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Origin of probabilities and their application to the multiverse

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We argue using simple models that all successful practical uses of probabilities originate in quantum fluctuations in the microscopic physical world around us, often propagated to macroscopic scales. Thus we claim there is no physically verified fully classical theory of probability. We comment on the general implications of this view, and specifically question the application of purely classical probabilities to cosmology in cases where key questions are known to have no quantum answer. We argue that the ideas developed here may offer a way out of the notorious measure problems of eternal inflation.

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I. INTRODUCTION

We use the concept of probability extensively in science, and very broadly in everyday life. Many probabilistic tools used to "quantify our ignorance" seem intuitive even to nonscientists. For example, if we consider the value of one bit which we know nothing about, we are inclined to assign probabilities to each value. Furthermore, it seems natural to give it a "50-50" chance of being 0 or 1. This everyday intuition is often believed to have deep theoretical justification based in "classical probability theory" (developed in famous works such as [1]).

Here we argue that the success of such intuition is fundamentally rooted in specific physical properties of the world around us. In our view the things we call "classical probabilities" can be seen as originating in the quantum probabilities that govern the microscopic world, suitably propagated by physical processes so as to be relevant on classical scales. From this perspective the validity of assigning equal probabilities to the two states of an unknown bit can be quantified by understanding the particular physical processes that connect quantum fluctuations in the microscopic world to that particular bit. The fact that we have simple beliefs about how to assign probabilities that do not directly refer to complicated processes of physical propagation is simply a reflection of the intuition we have built up by living in a world where these processes behave in a particular way. Our position has implications for how we use probabilities in general, but here we emphasize applications to cosmology which originally motivated our interest in this topic. Specifically, we question a number of applications of probabilities to cosmology that are popular today.

Many physicists view classical physics as something that emerges from a fundamentally quantum world under the right conditions (for example in systems large enough to have negligible quantum fluctuations and with suitable decohering behavior) without the need for new fundamental physics outside of the quantum theory.[1] Taking that point of view does not make the claims in this paper trivial ones. Yes, in that picture "all physics is fundamentally quantum," but here we focus specifically on the origin of randomness. Consider a classical computer well engineered to prevent quantum fluctuations of its constituent particles from affecting the classical steps of the computation. One could model a fluctuating classical system on such a computer (e.g. a gas of perfect classical billiards), but the fluctuations in such an idealized classical gas would indeed be classical ones. The appearance of a given fluctuation would reflect information already encoded in classical features of the initial state of the computation and would *not* come from quantum fluctuations of the particles making up the physical computer. We argue that the real physical world does not contain such perfectly isolated classical systems and that quantum uncertainty, not ignorance of classical information dominates probabilistic behavior we observe. (For the computer example just given, the quantum uncertainties will enter when setting up the initial state.)

In Bayesian language, the probability of a theory T being true given a data set D is computed by combining the probability of D given T ("$P(D|T)$") with the "prior probability" ($P(T)$) assigned to T. Often $P(T)$ will include other data combined in a similar way. Inputting new data over time produces a list of updated probabilities. The start of such a list always requires a "model uncertainty" (MU) prior that provides a personal statement about which model(s) you prefer. Expressions for $P(D|T)$ can be tested by statistical analysis of data and good scientists (discussing well designed experiments) should agree on how to compute $P(D|T)$. The MU prior is a personal choice which is not built from a scientifically rigorous process. The quantity $P(D|T)$ describes randomness in physical systems, whereas MU priors represent states of mind of

[1] We personally take this "fundamentally quantum" view but our arguments go through for some (but not all) other interpretations of quantum mechanics.

individual scientists. This paper only treats $P(D|T)$ probabilities, not MU priors. A further indication of the deep differences between $P(D|T)$ and MU priors is that the goal of science is to produce sufficiently high quality data (and sufficient consensus about the theories) that which MU priors the community are willing to take is of no consequence to the result. On the other hand, results will always depend strongly on at least some parts of $P(D|T)$.

