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$U(2)_A \times U(2)_V$-symmetric fixed point from the functional renormalization group

Mara Grahl

Institute for Theoretical Physics, Goethe University,
Max-von-Laue-Straße 1, D-60438 Frankfurt am Main, Germany

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The existence of a $U(2)_A \times U(2)_V$-symmetric fixed point in the chiral linear sigma model is confirmed using the functional renormalization group. Its stability properties and the implications for the order of the chiral phase transition of two-flavor QCD are discussed. Furthermore, several technical conclusions are drawn from the comparison with the results of resummed loop expansions.

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I. INTRODUCTION

Apart from other methods, our current understanding of QCD in the nonperturbative regime is strongly based on lattice gauge theory and effective models [1,2]. Despite all efforts the order of the chiral phase transition of QCD with two massless flavors has not been rigorously determined yet, and the interest in a reliable prediction remains strong. The cases of two massless or light flavors at vanishing baryonic chemical potential are of particular interest for lattice studies due to the comprehensive predictions of effective models [3–5]. The possible existence of a second-order chiral phase transition, as well as the corresponding universality class, can be investigated from the effective theory for the chiral condensate [6–18]. We can take into account the scalar mesons (σ and $\vec{a}_0$) as well as the pseudoscalar mesons (η and $\vec{\pi}$) by writing down the most general Lagrangian invariant under chiral symmetry. For the full symmetry, $U(2)_A \times U(2)_V \simeq U(1)_A \times U(1)_V \times [SU(2)/Z(2)]_L \times [SU(2)/Z(2)]_R$, this Lagrangian is given by [6,7,19–21] $\mathcal{L} = \frac{1}{2}\mathrm{Tr}(\partial_\mu \Phi^\dagger)(\partial_\mu \Phi) + U(\rho, \xi)$, where $\Phi = (\sigma + \mathrm{i}\eta)t_0 + \vec{t} \cdot (\vec{a}_0 + \mathrm{i}\vec{\pi})$, with t_a denoting the generators of $U(2)$ normalized such that $\mathrm{Tr}(t_a t_b) \equiv 1$ [13]. Furthermore,

$$\mathrm{Tr}\Phi^\dagger\Phi = \sum_i \phi_i^2 \equiv 2\rho, \qquad \phi_i \equiv \sigma, \vec{\pi}, \eta, \vec{a}_0,$$

$$\frac{1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)^2 - \rho^2 = (\sigma^2 + \vec{\pi}^2)(\eta^2 + \vec{a}_0^2) - (\sigma\eta - \vec{\pi}\cdot\vec{a}_0)^2 \equiv \xi.$$

We omit derivative couplings since we will only discuss the local-potential approximation (LPA, $Z = 1$) and, its minimal extension allowing for a field-independent wave-function renormalization factor Z (LPA'). We allow for a condensate only for the σ-field, so that we can use the truncation [12]

$$U(\rho, \xi) \equiv V(\rho) + W(\rho)\xi. \tag{1}$$

In this paper we focus on the case where the axial $U(1)_A$ symmetry has already been restored at the critical temperature T_c. Therefore, we do not take account of $U(1)_A$-breaking terms. For studies concerning the opposite scenario in which the anomaly remains present at T_c, we refer to Refs. [13–16,22–25]. The longstanding question of which of the two scenarios is actually realized is subject to an ongoing debate. The latest lattice results are quite controversial: whereas the case of restored anomaly is advocated by Refs. [3,26], the opposite scenario is favored by Refs. [4,27]. The predictions of effective theories for the chiral condensate are summarized in the following.

The existence of an IR-stable fixed point in the renormalization group (RG) flow of the effective theory for the order parameter is a necessary condition for a second-order phase transition to occur. If this scenario is realized or not depends on the initial values for the parameters in the UV limit determined by the underlying microscopic theory. Therefore, the RG analysis serves to either rule out the existence of a second-order phase transition or to confirm its possible existence.

