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Leptonic *CP* violating phase in the Yukawaon model

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In the so-called "Yukawaon" model, the (effective) Yukawa coupling constants $Y_f^{\rm eff}$ are given by vacuum expectation values (VEVs) of scalars Y_f (Yukawaons) with 3×3 components. In this brief article, we change VEV forms $\langle Y_f\rangle$ in the previous paper into a unified form. Therefore, parameter fitting for quark and lepton masses and mixings is revised. Especially, we obtain predicted values of neutrino mixing $\sin^2 2\theta_{13}$ and a leptonic CP violating phase δ_{CP}^{ℓ} that are consistent with the observed curve in the $(\sin^2 2\theta_{13}, \delta_{CP}^{\ell})$ reported by the T2K group recently.

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I. INTRODUCTION

Now measurement of CP violating phase δ_{CP}^{ℓ} in the lepton sector is within our reach because of the recent development of neutrino physics [1]. The measurement is very important to check quark and lepton mass matrix models currently proposed. At the same time, for model builders, it is urgently required to predict an explicit value of δ_{CP}^{ℓ} together with mixing value $\sin^2 2\theta_{13}$ based on their models. So, we estimate a value of δ_{CP}^{ℓ} based on the so-called Yukawaon model [2,3], which is a unified mass matrix model of quarks and leptons, and which is a kind of flavon model [4].

In the Yukawaon model, the (effective) Yukawa coupling constants $Y_f^{\rm eff}$ are given by vacuum expectation values (VEVs) of scalars Y_f (Yukawaons) with $(\mathbf{8}+\mathbf{1})$ of U(3) family symmetry:

$$(Y_f^{\rm eff})_i^{\ j} = \frac{y_f}{\Lambda}\langle Y_f\rangle_i^{\ j} \quad (f=u,d,\nu,e), \tag{1}$$

where Λ is a scale of the effective theory. In understanding flavor physics from the point of view of a non-Abelian family symmetry, the conventional Yukawa interactions explicitly break their family symmetry. It is only when the conventional Yukawa coupling constants are supposed to be given by Eq. (1) that we can build a model with an unbroken family symmetry.

The characteristic point of the Yukawaon model is the following point: The quark and lepton mass matrices are described by using only the observed values of charged lepton masses (m_e, m_μ, m_τ) as input parameters with family-number dependent values; thereby, we investigate whether we can describe all other observed mass spectra (quark and neutrino mass spectra) and mixings [the Cabibbo-Kobayashi-Maskawa [5] (CKM) mixing and the Pontecorvo-Maki-Nakagawa-Sakata [6] (PMNS) mixing] without using any other family number-dependent parameters. Here, the terminology "family number-independent parameters" means, for example, coefficients of a unit matrix $\mathbf{1}$, a democratic matrix X_3, and so on, where

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad X_3 = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{2}$$

In the previous paper, the form of $\langle Y_d\rangle$ in the down-quark sector has been supposed to be unnaturally different from those in other sectors. In this paper, we revise the form of $\langle Y_f\rangle$ so that it takes a unified form for all sectors as given in Eq. (3) in the next section. Accordingly, parameter fitting for quark and lepton masses and mixings is also revised as given in Secs. III and IV. Especially, it is shown in Sec. IV that we obtain predicted values for neutrino mixing $\sin^2 2\theta_{13}$ and a leptonic CP violating phase δ_{CP}^{ℓ} that are consistent with the observed curve in the $(\sin^2 2\theta_{13}, \delta_{CP}^{\ell})$ plane reported by the T2K group [7] recently.

II. MODELS

Hereafter, for convenience, we use the notation $\hat{A}$, A, and $\bar{A}$ for fields with $\mathbf{8}+\mathbf{1}$, $\mathbf{6}$, and $\mathbf{6}^*$ of U(3), respectively. Explicit forms of VEV relations among the Yukawaon in this paper are given by

$$\langle \hat{Y}_f\rangle_i^{\ j} = k_f[\langle\Phi_f\rangle_{ik}\langle\bar{\Phi}_f\rangle^{kj} + \xi_f \mathbf{1}_i^{\ j}] \quad (f=e,\nu,d,u), \tag{3}$$

