$\bar{B}_s^0 \to (\pi^0\eta^{(\prime)}, \eta^{(\prime)}\eta^{(\prime)})$ decays and the effects of next-to-leading order contributions in the perturbative QCD approach

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In this paper, we calculate the branching ratios and CP-violating asymmetries of the five $\bar{B}_s^0 \to (\pi^0\eta^{(\prime)}, \eta^{(\prime)}\eta^{(\prime)})$ decays, by employing the perturbative QCD (pQCD) factorization approach and with the inclusion of all currently known next-to-leading order (NLO) contributions. We find that (a) the NLO contributions can provide about 100% enhancements to the LO pQCD predictions for the decay rates of $\bar{B}_s^0 \to \eta\eta'$ and $\eta'\eta'$ decays, but result in small changes to $\mathrm{Br}(\bar{B}_s \to \pi^0\eta^{(\prime)})$ and $\mathrm{Br}(\bar{B}_s \to \eta\eta)$; (b) the newly known NLO twist-2 and twist-3 contributions to the relevant form factors can provide about 10% enhancements to the decay rates of the considered decays; (c) for $\bar{B}_s \to \pi^0\eta^{(\prime)}$ decays, their direct CP-violating asymmetries $\mathcal{A}_f^{\rm dir}$ could be enhanced significantly by the inclusion of the NLO contributions; and (d) the pQCD predictions for $\mathrm{Br}(\bar{B}_s \to \eta\eta^{(\prime)})$ and $\mathrm{Br}(\bar{B}_s \to \eta'\eta')$ can be as large as 4×10^{-5}, which may be measurable at LHCb or the forthcoming Super-B experiments.

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I. INTRODUCTION

As is well known, the studies for the mixing and decays of the B_s meson play an important role in testing the standard model (SM) and in searching for the new physics beyond the SM [1,2]. Some B_s meson decays, such as the leptonic decay $B_s^0 \to \mu^+\mu^-$ and the hadronic decays $B_s^0 \to (J/\Psi\phi, \phi\phi, K\pi, KK$, etc.), have been measured recently by the LHCb, ATLAS, and CMS collaborations [3–5].

In a very recent paper [6], we studied the $\bar{B}_s^0 \to (K\pi, KK)$ decays by employing the pQCD factorization approach with the inclusion of the NLO contributions [7–13] and found that the NLO contributions can interfere with the leading order (LO) part constructively or destructively for different decay modes and can improve the agreement between the SM predictions and the measured values for the considered decay modes [6]. The charmless hadronic two-body decays of the B_s meson, in fact, have been studied intensively by many authors by using rather different theoretical methods, such as the generalized factorization [14,15], the QCD factorization (QCDF) approach [16–18], and the pQCD factorization approach at the LO or partial NLO level [8,19–22]. In Refs. [7,9,10,13], the authors proved that the NLO contributions can play a key role in understanding the very large $\mathrm{Br}(B \to K\eta')$ [9,10], the so-called "$K\pi$-puzzle" [7,13], and the newly observed branching ratios and CP-violating asymmetries of $B_s \to K^+\pi^-$ and $B_s \to K^+K^-$ decays [3,4,6].

In this paper, we will calculate the branching ratios and CP-violating asymmetries of the five $B_s^0 \to (\pi^0, \eta^{(\prime)})\eta^{(\prime)}$ decays by employing the pQCD approach. We focus on the studies for the effects of various NLO contributions to the five $\bar{B}_s^0 \to (\pi^0\eta^{(\prime)}, \eta\eta, \eta\eta', \eta'\eta')$ decays, specifically those NLO twist-2 and twist-3 contributions to the form factors of $B_s^0 \to \pi, \eta^{(\prime)}$ transitions [11,12].

II. DECAY AMPLITUDES AT LO AND NLO LEVEL

As usual, we treat the B_s meson as a heavy-light system and consider it at rest for simplicity. Using the light-cone coordinates, we define the emitted meson M_2 as moving along the direction of $n = (1, 0, \mathbf{0}_{\rm T})$ and another meson M_3 the direction of $v = (0, 1, \mathbf{0}_{\rm T})$, and we also use x_i to denote the momentum fraction of the antiquark in each meson:

$$P_{B_s} = \frac{m_{B_s}}{\sqrt{2}}(1, 1, \mathbf{0}_{\rm T}), \qquad P_2 = \frac{M_{B_s}}{\sqrt{2}}(1, 0, \mathbf{0}_{\rm T}),$$
$$P_3 = \frac{M_{B_s}}{\sqrt{2}}(0, 1, \mathbf{0}_{\rm T}), \qquad k_1 = (x_1 P_1^+, 0, \mathbf{k}_{1\rm T}),$$
$$k_2 = (x_2 P_2^+, 0, \mathbf{k}_{2\rm T}), \qquad k_3 = (0, x_3 P_3^-, \mathbf{k}_{3\rm T}). \tag{1}$$

After making the integration over k_1^-, k_2^-, and k_3^+, we find the conceptual decay amplitude

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$$\mathcal{A} \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \cdot \mathrm{Tr}[C(t)\Phi_{B_s}(x_1,b_1)\Phi_{M_2}(x_2,b_2)\Phi_{M_3}(x_3,b_3) \times H(x_i,b_i,t)S_t(x_i)e^{-S(t)}], \quad (2)$$

where b_i is the conjugate space coordinate of $k_{i\mathrm{T}}$, $C(t)$ are the Wilson coefficients evaluated at the scale t, and Φ_{B_s} and Φ_{M_i} are wave functions of the B_s meson and the final state mesons. The Sudakov factor $e^{-S(t)}$ and $S_t(x_i)$ together suppress the soft dynamics effectively [23].