If the anomaly strength exceeds the cutoff scale, a phase transition of second order in the $O(4)$ universality class is predicted [6,13,28]. The case of small anomaly strength is subtle. The anomaly yields two independent quadratic mass terms. At mean-field level, it is evident that such a situation corresponds to a multicritical point with at least two relevant scaling variables. This is used as an argument in Ref. [14] to rule out a second-order phase transition with temperature being the only relevant scaling variable. However, in consistence with Refs. [13,20], we argue that the inclusion of fluctuations can, in principle, lead to an IR-stable fixed point corresponding to exactly such a scenario. Although associated with unphysical masses in the approximation considered, there in fact exists an (unphysical) $SU(2)_A \times U(2)_V$-symmetric, IR-stable fixed point exemplifying our consideration. This observation extends the critical reinvestigation of the standard criteria used for ruling out continuous transitions presented in Ref. [29]. The latter particularly points out that the irreducibility of a representation is not strictly ruling out a second-order phase transition associated with a single relevant scaling variable. In the absence of the anomaly, there is strong evidence from Refs. [14,17] for the existence of a second-order phase transition belonging to the $U(2)_V \times U(2)_A$ universality class. The existence and properties of the corresponding fixed point will be discussed in the remainder of this paper.

Reference [14] uses a resummed loop expansion at fixed spatial dimension, $D = 3$, based on the minimal subtraction scheme ($\overline{\mathrm{MS}}$) and the massive zero-momentum scheme (MZM) scheme, respectively. The discovered IR-stable, $U(2)_V \times U(2)_A$-symmetric fixed point corresponds to an anomalous dimension of $\eta \sim 0.12$. Previous studies in the framework of the ϵ-expansion failed to find the fixed point [6,10]. A plausible explanation is given in Ref. [14]: the fixed point only exists near $D = 3$. One might wonder, however, if the resummation scheme and the loop order also play a role. With our functional renormalization group (FRG) investigation presented in Sec. II, we demonstrate that the existence not only depends on the fixed spatial dimension but also on the way nonperturbative corrections are included.

II. FRG FIXED-POINT STUDY

Assuming a homogeneous condensate, and using the Litim regulator, the Wetterich equation for the potential (1) reads

$$\frac{\partial U_k}{\partial k} = \frac{2\pi^{D/2} k^{D+1} Z_k}{D\,\Gamma(D/2)(2\pi)^D}\left(1 - \frac{\eta}{2+D}\right)\sum_i \frac{1}{Z_k k^2 + M_i^2}, \quad (2)$$

where $\mathcal{L}_k = \frac{1}{2} Z_k Tr(\partial_\mu \Phi^\dagger)(\partial_\mu \Phi) + U_k$, with $\mathcal{L}_{k=\Lambda} = \mathcal{L}$ defining the bare Lagrangian in the UV limit. M_i^2 denote the eigenvalues of the mass matrix

$$M_{ij} \equiv \frac{\partial^2 U_k}{\partial \phi_i \partial \phi_j}, \quad i,j = 1,\ldots,8. \quad (3)$$

The anomalous dimension, η, is determined from the relation

$$\eta_k = -Z_k^{-1} k \frac{\partial Z_k}{\partial k}, \qquad \lim_{k\to 0} \eta_k = \eta. \quad (4)$$

