\mathrm{nG})$ and $C_{a\gamma}$ is defined as

$$C_{a\gamma} \equiv \left(\frac{E}{N} - \frac{2}{3}\frac{4+w}{1+w}\right)\frac{1+w}{w^{1/2}}, \tag{10}$$

where, for $w = 0.56$, $|C_{a\gamma}| \simeq 4$ for $E/N = 0$ (KSVZ model) and $|C_{a\gamma}| \simeq 1.49$ for $E/N = 8/3$ (DFSZ model). It is important to emphasize that Eq. (9) is valid when $M_{\rm QED} \ll M_a$ or

$$B_{\rm nG}^{1/3} x^{1/3}\left(\frac{T}{T_0}\right) \ll 1.23\times 10^9 \bar{m}_a^{1/3}. \tag{11}$$

On the other hand, we also need to calculate the axion mass at the resonance temperature $\bar{T}$, which is given by Eq. (8). Assuming that the interested temperature interval is coincident with the μ epoch, $2.88\times 10^5 \lesssim T/T_0 \lesssim 2\times 10^6$, the axion mass in this interval is

$$2.66\times 10^{-6}\ \mathrm{eV} \lesssim \bar{m}_a \lesssim 4.88\times 10^{-5}\ \mathrm{eV}. \tag{12}$$

So, as far as we limit our consideration for the magnetic field strength of the order $B_{\rm nG} \lesssim 10^3$ and axion mass range given by Eq. (12), we can safely use Eq. (9).

In the presence of μ distortion, we can expand the photon occupation number for $\mu \ll 1$ in power series, and using the fact that the leakage of photons is due to oscillations into axions, we get the following relation between P_a and μ:

$$P_a = \mu \frac{e^x}{e^x - 1}. \tag{13}$$

Using Eqs. (13), (9), and (8), we get the following relation between the magnetic field strength and the axion mass:

$$B_{\rm nG} = \frac{0.22}{\bar{m}_a C_{a\gamma}} \left(\frac{\mu e^x}{x(e^x - 1)} \right)^{1/2}. \tag{14}$$

We can see that Eq. (14) depends on the photon energy x, and tighter bound on $B_{\rm nG}$ or $\bar{m}_a$ is obtained for higher values of x. Indeed, using, for example, the energy range explored by COBE/FIRAS [3], $1.2 \le x \le 11.3$, we get a tighter limit on $B_{\rm nG}$ at $x = 11.3$:

$$B_{\rm nG} = 6.76 \times 10^{-2} \frac{\sqrt{\mu}}{\bar{m}_a C_{a\gamma}}. \tag{15}$$

Equation (15) is our main result, which connects three unknown parameters $\bar{m}_a$, $B_{\rm nG}$, and $C_{a\gamma}$, with the μ parameter that is determined by experiment. We may notice that for values of μ given by COBE [3] and PIXIE/PRISM [4] we have that the bound given by Eq. (11) is indeed well satisfied. We emphasize that our results in the resonant case (see Fig. 1), obtained by using Eq. (15), perfectly agree with the numerical solution of the quantum kinetic equation, Eq. (4), in the steady-state approximation.

Concluding, our main results are shown in Fig. 1, in which we present the exclusion and sensitivity limits on the magnetic field strength vs axion mass in the resonant case. In Fig. 1(a), the exclusion region in the case of COBE is shown for the KSVZ and DFSZ axion model. In Fig. 1(b), the sensitivity region of future space mission PIXIE is shown. If PIXIE will detect any spectral distortion in the CMB spectrum, that would be a potential signal of photon to axion oscillation.