For the considered B_s decays with a quark level transition $b \to q'$ with $q' = (d, s)$, the weak effective Hamiltonian H_{eff} can be written as [24]

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} V^*_{qq'} \left\{ [C_1(\mu)O_1^q(\mu) + C_2(\mu)O_2^q(\mu)] + \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right\}, \quad (3)$$

where $G_F = 1.16639 \times 10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi constant, V_{ij} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, $C_i(\mu)$ are the Wilson coefficients, and $O_i(\mu)$ are the four-fermion operators.

For the B_s^0 meson, we consider only the contribution of Lorentz structure

$$\Phi_{B_s} = \frac{1}{\sqrt{2N_c}} (\not{P}_{B_s} + m_{B_s})\gamma_5 \phi_{B_s}(\mathbf{k_1}), \quad (4)$$

with the distribution amplitude widely used in the literature [6,8,19,20,22]

$$\phi_{B_s}(x,b) = N_{B_s} x^2(1-x)^2 \exp\left[-\frac{M_{B_s}^2 x^2}{2\omega_{B_s}^2} - \frac{1}{2}(\omega_{B_s} b)^2\right], \quad (5)$$

where the parameter ω_{B_s} is a free parameter and we take $\omega_{B_s} = 0.50 \pm 0.05$ GeV for the B_s meson. For fixed ω_{B_s} and f_{B_s}, the normalization factor N_{B_s} can be determined through the normalization condition: $\int \frac{d^4k_1}{(2\pi)^4} \phi_{B_s}(\mathbf{k_1}) = f_{B_s}/(2\sqrt{6})$.

For the light π, K, η_q, and η_s, their wave functions are similar in form and can be defined as in Refs. [25–27]:

$$\Phi(P,x,\zeta) \equiv \frac{1}{\sqrt{2N_C}} \gamma_5 [\not{P}\phi^A(x) + m_0\phi^P(x) + \zeta m_0(\not{n}\not{v} - 1)\phi_P^T(x)], \quad (6)$$

where P and m_0 are the momentum and the chiral mass of the light mesons. When the momentum fraction of the quark (antiquark) of the meson is set to be x, the parameter ζ should be chosen as $+1$ (-1). The expressions of the relevant twist-2 $[\phi^A(x)]$ and twist-3 $[\phi^{P,T}(x)]$ distribution amplitudes of the mesons $M = (\pi, K, \eta_q, \eta_s)$ and the relevant chiral masses can be found easily in Refs. [6,10]. The relevant Gegenbauer moments a_i have been chosen as in Ref. [22]:

$$a_1^{\pi,\eta_q,\eta_s} = 0, \qquad a_2^{\pi,\eta_q,\eta_s} = 0.44 \pm 0.22. \quad (7)$$

The values of other parameters are $\eta_3 = 0.015$ and $\omega = -3.0$.

For the η-η' system, we use the traditional quark-flavor mixing scheme: the physical states η and η' are related to the flavor states $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$ through a single mixing angle ϕ,

$$\eta = \cos\phi\eta_q - \sin\phi\eta_s, \qquad \eta' = \sin\phi\eta_q + \cos\phi\eta_s. \quad (8)$$

The relation between the decay constants (f_q, f_s) and $(f_\eta^q, f_\eta^s, f_{\eta'}^q, f_{\eta'}^s)$, as well as the chiral enhancement m_0^q and m_0^s, has been defined, for example, in Ref. [10]. The parameters f_q, f_s, and ϕ have been extracted from the data [28]:

$$f_q = (1.07 \pm 0.02) f_\pi, \qquad f_s = (1.34 \pm 0.06) f_\pi, \qquad \phi = 39.3^\circ \pm 1.0^\circ, \quad (9)$$

with $f_\pi = 130$ MeV.

A. LO amplitudes

The five $B_s^0 \to \pi^0\eta^{(\prime)}, \eta\eta, \eta'\eta', \eta\eta'$ decays considered in this paper have been studied previously in Refs. [20,22] by employing the pQCD factorization approach at the leading order. The decay amplitudes as presented in Refs. [20,22] are confirmed by our recalculation. We here focus on the examination for the possible effects of all currently known NLO contributions to these five decay modes in the pQCD factorization approach. The relevant Feynman diagrams which may contribute to the considered B_s^0 decays at the leading order are illustrated in Fig. 1. We firstly show the LO decay amplitudes.