The flow equation for Z_k is derived from the second derivative of the effective action with respect to the fields and evaluated at the global minimum of the potential [30]. Using the Litim regulator and setting $D = 3$, in agreement with Ref. [31], we obtain

$$\eta_k = \frac{2}{3\pi^2[1+\bar{V}'_k(\bar{\rho}_{0,k})]^2}\left(\frac{4\bar{\rho}_{0,k}\bar{W}_k(\bar{\rho}_{0,k})^2}{[1+4\bar{W}_k(\bar{\rho}_{0,k})\bar{\rho}_{0,k}+\bar{V}'_k(\bar{\rho}_{0,k})]^2} + \frac{\bar{\rho}_{0,k}\bar{V}''_k(\bar{\rho}_{0,k})^2}{[1+\bar{V}'(\bar{\rho}_{0,k})+2\bar{\rho}_{0,k}\bar{V}''_k(\bar{\rho}_{0,k})]^2}\right), \quad (5)$$

where we introduced rescaled variables (labeled by a bar),

$$\bar{U} = k^{-D}U, \qquad \bar{\rho} = Zk^{2-D}\rho, \qquad \bar{\xi} = Z^2 k^{4-2D}\xi,$$
$$\bar{V} = k^{-D}V, \qquad \bar{W} = Z^{-2}k^{D-4}W,$$

and denoted the global minimum of U_k by ρ_0 (assuming $\xi_0 = 0$). We note that the rhs of Eq. (5) does not contain η due to our choice for the regulator.

In the following, we expand $\bar{V}(\bar{\rho})$ and $\bar{W}(\bar{\rho})$ in powers of $\bar{\rho}$ denoting the expansion coefficients by $\bar{p}_j$. We truncate the series at polynomial order $n \leq 24$ in the fields $\bar{\phi}_i$ [i.e., $\bar{V}$ up to $\mathcal{O}(\bar{\rho}^{12})$, $\bar{W}$ up to $\mathcal{O}(\bar{\rho}^{10})$] and refer to the LPA (LPA') in D spatial dimensions at truncation order n as $\mathrm{LPA}_D^{(n)}$ ($\mathrm{LPA}_D^{\prime(n)}$).

The flow equations for the $\bar{p}_j$ are derived similarly to Refs. [12,13], not listed explicitly here. To determine the stability properties of the fixed points, we analyze the flow in their neighborhood where it is governed by the linearized system. For this purpose we calculate the eigenvalues of the stability matrix

$$(S_{ij}) \equiv \left(\frac{\partial \beta_i}{\partial \bar{p}_j}\right)\Bigg|_{\bar{p}=\bar{p}*}, \quad (6)$$

where the fixed-point coordinates are denoted by $\{\bar{p}_i^*\}$, and the beta functions are given by $\beta_i(\bar{p}) \equiv k\partial_k \bar{p}_i$. In general one obtains n_s eigenvalues with positive real part, n_u with negative real part, and n_m with vanishing real part. The corresponding eigenvectors give rise to invariant subspaces of the parameter space inside which the flow stays if one starts within them [32]. In case of distinct eigenvalues, there is an n_s-dimensional *critical manifold* inside which the flow is attracted toward the fixed point in the infrared limit $k = 0$. Respectively, there exists an n_u-dimensional *unstable manifold* inside which the flow is repelled, and a n_m-dimensional *marginal manifold* inside which the flow has no direction at all. Here we note that complex valued eigenvalues (characteristic for a so-called *spiral fixed point*) always appear as conjugate pairs. Referring to the real and imaginary parts of the associated complex eigenvectors as eigenvectors, too, the critical manifold is spanned by n_s eigenvectors, the unstable manifold is spanned by n_u eigenvectors, and the marginal manifold is spanned by n_m eigenvectors. Therefore, if $n_m = 0$, one can reach the critical manifold by tuning n_u parameters starting anywhere in parameter space. Hence, a second-order phase transition with respect to a single scaling variable (temperature) can only exist if we have exactly $n_u = 1$. In this case we speak of an IR-stable fixed point.

