Universality in pure gravity mediation with vector multiplets

Jason L. Evans and Keith A. Olive

William I. Fine Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA

We consider models of pure gravity mediation in which scalar mass universality is imposed at the grand unified scale and gaugino masses are generated through loops. The minimal model requires a very restricted range for $\tan\beta \approx 2$–3 and scalar masses (set by the gravitino mass) of order 300 TeV–1.5 PeV in order to obtain a Higgs mass near 126 GeV. Here we augment the minimal model with one or more sets of vector multiplets (either a **10** and $\overline{\mathbf{10}}$ pair or one or more **5** and **5̄** pairs). If coupled to the minimal supersymmetric standard model Higgs, these allow for radiative electroweak symmetry breaking over a significantly larger range of $\tan\beta \approx 2$–40 and can fit the Higgs mass with much smaller values of the gravitino mass. In these models, the lightest supersymmetric particle is often the bino, and in order to satisfy the relic density constraint, the bino must be nearly degenerate with either the wino or gluino. In the models considered here, bino gluino coannihilations determine the relic density and since the two are nearly degenerate, LHC limits on the gluino mass are greatly relaxed allowing light relatively gravitinos and gluino masses well within the reach of the LHC.

I. INTRODUCTION

With the completion of Run I at the LHC, there is no hint of supersymmetry at mass scales $\tilde{m} \lesssim 1$ TeV [1]. As a result, simple models based on supergravity such as the constrained minimal supersymmetric standard model (CMSSM) [2] are being pushed to higher mass scales [3], taking away one of the arguments for low-energy supersymmetry. On the other hand, the necessity for higher mass scales, opens the door to model building and in particular allows for the construction of very simple models such as pure gravity mediation (PGM) [4–8]. In its simplest form [7], PGM with scalar mass universality contains one single free parameter, the gravitino mass, $m_{3/2}$ which sets the boundary condition for the scalar masses at some UV input scale, usually taken to be the grand unified (GUT) scale.

In the minimal model of PGM, one assumes a flat Kähler potential, and there is no tree level source for either gaugino masses or A terms. At one-loop, gaugino masses and A terms are generated through anomalies [9] and one expects $m_{1/2}, A_0 \ll m_0$ in these models, reminiscent of split supersymmetry [10]. Radiative electroweak symmetry breaking (EWSB) can be incorporated into the model at the expense of one additional parameter, c_H, associated with a Giudice-Masiero-like term [11–13] in the Kähler potential as described below. One can also easily trade c_H for the ratio of the two Higgs vacuum expectation values, $\tan\beta$, leaving the theory to be defined by $m_{3/2}$, $\tan\beta$ and the sign of the μ term. A similar particle spectrum was also derived in models with strong moduli stabilization [13–15].

A Higgs mass ≈ 126 GeV [16] is another phenomenological constraint that must be imposed on the model. In [7], it was shown that the minimal PGM model described above with scalar mass universality and radiative EWSB can account for the correct Higgs mass for gravitino masses in the range about 300–1500 TeV for a narrow range of $\tan\beta = 1.7$–2.5. Indeed, the determination of the Higgs mass provides one with a (model-dependent) upper limit on the scalar mass scale of order a PeV [17,18]. Because the lightest supersymmetric particle (LSP) in this theory is a wino with a nearly degenerate chargino, there is a lower limit on the scalar mass scale of about 80 TeV, needed to satisfy the experimental lower bound on the chargino mass [19]. A long lived chargino may be a tell tale signature of models of this type [4,5,15,20,21]. Dark matter may also be a natural consequence of this model as thermal wino dark matter with a relic density equal to the WMAP/Planck [22] determined value is expected when $m_{3/2} = 460$–500 TeV when $\mu < 0$ [7]. For lower $m_{3/2}$ or $\mu > 0$, either the dark matter comes from a source other than supersymmetry, or winos are produced nonthermally through moduli or gravitino decay [15,20,23–26].

The parameter space in PGM models can be significantly broadened [8] if Higgs mass universality at the GUT scale is not enforced as in nonuniversal Higgs mass models [27–29]. Simply allowing the Higgs soft masses to differ from the gravitino mass at the GUT scale frees up (to some extent) the restricted range on $\tan\beta$ and allows significantly lower values of the gravitino mass while still producing a Higgs mass of 126 GeV. The two Higgs soft masses may equal each other (one extra parameter) or differ (two extra parameters).

In this paper, we consider another generalization of PGM models, which maintains scalar mass universality. PGM is altered to include the contributions of additional vector representations. In particular, we consider the effects of

adding either pairs of **10** and $\overline{\textbf{10}}$'s and/or pairs of **5** and $\bar{\textbf{5}}$'s. The presence of these fields has multiple effects. They affect the running of the gauge couplings, primarily through the change in the beta functions. They also alter the anomaly mediated supersymmetry breaking (AMSB) contribution to gaugino masses as well as the threshold corrections to the gaugino masses and can lead to a much lighter(heavier) than expected gluino(wino). In the models considered, we often find that the LSP is a bino (rather than a wino as in minimal PGM models) and in order to satisfy the relic density constraint, the bino must be nearly degenerate with the gluino.[1] In this case, the LHC limits on the gluino mass [31] are significantly relaxed. If we also include a **5** and $\bar{\textbf{5}}$ the bino can also coannihilate with the wino.

Because these new vectorlike fields can couple to the Higgs through Yukawa couplings, they will affect the renormalization group running of the Higgs mass as well as the EWSB conditions. This will expand the allowed range of $\tan\beta$. Furthermore, these Yukawa couplings will further enhance the Higgs mass. Here we only consider the coupling of the **10** to the uplike MSSM Higgs with coupling y_t'. In this case, the minimization of the Higgs potential is performed as in the CMSSM and yields a solution for μ (and c_H), though the sign of μ is not determined by the solution. It is also possible to couple the new fields to the downlike MSSM Higgs with coupling y_b'. In this case, minimization may give rise to two distinct solutions with $|\mu_1| \neq |\mu_2|$. Here, however we will consider only cases which are affected by the new toplike Yukawa coupling and return to the possible effects of the bottomlike Yukawa elsewhere.

The paper is organized as follows. In the next section, we review and update PGM with scalar mass universality. In particular, we include a new calculation of the Higgs mass in split SUSY models [32] which corrects and updates the previous calculation [33]. We also enforce the experimental value of α_s at the weak scale at the expense of pure gauge coupling unification and examine the Higgs mass in this scenario. In Sec. III, we introduce the vector multiplets and describe our treatment of the running of the renormalization group equations (RGE)'s and loop corrections. In Sec. IV, we display some results for the Higgs and gaugino masses in this model. Our conclusions are given in Sec. V. Details of the calculations are collected in Appendixes A–D.

II. UPDATE ON UNIVERSAL PGM

As noted earlier, PGM models are based on minimal supergravity (mSUGRA) with a flat Kähler potential. The form [34–36] of the scalar potential in mSUGRA is given by

$$V = \left|\frac{\partial W}{\partial \phi^i}\right|^2 + (A_0 W^{(3)} + B_0 W^{(2)} + \text{H.c.}) + m_{3/2}^2 \phi^i \phi_i^*,$$

$$(1)$$

where the ϕ_i's are the low-energy fields, W is the low-scale superpotential,

$$W = (y_e H_1 L e^c + y_d H_1 Q d^c + y_u H_2 Q u^c) + \mu_0 H_1 H_2,$$

$$(2)$$

with the SU(2) indices being suppressed. $W^{(2)}$ and $W^{(3)}$ are the bilinear and trilinear superpotential terms. As one can see, the scalar masses are universal and are proportional to the gravitino mass. In addition, simple models of supersymmetry breaking impose $B_0 = A_0 - m_{3/2}$. If there is an R symmetry, and the gauge kinetic function, $h_{\alpha\beta} \propto \delta_{\alpha\beta}$ is independent of any supersymmetry breaking moduli with nonvanishing F terms, gaugino masses vanish at the tree level, and are generated at one loop through anomalies [9].

The remaining parameters, μ and $\tan\beta$, can be derived from the minimization of the Higgs potential. In general, obtaining the correct electroweak vacuum can be problematic unless one adds a Giudice-Masiero term [11–13],

$$\Delta K = c_H H_1 H_2 + \text{H.c.}, \qquad (3)$$

in the Kähler potential. Here, c_H is a constant and allows the μ and B terms to remain linearly independent, as in the CMSSM. In this way, both μ and c_H can be derived from the minimization of the Higgs potential, while the supergravity GUT scale boundary condition [36] $B_0 = A_0 - m_{3/2}$ is maintained:

$$\mu = \mu_0 + c_H m_{3/2}, \qquad (4)$$

$$B\mu = \mu_0(A_0 - m_{3/2}) + 2c_H m_{3/2}^2. \qquad (5)$$

Above, we have maintained our assumed flat Kähler potential with μ_0 being the μ term of the superpotential.[2] Recall that in PGM models, $A_0 \ll m_{3/2}$.

As the tree-level gaugino masses are essentially vanishing, the dominant source for gaugino masses comes from the one-loop anomaly mediated contributions [9], which are proportional to their one loop MSSM β functions, $\beta_1 = 11$, $\beta_2 = 1$, and $\beta_3 = -3$, giving

$$M_1 = \frac{33}{5}\frac{g_1^2}{16\pi^2} m_{3/2}, \qquad (6)$$

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[1]Similar conclusions were found in a related model which did not include the renormalization group evolution of couplings and masses, nor insist on radiative EWSB [30].

[2]To allow a GM term for the Higgs fields, the R charge of $H_u H_d$ must be zero. In this case, μ_0 must have the same R charge as the gravitino and could arise as some coupling times the gravitino mass. Therefore, the only source of R-symmetry breaking is the gravitino mass. In what follows, we will keep the gravitino as the only source of R-symmetry breaking.

$$M_2 = \frac{g_2^2}{16\pi^2} m_{3/2}, \qquad (7)$$

$$M_3 = -3\frac{g_3^2}{16\pi^2} m_{3/2}. \qquad (8)$$

Here, the subscripts of M_a, $(a = 1, 2, 3)$, correspond to the gauge groups of the Standard Model $U(1)_Y$, $SU(2)$ and $SU(3)$, respectively. Note that there are potentially large one-loop corrections to gaugino masses particularly at small $\tan\beta$ [37,38].

In [7], we followed the prescription detailed in [15] using the calculations in [33] to calculate the Higgs mass, m_h. Assuming gauge coupling unification, we found that the Higgs mass fell into the required range (124–128 GeV) for a narrow range in $\tan\beta \simeq 1.7$–2.5, and $m_{3/2} \simeq$ 300–1500 GeV, with m_h increasing as either $\tan\beta$ or $m_{3/2}$ are increased. Here, we update this result making several changes to the calculation. First, and most importantly, we fix α_s at the weak scale to its measured value taken here as $\alpha_s(M_Z) = .1180$. For example with $m_{3/2} = 1$ PeV and $\tan\beta = 2$, $m_h = 126.5$ GeV if we assume gauge coupling unification. However, in this case, $\alpha_s(M_Z) = 0.088$, far below the experimental value. Fixing $\alpha_s(M_Z)$ corresponds to an increase in α_s and as a consequence a decrease in the top quark Yukawa coupling, y_t, thus lowering m_h by a few percent. In addition, gauge coupling unification is lost as $\alpha_s(M_{GUT})$ is larger than $\alpha_1(M_{GUT}) = \alpha_2(M_{GUT})$ by about 3%. Secondly, we have improved our treatment of the gluino threshold in the running of α_s. With this improvement, m_h is found to be 122.5 GeV at the same

test point. Thirdly, we employ the recent calculations in [32] which correct some errors in the one-loop calculations quoted in [33] and include new two-loop contributions, but both of these changes make only a minor correction to the Higgs mass for the cases considered, as the dominant contribution is due to y_t.