II. THE PAGE PROBLEM

We outline the relevance of this question to cosmology using a simple toy model. It is commonplace in cosmology to contemplate a "multiverse" (e.g. in the context of "eternal inflation" [2]) in which many equivalent copies of a given observer appear in the theory. As pointed out by Page [3], even if one knew the full wave function for such a theory it would be impossible to make predictions about future observations using probabilities derived from that wave function. The problem arises because multiverse theories are expected to contain many copies of the observer (sometimes said to be in different "pocket universes") that are identical in terms of all current data, but which differ in details of their environments that affect outcomes of future experiments (e.g. experiments measuring neutrino masses or cosmological perturbations). In these theories it is impossible to construct appropriate projection operators to describe measurements where one does not know which part of the Hilbert space (i.e. which copy of us and our world) is being measured. Thus, the outcomes of future measurements are ill-posed quantum questions which cannot be answered within the theory.

To illustrate this problem consider a system comprised of two two-state subsystems called "A" and "B". The whole system is spanned by the four basis states constructed as products of basis states of the two subsystems: $\{|1\rangle^A|1\rangle^B, |1\rangle^A|2\rangle^B, |2\rangle^A|1\rangle^B, |2\rangle^A|2\rangle^B\}$. For the whole system in state $|\psi\rangle$, the probability assigned to measurement outcome "i" can be expressed as $\langle\psi|\hat{P}_i|\psi\rangle$ for a suitably chosen projection operator $\hat{P}_i$. One can readily construct projection operators corresponding to measuring system "A" in the "1" state (regardless of the state of the "B" subsystem):

$$\hat{P}_1^A \equiv (|1\rangle^A|1\rangle^{BB}\langle 1|^A\langle 1|) + (|1\rangle^A|2\rangle^{BB}\langle 2|^A\langle 1|). \qquad (1)$$

A similar operator $\hat{P}_1^B$ represents measurements of only subsystem "B". Operators such as $\hat{P}_{12} \equiv |1\rangle^A|2\rangle^{BB}\langle 2|^A\langle 1|$ represent measurements of *both* subsystems.

The problem arises because there is no projection operator that gives the probability of outcome "1" when the subsystem to be measured ("A" or "B") is undetermined. That is an ill-posed question in the quantum theory. Page emphasizes that this kind of question apparently needs to be addressed in order to make predictions in the multiverse, where our lack of knowledge about which pocket universe we occupy corresponds to "A" vs "B" not being determined in the toy model. Such ill-posed quantum questions exist in laboratory situations as well. We tend not to be concerned about these questions however, since there are also plenty of well-posed problems on which to focus our attention. Also, in the laboratory one might resolve the problem by adding a measurable "label" to the setup that does identify "A" vs "B". But such a resolution is believed not to be possible in many cosmological cases.

A natural response to this issue is to appeal to classical ideas about probabilities to "fill in the gap". In particular, if one could assign classical probabilities p_A and p_B for the measurement to be made on the respective subsystems, then one could answer the question posed above (the probability of the outcome "1" with the subsystem to be measured undetermined) by giving:

$$p_1 = p_A\langle\psi|\hat{P}_1^A|\psi\rangle + p_B\langle\psi|\hat{P}_1^B|\psi\rangle. \qquad (2)$$

Note that the values of p_A and p_B are *not* determined from $|\psi\rangle$, and instead provide additional information introduced to write Eq. (2). Although p_1 can be written as the expectation value of $\hat{P}_1 = p_A\hat{P}_1^A + p_B\hat{P}_1^B$, the operator $\hat{P}_1$ is not a projection operator ($\hat{P}_1\hat{P}_1 \neq \hat{P}_1$), confirming that p_1 does not give probabilities of fully quantum origin.

Authors who apply expressions like Eq. (2) to cosmology [4] do not claim this gives a quantum probability. Instead they appeal to classical notions of probability along the lines we have discussed at the start of this paper. Surely one successfully introduces classical probabilities such as p_A and p_B all the time in everyday situations to quantify our ignorance, so why should the same approach not be used in the cosmological case?