$$\langle\Phi_f\rangle_{ij} = k'_f\langle\Phi_0\rangle_{i\alpha}\langle\bar{S}_f\rangle^{\alpha\beta}\langle\Phi_0^T\rangle_{\beta j},$$
$$\langle\bar{\Phi}_f\rangle^{ij} = k'_f\langle\bar{\Phi}_0\rangle^{i\alpha}\langle S_f\rangle_{\alpha\beta}\langle\bar{\Phi}_0^T\rangle^{\beta j}, \quad (f=e,\nu), \tag{4}$$

$$\langle\bar{E}_u\rangle^{ik}\langle\Phi_u\rangle_{kl}\langle\bar{E}_u\rangle^{lj} = \langle\bar{\Phi}_0\rangle^{i\alpha}\langle S_u\rangle_{\alpha\beta}\langle\bar{\Phi}_0^T\rangle^{\beta j},$$
$$\langle E_u\rangle_{ik}\langle\bar{\Phi}_u\rangle^{kl}\langle E_u\rangle_{lj} = \langle\Phi_0\rangle_{i\alpha}\langle\bar{S}_u\rangle^{\alpha\beta}\langle\Phi_0^T\rangle_{\beta j}, \tag{5}$$

$$\langle\bar{P}_d\rangle^{ik}\langle\Phi_d\rangle_{kl}\langle\bar{P}_d\rangle^{lj} = \langle\bar{\Phi}_0\rangle^{i\alpha}\langle S_d\rangle_{\alpha\beta}\langle\bar{\Phi}_0^T\rangle^{\beta j},$$
$$\langle P_d\rangle_{ik}\langle\bar{\Phi}_d\rangle^{kl}\langle P_d\rangle_{lj} = \langle\Phi_0\rangle_{i\alpha}\langle\bar{S}_d\rangle^{\alpha\beta}\langle\Phi_0^T\rangle_{\beta j}, \tag{6}$$

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$$\langle S_f\rangle_{\alpha\beta} = (\mathbf{1} + a_f X_3)_{\alpha\beta}, \qquad \langle \bar{S}_f\rangle^{\alpha\beta} = (\mathbf{1} + a_f X_3)^{\alpha\beta}, \tag{7}$$

where $\langle E\rangle = \mathbf{1}$, and indices $\alpha, \beta, \ldots$ are of another family symmetry $\mathrm{U}(3)'$. We consider that the form (7) is due to a symmetry breaking $\mathrm{U}(3)' \to \mathrm{S}_3$ at $\mu = \Lambda'$. The ξ_f terms in Eq. (3) will be discussed later. Here, the VEV matrices $\hat{Y}_e$, $\hat{Y}_\nu$, $\hat{Y}_u$, and $\hat{Y}_d$ correspond to charged lepton mass matrix M_e, neutrino Dirac mass matrix M_{Dirac}, up-quark mass matrix M_u, and down-quark mass matrix M_d, respectively. Hereafter, we drop flavor-independent factors in those VEV matrices, because we deal with only mass ratios and mixings in this paper.

The VEV structures are essentially the same as in the previous paper [3]. However, we have done the following minor changes from the previous paper: (i) In the previous paper, $\langle \hat{Y}_d\rangle$ and $\langle \Phi_d\rangle$ were given by $\langle \hat{Y}_d\rangle = \langle \Phi_d\rangle\langle \bar{\Phi}_d\rangle$ and $\langle \Phi_d\rangle = \langle \Phi_0\rangle\langle \bar{S}_d\rangle\langle \Phi_0\rangle + \xi'_d\mathbf{1}$, respectively, differently from other sectors. However, it is unnatural that such a term $\xi'_d\mathbf{1}$ appears only in the VEV of Φ_d. In this paper, we remove the $\xi'_d\mathbf{1}$ term from the Φ_d and unify the appearance place of the $\mathbf{1}$ terms that appear in $\langle \hat{Y}_f\rangle$ common to all sectors as shown in Eq. (3). (ii) Along with the changing of the VEV structure in the down-quark sector, a phase matrix P_u in the previous paper is moved to the down-quark sector as shown in Eq. (6). For convenience, $\bar{E}$ in Eq. (5) and $\bar{P}_d$ in Eq. (6) were exchanged with $\bar{P}_u$ and $\bar{E}$ in the previous paper, respectively.