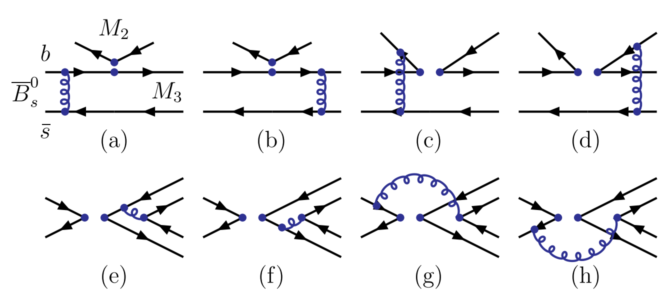

FIG. 1 (color online). Feynman diagrams which may contribute at leading order to $B_s^0 \to (\pi^0, \eta^{(\prime)})\eta^{(\prime)}$ decays.

For $\bar{B}_s^0 \to \pi^0\eta^{(\prime)}$ decays, the LO decay amplitudes are

$$\mathcal{A}(\bar{B}_s^0 \to \pi^0\eta) = \mathcal{A}(\bar{B}_s^0 \to \pi^0\eta_q)\cos\phi - \mathcal{A}(\bar{B}_s^0 \to \pi^0\eta_s)\sin\phi, \qquad (10)$$

$$\mathcal{A}(\bar{B}_s^0 \to \pi^0\eta') = \mathcal{A}(\bar{B}_s^0 \to \pi^0\eta_q)\sin\phi + \mathcal{A}(\bar{B}_s^0 \to \pi^0\eta_s)\cos\phi, \qquad (11)$$

with

$$\mathcal{A}(\bar{B}_s^0 \to \pi^0\eta_q) = \xi_u(f_{B_s}F_{a\eta_q}a_2 + M_{a\eta_q}C_2) - \frac{3}{2}\xi_t[f_{B_s}F_{a\eta_q}(a_7+a_9) + M_{a\eta_q}C_{10} + M^{P_2}_{a\eta_q}C_8], \qquad (12)$$

$$\sqrt{2}\mathcal{A}(\bar{B}_s^0 \to \pi^0\eta_s) = \xi_u(f_\pi F_{e\eta_s}a_2 + M_{e\eta_s}C_2) - \frac{3}{2}\xi_t[f_\pi F_{e\eta_s}(a_9-a_7) + M_{e\eta_s}(C_8+C_{10})], \qquad (13)$$

where $\xi_u = V_{ub}V^*_{us}$, $\xi_t = V_{tb}V^*_{ts}$, and a_i are the combinations of the Wilson coefficients C_i as defined, for example, in Ref. [10].

For $\bar{B}_s^0 \to \eta\eta, \eta\eta', \eta'\eta'$ decays, the LO decay amplitudes are

$$\sqrt{2}\mathcal{A}(\bar{B}_s^0 \to \eta\eta) = \cos^2\phi\mathcal{A}(\eta_q\eta_q) - \sin(2\phi)\mathcal{A}(\eta_q\eta_s) + \sin^2\phi\mathcal{A}(\eta_s\eta_s), \qquad (14)$$

$$\mathcal{A}(\bar{B}_s^0 \to \eta\eta') = [\mathcal{A}(\eta_q\eta_q) - \mathcal{A}(\eta_s\eta_s)]\cos\phi\sin\phi + \cos(2\phi)\mathcal{A}(\eta_q\eta_s), \qquad (15)$$

$$\sqrt{2}\mathcal{A}(\bar{B}_s^0 \to \eta'\eta') = \sin^2\phi\mathcal{A}(\eta_q\eta_q) + \sin(2\phi)\mathcal{A}(\eta_q\eta_s) + \cos^2\phi\mathcal{A}(\eta_s\eta_s), \qquad (16)$$

with

$$\mathcal{A}(\bar{B}_s^0 \to \eta_q\eta_q) = \xi_u M_{a\eta_q}C_2 - \xi_t M_{a\eta_q} \times \left(2C_4 + 2C_6 + \frac{1}{2}C_8 + \frac{1}{2}C_{10}\right), \qquad (17)$$

$$\sqrt{2}\mathcal{A}(\bar{B}_s^0 \to \eta_q\eta_s) = \xi_u(f_q F_{e\eta_s}a_2 + M_{e\eta_s}C_2) - \xi_t\left[f_q F_{e\eta_s}\left(2a_3 - 2a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9\right) + M_{e\eta_s}\left(2C_4 + 2C_6 + \frac{1}{2}C_8 + \frac{1}{2}C_{10}\right)\right], \qquad (18)$$

$$\mathcal{A}(\bar{B}_s^0 \to \eta_s\eta_s) = -2\xi_t\left[f_s F_{e\eta_s}\left(a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10}\right) + (f_s F^{P_2}_{e\eta_s} + f_{B_s}F^{P_2}_{a\eta_s})\left(a_6 - \frac{1}{2}a_8\right) + (M_{e\eta_s} + M_{a\eta_s}) \times \left(C_3 + C_4 + C_6 - \frac{1}{2}C_8 - \frac{1}{2}C_9 - \frac{1}{2}C_{10}\right)\right]. \qquad (19)$$