Our view is that the two cases are completely different. We believe that in every situation where we use "classical" probabilities successfully to describe physical randomness these probabilities could in principle be derived from a wave function describing the full physical situation. In this context classical probabilities are just ways to estimate quantum probabilities when calculating them directly is inconvenient. Our extensive experience using classical probabilities in this way (really quantifying our *quantum* ignorance) cannot be used to justify the use of classical probabilities in situations where quantum probabilities have been clearly shown to be ill-defined and uncomputable. Translating the formal framework from one situation to the other is not an extrapolation but the creation of a brand new conceptual framework that needs to be justified on its own.[2]

[2]Cooperman [5] has explored the interpretation of these matters in the context of the positive operator valued measure formalism. In our view this does not really resolve the problem, since one has to introduce new probabilities equivalent to p_A and p_B in an equally *ad hoc* way. We definitely do agree with the connections he draws to the standard treatment of identical particles, which we find quite intriguing.

TABLE I. The number of collisions, [n_Q from Eq. (4)] before quantum uncertainty dominates, evaluated for physical systems modeled as a "gas" of billiards with different properties. Values $n_Q < 1$ indicate that quantum fluctuations are so dominant that Eq. (4) breaks down. All randomness in these quantum dominated systems is fundamentally quantum in nature.

	r(m)	l (m)	m (kg)	$\bar{v}$ (m/s)	$\lambda\!\!\!^{-}_{dB}$ (m)	$\Delta b(m)$ (m)	n_Q
Nitrogen at STP (Air)	1.6×10^{-10}	3.4×10^{-07}	4.7×10^{-26}	360	6.2×10^{-12}	2.9×10^{-9}	−0.3
Water at body temp	3.0×10^{-10}	5.4×10^{-10}	3.0×10^{-26}	460	7.6×10^{-12}	1.3×10^{-10}	0.6
Billiards game	0.029	1	0.16	1	6.6×10^{-34}	5.1×10^{-17}	8
Bumper car ride	1	2	150	0.5	1.4×10^{-36}	3.4×10^{-18}	25

We are only challenging the ad hoc introduction of classical probabilities such as p_A and p_B. We are not criticizing the use of standard ideas from probability theory to manipulate and interpret probabilities that have a physical origin. Of course we never know the wave function completely (and thus often write states as density matrices). Our claim is that probabilities are only proven and reliable tools if they have clear values determined from the quantum state, despite our uncertainties about it.

III. BILLIARDS

We next use simple calculations to argue that it is realistic to expect all probabilities we normally use to have a quantum origin. Consider a gas of idealized billiards with radius r, mean free path l, average speed $\bar{v}$, and mass m. If two of these billiards approach each other with impact parameter b, the uncertainties in the transverse momentum ($\delta p_\perp$) and position ($\delta x_\perp$) contribute to an uncertainty in the impact parameter given by

$$\Delta b = \delta x_\perp + \frac{\delta p_\perp}{m}\Delta t = \sqrt{2}\left(a + \frac{\hbar}{2a}\frac{l}{m\bar{v}}\right), \qquad (3)$$

where the second equality is achieved using $\Delta t = l/\bar{v}$ and assuming a minimum uncertainty wave packet of width a in each transverse direction. The value of Δb is minimized by $a = \sqrt{\hbar l/(2m\bar{v})} \equiv \sqrt{l\lambda\!\!\!^{-}_{dB}/2}$. We will show that Δb is significant even when minimized.

The local nature of subsequent collisions creates a distribution of entangled localized states reflecting the range of possible collision points implied by Δb. We estimate the width of this distribution as it fans out toward the next collision by classically propagating collisions that occur at either side of the range Δb. (Neglecting additional quantum effects increases the robustness of our argument.) The geometry of the collision amplifies uncertainties in a manner familiar from many chaotic processes [6,7]. The quantity $\Delta b_n = \Delta b(1 + (2l)/r)^n$ gives the uncertainty in b after n collisions.

Setting $\Delta b_n = r$ and solving for n determines n_Q, the number of collisions after which the quantum spread is so large that there is significant quantum uncertainty as to which billiard takes part in the next collision:

$$n_Q = -\frac{\log(\frac{\Delta b}{r})}{\log(1 + \frac{2l}{r})}. \qquad (4)$$

For Table I we evaluated Eq. (4) with different input parameters chosen to represent various physical situations.[3]

Table I shows that water and air are so dominated by quantum fluctuations that $n_q < 1$, indicating the breakdown of Eq. (4), but all the more strongly supporting our view that *all* randomness in these systems is fundamentally quantum. This result strongly indicates that if one were able to fully model the molecules in these macroscopic systems one would find that the intrinsic quantum uncertainties of the molecules, amplified by processes of the sort we just described, would be fully sufficient to account for all the fluctuations. One would not be required to "quantify our ignorance" using classical probability arguments to fully understand the system. For example, the Boltzmann distribution for one of these systems in a thermal state should really be derivable as a feature dynamically achieved by the wave function without appeal to formal arguments about equipartition, etc.