Neutrino mass matrix M_ν is given by a seesaw type

$$(M_\nu)^{ij} = \langle \hat{Y}_\nu^T\rangle^i_k \langle Y_R^{-1}\rangle^{kl}\langle \hat{Y}_\nu\rangle^j_l, \tag{8}$$

as in the previous paper [3], where

$$\langle Y_R\rangle_{ij} = \langle \hat{Y}_e\rangle_i^k\langle \Phi_u\rangle_{kj} + \langle \Phi_u\rangle_{ik}\langle \hat{Y}_e^T\rangle^k_j. \tag{9}$$

In general, we can choose either one in two cases, (a) $\langle \bar{A}\rangle = \langle A\rangle^*$ or (b) $\langle \bar{A}\rangle = \langle A\rangle$, for VEV matrices $\langle A\rangle$ and $\langle \bar{A}\rangle$ under the D-term condition. We assume the type (b) for Φ_f and S_f, while we assume the type (a) for P_d:

$$\begin{aligned}\langle P_d\rangle &= v_P \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, 1),\\ \langle \bar{P}_d\rangle &= v_P \mathrm{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1).\end{aligned} \tag{10}$$

In order to distinguish each Yukawaon from the others, we assume that $\hat{Y}_f$ have different R charges from each other together with considering R-charge conservation [a global U(1) symmetry in $N = 1$ supersymmetry (SUSY)]. The R-charge assignments are essentially not changed from the previous paper [3] except for E_u and P_d.

Since we consider that the charged lepton mass matrix is the most fundamental one, we assume $a_e = 0$ and $\xi_e = 0$. Then, $\langle \Phi_0\rangle$ is expressed as follows:

$$\langle \Phi_0\rangle = \langle \bar{\Phi}_0\rangle \equiv \mathrm{diag}(x_1, x_2, x_3) \propto \mathrm{diag}(m_e^{1/4}, m_\mu^{1/4}, m_\tau^{1/4}), \tag{11}$$

from the D-term condition, where x_i are real and those are normalized as $x_1^2 + x_2^2 + x_3^2 = 1$.

Now let us give a brief review of the derivation of ξ_f terms. We assume the following superpotential for $\hat{Y}_f$ ($f = \nu, e, u, d$), with introducing flavons $\hat{\Theta}_f$,

$$\begin{aligned}W_{\hat{Y}} = \sum_{f=\nu,e,u,d} &[(\mu_f(\hat{Y}_f)_i^j + \lambda_f(\Phi_f)_{ik}(\bar{\Phi}_f)^{kj})(\hat{\Theta}_f)^i_j \\ &+ (\mu'_f(\hat{Y}_f)^i_i + \lambda'_f(\Phi_f)_{ik}(\bar{\Phi}_f)^{ki})(\hat{\Theta}_f)^j_j].\end{aligned} \tag{12}$$

(Here, we have assumed that only $\hat{\Theta}_f$ can be allowed to appear as a form $\mathrm{Tr}[\hat{\Theta}]$ in the superpotential.) Then a SUSY vacuum condition $\partial W_{\hat{Y}}/\partial\hat{\Theta}_f = 0$ leads to VEV relation

$$\langle \hat{Y}_f\rangle = \langle \Phi_f\rangle\langle \bar{\Phi}_f\rangle + \xi_f\mathbf{1}, \tag{13}$$

where

$$\begin{aligned}\xi_f &= -\frac{\mu'_f}{\mu_f}\left(\mathrm{Tr}[\langle \hat{Y}_f\rangle] + \frac{\lambda'_f}{\mu'_f}\mathrm{Tr}[\langle \Phi_f\rangle\langle \bar{\Phi}_f\rangle]\right)\\ &= -\frac{\lambda_f/\mu_f - \lambda'_f/\mu'_f}{1 - 3\mu'_f/\mu_f}\mathrm{Tr}[\langle \Phi_f\rangle\langle \bar{\Phi}_f\rangle].\end{aligned} \tag{14}$$

Here, we have assumed that all VEVs of flavons $\hat{\Theta}$ take $\langle \hat{\Theta}\rangle = 0$, so that SUSY vacuum conditions for other flavons do not bring any additional VEV relations. As seen in Eq. (14), if $\langle \Phi_f\rangle$ is complex, then the coefficient ξ_f becomes complex, too. Although the derivation discussed above was given in the previous work [3], we considered that the effect of the phase of ξ_ν is negligibly small, so that we treated ξ_ν as a real parameter approximately in the previous work. However, in this paper, we found that the phase of ξ_ν affects not a little on our parameter fitting.