In general, is not possible to give definite limits on B and m_a since none of them is known exactly and moreover only limits (in the case of COBE, upper limit) on the μ parameter exist, which relates both. Nevertheless, we can outline important conclusions considering the upper limits of all of them. We can base our arguments by simply focusing on Eq. (15). First, based on the limit on μ from COBE, we can limit the axion mass, if we know the limit on B. For instance, in the case of the KSVZ axion model and a homogeneous magnetic field with strength $B \lesssim 3.2$ nG, we obtain from Eq. (15) that 4.8×10^{-5} eV $\lesssim m_a$. The limit on magnetic field strength is by a factor 1.2 stronger than that found for a uniform and anisotropic magnetic field in Ref. [19] and is by a factor 3.2 weaker than that found in Ref. [20], from the Faraday rotation of the Layman α forest. It is interesting to note that the upper limit $B \lesssim 3.2$ nG is very close to the limit found in Ref. [24] ($B \lesssim 3.1$–3.2 nG) from the CMB temperature cross-correlation spectra, TT and TE of WMAP five-year data for the case of stochastic

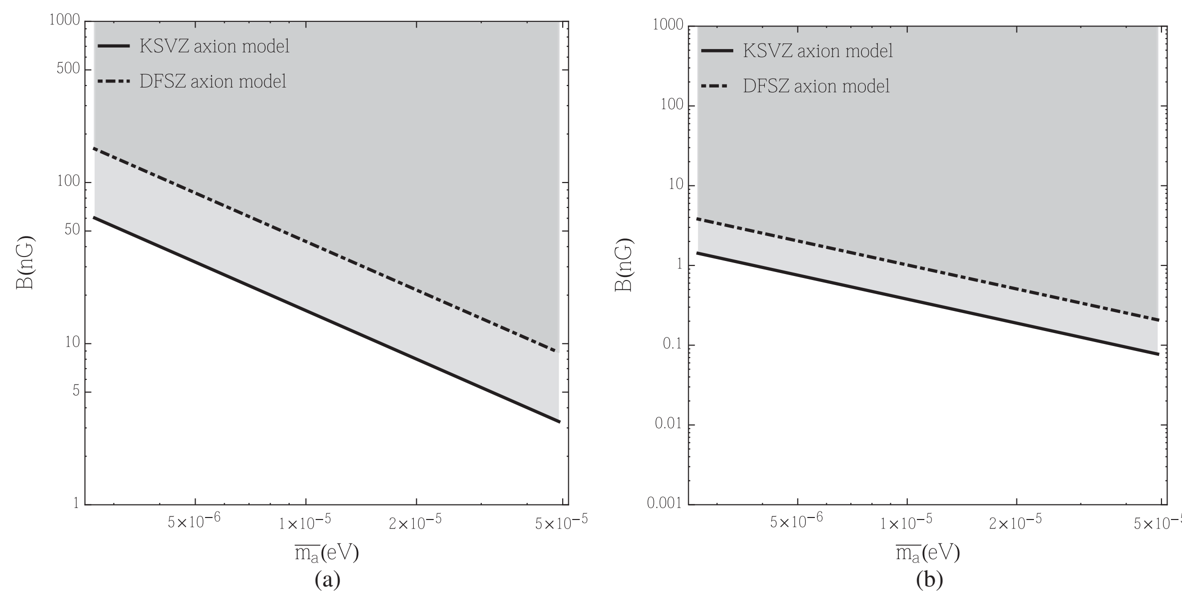

FIG. 1. Exclusion and sensitivity plot for the axion parameter space $B - \bar{m}_a$ in the *resonant case* due to μ distortion for $\bar{m}_a = 2.66 \times 10^{-6} - 4.88 \times 10^{-5}$ eV. In (a), the exclusion plot for the COBE [3] upper limit on μ is shown, and in (b), the sensitivity region of PIXIE/PRISM [4], based on the expected sensitivity on the μ parameter is shown. In both figures, the region above the solid line corresponds to the KSVZ axion model ($|C_{a\gamma}| \simeq 4$), and the region the above dotted-dashed line corresponds to the DFSZ axion model ($|C_{a\gamma}| \simeq 1.49$).

magnetic field with comoving coherence length scale $\lambda_B \simeq 1$ Mpc and field spectral index $n_B \simeq 1.6$ (blue magnetic field spectrum).

