PHYSICAL REVIEW D **90,** 117902 (2014)

Interpretation of an “edge” in proton-proton scattering

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(Received 5 November 2014; published 22 December 2014)

A study of proton-proton collisions at very high energy has revealed a “black disk,” whose radius grows with the logarithm of the center-of-mass energy, surrounded by an edge of approximately constant width 1 fm. We interpret this behavior as the maximum length of a QCD string connecting the color triplet and antitriplet components of the proton, and we propose further tests of this explanation.

DOI: 10.1103/PhysRevD.90.117902 PACS numbers: 11.80.Fv, 13.85.Lg, 13.85.Dz, 13.75.Cs

It has been known for over 50 years that the total proton-proton cross section cannot grow with energy faster than $\ln^2 s$, where s is the square of the center-of-mass energy [1,2]. Recently fits to pp and $\bar{p}p$ cross sections up to Tevatron energies have indicated a structure consisting of a black disk whose radius grows as $\ln\sqrt{s}$, surrounded by an edge of an approximately constant thickness of ~1 fm [3]. This behavior should be interpreted as a property of the two-body scattering amplitude as a function of the impact parameter. These fits describe data both from the CERN Large Hadron Collider (LHC) and from cosmic ray interactions up to $\sqrt{s} = 80$ TeV [4–6].

The purpose of this paper is to interpret the energy-independent edge as the extent to which a color-triplet constituent of the proton can separate from the remainder (which will be an antitriplet) before the QCD string connecting them breaks. This provides a fuzzy “edge” to the proton, or, for that matter, to any hadron which can dissociate into a 3-3^* virtual pair. We begin by recalling the arguments for a universal “string-breaking” length, apply them to proton-proton scattering, and propose further tests of this prediction of universal behavior.

A heavy quark Q and a heavy antiquark $\bar{Q}$ may be viewed as connected by a color-triplet QCD string, as shown in Fig. 1. When Q and $\bar{Q}$ are pulled apart by a sufficient distance, the QCD string connecting them will contain sufficient energy to permit the production of a light quark-antiquark pair $\bar{q}q$, with $\bar{q}$ connected to Q and q connected to $\bar{Q}$ by separate QCD strings.

The flavor independence of the $Q\bar{Q}$ potential was demonstrated some time ago for charmonium and bottomonium systems [7,8]. Additional recent supporting evidence has come from the observation of an excited $\bar{b}c$ system separated from the ground state B_c by an amount between the (roughly equal) 2S–1S spacings of charmonium and bottomonium [9]. Thus it suffices to base an argument on the bottomonium potential, which contains the most levels below flavor threshold and thus for which the most detailed information is available [10]. Using an approximate simple form for the interquark potential [11], and taking note of the bottomonium flavor threshold of 10.58 GeV, it was found that the b-$\bar{b}$ separation for which the string between them breaks lay between 1.4 and 1.5 fm. At that time unquenched lattice QCD was just entering its infancy, so no corresponding lattice result was available.

The situation has now changed. A detailed QCD lattice calculation with two light flavors [12] estimates a threshold separation $r_c = (1.13 \pm 0.10 \pm 0.10)$ fm, including all systematic errors. One expects the addition of the strange quark flavor to have little effect as the B_s-$\bar{B}_s$ threshold lies considerably higher than the B-$\bar{B}$ threshold of 10.56 GeV.

How does this relate to the thickness of the edge seen in Ref. [3]? One imagines any hadron to consist of quarks, antiquarks, and gluons, each of which can be imagined to be on a “leash” connecting them to the rest of the hadron. Color triplets will be connected to antitriplet remnants; color antitriplets will be connected to triplet remnants; and gluons will be connected to color octet remnants. We shall assume that the color octet string connecting a gluon with the rest of the proton has greater tension and thus plays less of a role in determining the proton’s outer edge. Then the maximum radius of the proton, no matter at what energy,