In Fig. 1, we show the updated calculation of the Higgs mass as a function of $\tan\beta$ (left) and $m_{3/2}$ (right). In the left panel, we see that for each value of $m_{3/2}$, the Higgs mass rises as $\tan\beta$ is increased. At some point, the increase is very sudden as the derived value of μ^2 goes to 0, and we lose the ability to achieve successful radiative EWSB. As μ is decreased the Higgsinos become lighter and there are additional contributions to the running of the Higgs quartic coupling. As a result, the Higgs mass is largest for points which corresponds to the focus point region of the CMSSM [39]. For low values of $\tan\beta$, the top quark Yukawa diverges during the running of the RGEs and that region is shown as shaded.

In the right panel of Fig. 1, we see the behavior of the Higgs mass for fixed $\tan\beta$ as a function of the gravitino mass. Once again, as $m_{3/2}$ is increased, the solution for μ is driven smaller and the Higgs mass in increased. Beyond the point where the curve appears to go vertical, there is no radiative EWSB.

In comparison with the results in [7], while the Higgs mass is typically lower, the qualitative conclusions are unchanged. For large $m_{3/2} \sim \mathcal{O}(1)$ PeV, and in a narrow range for $\tan\beta$, a Higgs mass of 126 GeV is possible. The results for the remaining particle spectrum such as the gaugino sector are unchanged.
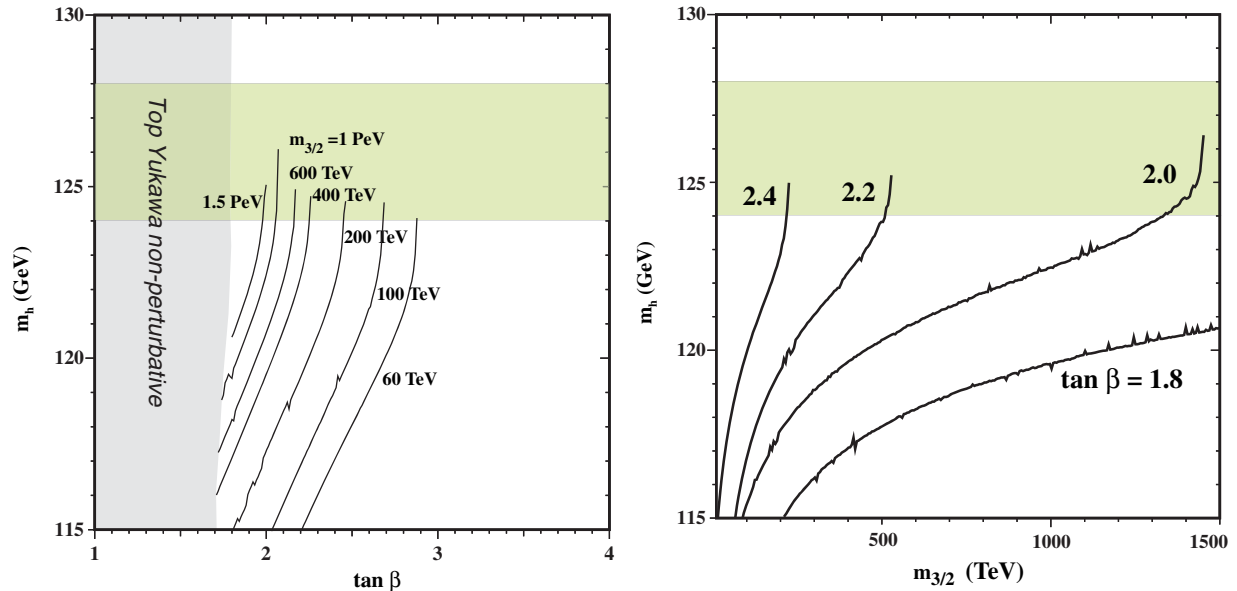

FIG. 1 (color online).   The light Higgs mass as a function of $\tan\beta$ (left) and $m_{3/2}$ (right). The LHC range (including an estimate of theoretical uncertainties) of $m_h = 126 \pm 2$ GeV is shown as the pale green horizontal band. The different curves correspond to different values of $m_{3/2}$ between 60 and 1500 TeV as marked. In the right panel, the curves correspond to four values of $\tan\beta$ between 1.8 and 2.4 as marked.

III. ADDING VECTOR MULTIPLETS

It is well known that adding (light) vectorlike states to supersymmetric theories with anomaly meditation [9] can help resolve the problem of tachyonic sleptons [40]. While the this problem is inherently absent in PGM models, the presence of such vectorlike states affects the low- energy theory in several fundamental ways. Additional fields with Standard Model charges will affect the running of the gauge couplings, and as such will directly affect the pattern of gaugino masses in AMSB [6,30,41]. Here, we show that coupling vectorlike fields to the MSSM Higgs not only affects the running of the gauge couplings, but also the soft mass parameters associated with the two Higgs doublets and can greatly ease the problem of radiative EWSB. Indeed, we are able to find solutions for a wide range of values of $\tan\beta$, greatly easing the problem of obtaining a Higgs mass in the desired LHC range.

We begin by including additional states labeled $\Phi, \bar{\Phi}$ which are in either a **5** and $\bar{\textbf{5}}$ or **10** and $\overline{\textbf{10}}$ representation of SU(5). In its simplest form, the theory need not contain any superpotential interactions involving the new fields, but have only the following Kähler potential:

$$K = |\Phi|^2 + |\bar{\Phi}|^2 + C(\Phi\bar{\Phi} + \text{H.c.}). \qquad (9)$$

In PGM, supersymmetry breaking will generate universal scalar masses for these fields in addition to mass terms which arise from the Giudice-Masiero term included in K. The latter leads to an effective μ term with $\mu = Cm_{3/2}$ and $B\mu = 2Cm_{3/2}^2$. The mass matrix for the scalars associated with Φ and $\bar{\Phi}$ is

$$M^2 = m_{3/2}^2 \begin{bmatrix} cc(C^2+1) & 2C \\ 2C & (C^2+1) \end{bmatrix}, \qquad (10)$$

evaluated here at the input GUT scale.

As an example, let us first consider the case where Φ is given by a **10** representation of SU(5). In this case, gauge invariance would allow a superpotential coupling of the **10** to the MSSM Higgses which are found in a $\textbf{5}_u$ and $\bar{\textbf{5}}_d$ representation,

$$W = y_t'\textbf{5}_u\textbf{10}\,\textbf{10} + y_b'\bar{\textbf{5}}_d\overline{\textbf{10}}\,\overline{\textbf{10}}. \qquad (11)$$

Since the colored components of the $\textbf{5}_u$ and $\bar{\textbf{5}}_d$ Higgses should have GUT scale masses and decouple from the theory, this reduces to

$$W = y_t'H_uQU + y_b'H_d\bar{Q}\,\bar{U}, \qquad (12)$$

where Q, U, and E make up the components of the **10**. The Giudice-Masiero term for the **10** and $\overline{\textbf{10}}$ gives a constraint on the R charge, $R_{\textbf{10}} + R_{\overline{\textbf{10}}} = 0$. Because of this, R symmetry requires one of either y_t' or y_b' to be zero unless there is additional R-symmetry breaking beyond the gravitino mass.

Furthermore, for $y_b' = 0$ the combination $-\frac{1}{2}\frac{V_{1L_2}}{v_2}\tan^2\beta + \frac{1}{2}\frac{V_{1L_1}}{v_1}$ (for definitions of the expression and more details see Appendix C) is even in μ, containing terms μ^0 and μ^2 only. This gives the typical and relatively simple solution for μ^2. However, when $y_b' \neq 0$ there is also a linear term in μ which allows for the possibility of two distinct solutions of μ, i.e. $|\mu_1| \neq |\mu_2|$. We leave the analysis of this theory for future work and from here on we take $y_b = 0$.

To include this sector of the theory, we must input a new Yukawa coupling, y_t' at the GUT scale along with C and the soft masses which are set at their universal value given by $m_{3/2}$,

$$m_Q^2 = m_{\bar{Q}}^2 = m_U^2 = m_{\bar{U}}^2 = m_{3/2}^2. \qquad (13)$$

As noted above and in Appendix A, the Giudice-Masiero term induces a μ term as well as a supersymmetry breaking B term given by

$$\mu_Q = \mu_U = C_{10}m_{3/2} \qquad (14)$$

$$b_Q = b_U = 2C_{10}m_{3/2}^2. \qquad (15)$$

These quantities are then also run down to the weak scale using the β functions given in Appendix A. Also given in Appendix A are the contributions to the MSSM β functions which are affected by the new terms. We have neglected the running of the B terms as their beta functions are proportional to gaugino masses and A terms, both of which are small compared with the gravitino mass. The physical masses of the new states are determined by the diagonalization of the mass matrices given in Appendix B.

The dominant contribution to the fermion masses come from the Giudice-Masiero term and are

$$M_f = C_{10}m_{3/2} \qquad (16)$$

By comparing the fermion and boson masses, we see that in the limit that $C_{10} \gg 1$, the boson and fermions become degenerate.