This argument that the randomness in collections of molecules in the world around us has a fully quantum origin lies at the core of our case. We expect that all practical applications of probabilities can be traced to this intrinsic randomness in the physical world. As an illustration, we next trace the randomness of a coin flip to Brownian motion of polypeptides in the human nervous system.

IV. COIN FLIP

Randomness in a coin flip comes from a lack of correlation between the starting and ending coin positions. The signal triggering the flip travels along human neurons which have an intrinsic temporal uncertainty of $\delta t_n \approx 1$ ms [9]. It has been argued that fluctuations in the number of open neuron ion channels can account for the observed values of δt_n [9]. These molecular fluctuations are due to random Brownian motion of polypeptides in their

[3] Raymond [8] presents similar result, applied only to actual billiards. He also makes some general points about the implications of his result that overlap with some of the points we are making here.

surrounding fluid. Based on our assessment that the probabilities for fluctuations in water are fundamentally quantum, we argue that the value of δt_n realized in a given situation is also fundamentally quantum. Quantum fluctuations in the water drive the motion of the polypeptides, resulting in different numbers of ion channels being open or closed at a given moment in each instance realized from the many quantum possibilities.

Consider a coin flipped and caught at about the same height, by a hand moving at speed v_h in the direction of the toss and with a flip imparting an additional speed v_f to the coin. A neurological uncertainty in the time of the flip, δt_n, results in a change in flight time $\delta t_f = \delta t_n \times v_h/(v_h + v_f)$. A similar catch time uncertainty gives a total flight time uncertainty $\delta t_t = \sqrt{2}\delta t_f$. A coin flipped upward by an impact at its edge has a rotation frequency $f = 4v_f/(\pi d)$ where d is the coin diameter. The uncertainty in the number of spins is $\delta N = f \delta t_t$. Using $v_h = v_f = 5$ m/s and $d = 0.01$ m (and $\delta t_n = 1$ ms) gives $\delta N = 0.5$, enough to make the outcome of the coin toss completely dependent on the time uncertainty in the neurological signal which we have argued is fully quantum.

No doubt we have neglected significant factors in modeling the coin flip. The point here is that even with all our simplifications, we have a plausibility argument that the outcome of a coin flip is truly a quantum measurement (really, a Schrödinger cat) and that the 50-50 outcome of a coin toss may in principle be derived from the quantum physics of a realistic coin toss with no reference to classical notions of how we must "quantify our ignorance." Estimates such as this one illustrate how the quantum nature of fluctuations in the gasses and fluids around us can lead to a fundamental quantum basis for probabilities we care about in the macroscopic world.

V. DIGITS OF Π

The view that all practical applications of probabilities are based on physical quantum probabilities seems a challenging proposition to verify. As we have illustrated with the coin flip, the path from microscopic quantum fluctuations to macroscopic phenomena is complicated to track. And there are endless cases to check (rolling dice, choosing a random card etc.), most also too complicated to work through conclusively. So arguing our position on a case-by-case basis is certainly an impractical task.

On the other hand, our ideas are very easy to falsify. All one needs is one illustration of a case where classical notions of probability are useful in a physical system that is fully isolated from the quantum fluctuations. Once the practical value of purely classical probabilities is established there is no reason it should not be applicable to other situations. One idea for such a counterexample was proposed by Carroll [10]. One could place bets on, say, the value of the millionth digit of π. Since the digits of π are believed to be random [11] one should be able to use this apparently purely classical notion to win bets. While on the face of it this appears to be an ideal counterexample, further scrutiny reveals an essential quantum role.