In the $\mathrm{LPA}_{D=3}^{(6)}$ we find an unstable $O(8)$-symmetric fixed point with stability-matrix eigenvalues $\{12.925, 8.125, 1.509, -1.380, -0.503\}$; two IR-stable, $U(2)_A \times U(2)_V$-symmetric spiral fixed points, $A^{(6)}$ and $\tilde{A}^{(6)}$, respectively, with eigenvalues $\{15.660, 0.625 + 3.534\,\mathrm{i}, 0.625 - 3.534\,\mathrm{i}, 1.631, -1.374\}$ and $\{13.222, 1.188 + 2.148\,\mathrm{i}, 1.188 - 2.148\,\mathrm{i}, -1.511, 1.373\}$, respectively; and the Gaussian fixed point with eigenvalues $\{-2, -1, -1, 0, 0\}$. $A^{(6)}$ is associated with physical (i.e., non-negative) mass-matrix eigenvalues, whereas $\tilde{A}^{(6)}$ is not. Their existence is highly nontrivial since they do not exist at quartic truncation order, neither in the LPA [12,18] nor in the LPA' [31].

The $O(8)$-symmetric fixed point remains IR unstable for all $D(\neq 4)$ and becomes marginal at $D = 4$. As expected, the Gaussian fixed point becomes IR stable for $D > 4$. At $D = 0$ the eigenvalues for $A^{(6)}$ are given by $(70.229, 2.407 + 16.070\mathrm{i}, 2.407 - 16.070\mathrm{i}, 10.398, -2)$. The fixed point remains IR stable below the critical dimension $D_c \sim 3.69$ at which the real part of the complex eigenvalues changes sign, $(4.402, -1.767, -0.002 + 0.955\ \mathrm{i}, -0.002 - 0.955\ \mathrm{i}, 0.399)$, and it remains unstable above. Similarly, $D_c \sim 3.65$ for $\tilde{A}^{(6)}$. The relatively large $\epsilon = 4 - D_c \sim 0.3$ provides further evidence that the ϵ-expansion is not capable of finding an $U(2)_A \times U(2)_V$-symmetric fixed point [14].

All fixed points are also present in the $\mathrm{LPA}_D'^{(6)}$ with slightly different coordinates and eigenvalues. The $U(2)_A \times U(2)_V$-symmetric fixed points now become IR unstable. One at $D \sim 3.65$ and the other one at $D \sim 3.62$.

The occurrence of the marginal eigenvalues for the Gaussian fixed point can be explained as follows. In general the beta functions for a rescaled mass parameter $\bar{m}^2$, a rescaled quartic coupling $\bar{\lambda}_4$, and a rescaled sextic coupling $\bar{\lambda}_6$, respectively, are given by

$$\begin{aligned} \beta_{m^2} &= (-2 + \eta)\bar{m}^2 + f_2(\bar{p}), \\ \beta_4 &= (D - 4 + 2\eta)\bar{\lambda}_4 + f_4(\bar{p}), \\ \beta_6 &= (2D - 6 + 3\eta)\bar{\lambda}_6 + f_6(\bar{p}), \end{aligned} \tag{7}$$

where the $f_i(\bar{p})$ denote nonlinear functions of the rescaled parameters. Since these functions as well as the anomalous dimension, η, vanish at the Gaussian fixed point, we can conclude that (for $D = 3$) $\bar{m}^2$ and $\bar{\lambda}_4$ are relevant parameters with respect to this fixed point. They yield stability matrix eigenvalues -2 and -1, respectively. Similarly, the sextic coupling contributes a vanishing eigenvalue at the Gaussian fixed point, and higher-order couplings yield positive eigenvalues.

For $n \geq 8$ several $U(2)_A \times U(2)_V$-symmetric fixed points exist. In Fig. 1 we only show those which are relevant for the discussion of the IR-stable ones. We were able to distinguish the individual fixed points unambiguously from each other since their coordinates and eigenvalues change little when proceeding to the next higher order. We also plot the values for the critical exponent ν reported in Ref. [14] ($\nu \sim 0.71$ for the MZM scheme, $\nu \sim 0.76$ for the $\overline{MS}$ scheme). The critical exponent associated with $A^{(6)}$ ($\nu \sim 1/1.374 \sim 0.728$ for the LPA, $\nu \sim 1/1.361 \sim 0.735$ for the LPA') is in unexpectedly good agreement. The value for the anomalous dimension in the LPA' is significantly smaller compared to the result of Ref. [14] ($\eta \sim 0.0334$ compared to $\eta \sim 0.12$). The IR-stable $A^{(n>6)}$ only exists in the LPA' but disappears for $n > 14$. Another $U(2)_A \times U(2)_V$-symmetric, IR-unstable fixed point (with