In PGM models, gaugino masses are given by their anomaly mediated contribution, and when we include new vectorlike multiplets, the β functions for the gaugino masses are affected. The one-loop RGEs for the gauge couplings above the SUSY scale, are altered to be

$$\beta_1 = \beta_{\text{MSSM}} + \frac{5}{3}(N_{5+\bar{5}} + 3N_{10+\bar{1}0})$$

$$\beta_2 = \beta_{\text{MSSM}} + N_{5+\bar{5}} + 3N_{10+\bar{1}0}.$$

$$\beta_3 = \beta_{\text{MSSM}} + N_{5+\bar{5}} + 3N_{10+\bar{1}0} \qquad (17)$$

At two loops the RGEs are modified as well and these contributions are given in Appendix A. The expression
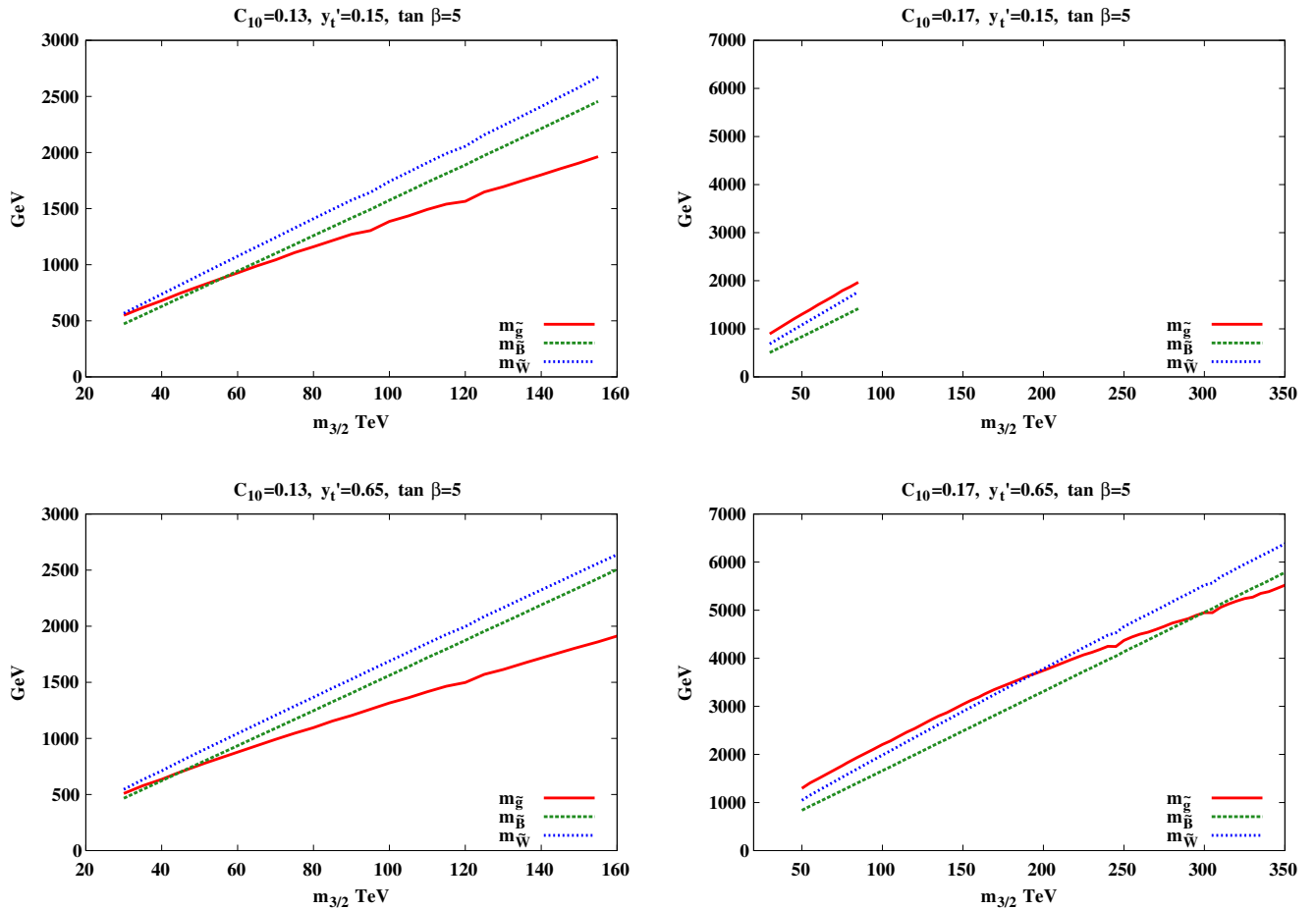
FIG. 2 (color online). Gaugino masses as a function of the gravitino mass for fixed $y'_t = 0.15$ (upper panels), $y'_t = 0.65$ (lower panels) and fixed $C_{10} = 0.13$ (left panels), $C_{10} = 0.17$ (right panels). $\tan \beta = 5$ in all four panels. The gluino mass is shown as a red solid line, the wino mass by a blue dotted line, and the bino mass by a green dashed line.

above for the β function is valid in the supersymmetric regime where boson and fermions are nearly degenerate and smaller than the renormalization scale Q. As the scale Q drops below the masses associated with the vectorlike fields, their contributions are removed from the β functions. Since fermions and bosons contribute differently to the β function, they will be integrated out differently. As we pass the scale where the fermions are integrated out, we remove $(2/3)$ of the above contribution due to $\mathbf{5}, \mathbf{\bar{5}}, \mathbf{10}$, and $\mathbf{\overline{10}}$'s in the one-loop beta function. The two different scalars have different masses and are decoupled at different thresholds subtracting for each $(1/6)$ of the total as Q drops below their mass.

Radiative electroweak symmetry breaking is also affected by the presence of the new vectorlike states. As seen in Eq. (A21) in Appendix A, the new Yukawa coupling y'_t contributes to the running of the uplike Higgs soft mass, m_{H_u} in an analogous way to the top quark Yukawa coupling, driving it negative as one runs down from the GUT scale. This makes it easier to find solutions to the Higgs minimization equations, and allows for larger values of $\tan \beta$. Perhaps more importantly however, the new vectorlike states

contribute to the one loop corrected Higgs potential. As discussed in Appendix C, the $(Q, \bar{Q}, U, \bar{U}$ mass matrices are nontrivial and contribute to the solutions for μ and c_H).

In addition, we include threshold corrections to neutralino, chargino, and gluino masses. These are handled in a similar way to MSSM corrections given in [37,38].

Finally, we comment on the effect of the vectorlike states on the calculation of the Higgs mass. As noted earlier, we follow the MSSM calculation outlined in [32]. However, as explained in Appendix D, we include new one-loop contributions to the Higgs quartic coupling. Because the fermions and bosons of the additional vectorlike states both have masses similar to $m_{3/2}$, these corrections can be implemented as one-loop threshold corrections at the scale M_{SUSY}. Because the threshold corrections do not affect y_t, they will have little effect on the running of the Higgs quartic coupling and make only an additive correction. These corrections tend to increase the Higgs mass by a few percent. However, for larger values of the GM term the fermion masses are larger than the boson masses and these corrections will suppress the Higgs mass.
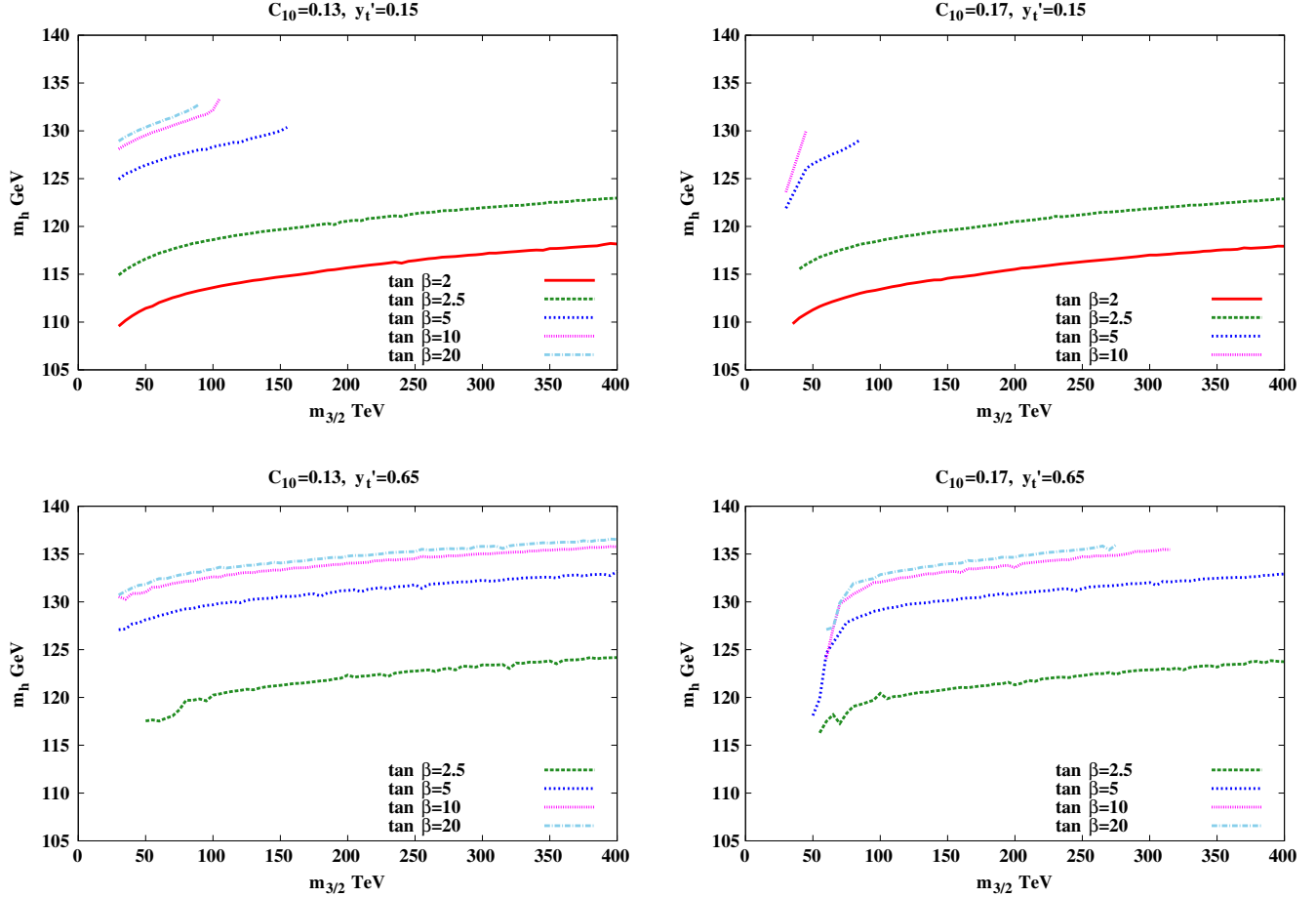
FIG. 3 (color online).   The Higgs mass as a function of the gravitino mass for fixed $y'_t = 0.15$ (upper panels), $y'_t = 0.65$ (lower panels) and fixed $C_{10} = 0.13$ (left panels), $C_{10} = 0.17$ (right panels). Four to five values of $\tan\beta$ are chosen: 2 (solid red); 2.5 (green dashed); 5 (blue short dashed); 10 (violet dotted); and 20 (cyan dot-dashed).

IV. RESULTS

As discussed earlier, the inclusion of new vectorlike states, affects the gaugino and Higgs masses as well as the allowed ranges of the two PGM input parameters, $m_{3/2}$ and $\tan\beta$. In this section, we display the resulting gaugino and Higgs masses as a function of the two PGM parameters as well as the new GM couplings $C_{10,5}$ and the Yukawa coupling y'_t. As noted earlier, we ignore the effects of the downlike coupling, y'_b.

A. Adding a 10 and $\overline{\mathbf{10}}$

In this section, we will restrict our attention to the case with one additional **10** and $\overline{\mathbf{10}}$ pair. Additional **10** and $\overline{\mathbf{10}}$ pairs would induce nonperturbative running in the strong gauge coupling.

We begin the discussion of the particle spectrum with the parameter dependence of the gaugino masses. As discussed above, the inclusion of vectorlike multiplets modify the well-known anomaly mediated relations between the gaugino masses [6,30,41]. In Fig. 2, we show results for the gaugino masses as a function of the gravitino with $\tan\beta = 5$ for fixed

values of the Giudice-Masiero term, $C_{10} = 0.13$, 0.17 and toplike Yukawa, $y'_t = 0.15$, 0.65. As one can see, all of the gaugino masses are approximately linearly dependent on the gravitino mass and there is little dependence on $\tan\beta$. Parameter values have been chosen such that there is (in most cases) a region where the bino is the LSP and nearly degenerate with one of the two other gauginos. In this case, we were only able to find regions with bino-gluino degeneracy which is sufficient for controlling the relic density through coannihilations [42]. While the gaugino mass spectrum is only weakly dependent on y'_t, there is a relatively strong dependence on C_{10} as we now explain.