Let's phrase this problem more systematically: One expects that if you finds someone who thinks the digits of π are not randomly distributed, you can make money betting against them. Or equivalently, the expected payout P_π is zero if betting with someone who *does* think the digits are random. A simple formula for such a payout is given by

$$P_\pi = \lim_{N_{\rm tot}\to\infty} \frac{1}{N_{\rm tot}} \sum_{\{i\}} (N_\pi^i - 4.5) = 0, \tag{5}$$

where $\{i\}$ is the ensemble (of size $N_{\rm tot}$) of the digits chosen and N_π^i is the actual value of the ith digit of π. The result depends entirely on the choice of ensemble. With enough knowledge of π one can come up with ensembles that give any answer you like (for example that only ever select the digit "1"), despite all the randomness "intrinsic" to π (and in fact *because* the properties of π are classical and knowable). Thus we argue that the outcomes of such bets are all about the ensemble selected, and the choice of the ensemble is the only source of randomness in the entire activity.

The reason the initial idea of betting on π is so compelling is that no one ever thinks an ensemble will be chosen with attention to the actual values of the digits of π. One can see how quantum mechanics comes in by scrutinizing the process of coming up with ensembles. It could be through the human neurons used in selecting a classical random number seed,[4] or through something more systematic like a roulette wheel. Again this falls in the category where one counterexample could ruin the argument, but so far we have not found one. The bet really is about the lack of correlation between the digit selection and the digit value and we argue it is quantum processes such as those discussed here that are being counted on to create the lack of correlation that is crucial to the fairness of the bet.

Our analysis depends crucially on seemingly "accidental" levels of quantum noise in the physical world. Our point is that accidental or not, we count on this quantum noise to produce the uncorrelated microscopic states that lie at the heart of our understanding of randomness and probabilities in the world around us. Extending this understanding to domains where quantum noise cannot play this role is not at all straightforward. Discussions of the nonrandom behaviors of classical random number generators (such as in [12]) underscore the difficulty of even imagining a classical source of randomness with the necessary lack of correlations.

[4]Similarly, the involvement of neurons etc. with the initial setup prevents the classical computer example in Sec. I from being a counterexample.

VI. TOWARD A SOLUTION OF COSMIC MEASURE PROBLEMS

So far we have used our ideas about probability to critique the introduction of purely classical probabilities into cosmological theories, which is an approach advocated by others [4]. In this section we use the ideas introduced here to work out our own approach to probabilities in the multiverse. We embrace the idea advocated above, that fundamentally classical probabilities have no place in cosmological theories, and declare that questions that seem to require classical probabilities for answers simply are not answered in that theory. We are basically advocating a more strict discipline about which questions are actually addressed by a given theory.[5] Then one can ask if there are multiverse theories with sufficient predictive power to remain viable after this discipline is imposed. Our first assessment of this question suggests that imposing this discipline may reduce or completely eliminate the notorious measure problems of eternal inflation and the multiverse.

One challenge one faces when exploring this matter is the fact that most discussions of eternal inflation and the multiverse are approached in a semiclassical manner (for example assuming well-defined classical spatial slices of infinite extent). A more careful attempt to identify the full quantum nature of the picture may point to additional ways proper quantum probabilities are assigned. We will not try to address that aspect of the question here, and really just take a first look at the impact of hewing to our proposed probability discipline.

A general point immediately becomes clear: We are used to linking counting with probabilities, but such connections are not always direct or relevant. Counting up the heads and tails in a long string of coin flips *is* connected with proper quantum probabilities. Starting with our results of Sec. IV one can see that a specific quantum probability is assigned to each different possible heads/tails count, and thus counting can be tied in to well-defined quantum probabilities for that system. However, the fact that one cosmology may have three pocket universes of type A, while another may have 10^{100} does not make a difference, because as we discussed in Sec. II, no quantum probabilities can be constructed to determine which among different (equivalent so far) observers you might be. While these numbers (by analogy with the flips of multiple coins) may be linked to global properties of the state, they cannot by used to determine which among equivalent patches a given observer occupies.

The insight that counting of observers in itself is insufficient to lead to proper probabilities leads to some interesting conclusions. One is immediately drawn to the question of "volume factors" that give large volume regions more weight than small ones. To the extent that volume factors are only a stand-in for counting observers we regard such counting as meaningless because it cannot be related to true quantum probabilities.