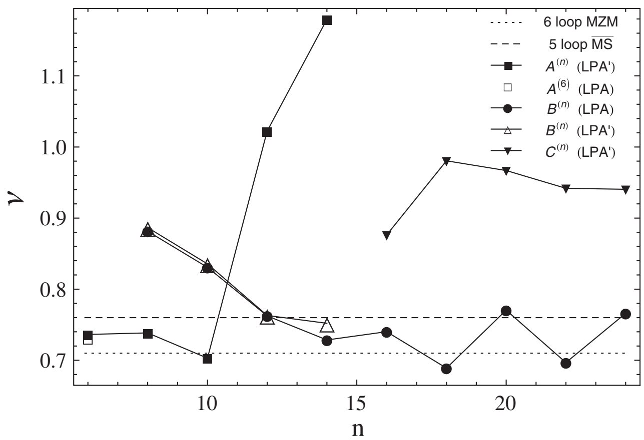

FIG. 1. Critical exponent ν for several $U(2)_A \times U(2)_V$-symmetric fixed points at different polynomial truncation order n. $D = 3$.

three relevant eigenvalues), $B^{(n)}$, shows oscillatory behavior around $\nu \sim 0.73$ for $n \geq 12$ in the LPA. The $B^{(n<18)}$ are nonspiral, and $B^{(18)}$ contains one irrelevant eigenvalue with a small imaginary part which continuously grows for $n > 18$. $B^{(n)}$ also exists in the LPA' between $8 \leq n \leq 14$. Its successor at $n = 16$ is another $U(2)_A \times U(2)_V$-symmetric, IR-unstable spiral fixed point, $C^{(16)}$, with coordinates close to those of $B^{(14)}$.

In the remainder of this section, we will argue why the LPA' remains inconclusive, pointing out general differences between the FRG and other RG approaches first. For a more fundamental comparison between both approaches, we refer to Refs. [33,34].

In the framework of the ϵ-expansion or other loop expansions at fixed spatial dimension D, one usually argues that also in case of non-Gaussian fixed points the canonical scaling dimension determines if a coupling can affect stability [35]. Accordingly, depending on the sign of their canonical scaling dimension, one speaks of relevant, marginal, and irrelevant parameters. Obviously, especially marginal eigenvalues are sensitive to the loop order. Therefore, one has to consider the possibility that higher-order loop corrections change the marginal eigenvalue into a nonvanishing one. It is important to note that if a marginal eigenvalue for a certain fixed point turns nonzero at higher order this can also change the stability properties of the other fixed points. This is for example the case in the $O(N = 4)$ model with di-icosahedral anisotropy [36]. In the presence of an anisotropy, the $O(4)$-symmetric fixed point acquires a marginal eigenvalue at one-loop order in the ϵ-expansion, whereas the anisotropic fixed point is IR unstable. At two-loop order, however, the anisotropic fixed point can become the IR-stable one. We reinvestigated the situation using the FRG in LPA and found that the anisotropic fixed point also becomes IR stable when going beyond the quartic truncation order [20].

However, a change of stability can occur even in the absence of any marginal eigenvalues. A famous example is the $O(N)$ model with cubic anisotropy for $D = 3$ [37,38]. The model exhibits an $O(N)$-symmetric (isotropic) fixed point as well as a cubic fixed point. For $N > N_c$ the cubic fixed point is the IR-stable one, the isotropic fixed point being IR unstable, and vice versa for $N < N_c$. The value for N_c depends on the loop order as well as on the resummation scheme and is still under debate.