From Eq. (A1), the contribution of a single **10** and $\overline{\mathbf{10}}$ pair, would yield $\beta_1 = 16$, $\beta_2 = 4$, and $\beta_3 = 0$ and the anomaly mediated contribution to the gluino mass is zero. Now, the gaugino masses are modified slightly by two-loop effects, but the most significant correction comes from the one-loop threshold corrections [37,38]. This is particularly true when one includes the corrections due to a **10** and $\overline{\mathbf{10}}$ pair alone because $\beta_3 = 0$. The C_{10} dependence of the gaugino masses is sourced in the scalar and fermionic
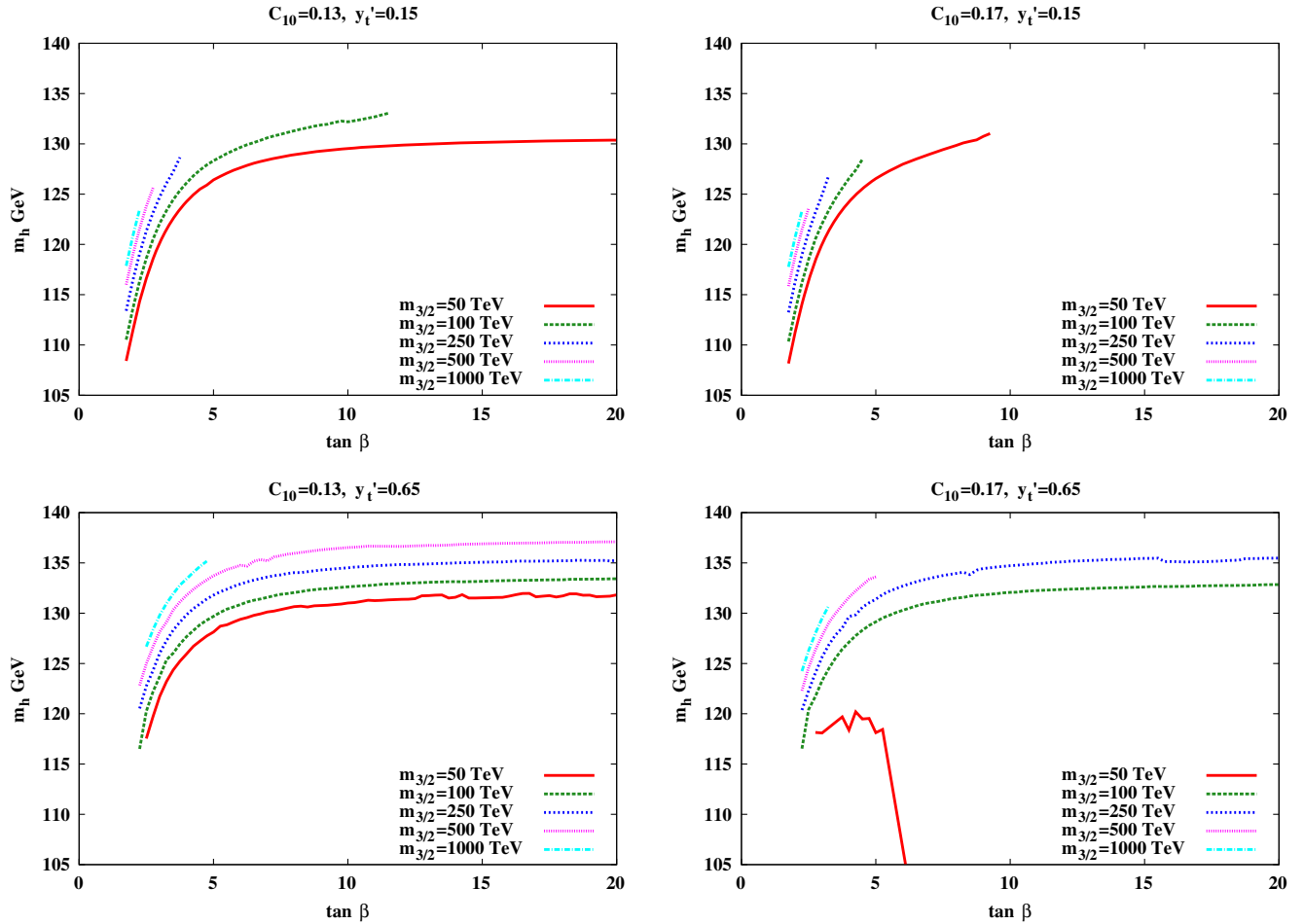
FIG. 4 (color online).   The Higgs mass as a function of $\tan\beta$ for fixed $y'_t = 0.15$ (upper panels), $y'_t = 0.65$ (lower panels) and fixed $C_{10} = 0.13$ (left panels), $C_{10} = 0.17$ (right panels). Five values of $m_{3/2}$ are chosen: 50 TeV (solid red); 100 TeV (green dashed); 250 TeV (blue short dashed); 500 TeV (violet dotted); and 1 PeV (cyan dot-dashed).

masses of the **10** and $\overline{\mathbf{10}}$. Recall that the masses of the **10** and $\overline{\mathbf{10}}$ (scalars and fermions) are dependent on C_{10} through μ_Q and b_Q for the scalars (see Eqs. (14) and (15) and Appendix B) and the fermion masses are directly proportional to C_{10} [see Eq. (16)] and so the threshold corrections are strongly dependent on C_{10} and in some regions of parameters space proportional to C_{10}. There is also a weak dependence of the gluino mass on $m_{3/2}$ due to running. For large $m_{3/2}$, the RG running is terminated at a higher scale and the μ_i are less enhanced. Because the bino and wino anomaly mediated masses are nonzero, their scaling is less dependent on C_{10}. For smaller values of C_{10} and larger values of $m_{3/2}$, the threshold corrections to the gluino mass are insufficient and the gluino becomes the LSP. This portion of the parameter space is, of course, excluded. Thus we have an upper bound on the gravitino mass and as such an upper bound on the sparticle mass spectrum.

As one can see from Fig. 2, there is only a limited range in C_{10} where the mass spectrum is acceptable. At $C_{10} = 0.13$, the degeneracy point (and upper limit on $m_{3/2}$) occurs at relatively low gravitino mass, around $m_{3/2} \sim 50$ TeV.

For this value of C_{10}, degeneracy occurs when $m_{\tilde{B}} \lesssim m_{\tilde{g}} \sim 800$ GeV. While this is below the nominally quoted LHC lower limit on the gluino mass, these limits are greatly relaxed when the neutralino and gluino are nearly degenerate as is the case here. When C_{10} is lowered to 0.11, the degeneracy point occurs at $m_{3/2} \approx 20$ TeV and the gaugino masses are only about 300 GeV. In the upper right panel of Fig. 2, there is no cross over between the bino and gluino and the bino is always the LSP leading to an excessive relic density. EWSB fails before the gluino mass drops below the bino mass. Without the assistance of larger y'_t, EWSB fails for larger values of the gravitino mass. At higher y'_t, as in the lower right panel of Fig. 2, radiative EWSB is extended to higher gravitino mass and we find a degeneracy point around $m_{3/2} \sim 300$ TeV and $m_{\tilde{B}} \lesssim m_{\tilde{g}} \sim 5$ TeV, just outside the reach of the LHC. Raising C_{10} further, impedes the possibility of radiative electroweak symmetry breaking unless y'_t is increased. However, as C_{10} is further increased the scalars in the **10** and $\overline{\mathbf{10}}$ run tachyonic and the model breaks down.

In Fig. 3, we show the calculated Higgs mass as a function of the gravitino mass for fixed values of C_{10} and y'_t
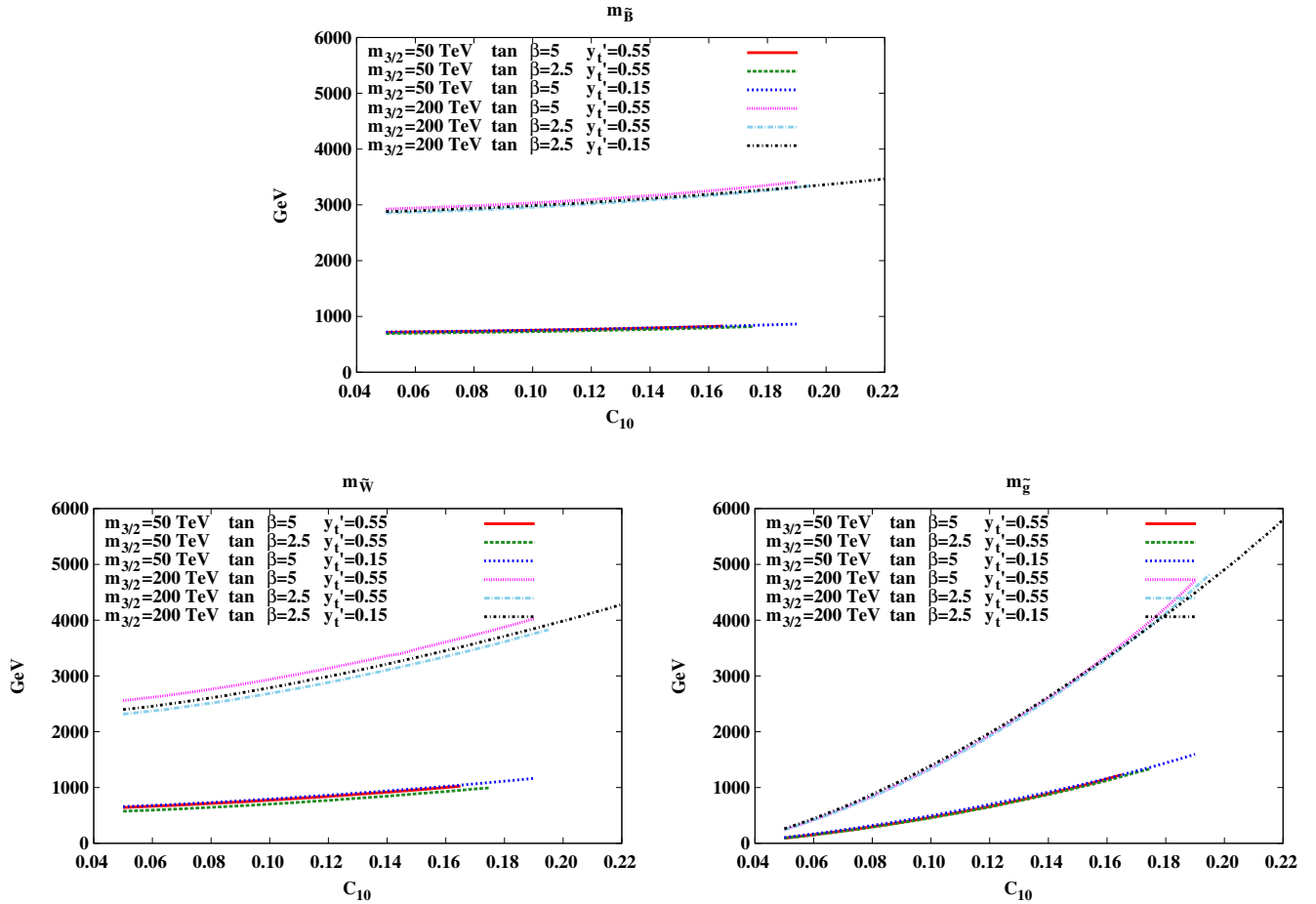
FIG. 5 (color online).   The gaugino masses, $m_{\tilde{B}}$ (upper), $m_{\tilde{W}}$ (lower left), $m_{\tilde{g}}$ (lower right) as a function of C_{10}, for combinations of $m_{3/2} = 50, 200$ TeV, $\tan\beta = 2, 5$, and $y'_t = 0.15, 0.55$.