The individual decay amplitudes $(F_{eM_3}, F^{P2}_{eM_3}, \ldots)$ in Eqs. (12), (13), (17)–(19) are obtained by evaluating the Feynman diagrams in Fig. 1 analytically. Here $(F_{eM_3}, F^{P2}_{eM_3})$ and $(M_{eM_3}, M^{P2}_{eM_3})$ come from the evaluations of Figs. 1(a), 1(b) and Figs. 1(c), 1(d), respectively; while $(F_{aM_3}, F^{P2}_{aM_3})$ and $(M_{aM_3}, M^{P2}_{aM_3})$ are obtained by evaluating Figs. 1(e), 1(f) and Figs. 1(g), 1(h), respectively. One can find the expressions of all these decay amplitudes, for example, in Refs. [20,22]. For the sake of the reader, we show F_{eM_3} and $F^{P2}_{eM_3}$ explicitly here:

$$\begin{aligned}F_{eM_3} = 8\pi C_F M^4_{B_s}\int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_s}(x_1,b_1) \\ \cdot \{[(1+x_3)\phi^A_3(x_3) + r_3(1-2x_3)(\phi^P_3(x_3)+\phi^T_3(x_3))] \\ \cdot\alpha_s(t^1_e)h_e(x_1,x_3,b_1,b_3)\exp[-S_{ab}(t^1_e)] + 2r_3\phi^P_3(x_3) \\ \cdot\alpha_s(t^2_e)h_e(x_3,x_1,b_3,b_1)\exp[-S_{ab}(t^2_e)]\},\end{aligned} \qquad (20)$$

$$\begin{aligned}F^{P_2}_{eM_3} = 16\pi C_F M^4_{B_s}\int_0^1 dx_1 dx_3 \\ \times\int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_s}(x_1,b_1) r_2 \\ \cdot\{[\phi^A_3(x_3) + r_3(2+x_3)\phi^P_3(x_3) - r_3 x_3\phi^T_3(x_3)] \\ \cdot\alpha_s(t^1_e)h_e(x_1,x_3,b_1,b_3)\exp[-S_{ab}(t^1_e)] \\ +2r_3\phi^P_3(x_3)\alpha_s(t^2_e)h_e(x_3,x_1,b_3,b_1)\exp[-S_{ab}(t^2_e)]\},\end{aligned} \qquad (21)$$

where $C_F = 4/3$ is the color factor, and $r_2 = m_0^{M_2}/M_{B_s}$ and $r_3 = m_0^{M_3}/M_{B_s}$ with the chiral mass m_0 for final state mesons M_2 and M_3. The explicit expressions of the hard energy scales (t^1_e, t^2_e), the hard function h_e, and the Sudakov factor $\exp[-S(t)]$ can be found, for example, in Refs. [20,22].

B. NLO contributions

After many years' efforts, almost all NLO contributions in the pQCD approach become available now:

(a) The NLO Wilson coefficients $C_i(\mu)$ with $\mu \approx m_b$ [24] and the strong coupling constant $\alpha_s(\mu)$ at two-loop level.

(b) The NLO vertex corrections (VC) [16], the NLO contributions from the quark loops (QL) [7] or from the chromomagnetic penguin (MP) operator O_{8g} [29].

The relevant Feynman diagrams are shown in Figs. 2(a)–2(h).

(c) The NLO twist-2 and twist-3 contributions to the form factors of $B \to P$ transitions (here P refers to the light pseudoscalar mesons) [11,12]. Based on the $SU(3)$ flavor symmetry, we will extend directly the formulas for NLO contributions to the form factors of $B \to P$ transition as given in Refs. [11,12] to the cases for $B_s \to P$ transitions.

In this paper, we adopt the relevant formulas for all currently known NLO contributions directly from Refs. [6,7,10–12,16,29] without further discussion about the details. The still missing part of the NLO contributions in the pQCD approach is the calculation for the NLO corrections to the LO hard spectator and annihilation diagrams. But from the comparative studies for the LO and NLO contributions from different sources in Refs. [10,13], we believe that those still unknown NLO contributions are most possibly the higher order corrections to the small LO quantities, and therefore can be neglected safely.

According to Refs. [7,16], the vertex corrections can be absorbed into the redefinition of the Wilson coefficients by adding a vertex function $V_i(M)$ to them. The expressions of the vertex functions $V_i(M)$ can be found easily in Refs. [7,16]. The NLO "QL" and "MP" contributions are a kind of penguin correction with the insertion of the four quark operators and the chromomagnetic operator O_{8g}, respectively, as shown in Figs. 2(e)–2(f) and 2(g)–2(h). For the $b \to s$ transition, the relevant effective Hamiltonian $H_{\rm eff}^{ql}$ and $H_{\rm eff}^{mp}$ can be written as the following form:

$$H_{\rm eff}^{(ql)} = -\sum_{q=u,c,t}\sum_{q'}\frac{G_F}{\sqrt{2}}V_{qb}^* V_{qs}\frac{\alpha_s(\mu)}{2\pi}C^q(\mu, l^2) \times (\bar{b}\gamma_\rho(1-\gamma_5)T^a s)(\bar{q}'\gamma^\rho T^a q'), \tag{22}$$