This insight also relates to the "young universe" or "end of time" problem [14,15], which can be sketched as follows: If one regulates the cosmology with a time cutoff, inflation guarantees that most pocket universes will be produced close to the cutoff. Then the time cutoff shows up at early times (relative to their time of production which is under strong pressure to happen late) for most pocket universes. This problem persists even as one pushes the time cutoff out to infinity. But there is no evidence that this counting has anything to do with probabilities predicted by the theory which are relevant to an observer. There is no sign that such theories are able to assign a true quantum probability to the time when a particular observer's pocket universe was produced. One is simply looking at different pocket universes, and which one we occupy is not determined by the theory.

Our position appears to offer significant implications for the Boltzmann brain problem [16–18]. For our purposes here, this problem is simply the case where pathological observers, called Boltzmann brains or BBs, vastly outnumber realistic ones. (The pathology of the BBs is that they match all the data we have so far, but the next moment experience catastrophic breakdown of physicality, experiencing a rapid heat death.) Again, we claim here that counting numbers of BBs vs realistic observers cannot be related to quantum probabilities predicting which an observer is more likely to experience. Thus, as long as there is at least one realistic pocket universe, there will be no BB problem, no matter how many BBs are produced in the theory.

Now let us look at this matter from a slightly different point of view. The real problem arises when one does not know which part of the Hilbert space one is about to measure. However, if one just takes one piece of the Hilbert space in an eternally inflating universe, that patch alone will have probabilities of tunneling into pocket universe A or B, and perhaps many other outcomes as well. If one simply traces out the rest of the Hilbert space, one will have a density matrix for what is going on in that patch. With that one *can* take expectation values of operators, without introducing classical probabilities to determine which pocket you are in. To the extent that the BB problem can be phrased in this way (in terms of a quantum branching into BBs vs realistic cosmologies in a given patch), we expect the BB problem will remain if realistic cosmologies are sufficiently suppressed.[6] And if all patches are the same

[5] Although here we focus on cosmology, it appears that our approach is relevant to other areas where there is confusion about how to assign probabilities, such as the "sleeping beauty problem"[13].

[6] In [16] one of us (A. A.) treats BBs in the traditional counting language in toy models. However, we expect that with a bit more realism the kind of quantum chaos discussed in this paper would allow those BB discussions to go over nicely into the (more legitimate) quantum branching form described here, without changing the conclusions in [16].

(as may well be the case for highly symmetric theories such as eternal inflation) then it does not really matter what patch you are in. The answer will still be the same.

While we have yet to offer a rigorous demonstration, this set of ideas seem promising to us as a way out of the measure problems in cosmology. A more formal way to describe this picture is that if one does consider a theory with multiple possible locations for the observer, one would be obliged to give a "prior" on which location we occupy. These priors would look very much the same as the classical probabilities that show up for example in Eq. (2). However, by viewing these probabilities as priors, our agenda would be to reach a point where their values do not matter to our answers.[7] It would appear that for sufficiently symmetric theories, independence from these priors would be easy to achieve. Also, if certain observables are sufficiently correlated, the measurement of one (which itself did not have a prediction for the outcome due to dependence on priors) could then lead to predictions for the other observable. Both of these pictures outlined here could lead to a substantial level of predictive power, despite the restrictions imposed by our probability discipline.

VII. CONCLUSIONS

In summary, we have argued that all successful applications of probability to describe nature can be traced to quantum origins. Because of this, there has not been any systematic validation of purely classical probabilities, even though we appear to use them all the time. These matters are of particular importance in multiverse theories where truly classical probabilities are used to address critical questions not addressed by the quantum theory. Such applications of classical probabilities need to be built systematically on separate foundations and not be thought of as extensions of already proven ideas. We have yet to see purely classical probabilities motivated and validated in a compelling way, and thus are skeptical of multiverse theories that depend on classical probabilities for their predictive power. Fundamentally finite cosmologies [19] that do not have duplicate observers do not require classical probabilities. These seem to be a more promising path.