III. PARAMETER FITTING

General: We summarize our mass matrices M_f ($\langle Y_f\rangle$) as follows:

$$\begin{aligned}Y_e &= \Phi_e\bar{\Phi}_e + \xi_e\mathbf{1},\\ \Phi_e &= \bar{\Phi}_e = \Phi_0(\mathbf{1} + a_e X_3)\Phi_0, \quad (a_e = 0, \xi_e = 0),\end{aligned} \tag{15}$$

$$Y_\nu = \Phi_\nu\bar{\Phi}_\nu + \xi_\nu e^{i\beta_\nu}\mathbf{1}, \quad \Phi_\nu = \bar{\Phi}_\nu = \Phi_0(\mathbf{1} + a_\nu e^{i\alpha_\nu}X_3)\Phi_0, \tag{16}$$

$$Y_u = \Phi_u\bar{\Phi}_u + \xi_u\mathbf{1}, \qquad \Phi_u = \bar{\Phi}_u = \Phi_0(\mathbf{1} + a_u X_3)\Phi_0, \tag{17}$$

$$\begin{aligned}Y_d &= \Phi_d\bar{\Phi}_d + \xi_d e^{i\beta_d}\mathbf{1}, \qquad \Phi_d = P_d^*\Phi_0(\mathbf{1} + a_d e^{i\alpha_d}X_3)\Phi_0 P_d^*,\\ \bar{\Phi}_d &= P_d\Phi_0(\mathbf{1} + a_d e^{i\alpha_d}X_3)\Phi_0 P_d,\end{aligned} \tag{18}$$

$$M_\nu = Y_\nu Y_R^{-1} Y_\nu, \qquad Y_R = Y_e \Phi_u + \Phi_u Y_e. \tag{19}$$

Here, for convenience, we have dropped the notations "$\langle$", "$\rangle$" and "$\hat{}$". Since we are interested only in the mass ratios and mixings, we use dimensionless expressions $\Phi_0 = \mathrm{diag}(x_1, x_2, x_3)$ (with $x_1^2 + x_2^2 + x_3^2 = 1$), $P_d = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$, and $E = \mathbf{1} = \mathrm{diag}(1, 1, 1)$. Therefore, the parameters a_e, a_ν, ... are redefined by Eqs. (15)–(19).

Since the parameters a_f in Eq. (7) can be complex in general, we denote a_f as $a_f e^{i\alpha_f}$ by real parameters (a_f, α_f). The VEV structure of Y_u in the present paper is practically unchanged from the previous paper [3], so that we inherit the numerical results in the up-quark sector in the previous work by assuming $\alpha_u = 0$. Since we choose α_ν and α_d as $\alpha_\nu \neq 0$ and $\alpha_d \neq 0$, we have $\beta_\nu \neq 0$ and $\beta_d \neq 0$ according to Eq. (14). We have denoted ξ_ν and ξ_d in Eq. (3) as $\xi_\nu e^{i\beta_\nu}$ and $\xi_d e^{i\beta_d}$, respectively, in Eqs. (16) and (18). Of course, the parameters β_f are fixed by the values (a_f, α_f), so that β_f are not free parameters.

The explicit values of the parameters (x_1, x_2, x_3) are fixed by Eq. (11) as

$$(x_1, x_2, x_3) = (0.115144, 0.438873, 0.891141), \tag{20}$$

where we have normalized x_i as $x_1^2 + x_2^2 + x_3^2 = 1$. Therefore, in the present model, except for the parameters (x_1, x_2, x_3), we have ten adjustable parameters, $(a_\nu, \alpha_\nu, \xi_\nu)$, (a_u, ξ_u), (a_d, α_d, ξ_d), and (ϕ_1, ϕ_2) for the 16 observable quantities (six mass ratios in the up-quark, down-quark, and neutrino sectors, four CKM mixing parameters, and $4 + 2$ PMNS mixing parameters).