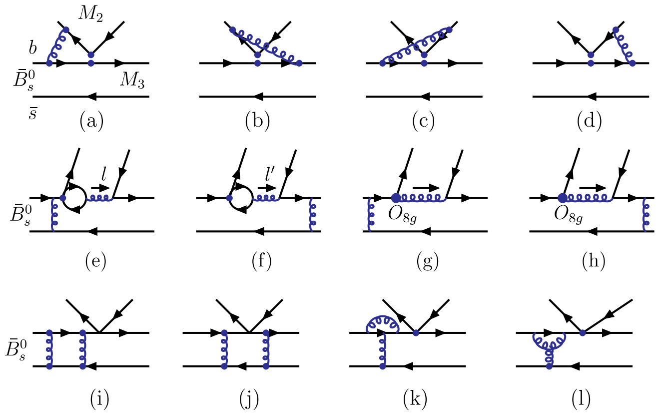

FIG. 2 (color online). Typical Feynman diagrams for NLO contributions: the vertex corrections (a)–(d); the quark loops (e)–(f), the chromomagnetic penguin contributions (g)–(h), and the NLO twist-2 and twist-3 contributions to $B_s \to P$ transition form factors (i)–(l).

$$H_{\rm eff}^{mp} = -\frac{G_F}{\sqrt{2}}\frac{g_s}{8\pi^2}m_b V_{tb}^* V_{ts} C_{8g}^{\rm eff}\bar{s}_i \sigma^{\mu\nu}(1+\gamma_5)T_{ij}^a G_{\mu\nu}^a b_j, \tag{23}$$

where l^2 is the invariant mass of the gluon which attaches the quark loops in Figs. 2(e)–2(f), and the functions $C^q(\mu, l^2)$ can be found in Refs. [7,9]. The $C_{8g}^{\rm eff}$ in Eq. (23) is the effective Wilson coefficient with the definition of $C_{8g}^{\rm eff} = C_{8g} + C_5$ [7].

By analytical evaluations, we find that (a) the decay modes $B_s^0 \to \pi^0\eta^{(\prime)}, \eta_q\eta_q$, and $\eta_q\eta_s$ do not receive the NLO contributions from the quark-loop and the magnetic-penguin diagrams; and (b) only the $B_s^0 \to \eta_s\eta_s$ decay mode gets the NLO contributions from the quark-loop diagrams and the O_{8g} operator:

$$\begin{aligned}\mathcal{M}_{\eta_s\eta_s}^{(ql)} = &-16 m_{B_s}^4 \frac{C_F{}^2}{\sqrt{2N_c}}\int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_s}(x_1)\{[(1+x_3)\phi_{\eta_s}^A(x_2)\phi_{\eta_s}^A(x_3)\\ &+2r_{\eta_s}\phi_{\eta_s}^P(x_2)\phi_{\eta_s}^A(x_3) + r_{\eta_s}(1-2x_3)\phi_{\eta_s}(x_2)(\phi_{\eta_s}^P(x_3)+\phi_{\eta_s}^T(x_3))]\\ &\cdot \alpha_s^2(t_a)\cdot h_e(x_1,x_3,b_1,b_3)\cdot \exp[-S_{ab}(t_a)]C^{(q)}(t_a,l^2)\\ &+2r_{\eta_s}\phi_{\eta_s}^A(x_2)\phi_{\eta_s}^P(x_3)\cdot\alpha_s^2(t_b)\cdot h_e(x_3,x_1,b_3,b_1)\cdot\exp[-S_{ab}(t_b)]C^{(q)}(t_b,l'^2)\},\end{aligned} \tag{24}$$

$$\begin{aligned}\mathcal{M}_{\eta_s\eta_s}^{(mp)} = &-32 m_{B_s}^6 \frac{C_F{}^2}{\sqrt{2N_c}}\int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 b_3 db_3 \phi_{B_s}(x_1)\\ &\times\{[(1-x_3)[2\phi_{\eta_s}^A(x_3)+r_{\eta_s}(3\phi_{\eta_s}^P(x_3)+\phi_{\eta_s}^T(x_3))+r_{\eta_s}x_3(\phi_{\eta_s}^P(x_3)-\phi_{\eta_s}^T(x_3))]\phi_{\eta_s}^A(x_2)\\ &-r_{\eta_s}x_2(1+x_3)(3\phi_{\eta_s}^P(x_2)-\phi_{\eta_s}^T(x_2))\phi_{\eta_s}^A(x_3)]\cdot\alpha_s^2(t_a)h_g(x_i,b_i)\cdot\exp[-S_{cd}(t_a)]C_{8g}^{\rm eff}(t_a)\\ &+4r_{\eta_s}\phi_{\eta_s}^A(x_2)\phi_{\eta_s}^P(x_3)\cdot\alpha_s^2(t_b)\cdot h_g'(x_i,b_i)\cdot\exp[-S_{cd}(t_b)]C_{8g}^{\rm eff}(t_b)\},\end{aligned} \tag{25}$$

where the terms proportional to small quantity $r_{\eta_s}^2$ are not shown explicitly. The expressions for the hard functions (h_e, h_g), the functions $C^{(q)}(t_a, l^2)$ and $C^{(q)}(t_b, l'^2)$, the Sudakov functions $S_{ab,cd}(t)$, the hard scales $t_{a,b}$, and the effective Wilson coefficients $C_{8g}^{\rm eff}(t)$ can be found easily, for example, in Refs. [6,7,10].