For the DFSZ axion model, the upper limit for a uniform magnetic field is $B \lesssim 9$ nG, which is by a factor 2.5 weaker than the KSVZ axion model for the same axion mass; see Fig. 1(a). This upper limit on the magnetic field strength for the DFSZ axion model would produce a larger temperature anisotropy with respect to the observed one and makes the DFSZ axion model disfavored with respect to the KSVZ axion model. PIXIE/PRISM are more sensitive than COBE and in principle can better confine the axion parameter space with respect to COBE; see the Fig. 1(b) limits with respect to it. In particular, in the case of the detection of spectral distortions, and assuming that the strength of the magnetic field is close to its canonical value, $B \sim 1$ nG (for a uniform magnetic field with coherence length of the Hubble horizon), it would be an extremely important signature of axions in the mass range, $\bar{m}_a \simeq 2$–$3\ \mu$eV.

The ADMX collaboration [25] excluded all axion models of being dark matter in the mass region 3.3–3.5 μeV. This mass range lies in the axion mass range considered in this paper; see Eq. (12). Thus, it would be possible to use the ADMX limits on the axion mass to constrain the magnetic field strength. For example, considering the limit $3.5\ \mu\text{eV} \lesssim m_a$, we find the magnetic field strength to be (in the case of COBE) $B \lesssim 53$ nG for the KSVZ axion model and $B \lesssim 141$ nG for the DFSZ axion model. In the case of PIXIE/PRISM, we would have $B \sim 1$ nG for the KSVZ axion model and $B \sim 2.7$ nG for the DFSZ axion model. However, knowing the upper and/or the lower limit for the axion mass, it allows us to constrain only the magnetic field strength. In this case, the above limits are valid for a uniform magnetic field with a coherence length of at least comparable to the horizon scale during the μ epoch, $\lambda_B^\mu \sim H^{-1}(z_\mu)$ or $\lambda_B^\mu(z_\mu) \sim 3.8$ pc (or $\lambda_B^\mu \sim 1.3$ Mpc at present), where the redshift corresponding to the resonant axion mass $\bar{m}_a \simeq 3.5\ \mu$eV during the μ epoch is $z_\mu \simeq 3.44 \times 10^5$; see Eq. (8).

The derived limits for a uniform magnetic field with coherence length comparable with the horizon scale are in general stronger than those found from the temperature anisotropy [19] and slightly weaker than those found from the Faraday rotation [20], at smaller coherence length scales. Indeed, at the coherence length scale $\lambda_B \sim 1$ Mpc, the Faraday rotation of the Lyman α forest gives $B \lesssim 10$ nG [20], which is by a factor 5.3 stronger than the limit found for the KSVZ axion model and by a factor 14.1 stronger than the DFSZ axion model (using the ADMX limit on the axion mass and the COBE limit on the μ parameter). The limits on the axion mass found here, in general, are of the same order of magnitude, with the limits found by the misalignment mechanism; see Refs. [26] and [10]. Indeed, the lower limit $4.8 \times 10^{-5}\ \text{eV} \lesssim m_a$ for $B \lesssim 3.2$ nG is very close to that found in Ref. [27], $m_a \lesssim 76$–$82\ \mu$eV, for CDM axions. According to Ref. [27], an axion within this mass range would explain all dark matter contents in the Universe without requiring other candidates. In our case, an axion in the mass range $m_a \lesssim 7.6$–$8.2\ \mu$eV would make nonresonant oscillation into CMB photons during the μ epoch. If the misalignment mechanism limits are used instead of the ADMX limit, for the axion mass range (nonresonant oscillation) coincident with those in Ref. [27], the strength of the homogeneous magnetic field at $\lambda_B \sim 1$ Mpc would be between 1.4×10^3 and 1.6×10^3 nG, depending on the nonresonant axion mass. These limits are weaker than those found from the Faraday rotation of the Lyman α forest and are comparable with the limits found from a homogeneous Universe; see Ref. [20].

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