Quark mass ratios: First, we fix the parameter values (a_u, ξ_u) from the observed up-quark mass ratios [8] $r_{12}^u \equiv (m_u/m_c)^{1/2} = 0.045^{+0.013}_{-0.010}$ and $r_{23}^u \equiv (m_c/m_t)^{1/2} = 0.060 \pm 0.005$ at $\mu = m_Z$ [8] as follows:

$$(a_u, \xi_u) = (-1.4715, -0.001521). \tag{21}$$

Of course, we obtain the same values as those in the previous paper.

Next, we try to fix the parameters (a_d, α_d, ξ_d) in the down-quark sector by using input parameters [8] $r_{12}^d \equiv m_d/m_s = 0.053^{+0.005}_{-0.003}$ and $r_{23}^d \equiv m_s/m_b = 0.019 \pm 0.006$. However, since we have three parameters for two input values m_d/m_s and m_s/m_b, we cannot fix our three parameters. It is more embarrassing that there is no solution of $m_s/m_b \sim 0.019$ in the (a_d, α_d, ξ_d) parameter region. Nevertheless, we found that the minimal value of m_s/m_b is $m_s/m_b \sim 0.03$ at $(a_d, \alpha_d, \xi_d) \sim (-1.5, 16°, 0.004)$, which can give a reasonable value of m_d/m_s at the same time, too. Therefore, we take the following values:

$$(a_d, \alpha_d, \xi_d) = (-1.4735, 15.7°, 0.00400), \tag{22}$$

which leads to predictions $r_{12}^d = 0.0597$ and $r_{23}^d = 0.0312$. Note that the value $r_{23}^d = 0.0312$ is considerably large compared with $r_{23}^d \simeq 0.019$ by Xing *et al.* [8], while the value is consistent with $r_{23}^d \simeq 0.031$ by Fusaoka and Koide [9]. The values $m_d(\mu)$ and $m_s(\mu)$ are estimated at a lower energy scale, $\mu \sim 1$ GeV, so that we consider that the ratio r_{12}^d at $\mu = M_Z$ is reliable. On the other hand, the value $m_b(\mu)$ is extracted at a different energy scale $\mu \sim 4$ GeV from $\mu \sim 1$ GeV, so that the value $m_b(M_Z)$ is affected by the prescription of threshold effects at $\mu = m_t$, while the value $m_s(M_Z)$ is affected by those at $\mu = m_c$, $\mu = m_b$, and $\mu = m_t$. We consider that as for the ratio r_{23}^d at $\mu = M_Z$ the value is still controversial. Anyhow, we have fixed three parameters (a_d, α_d, ξ_d) only from two values m_d/m_s and m_s/m_b.

CKM mixing: The purpose of the present paper is to discuss PMNS parameters, especially *CP* violating phase δ_{CP}^ℓ. However, since our model is to give unified description of quarks and leptons, for reference, we give results of CKM parameter fitting, too.

Since the parameters (a_u, ξ_u) and (a_d, α_d, ξ_d) have been fixed by the observed quark mass ratios, the CKM mixing matrix elements $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, and $|V_{td}|$ are functions of the remaining two parameters ϕ_1 and ϕ_2 defined by Eq. (10). We use the observed CKM mixing matrix elements [10] $|V_{us}| = 0.2254 \pm 0.0006$, $|V_{cb}| = 0.0414 \pm 0.0012$, $|V_{ub}| = 0.00355 \pm 0.00015$, and $|V_{td}| = 0.00886^{+0.00033}_{-0.00032}$. (Two of those are used as input values in the present analysis, and the remaining two are our predictions as references.) All the experimental CKM parameters are satisfied by fine-tuning the parameters ϕ_1 and ϕ_2 as

$$(\phi_1, \phi_2) = (-42.0°, -15.1°), \tag{23}$$

which leads to the numerical results as follows: $|V_{us}| = 0.2255$, $|V_{cb}| = 0.0429$, $|V_{ub}| = 0.00359$, and $|V_{td}| = 0.00928$ with $\delta_{CP}^\ell = 73.0°$. In spite of our aim described in Sec. I, we are forced to introduce family number-dependent parameters (ϕ_1, ϕ_2) in the present model, too, the same as in the previous model [3]. Model building without using parameter (ϕ_1, ϕ_2) is left to our future task.