and four or five values of $\tan\beta$. Curves which end abruptly at large $m_{3/2}$ do so due to the absence of EWSB. Recall that our requirement that $m_{\tilde{g}} \approx m_{\tilde{B}}$ from the cosmological relic density constraint, implies that for low C_{10}, we must have $m_{3/2} \lesssim 50$ TeV. In the left panels of Fig. 3, we can read off which values of $\tan\beta$ are needed to obtain the correct Higgs mass for $m_{3/2} \lesssim 50$ TeV. At large C_{10} with $y'_t = 0.65$, bino-gluino degeneracy required $m_{3/2} \approx 300$ TeV, which in turn requires lower values of $\tan\beta \lesssim 5$. At large y'_t, values of $\tan\beta$ as low as 2 are not allowed. The RG running of the top Yukawa are altered by y'_t, and the top Yukawa coupling will become nonperturbative for the combination of large values of y'_t and small $\tan\beta$.

In Fig. 4, we show the complementary plots of the calculated Higgs mass as a function of $\tan\beta$ for fixed values of C_{10} and y'_t and five values of the gravitino mass.

To see more explicitly the dependence of the sparticle masses on the Giudice-Masiero coupling, C_{10}, we show in Fig. 5 the dependence of the gaugino masses as a function of C_{10}, and in Fig. 6 the dependence of the Higgs mass as a function of C_{10}. As one clearly sees, the gaugino masses are predominantly sensitive to the gravitino mass and the six curves break up into two groups of three depending on the

two values of $m_{3/2}$ chosen. One also sees the strong dependence of the gluino mass on C_{10}. This is crucial since the addition of a **10** and $\overline{\mathbf{10}}$ pair cancels the MSSM value of β_3 and the gluino is *a priori* very light in this model. Indeed when C_{10} is small, we see that the increase in the gluino mass is relatively modest when increasing the gravitino mass from 50 to 200 TeV. At larger C_{10} the gluino's dependence on $m_{3/2}$ becomes comparable to the other gaugino masses.

The Higgs mass as shown in Fig. 6 is relatively insensitive to C_{10} and we see much stronger dependences on both $m_{3/2}$ and $\tan\beta$. We do see, however, a sharp drop in m_h above a critical value in C_{10}. At sufficiently large C_{10}, the fermion masses given by Eq. (16) become large and begin to cancel the one-loop contribution to m_h from the scalars. When $\tan\beta = 5$ and $m_{3/2}$ is large (as in the lower right panel), this cancellation occurs after we lose the ability to achieve radiative EWSB.

B. Adding a 10 and $\overline{\mathbf{10}}$ plus a 5 and $\bar{\mathbf{5}}$

In this section, we consider the consequences of adding a **5** and $\bar{\mathbf{5}}$ pair. We will again give these fields a GM term in the Kähler potential. However, without an additional singlet
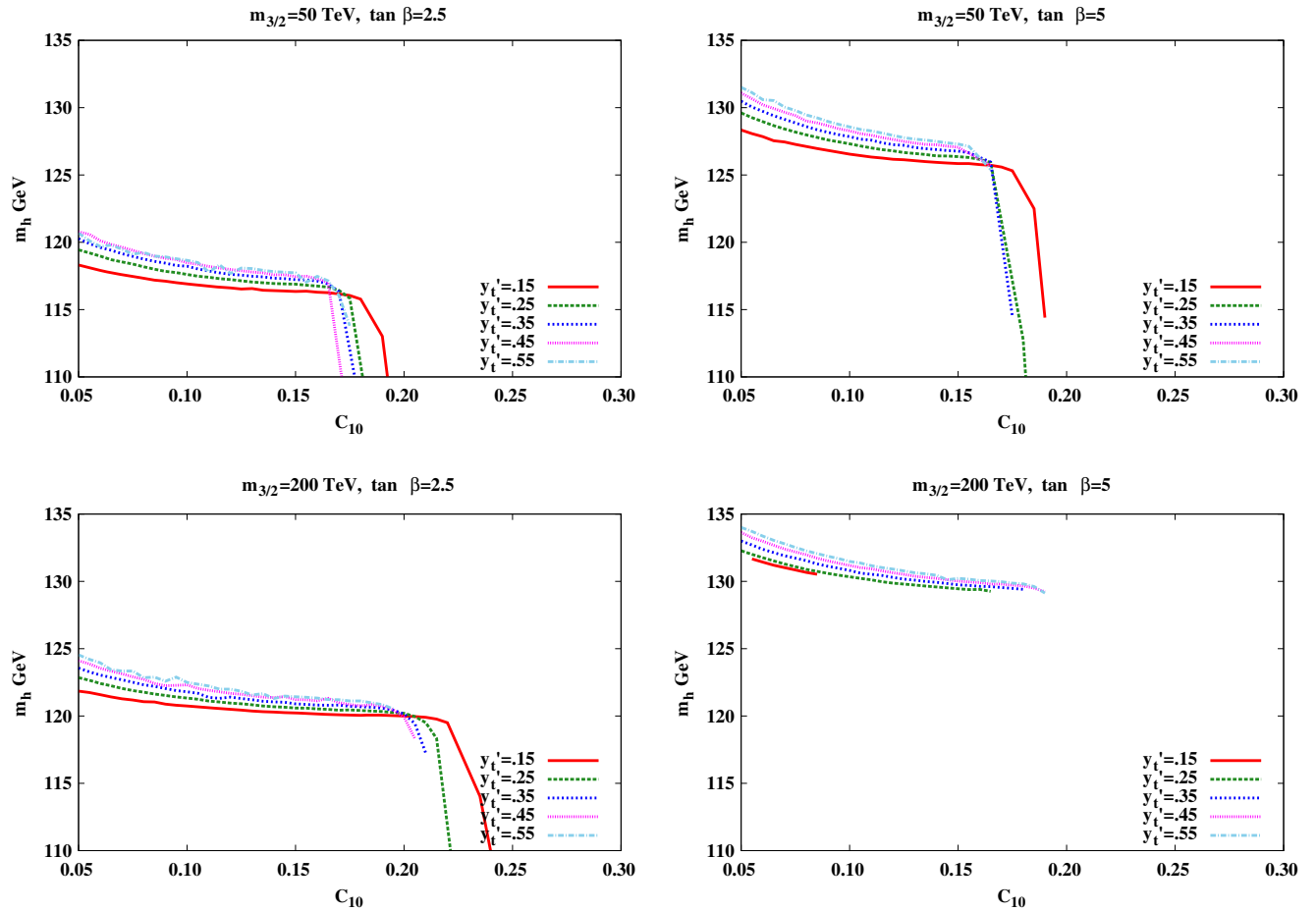
FIG. 6 (color online).   The Higgs mass as a function of C_{10} for fixed $m_{3/2} = 50$ TeV (upper panels), $m_{3/2} = 200$ TeV (lower panels) and fixed $\tan\beta = 2.5$ (left panels), $\tan\beta = 5$ (right panels). Five values of y'_t are chosen: 0.15 (solid red); 0.25 (green dashed); 0.35 (blue short dashed); 0.45 (violet dotted); and 0.55 (cyan dot-dashed).

or some mixing with a **10,** these fields cannot couple to the Higgs fields.[3]

In Fig. 7, we have plotted the gaugino masses with respect to C_5, the GM term for the **5** and **5̄** for fixed $m_{3/2} = 100$ TeV, $\tan\beta = 3$, and $y'_t = 0.07$ for four choices of $C_{10} = 0.02, 0.04, 0.06,$ and 0.08. Note that the preferred ranges of y'_t and C_{10} are both lower in this case due to the additional running induced by the addition of the **5** and **5̄**. As is expected, the gaugino masses all increase with C_5. However, these figures show features of the scaling with C_5 that has been previously neglected in other works [6,30,41]. In previous analyses, the running of the μ_i had been ignored. At the GUT scale these masses are universal. However, as they are run down to the SUSY breaking scale their masses diverge. Since the running of supersymmetric parameters are proportional to anomalous dimensions, as discussed in Appendix A, the μ_D of the **5** and **5̄** will run

differently than the μ_L. In fact, the beta function of μ_D has a piece proportional to the strong coupling and so is much more enhanced than μ_L. Now in the limit, $\mu_{D,L}/m_{3/2} \gg 1$, the gaugino masses become independent of $\mu_{D,L}$ and scale only with $m_{3/2}$. This behavior can be seen in Fig. 7 for $C_5 \gtrsim 0.6$ for wino and $C_5 \gtrsim 0.3$ for the gluino. Again, this levelling out occurs at different values of C_5 because the supersymmetric masses run differently. The bino mass is dependent on both μ_L and μ_D. Because of this, it has three different regions of scaling with respect to C_5. For $C_5 \lesssim 0.3$, it is increasing most quickly because it is scaling with respect to both μ_L and μ_D. However, once $C_5 \gtrsim 0.3$ the scaling of the bino mass with μ_D disappears and it now only scales with μ_L. Above $C_5 \sim 0.6$, the scaling with μ_L disappears and its mass only scales with $m_{3/2}$.