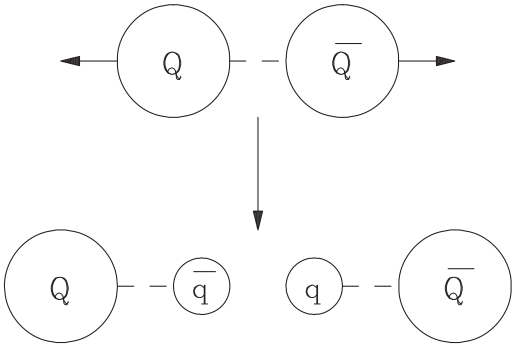

FIG. 1. Top: Heavy quarks Q and $\bar{Q}$ connected by a QCD string (dashed line). When pulled sufficiently apart (bottom) it is energetically favorable for Q to become connected to a light antiquark $\bar{q}$ and $\bar{Q}$ to become connected to a light quark q by separate strings.

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will fluctuate from a value R to a value $R + \Delta$ where Δ corresponds to the maximum length of a color-triplet QCD string.

The proton-proton scattering amplitude a at high energy is predominantly imaginary, $a = (\eta - 1)/(2i)$, where $\eta = 0$ corresponds to maximum absorption and $\eta = 1$ corresponds to no scattering. The total and elastic cross sections may be expressed as integrals over an impact parameter b as

$$\sigma_{\mathrm{tot}} = 8\pi \int_0^\infty (1-\eta) b db/2,$$
$$\sigma_{\mathrm{el}} = 8\pi \int_0^\infty (1-\eta)^2 b db/4. \tag{1}$$

A simplified model of the behavior noted in Ref. [3] may be constructed for an imaginary scattering amplitude with $\eta = 0$ for an impact parameter $b < R$, $\eta = (b - R)/\Delta$ for $R \le b \le R + \Delta$, and $\eta = 1$ for $b > R + \Delta$. For $\eta = 0$, the protons overlap substantially, and the scattering amplitude a is strongly absorptive. The transition region of width Δ is to be identified with the string-breaking distance. Then for $\Delta \ll R$,

$$\begin{aligned}\sigma_{\mathrm{tot}} - 2\sigma_{\mathrm{el}} &= 4\pi \int_R^{R+\Delta} b db \eta (1-\eta) \\ &\simeq 4\pi R \Delta \int_0^1 \eta(1-\eta) d\eta \\ &= 2\pi R\Delta/3.\end{aligned} \tag{2}$$

This quantity is denoted $\pi R t$ in Ref. [3], where $t \simeq 1.1$ fm is interpreted as the thickness of the edge. Thus, in our language, $\Delta = 3t/2 \simeq 1.6$ fm, close to the original estimate of Ref. [10] for the string-breaking distance.

The question is asked in Ref. [3]: How universal is this behavior? To test it, one would have to investigate other systems besides proton-proton collisions. But proton-proton collisions can be used to generate effective pion beams, as in the reactions $pp \to n\pi^+ p$ [13] and $pp \to \Delta^{++} X$ [13,14], where X has the quantum numbers of $\pi^- p$. We thus advocate studying the latter reaction at the LHC with the decay $\Delta^{++} \to \pi^+ p$ detected at low momentum transfer from the initial proton, in such a way that the effect of the pion pole can be isolated. As the pion also can be separated into a color triplet and a color antitriplet, we predict that pion-nucleon scattering will exhibit the same edge behavior as seen in proton-proton scattering.

Another possibility for studying very-high-energy elastic cross sections would be the electroproduction on protons at low Q^2 of the vector mesons (ρ, ω, ϕ), which would occur in the ratio 9:1:2 by virtue of the corresponding quark charges. However, it would be more challenging to separate the total cross sections of individual vector mesons on protons from such data.

I thank Martin Block, Andreas Kronfeld, Wolfgang Ochs, and Leo Stodolsky for discussions; the W. and E. Heraeus Foundation for partial support during a workshop at Oberwölz, Austria; the Max-Planck-Insitüt für Physik for hospitality during part of this investigation, and the Physics Department of the University of Chicago for partial travel support. This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG02-13ER41598.

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