We are not the only ones who regard quantum probabilities as most fundamental (e.g. [20]), but there are also opposing views.[8] In addition to the case already discussed where classical probabilities are introduced in multiverse theories to enhance predictive power (such as in [4]), some theories insert classical ideas for other reasons, often in hopes of allaying interpretational concerns (e.g. [22,24,25]). The arguments presented here make us generally doubtful of such classical formulations, since our analysis reinforces the fundamental role of quantum theory in our overall understanding of probabilities. Perhaps some of these alternate theories integrate the classical ideas sufficiently tightly with the quantum piece that the everyday tests we have discussed could just as well be regarded as tests of the classical ideas in the alternate theory. However, such logic seems overly complex to us, and we prefer the simpler interpretation that the strong connection between all our experiences with probabilities and the quantum world means the quantum theory really is the defining physical theory of probabilities. We have offered suggestions that sticking only to quantum probabilities to make predictions in the multiverse may not be all that debilitating to the predictive power of multiverse theories and may actually offer a solution to the notorious measure problems of eternal inflation.

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[7]Note that while formally these priors look the same as the classical probabilities discussed in [4], those authors emphasize cases where results *do* depend in a fundamental way on the values chosen for the classical probabilities. So they are not really treating their classical probabilities as prior probabilities, the values of which should ultimately not be important.

[8]There are also some papers where the degree overlap is not so clear. Vilenkin appears to focus on quantum probabilities in [21], but then also seems to embrace a fundamentally classical picture similar to that advocated in [22]. Some aspects of [23] also seem to overlap, although other things (such as the emphasis on holography) seem very different, so it is hard to tell the overall degree of agreement

[1] P. S. Laplace, *Essai Philosphique Sur Les Probabilités* (Bachelier, Paris, 1825).
[2] A. H. Guth, J. Phys. **A40**, 6811 (2007).
[3] D. N. Page, J. Cosmol. Astropart. Phys. 07 (2009) 008.
[4] M. Srednicki and J. Hartle, Phys. Rev. D **81**, 123524 (2010); D. N. Page, arXiv:1203.1837.
[5] J. H. Cooperman, J. Cosmol. Astropart. Phys. 02 (2011) 014.
[6] G. D. Birkhoff, *Dynamical Systems* (American Mathematical Society, Providence, 1927).
[7] W. H. Zurek and J. P. Paz, Phys. Rev. Lett. **72**, 2508 (1994).
[8] D. J. Raymond, Am. J. Phys. **35**, 102 (1967).
[9] A. A. Faisal, L. P. J. Selen, and D. M. Wolpert, Nat. Rev. Neurosci. **9**, 292 (2008).
[10] S. Carroll, PCTS workshop on inflation, 2011.
[11] D. H. Bailey, S. M. Plouffe, P. B. Borwein, and J. M. Borwein, Mathematical Intelligencer **19**, 50 (1997).
[12] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in FORTRAN: The Art of Scientific Computing* (Cambridge University Press, Cambridge, England, 1996).
[13] A. Elga, Analysis **60**, 143 (2000).
[14] R. Bousso, B. Freivogel, S. Leichenauer, and V. Rosenhaus, Phys. Rev. D **83**, 023525 (2011).
[15] A. H. Guth and V. Vanchurin, arXiv:1108.0665.
[16] A. Albrecht, arXiv:1401.7309.
[17] A. Albrecht and L. Sorbo, Phys. Rev. D **70**, 063528 (2004).
[18] D. N. Page, Phys. Rev. D **78**, 063536 (2008).
[19] T. Banks and W. Fischler, arXiv:astro-ph/0307459; A. Albrecht, Phys. Rev. Lett. **107**, 151102 (2011).
[20] D. Deutsch, Proc. R. Soc. A **455**, 3129 (1999); D. Wallace, in *Many Worlds? Everett, Quantum Theory, and Reality* (Oxford University Press, New York, 2010); W. H. Zurek, Phys. Rev. Lett. **106**, 250402 (2011); R. Bousso and L. Susskind, Phys. Rev. D **85**, 045007 (2012).
[21] A. Vilenkin, J. Cosmol. Astropart. Phys. 05 (2014) 005.
[22] A. Aguirre, M. Tegmark, and D. Layzer, Phys. Rev. D **84**, 105002 (2011).
[23] Y. Nomura, Found. Phys. **43**, 978 (2013).
[24] G. 't Hooft, AIP Conf. Proc. **1446**, 341 (2010).
[25] S. Weinberg, Phys. Rev. A **85**, 062116 (2012).