The NLO twist-2 and twist-3 contributions to the form factors of $B \to \pi$ transition have been calculated very recently in Refs. [11,12]. Based on the $SU(3)$ flavor symmetry, we extend the formulas of NLO contributions for the $B \to \pi$ transition form factor as given in Refs. [11,12] to the cases for $B_s \to (\pi, \eta_q, \eta_s)$ transition form factors directly, after making appropriate replacements for some parameters. The NLO form factor $f^+(q^2)$ for $B_s \to \pi$ transition, for example, can be written in the form of

$$\begin{aligned} f^+(q^2)|_{\rm NLO} &= 8\pi m_{B_s}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \\ &\times \Big\{ r_\pi [\phi_\pi^P(x_2) - \phi_\pi^T(x_2)] \cdot \alpha_s(t_1) \cdot e^{-S_{B_s\pi}(t_1)} \cdot S_t(x_2) \cdot h(x_1, x_2, b_1, b_2) \\ &+ \left[(1 + x_2\eta)(1 + F_{\rm T2}^{(1)}(x_i, \mu, \mu_f, q^2))\phi_\pi^A(x_2) + 2r_\pi\left(\frac{1}{\eta} - x_2\right)\phi_\pi^T(x_2) - 2x_2 r_\pi \phi_\pi^P(x_2) \right] \\ &\cdot \alpha_s(t_1) \cdot e^{-S_{B_s\pi}(t_1)} \cdot S_t(x_2) \cdot h(x_1, x_2, b_1, b_2) \\ &+ 2r_\pi \phi_\pi^P(x_2)(1 + F_{\rm T3}^{(1)}(x_i, \mu, \mu_f, q^2)) \cdot \alpha_s(t_2) \cdot e^{-S_{B_s\pi}(t_2)} \cdot S_t(x_2) \cdot h(x_2, x_1, b_2, b_1) \Big\}, \end{aligned} \tag{26}$$

where $\eta = 1 - q^2/m_{B_s}^2$ with $q^2 = (P_{B_s} - P_3)^2$ and P_3 is the momentum of the meson M_3 which absorbed the spectator light quark of the B meson, μ (μ_f) is the renormalization (factorization) scale, the hard scales $t_{1,2}$ are chosen as the largest scales of the propagators in the hard b-quark decay diagrams [11,12], and the function $S_t(x_2)$ and the hard function $h(x_i, b_j)$ can be found in Refs. [11,12]. And finally the NLO factors $F_{\rm T2}^{(1)}(x_i, \mu, \mu_f, q^2)$ and $F_{\rm T3}^{(1)}(x_i, \mu, \mu_f, q^2)$ which describe the NLO twist-2 and twist-3 contribution to the form factor $f^{+,0}(q^2)$ of the $B_s \to \pi$ transition can be found in Refs. [6,11,12]. For $B_s \to \pi$ transition, for example, these two factors can be written as

$$\begin{aligned} F_{\rm T2}^{(1)} &= \frac{\alpha_s(\mu_f) C_F}{4\pi} \left[\frac{21}{4} \ln\frac{\mu^2}{m_{B_s}^2} - \left(\frac{13}{2} + \ln r_1\right) \ln\frac{\mu_f^2}{m_{B_s}^2} + \frac{7}{16}\ln^2(x_1 x_2) + \frac{1}{8}\ln^2 x_1 \right. \\ &+ \frac{1}{4}\ln x_1 \ln x_2 + \left(-\frac{1}{4} + 2\ln r_1 + \frac{7}{8}\ln\eta\right)\ln x_1 + \left(-\frac{3}{2} + \frac{7}{8}\ln\eta\right)\ln x_2 \\ &\left. + \frac{15}{4}\ln\eta - \frac{7}{16}\ln^2\eta + \frac{3}{2}\ln^2 r_1 - \ln r_1 + \frac{101\pi^2}{48} + \frac{219}{16} \right], \end{aligned} \tag{27}$$