In comparison to loop expansions, the stability matrix eigenvalues are much more sensitive to the polynomial truncation order in the FRG formalism. Using the FRG, the accuracy of the critical exponents heavily depends on irrelevant couplings [39]. This is explained by the fact that fluctuations are taken into account differently in both approaches. Irrelevant couplings can be safely ignored in the loop expansion, and nonperturbative effects are captured by using resummation. In contrast, if we were able to solve the FRG equation without truncating the effective action, we would obtain exact results. In the LPA at quartic truncation order, however, one generically reproduces the one-loop ϵ-expansion results when setting the mass parameter to zero [12,30].

Apart from the stability properties, also the existence of fixed points can depend on the truncation, as demonstrated above for the $N_f = 2$ linear sigma model. This was also observed applying the Wegner–Houghton equation to the $U(3)_A \times U(3)_V$-symmetric model [40].

Our conclusions are as follows. One cannot trust an approximation scheme only because no marginal eigenvalues appear for the non-Gaussian fixed points. It is important to investigate the convergence of the results with respect to the truncation order (polynomial, derivative-expansion, and loop order, respectively). The persistent occurrence of $U(2)_A \times U(2)_V$-symmetric fixed points at high truncation order in the LPA as well as in LPA', together with the comparison of our results for ν with those of resummed loop expansions, provides further evidence for their physical relevance. To decide on the stability, however, one needs to study the convergence of results beyond the LPA' taking into account derivative couplings.

III. CONCLUSIONS

We further investigated the possibility that the two-flavor chiral phase transition can be of second order in the absence of the axial anomaly, using the FRG method with the Litim regulator in the LPA as well as in the LPA'.

We found two IR-stable, $U(2)_A \times U(2)_V$-symmetric fixed points at polynomial truncation order $n = 6$, one of them associated with unphysical masses. The value for the critical exponent, $\nu \sim 0.73$, calculated for the one associated with physical masses, $A^{(6)}$, is in unexpectedly good agreement with the result of Ref. [14]. At least for $12 \leq n \leq 24$ in the LPA, another (IR-unstable) $U(2)_A \times U(2)_V$-symmetric fixed point, $B^{(n)}$, shows oscillatory behavior around $\nu \sim 0.73$.

The fact that an $U(2)_A \times U(2)_V$-symmetric fixed point appears by simply including sextic invariants demonstrates that its existence not only depends on the spatial dimension but also on the way nonperturbative corrections are taken into account. In the framework of a resummed perturbative expansion, this concerns the resummation scheme and the perturbative order.

Our main conclusion is that the stability of the $U(2)_A \times U(2)_V$-symmetric fixed point of the linear sigma model remains unclear since the fixed-point structure changes significantly when going from the LPA to the LPA'. Also the dependence on the regulator should be carefully investigated. Since the fixed-point structure of the dimensionally reduced theory controls the behavior near T_c [41], previous finite-temperature studies [18,20,31] remain inconclusive, too. Based on the observation that the fixed points $A^{(n\leq 10)}$ and $B^{(n>10)}$ yield a value for ν close to 0.73, we speculate that both fixed points merge at higher order in the derivative expansion.

Finally, the simultaneous occurrence of two IR-stable fixed points (although one of them is unphysical) is interesting regarding the universality hypothesis. The example illustrates that, in principle, it is possible that two systems sharing (a) the same spatial dimension, (b) the same number of order parameter components, and (c) the same symmetry properties can be attracted to different IR-stable fixed points (here $A^{(6)}$ and $\tilde{A}^{(6)}$, respectively). However, we state clearly that the given example has to be regarded as an artifact of the utilized truncation. A similar situation, although to our knowledge not strictly ruled out, is commonly not believed to appear in a physical setting.

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