In Fig. 8, we show two sets of gaugino masses for $m_{3/2} = 50, 100$ and $\tan\beta = 3, 3.5$, respectively. Here, we see explicitly the strong dependence of the gaugino masses on $m_{3/2}$. The value of $\tan\beta$ is adjusted to obtain the correct value of m_h. At very low $C_5 \lesssim 0.1$, we have a gluino LSP. However, very quickly as C_5 is increased, the LSP becomes the wino. Indeed, from this figure, we see that by including
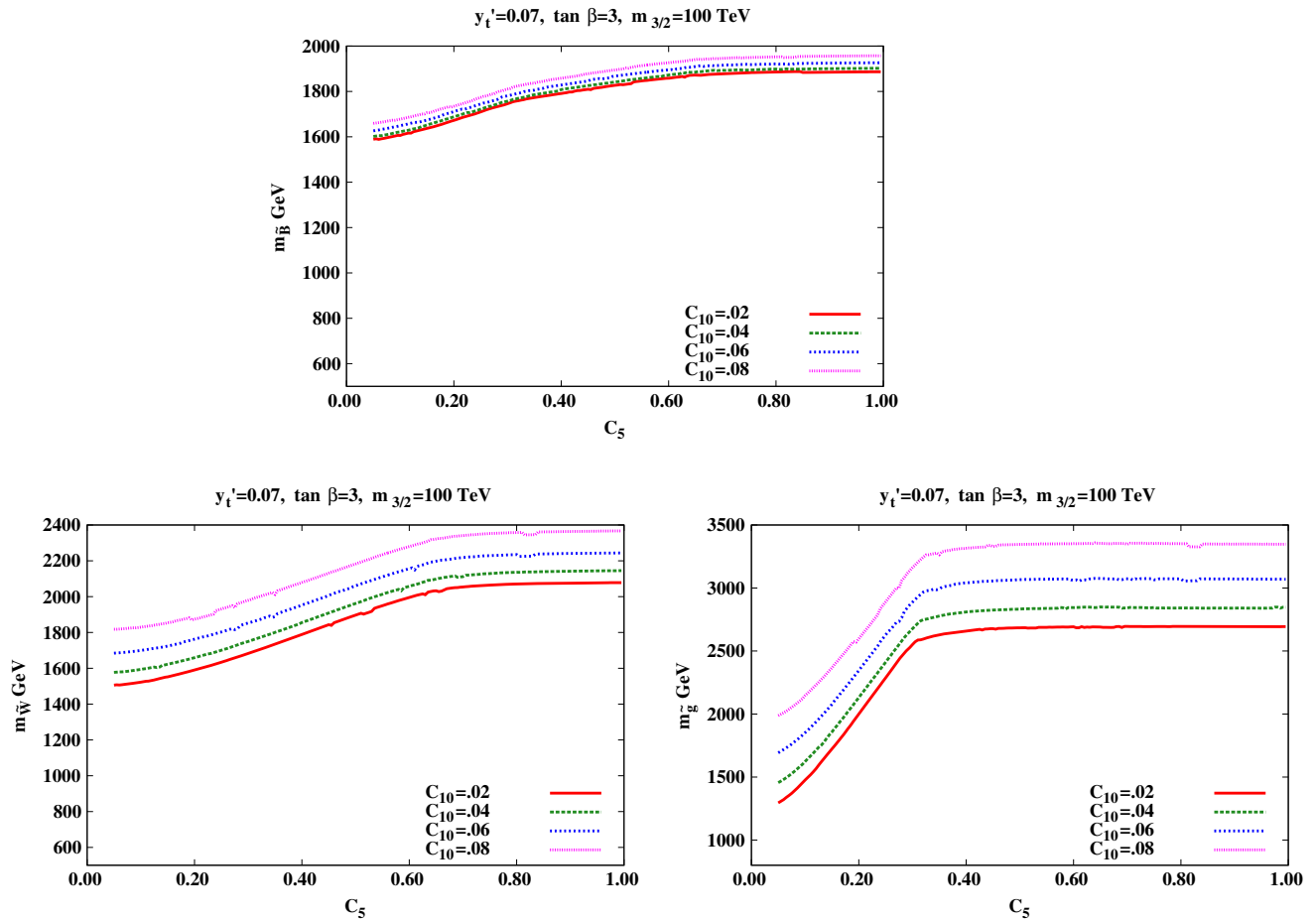
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[3]Through the operator $5_u\overline{\mathbf{105}}$, this field could interact with the up Higgs and slightly change the phenomenology. However, this case would not be significantly different from what we have already considered and could lead to flavor problems.

FIG. 7 (color online). The gaugino masses, $m_{\tilde{B}}$ (upper), $m_{\tilde{W}}$ (lower left), $m_{\tilde{g}}$ (lower right) as a function of C_{10}, for $m_{3/2} = 100$ TeV, $\tan\beta = 3$ and $y'_t = 0.07$ for different values of C_{10}.

a **5** and **5̄** we get regions of parameter space where the dark matter density is realized through bino-wino coannihilation around $C_5 \sim 0.4$. At larger values of C_5, the LSP is a bino and without the benefit of coannihilation, the relic density of dark matter would be too large.

Finally, since the **5** and **5̄** do not couple directly to the Higgs fields they will have minimal effect on the Higgs mass. However, if any of the masses of the **5** and **5̄** are below M_{SUSY}, they will alter the running of the gauge couplings as well as the top Yukawa coupling. However, this is effectively a two-loop effect and is very minor.

V. SUMMARY

The initial run of the LHC, which saw no definitive signs of supersymmetric particles and found a rather large Higgs mass, has given credence to models with split supersymmetrylike mass spectra. The simplest of these models, Universal PGM, has a very restricted hierarchy of gaugino masses generated by anomaly mediation. The dark matter candidate is the wino, which has been under scrutiny [43]. It also requires a rather large value of $m_{3/2}$ in order to generate a sufficiently heavy Higgs mass. At large $m_{3/2}$,

the gaugino masses may be well beyond the reach of the LHC.

Generating corrections to this very restrictive spectrum of gauginos is rather nontrivial. However, Supergravity does
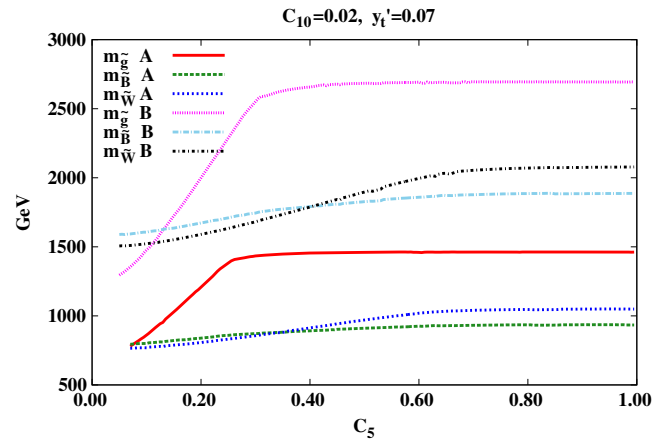

FIG. 8 (color online). The gaugino masses, $m_{\tilde{g}}$, $m_{\tilde{B}}$, and $m_{\tilde{W}}$ as a function of C_5. A) Gaugino masses for $m_{3/2} = 50$ TeV and $\tan\beta = 3.5$. B) Gaugino masses for $m_{3/2} = 100$ TeV and $\tan\beta = 3$. In all cases, $C_{10} = 0.02$ and $y'_t = 0.07$.

offer one rather simple mechanism for generating additional mass contributions to the gaugino masses. If vectorlike multiplets of $SU(5)$ with a Giudice-Masiero term in the Kähler potential are added, the gaugino mass relations can be altered. They are altered in two ways. First, the additional anomaly mediated contribution to the gauginos arising from an enhanced β function is never subtracted off by threshold corrections as the theory drops below the scale of the **10** and $\overline{\mathbf{10}}$. This is due to the sign of the B term for the **10** and $\overline{\mathbf{10}}$ which is generated by the Giudice-Masiero term. Secondly, if the Giudice-Masiero term in the Kähler potential is large, the threshold corrections to the gauginos will also be large and further increase the masses of gauginos.

In this paper, we have considered a generalization of PGM which includes an additional **10** and $\overline{\mathbf{10}}$ and **5** and $\bar{\mathbf{5}}$. These fields change many aspects of the model. First, they alter the gaugino mass spectra in a nontrivial way, opening the door for other (nonwino) dark matter candidates. Secondly, these fields can couple to the MSSM Higgs bosons. These couplings aid EWSB and open up the parameter space of $\tan\beta$.

The simplest of these models includes an additional **10** and $\overline{\mathbf{10}}$ pair. In this case, with the **10** coupled to the MSSM uplike Higgs, it is possible to achieve radiative EWSB for $\tan\beta = 2 \sim 40$. Because $\tan\beta$ is allowed to be much larger than in the simple universal PGM case, $m_{3/2}$ can be taken much smaller. The gluino mass is suppressed because $m_{3/2}$ is smaller and $\beta_3 = 0$. As a result, the gauginos maybe within reach of the LHC. The simplest dark matter candidate is the bino whose relic density is suppressed by coannihilating with gluino. To get sufficient suppression, the bino and gluino need to be rather degenerate. Because of this degeneracy, the LHC constraints on the gluino are relaxed. These models also tend to have an upper bound on the gravitino mass because the gluino becomes the LSP for larger values of $m_{3/2}$.

Adding an additional **5** and $\bar{\mathbf{5}}$, removes the upper bound on the gravitino mass since the gluino now scales more drastically with $m_{3/2}$. It also jumbles up the mass hierarchies of the gauginos, and we now have dark matter candidates coming from bino and wino coannihilation. Also the wino can again be the LSP for intermediate values of C_5. This case also highlights the effects of RG running on the gaugino mass spectrum which can have significant effects.

ACKNOWLEDGMENTS

APPENDIX A: ONE-LOOP β FUNCTIONS

In this and subsequent Appendixes, we will restrict our attention to the contributions of the **10** and $\overline{\mathbf{10}}$ only. The contributions due to the **5** and $\bar{\mathbf{5}}$ can be found in an analogous manner. At one-loop, the RGEs can be found from the anomalous dimensions and their analytic continuation into superspace. Starting with the Yukawa couplings, we have the formula for the anomalous dimension,

$$\gamma_i^j = \frac{1}{32\pi^2}\left(y_{ilm}y^{jlm} - 4\sum_\alpha C_\alpha(\Phi_i)g_\alpha^2\right), \quad \text{(A1)}$$

where the y_{ilm} are Yukawa couplings and C_α is the quadratic Casimir associated with the gauge group labeled by α. g_α is the gauge coupling. The beta function for the Yukawa coupling is

$$\beta(y_{ijk}) = \gamma_i^n y_{njk} + \gamma^n y_{ink} + \gamma^n y_{ijn}. \quad \text{(A2)}$$

For convenience we list the hypercharge of the different states:

$$Q: \frac{Y}{2} = \frac{1}{6} \quad U: \frac{Y}{2} = -\frac{2}{3} \quad H_u: \frac{Y}{2} = \frac{1}{2} \quad E := 1. \quad \text{(A3)}$$

Now the anomalous dimensions of the fields $Q, U, \bar{Q}, \bar{U}$ are

$$\gamma_Q = \frac{1}{16\pi^2}\left(|y_t'|^2 - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{1}{18}g_Y^2\right) \quad \text{(A4)}$$

$$\gamma_U = \frac{1}{16\pi^2}\left(2|y_t'|^2 - \frac{8}{3}g_3^2 - \frac{8}{9}g_Y^2\right) \quad \text{(A5)}$$

$$\gamma_E = -\frac{1}{8\pi^2}g_Y^2 \quad \text{(A6)}$$

$$\gamma_{H_u} = \frac{1}{16\pi^2}\left(3|y_t'|^2 + 3|y_t|^2 - \frac{3}{2}g_2^2 - \frac{1}{2}g_Y^2\right) \quad \text{(A7)}$$

and the anomalous dimensions for H_d, $\bar{Q}$, $\bar{U}$, and $\bar{E}$ can be found by taking $y_t \to y_b$ and $y_t' \to y_b'$ in the anomalous dimensions for H_u, Q, U, and E, respectively, and we have neglected the contribution of the τ Yukawa coupling. The anomalous dimensions for the MSSM fields with the same gauge symmetries can be found by taking $y_t' \to y_t$. Since the anomalous dimensions are diagonal, we get

$$\beta(y_t') = y_t'(\gamma_Q + \gamma_U + \gamma_{H_u})$$
$$= \frac{y_t'}{16\pi^2}\left(6|y_t'|^2 + 3|y_t|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{9}g_Y^2\right)$$
$$\text{(A8)}$$

$$\beta(y_t) = y_t(\gamma_{Q_{SM}} + \gamma_{U_{SM}} + \gamma_{H_u})$$
$$= \frac{y_t}{16\pi^2}\left(6|y_t|^2 + 3|y_t'|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{9}g_Y^2\right)$$
$$\text{(A9)}$$

$$\beta(y'_b) = y'_b(\gamma_{\bar{Q}} + \gamma_{\bar{U}} + \gamma_{H_d})$$
$$= \frac{y'_b}{16\pi^2}\left(6|y'_b| + 3|y_b|^2 - \frac{16}{3}g_3^2 - 3g_3^2 - \frac{13}{9}g_Y^2\right)$$

(A10)

$$\beta(y_b) = y'_b(\gamma_{Q_{SM}} + \gamma_{D_{SM}} + \gamma_{H_d})$$
$$= \frac{y_b}{16\pi^2}\left(6|y_b| + 3|y'_b|^2 - \frac{16}{3}g_3^2 - 3g_3^2 - \frac{7}{9}g_Y^2\right).$$

(A11)

The MSSM running of y_τ will also be affected because it depends on γ_{H_u}. From examining the expression for γ_{H_u}, we see that this will give an additional contribution to the running of y_τ of $3|y'_b|^2$.