IV. PARAMETER FITTING IN THE PMNS MIXING AND *CP* VIOLATING PHASE δ_{CP}^ℓ

We have already fixed our seven parameters as Eqs. (21)–(23). The remaining free parameters are only $(a_\nu, \alpha_\nu, \xi_\nu)$ in the Dirac neutrino sector. We determine the parameter values of $(a_\nu, \alpha_\nu, \xi_\nu)$ as follows:

$$(a_\nu, \alpha_\nu, \xi_\nu) = (-3.54, -18.0°, -0.0238), \tag{24}$$

which are obtained so as to reproduce the observed values [10] of the following PMNS mixing angles and R_ν:

$$\sin^2 2\theta_{12} = 0.846 \pm 0.021,$$
$$\sin^2 2\theta_{13} = 0.093 \pm 0.008, \tag{25}$$

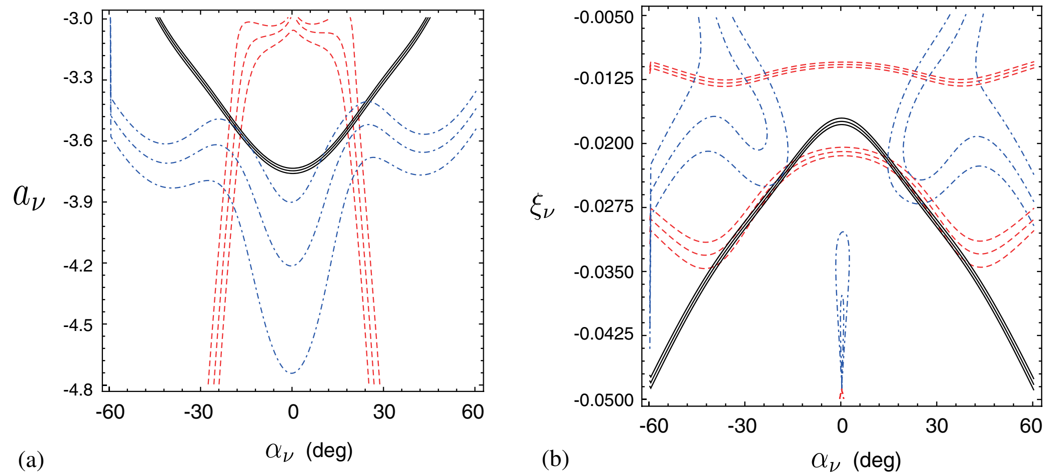

FIG. 1 (color online). Contour curves of the observed center, upper, and lower values of the lepton mixing parameters $\sin^2 2\theta_{12}$ (dashed), $\sin^2 2\theta_{13}$ (dot dashed), and the neutrino mass squared difference ratio R_ν (solid). (a): We draw the curves in the (α_ν, a_ν) plane by taking $\xi_\nu = -0.0238$. (b): We draw the curves in the (α_ν, ξ_ν) plane by taking $a_\nu = -3.54$.

$$R_\nu \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{m^2_{\nu 2} - m^2_{\nu 1}}{m^2_{\nu 3} - m^2_{\nu 2}} = \frac{(7.53 \pm 0.18) \times 10^{-5}\ \text{eV}^2}{(2.44 \pm 0.06) \times 10^{-3}\ \text{eV}^2} = (3.09 \pm 0.15) \times 10^{-2}. \quad (26)$$

We show the a_ν and α_ν dependencies of the PMNS mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and R_ν in Figs. 1(a)–1(b), respectively. It is found that R_ν is very sensitive to a_ν.