$$\begin{aligned} F_{\rm T3}^{(1)} &= \frac{\alpha_s(\mu_f) C_F}{4\pi} \left[\frac{21}{4} \ln\frac{\mu^2}{m_{B_s}^2} - \frac{1}{2}(6 + \ln r_1) \ln\frac{\mu_f^2}{m_{B_s}^2} + \frac{7}{16}\ln^2 x_1 - \frac{3}{8}\ln^2 x_2 \right. \\ &+ \frac{9}{8}\ln x_1 \ln x_2 + \left(-\frac{29}{8} + \ln r_1 + \frac{15}{8}\ln\eta\right)\ln x_1 + \left(-\frac{25}{16} + \ln r_2 + \frac{9}{8}\ln\eta\right)\ln x_2 \\ &\left. + \frac{1}{2}\ln r_1 - \frac{1}{4}\ln^2 r_1 + \ln r_2 - \frac{9}{8}\ln\eta - \frac{1}{8}\ln^2\eta + \frac{37\pi^2}{32} + \frac{91}{32} \right], \end{aligned} \tag{28}$$

where $r_i = m_{B_s}^2/\xi_i^2$ with the choice of $\xi_1 = 25 m_{B_s}$ and $\xi_2 = m_{B_s}$. For the considered $B_s \to (\pi^0, \eta^{(\prime)})\eta^{(\prime)}$ decays, the large recoil region corresponds to the energy fraction $\eta \sim O(1)$. We also set $\mu = \mu_f = t$ in order to minimize the NLO contribution to the form factors [12,30].

III. NUMERICAL RESULTS

In the numerical calculations, the following input parameters (here the masses, decay constants, and QCD scales are in units of GeV) will be used [31,32]:

$$\begin{aligned} &\Lambda_{\overline{\rm MS}}^{(5)} = 0.225, \quad f_{B_s} = 0.23, \quad f_\pi = 0.13, \\ &m_{B_s} = 5.37, \quad m_\eta = 0.548, \quad m_{\eta'} = 0.958, \\ &m_0^\pi = 1.4, \quad \tau_{B_s^0} = 1.497 \text{ ps}, \quad m_b = 4.8, \\ &M_W = 80.41. \end{aligned} \tag{29}$$

For the CKM matrix elements, we adopt the Wolfenstein parametrization and use the following CKM parameters: $\lambda = 0.2246$, $A = 0.832$, $\bar{\rho} = 0.130 \pm 0.018$, and $\bar{\eta} = 0.350 \pm 0.013$.

Taking $B_s \to \pi$ transition as an example, we calculate and present the pQCD predictions for the form factors $F_0^{\bar{B}_s^0\to\pi}(0)$ at the LO and NLO level, respectively:

$$F_0^{\bar{B}_s^0\to\pi}(0) = \begin{cases} 0.22 \pm 0.05, & \text{LO}, \\ 0.24 \pm 0.05, & \text{NLO}, \end{cases} \tag{30}$$

where the error comes from the uncertainty of $\omega_{B_s} = 0.50 \pm 0.05$ GeV, $f_{B_s} = 0.23 \pm 0.02$ GeV, and the Gegenbauer moments $a_2^\pi = 0.44 \pm 0.22$. Explicit calculations tell us that the NLO twist-2 enhancement to the full LO prediction is around 25%, but it is largely canceled by the negative NLO twist-3 contribution and finally led to a small total enhancement (about $7\% \sim 9\%$) to the full LO prediction, as predicted in Ref. [12].

For the considered five $\bar{B}_s^0$ decays, the CP-averaged branching ratios can be written in the following form:

$$\mathrm{Br}(B_s^0 \to f) = \frac{G_F^2 \tau_{B_s}}{32\pi m_{B_s}} \frac{1}{2} [|\mathcal{A}(\bar{B}_s^0 \to f)|^2 + |\mathcal{A}(B_s^0 \to \bar{f})|^2], \tag{31}$$

where τ_{B_s} is the lifetime of the B_s^0 meson.

In Table I, we list the pQCD predictions for the CP-averaged branching ratios of the considered B_s^0 decays. The label "NLO-I" means that all currently known NLO contributions are taken into account except for those to the form factors. As a comparison, we also show the central values of the LO pQCD predictions as given in Ref. [22], the partial NLO predictions in Ref. [8], and the QCDF predictions in Ref. [16] in the last three columns of Table I. The main theoretical errors come from the uncertainties of the various input parameters, such as $\omega_{B_s} = 0.50 \pm 0.05$, $f_{B_s} = 0.23 \pm 0.02$ GeV and $a_2^\pi = 0.44 \pm 0.22$. The total errors of our pQCD predictions are obtained by adding the individual errors in quadrature.

From the numerical results as listed in Table I, one can observe the following points:

(i) For $\bar{B}_s^0 \to (\pi^0\eta, \pi^0\eta', \eta\eta)$ decays, the NLO enhancements to the full LO predictions are small in size: less than 30%. For $\bar{B}_s^0 \to (\eta\eta', \eta'\eta')$ decays, however, the NLO enhancements can be as large as 100%. The branching ratios at the order of 4×10^{-5} should be measured at LHCb or Super-B factory experiments.

(ii) By comparing the numerical results as listed in the third (NLO-I) and fourth (NLO) column, one can see that the NLO contributions to the form factors alone can provide ~10% enhancement to the branching ratios.