The beta functions for the masses can be found from the expression

$$\frac{d}{dt}(m^2)^j_i = \gamma^{l*}_i(m^2)^j_l + \gamma^j_l(m^2)^l_i + 2\gamma^{(2)j}_i$$
$$+ \frac{2g_Y^2}{16\pi^2}\delta^j_i\frac{Y}{2}\mathbf{Tr}\left(\frac{Y}{2}m^2\right),$$

(A12)

where

$$\gamma^{(2)j}_i = \frac{1}{16\pi^2}\left(y_{ikl}(m^2)^l_n y^{jkn} + \frac{1}{2}A^*_{ikl}A^{jkl}\right.$$
$$\left. - 2\sum_\alpha g_\alpha^2 C_\alpha(\Phi_i)(2|M_\alpha|^2\delta^j_i - (m^2)^j_i)\right).$$

(A13)

Here the A_{ikl} are A terms and M_α are gaugino masses.

For H_u we have

$$\gamma^{(2)}_{H_u} = \frac{1}{16\pi^2}\left(3|y_t|^2(S_t - m^2_{H_u}) + 3|y'_t|^2(S_{t'} - m_{H_u})\right.$$
$$\left. - 3g_2^2|M_2|^2 - g_y^2|M_1|^2 + \left(\frac{3}{2}g_2^2 + \frac{1}{2}g_Y^2\right)m^2_{H_u}\right),$$

(A14)

where

$$S_t = m^2_{\tilde{t}_L} + m^2_{\tilde{t}_R} + m^2_{H_u} + |A_t|^2$$

(A15)

$$S_{t'} = m^2_Q + m^2_U + m^2_{H_u} + |A'_t|^2,$$

(A16)

$\gamma^{(2)}$ for Q is

$$\gamma^{(2)}_Q = \frac{1}{16\pi^2}\left(|y'_t|^2(S_{t'} - m^2_Q) - \frac{16}{3}g_3^2|M_3|^2 - 3g_2^2|M_2|^2\right.$$
$$\left. - \frac{1}{9}g_Y^2|M_1|^2 + \left(\frac{8}{3}g_3^2 + \frac{3}{2}g_2^2 + \frac{1}{18}g_Y^2\right)m^2_Q\right),$$

(A17)

and for U it is

$$\gamma^{(2)}_U = \frac{1}{16\pi^2}\left(2|y'_t|^2(S_{t'} - m^2_U) - \frac{16}{3}g_3^2|M_3|^2\right.$$
$$\left. - \frac{16}{9}g_Y^2|M_1|^2 + \left(\frac{8}{3}g_3^2 + \frac{8}{9}g_Y^2\right)m^2_U\right).$$

(A18)

Now the anomalous dimensions are again diagonal, so we can simplify the RGEs to

$$\frac{d}{dt}(m^2)^j_i = 2\gamma^j_l(m^2)^l_i + 2\gamma^{(2)j}_i + \frac{2g_Y^2}{16\pi^2}\delta^j_i\frac{Y}{2}S,$$

(A19)

where

$$S = \mathbf{Tr}\left(\frac{Y}{2}m^2\right).$$

(A20)

The β functions are then

$$\beta(m^2_{H_u}) = \frac{1}{8\pi^2}\left(3|y_t|^2 S_t + 3|y'_t|^2 S_{t'} - 3g_2^2|M_2|^2\right.$$
$$\left. - g_Y^2|M_1|^2 + \frac{1}{2}g_Y^2 S\right);$$

(A21)

$$\beta(m^2_Q) = \frac{1}{8\pi^2}\left(|y'_t|^2 S_{t'} - \frac{16}{3}g_3^2|M_3|^2 - 3g_2^2|M_2|^2\right.$$
$$\left. - \frac{1}{9}g_Y^2|M_1|^2 + \frac{1}{6}g_Y^2 S\right);$$

(A22)

$$\beta(m^2_U) = \frac{1}{8\pi^2}\left(2|y'_t|^2 S_{t'} - \frac{16}{3}g_3^2|M_3|^2\right.$$
$$\left. - \frac{16}{9}g_Y^2|M_1|^2 - \frac{2}{3}g_Y^2 S\right).$$

(A23)

The β functions for $m^2_{H_d}, m^2_Q$, and m^2_U can be obtained from those for $m^2_{H_u}, m^2_Q$, and m^2_U, with the transformations, $y_t \to y_b, y'_t \to y'_b, S \to -S, S_t \to S_b$, and $S'_t \to S'_b$.

Next, we calculate the β function for the supersymmetric masses. Because the **10** and $\overline{\mathbf{10}}$ break up into MSSM-like fields after the GUT breaking, they will each have there own effective μ term in the superpotential[4] of the form

$$W = \mu_Q Q\bar{Q} + \mu_U U\bar{U} + \mu_E E\bar{E}.$$

(A24)

The beta functions for these masses can simply be found from the expressions

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[4]These terms actually arise from the Kähler potential via the Giudice-Masiero mechanism and have input values given by $\mu_i = C_{10}m_{3/2}$.

$$\beta(\mu_Q) = \mu_Q(\gamma_Q + \gamma_{\bar{Q}}), \tag{A25}$$

$$\beta(\mu_U) = \mu_Q(\gamma_U + \gamma_{\bar{U}}), \tag{A26}$$

$$\beta(\mu_E) = \mu_E(\gamma_E + \gamma_{\bar{E}}), \tag{A27}$$

which give

$$\beta(\mu_Q) = \frac{1}{16\pi^2}\left(|y_t'|^2 + |y_b'|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{1}{9}g_Y^2\right)\mu_Q, \tag{A28}$$

$$\beta(\mu_U) = \frac{1}{16\pi^2}\left(2(|y_t'|^2 + |y_b'|^2) - \frac{16}{3}g_3^2 - \frac{16}{9}g_Y^2\right)\mu_U, \tag{A29}$$

$$\beta(\mu_E) = -\frac{1}{4\pi^2}g_Y^2\mu_E. \tag{A30}$$

Finally, for completeness, we give the two-loop contributions to the gauge coupling β functions which can be written as

$$\beta_a^{(2)} = \frac{g_a^3}{(16\pi^2)^2}B_{ab}^{(2)}g_b^2, \tag{A31}$$

where in the MSSM,

$$B_{ab}^{(2)} = \begin{bmatrix} \frac{199}{9} & 9 & \frac{88}{3} \\ 3 & 25 & 24 \\ \frac{11}{3} & 9 & 14 \end{bmatrix}, \tag{A32}$$

which can be decomposed into the pieces coming from the **10** and $\bar{\mathbf{5}}$ representations. The contribution to $B_{ab}^{(2)}$ from the **10** is

$$B_{ab}^{(10)} = \begin{bmatrix} \frac{115}{18} & \frac{1}{2} & 8 \\ \frac{1}{6} & \frac{21}{2} & 8 \\ 1 & 3 & 17 \end{bmatrix}. \tag{A33}$$

The contribution from the $\bar{\mathbf{5}}$ is

$$B_{ab}^{(5)} = \begin{bmatrix} \frac{35}{54} & \frac{3}{2} & \frac{16}{9} \\ \frac{1}{2} & \frac{7}{2} & 0 \\ \frac{2}{9} & 0 & \frac{17}{3} \end{bmatrix}. \tag{A34}$$

Each Higgs contributes

$$B_{ab}^{(H)} = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} & 0 \\ \frac{1}{2} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{A35}$$

There is also contribution from gauginos which is given by $B_{ab}^A = \mathrm{diag}(0, -24, -54)$. Using these we see that we get

$$B_{ab}^{(2)} = B_{ab}^A + 3(B_{ab}^{(10)} + B_{ab}^{(5)}) + 2B_{ab}^{(H)}. \tag{A36}$$

Since the contribution to the RGEs from a **10** is the same as a $\overline{\mathbf{10}}$ and **5** is the same as a $\bar{\mathbf{5}}$, we can decompose the two-loop RGEs as follows:

$$B_{ab}^{2\mathrm{tot}} = B_{ab}^{(2)} + 2N_{10+\bar{1}0}B_{ab}^{(10)} + 2N_{5+\bar{5}}B_{ab}^{(5)}. \tag{A37}$$

APPENDIX B: MASS MATRICES

The soft masses and μ terms are run down to the weak scale and evaluated at the scale M_Q and M_U determined iteratively using the mass matrices for these fields,

$$M_Q^2 = \begin{pmatrix} m_Q^2 + \mu_Q & b_Q \\ b_Q & m_{\bar{Q}}^2 + \mu_Q^2 \end{pmatrix}$$

$$M_U^2 = \begin{pmatrix} m_U^2 + \mu_U & b_U \\ b_U & m_{\bar{U}}^2 + \mu_U^2 \end{pmatrix}. \tag{B1}$$

These matrices are diagonalized using the rotation matrices,

$$U_U = \begin{pmatrix} \cos\beta_U & -\sin\beta_U \\ \sin\beta_U & \cos\beta_U \end{pmatrix}$$

$$U_Q = \begin{pmatrix} \cos\beta_Q & -\sin\beta_Q \\ \sin\beta_Q & \cos\beta_Q \end{pmatrix}, \tag{B2}$$

where

$$\tan\beta_Q = \frac{m_{\bar{Q}}^2 - m_Q^2 + \sqrt{(m_Q^2 - m_{\bar{Q}}^2)^2 + 4|b_Q|^2}}{2|b_Q|} \tag{B3}$$

$$\tan\beta_U = \frac{m_{\bar{U}}^2 - m_U^2 + \sqrt{(m_Q^2 - m_{\bar{Q}}^2)^2 + 4|b_U|^2}}{2|b_U|}. \tag{B4}$$

Now we use these mixing matrices and rotate the fields to

$$\begin{pmatrix} Q_+ \\ Q_- \end{pmatrix} = U_Q \begin{pmatrix} Q \\ \bar{Q}^\dagger \end{pmatrix} \quad \begin{pmatrix} U_+ \\ U_- \end{pmatrix} = U_U \begin{pmatrix} U \\ \bar{U}^\dagger \end{pmatrix}. \tag{B5}$$

APPENDIX C: THE HIGGS POTENITAL

The possibility of incorporating radiative electroweak symmetry breaking requires viable solutions to the minimization of the Higgs potential. In this Appendix we outline the effect of the new vectorlike multiplets in the one-loop corrected Higgs potential.