As seen in Fig. 2, we obtain two solutions, which are consistent with the neutrino data except for the data of δ^ℓ_{CP}. However, as seen in the best fit curve on the $(\sin^2 2\theta_{13}, \delta^\ell_{CP})$ plane in Fig. 5 in the resent T2K article [7], the solution with $0 < \delta^\ell_{CP} < \pi$ is obviously ruled out. Therefore, we adopt the solution with $-\pi < \delta^\ell_{CP} < 0$ in our model. Then we obtain the predictions of our model,

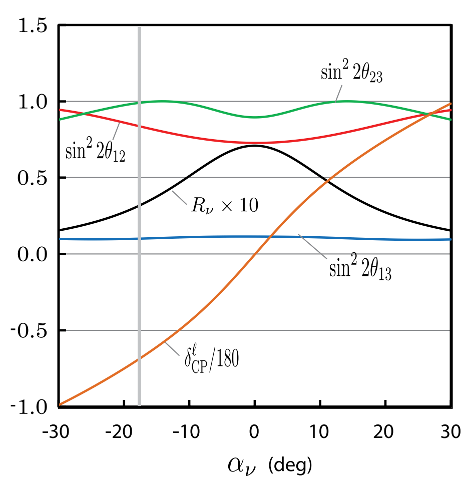

FIG. 2 (color online). α_ν dependence of the lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, R_ν, and the leptonic CP violating phase δ^ℓ_{CP}. We draw the curves of those as functions of α_ν for the case of $\xi_\nu = -0.0238$ by taking $a_\nu = -3.54$ (solid).

$$R_\nu = 0.0310, \quad \sin^2 2\theta_{12} = 0.837, \quad \sin^2 2\theta_{23} = 0.988,$$
$$\sin^2 2\theta_{13} = 0.0987, \quad \delta^\ell_{CP} = -125^\circ. \quad (27)$$

We can predict neutrino masses, for the parameters given by (21) and (24), as follows:

$$m_{\nu 1} \simeq 0.00037\,\text{eV}, \quad m_{\nu 2} \simeq 0.00868\,\text{eV}, \quad m_{\nu 3} \simeq 0.0501\,\text{eV}, \quad (28)$$

by using the input value [10] $\Delta m^2_{32} \simeq 0.00244\ \text{eV}^2$. We also predict the effective Majorana neutrino mass [11] $\langle m \rangle$ in the neutrinoless double beta decay as

$$\langle m \rangle = |m_{\nu 1}(U_{e1})^2 + m_{\nu 2}(U_{e2})^2 + m_{\nu 3}(U_{e3})^2| \simeq 6.0 \times 10^{-3}\ \text{eV}. \quad (29)$$

Our model predicts $\delta^\ell_{CP} = -125^\circ$ for the Dirac CP violating phase in the lepton sector, which indicates a relatively large CP violating effect in the lepton sector.

V. CONCLUDING REMARKS

We have tried to describe quark and lepton mass matrices by using only the observed values of charged lepton masses (m_e, m_μ, m_τ) as input parameters with family number-dependent values, except for P_d defined by Eq. (10). Thereby, we have investigated whether we can describe all other observed mass spectra (quark and neutrino mass spectra) and mixings (CKM and PMNS mixings) without using any other family number-dependent parameters. In conclusion, we have obtained reasonable results. We have predicted the CP violating phase in the lepton sector as $\delta^\ell_{CP} \simeq -125^\circ$ and

$\sin^2 2\theta_{13} \simeq 0.099$ in Eq. (27), which are consistent with the observed curve in the $(\sin^2 2\theta_{13}, \delta^{\ell}_{CP})$ plane that has been reported by the T2K group [7]. (The predicted value of δ^{ℓ}_{CP} in the previous paper was $\delta^{\ell}_{CP} = -26°$.)

The origin of the CP violation in the lepton sector is in the phase factor α_ν in the Dirac neutrino mass matrix (16). Note that we have taken $\alpha_f = 0$ ($f = e, u$) for economy of the parameters. However, we have been obliged to accept $\alpha_\nu \neq 0$ in order to fit the observed value of $\sin^2 2\theta_{13}$.

Although the present model is a minor improved version of the previous paper [3], the predicted value of δ^{ℓ}_{CP} has been changed into a more detectable value in near future neutrino observations, and it is consistent with the recent T2K result [7]. We expect that the value of δ^{ℓ}_{CP} will be confirmed by near future observations.

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