The Higgs potential can be written as

$$V_T = m_1^2 v_1^2 + m_2 v_2^2 - B v_1 v_2 + D + V_{1L}, \quad \text{(C1)}$$

where

$$D = \frac{g_1^2 + g_2^2}{8} (v_1^2 - v_2^2)^2, \quad \text{(C2)}$$

B is the MSSM supersymmetry bilinear mass term, and V_{1L} is the Coleman-Weinberg potential. Here $v_{1(2)}$ is understood to be the vacuum expectation value of $H_{d(u)}$. The derivatives of the potential with respect to v_1 and v_2 can be easily combined to give a solution for B:

$$2B = (m_1^2 + m_2^2) \sin 2\beta + \frac{\sin 2\beta}{2} \left(\frac{D_1 + V_{1L_1}}{v_1} + \frac{D_2 + V_{1L_2}}{v_2} \right), \quad \text{(C3)}$$

where the subscripts i on D and V_{1L} represent derivatives with respect to v_i. The combination $\frac{V_{T_2}}{v_2} \tan^2\beta - \frac{V_{T_1}}{v_1}$ can be rearranged to solve for $v^2 = v_1^2 + v_2^2$:

$$v^2 = \frac{4}{(g_1^2 + g_2^2)(\tan^2\beta - 1)}$$
$$\times \left(m_1^2 - m_2^2 \tan^2\beta - \frac{1}{2} \frac{V_{1L_2}}{v_2} \tan^2\beta + \frac{1}{2} \frac{V_{1L_1}}{v_1} \right). \quad \text{(C4)}$$

Now the Coleman-Weinberg potential can be written as

$$V_{1L} = \frac{m^4}{32\pi^2} \left(\ln\left(\frac{m^2}{Q^2} \right) - \frac{3}{2} \right) \quad \text{(C5)}$$

for each mass eigenstate of the theory. This is well known in the MSSM, but the introduction of vectorlike multiplets requires the diagonalization of a new 4×4 mass matrix for the case of a **10** and $\overline{\textbf{10}}$ written in the $(\bar{Q}, Q^\dagger, \bar{U}, U^\dagger)$ basis:

$$M_{10}^2 = \begin{bmatrix} m_{\bar{Q}}^2 + \mu_Q^2 & b_Q & 0 & \mu_Q v_2 y_t' \\ b_Q & v_2^2 y_t'^2 + m_Q^2 + \mu_Q^2 & \mu_U v_2 y_t' & v_1 \mu y_t' \\ 0 & v_2 y_t' \mu_U & m_{\bar{U}}^2 + \mu_U^2 & b_U \\ v_2 y_t' \mu_Q & y_t' v_1 \mu & b_U & v_2^2 y_t'^2 + m_U^2 + \mu_U^2 \end{bmatrix}. \quad \text{(C6)}$$

Here we have set $y_b' = 0$ for simplicity. Upon diagonalization, derivatives of the eigenmasses can be taken with respect to v_1 and v_2.

There is in addition a contribution to V_{1L} from the fermionic states which have the following mass matrix in the $(Q, U, \bar{Q}, \bar{U})$ basis:

$$M_{\tilde{10}} = \begin{pmatrix} 0 & v_2 y_t' & \mu_Q & 0 \\ v_2 y_t' & 0 & 0 & \mu_U \\ \mu_Q & 0 & 0 & 0 \\ 0 & \mu_U & 0 & 0 \end{pmatrix}. \quad \text{(C7)}$$

Once again, derivatives of the eigenmasses with respect to v_1 and v_2 are needed in order to evaluate Eq. (C4). Recall that fermionic states contribute to V_{1L} with the opposite sign relative to the bosonic states.

Finally we note that when $y_b' = 0$, the combination $-\frac{1}{2} \frac{V_{1L_2}}{v_2} \tan^2\beta + \frac{1}{2} \frac{V_{1L_1}}{v_1}$ is even in μ (containing terms, μ^0 and μ^2 only), allowing for a relatively simple solution for μ^2. When $y_b' \neq 0$ there is also a linear term in μ which allows for the possibility of two solutions of μ with $|\mu_1| \neq |\mu_2|$. But we do not discuss this case any further here.

APPENDIX D: THE HIGGS QUARTIC COUPLING

The new fields will affect the quartic Higgs coupling and we compute this contribution here. First, we sort the interactions into quartic and trilinear terms and only keep interactions with the Higgs fields in them. The quartic couplings interactions are

$$-\mathcal{L}_4 = |y_t|^2 |H_u Q|^2 + |y_t|^2 |H_u U|^2 \quad \text{(D1)}$$

$$= |y_t|^2 (|H_u U_{U_{1i}} U_i|^2 + |H_u U_{Q_{1i}} Q_i|^2). \quad \text{(D2)}$$

The trilinear couplings are

$$-\mathcal{L}_3 = y_t' \mu_Q (U_{Q_{2i}}^\dagger U_{U_{1j}} H_u Q_i U_j + \text{H.c.}) + y_t' \mu_U (U_{U_{2j}}^\dagger U_{Q_{1i}} H_u Q_i U_j + \text{H.c.}) = y_t' M_{ij} H_u Q_i U_j, \quad \text{(D3)}$$

where

$$M_{ij} = \mu_Q U_{Q_{2i}}^\dagger U_{U_{1j}} + \mu_U U_{U_{2j}}^\dagger U_{Q_{1i}}. \quad \text{(D4)}$$

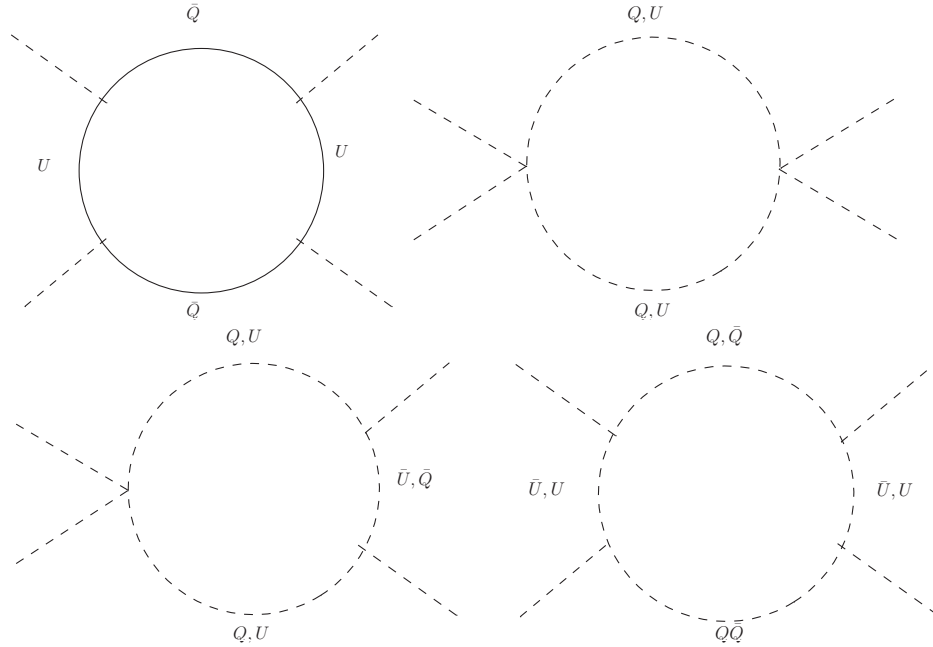The fermion interactions are simple and take the form

FIG. 9.   The diagrams contributing to the Higgs quartic coupling.

$$-\mathcal{L}_f = y_t H_u \tilde{Q}\, \tilde{U}. \tag{D5}$$

There are four diagrams that then contribute to the Higgs quartic coupling. These are found in Fig. 9.

Now the contribution from the fermion graph is

$$\Gamma^F = -\frac{4N_C|y_t|^4}{(\mu_Q^2 - \mu_U^2)^2}\left(\mu_Q^4 B_0(0, mu_Q, \mu_Q)\right.$$
$$\left.+ \mu_U^4 B_0(0, \mu_Q, \mu_Q) - 2\mu_Q^2\mu_U^2 B_0(0, \mu_Q, \mu_U)\right). \tag{D6}$$

The contribution from the diagram with quartic scalar interactions only gives

$$\Gamma^{SS} = 2N_C|y_t|^4(|U_{U_{1j}}|^2|U_{U_{1i}}|^2 B_0(0, m_{U_i}, m_{U_j})$$
$$+ |U_{Q_{1i}}|^2|U_{Q_{1j}}|^2 B_0(0m_{Q_i}, m_{Q_j})). \tag{D7}$$

Although it is not shown, the infinities of Γ^F and Γ^{SS} cancel and these are the only infinities that appear.

The contribution from the diagram with quartic scalar couplings and trilinear couplings gives

$$4N_C|y_t|^4(U^*_{U_{1j}}M^T_{ij}M^*_{jk}U_{U_{1k}}C(m_{U_j}, m_{Q_j}, m_{U_k})$$
$$+ U_{Q_{1j}}M_{ij}M^\dagger_{jk}U^*_{Q_{1k}}C(m_{Q_j}, m_{U_j}, m_{Q_k})). \tag{D8}$$

Last, we give the contribution for all trilinear couplings which gives

$$\Gamma^{4T} = 4N_C|y_t|^4(M_{ij}M^\dagger_{jk}M_{kl}M^\dagger_{\ell i}D(m_{Q_i}, m_{U_j}, m_{Q_k}, m_{U_\ell})). \tag{D9}$$

We have defined the above expressions in terms of the Passarino-Veltman functions which are

$$B_0(0, m_1, m_2) = \int \frac{d^{4-\epsilon}p}{(2\pi)^4} \frac{1}{(p^2 - m_1^2)(p^2 - m_2^2)} \tag{D10}$$

$$C(m_1, m_2, m_3) = \int \frac{d^{4-\epsilon}p}{(2\pi)^4} \frac{1}{(p^2 - m_1^2)(p^2 - m_2^2)(p^2 - m_3^2)} \tag{D11}$$

$$D(m_1, m_2, m_3, m_4) = \int \frac{d^{4-\epsilon}p}{(2\pi)^4} \frac{1}{(p^2 - m_1^2)(p^2 - m_2^2)(p^2 - m_3^2)(p^2 - m_4^2)}, \tag{D12}$$

with the infinities subtracted off in $B0$. This gives a one-loop correction to the Higgs quartic coupling of

$$\delta\lambda_{\text{eff}} = -\frac{1}{2}(\Gamma^F + \Gamma^{SS} + \Gamma^{STT} + \Gamma^{4T}). \tag{D13}$$

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