(iii) The pQCD predictions agree with the QCDF predictions within 1 standard deviation. The pQCD predictions given in some previous works [8,22] are confirmed by our new calculations. Some differences between the central values are induced by the different choices of some input parameters, such as the Gegenbauer moments and the CKM matrix elements.

(iv) The main theoretical errors are coming from the uncertainties of input parameters $\omega_{B_s} = 0.50 \pm 0.05$, $f_{B_s} = 0.23 \pm 0.02$ GeV, and $a_2^\pi = 0.44 \pm 0.22$. The total theoretical error is in general around 30% to 50%.

Now we turn to the evaluations of the CP-violating asymmetries of the five considered decay modes. In the B_s system, we expect a much larger decay width difference: $\Delta\Gamma_s/(2\Gamma_s) \sim -10\%$ [31]. Besides the direct CP violation $\mathcal{A}_f^{\mathrm{dir}}$, the CP-violating asymmetries S_f and H_f are defined as usual [8,22]:

$$\mathcal{A}_f^{\mathrm{dir}} = \frac{|\lambda|^2 - 1}{1 + |\lambda|^2}, \qquad \mathcal{S}_f = \frac{2\mathrm{Im}[\lambda]}{1 + |\lambda|^2}, \qquad \mathcal{H}_f = \frac{2\mathrm{Re}[\lambda]}{1 + |\lambda|^2}. \tag{32}$$

They satisfy the normalization relation $|\mathcal{A}_f|^2 + |\mathcal{S}_f|^2 + |\mathcal{H}_f|^2 = 1$, while the parameter λ is of the form

$$\lambda = \eta_f e^{2i\epsilon} \frac{A(\bar{B}_s^0 \to f)}{A(B_s^0 \to \bar{f})}, \tag{33}$$

where η_f is $+1(-1)$ for a CP-even (CP-odd) final state f and $\epsilon = \arg[-V_{ts}V_{tb}^*]$ is very small in size.

The pQCD predictions for the direct CP asymmetries $\mathcal{A}_f^{\mathrm{dir}}$ and the mixing-induced CP asymmetries S_f and H_f of the considered decay modes are listed in Tables II and III. In these tables, the label "LO" means the LO pQCD

TABLE I. The pQCD predictions for the branching ratios (in units of 10^{-6}) of the considered five $\bar{B}_s^0$ decays. As a comparison, we also list the theoretical predictions as given in Refs. [8,16,22], respectively.

Mode	LO	NLO-I	NLO	LO [22]	NLO-I [8]	QCDF [16]
$\bar{B}_s^0 \to \pi^0\eta$	0.05	0.05	0.06 ± 0.03	0.05	0.03	0.08
$\bar{B}_s^0 \to \pi^0\eta'$	0.10	0.11	0.13 ± 0.06	0.11	0.08	0.11
$\bar{B}_s^0 \to \eta\eta$	10.1	9.9	$10.6^{+3.8}_{-2.7}$	8.0	10.0	15.6
$\bar{B}_s^0 \to \eta\eta'$	27.5	38.4	$41.4^{+16.4}_{-12.0}$	21.0	34.9	54.0
$\bar{B}_s^0 \to \eta'\eta'$	20.5	37.7	$41.0^{+17.5}_{-13.4}$	14.0	25.2	41.7

TABLE II. The pQCD predictions for the direct CP asymmetries (in %) of the five $\bar{B}_s^0$ decays. The meanings of the labels are described in the text.

Mode	LO	+VC	+QL	+MP	NLO	pQCD [22]	QCDF [16]
$\bar{B}_s^0 \to \pi^0\eta$	$-2.5^{+8.9}_{-7.8}$	39.8	…	…	$40.3^{+5.4}_{-7.5}$	$-0.4^{+0.3}_{-0.3}$	…
$\bar{B}_s^0 \to \pi^0\eta'$	$24.7^{+0.3}_{-1.0}$	52.7	…	…	$51.9^{+2.9}_{-3.3}$	$20.6^{+3.4}_{-2.9}$	$27.8^{+27.2}_{-28.8}$
$\bar{B}_s^0 \to \eta\eta$	$-0.2^{+0.3}_{-0.2}$	-2.2	1.7	-1.8	$-2.3^{+0.5}_{-0.4}$	$-0.6^{+0.6}_{-0.5}$	$-1.6^{+2.4}_{-2.4}$
$\bar{B}_s^0 \to \eta\eta'$	-1.1 ± 0.1	-1.0	0.1	-0.1	-0.2 ± 0.2	$-1.3^{+0.1}_{-0.2}$	$0.4^{+0.5}_{-0.4}$
$\bar{B}_s^0 \to \eta'\eta'$	1.4 ± 0.2	1.5	2.7	2.8	2.8 ± 0.4	$1.9^{+0.4}_{-0.5}$	$2.1^{+1.3}_{-1.4}$

TABLE III. The pQCD predictions for the mixing-induced CP asymmetries (in %) S_f (the first row) and H_f (the second row). The meanings of the labels are the same as in Table II.

Mode	LO	+VC	+QL	+MP	NLO	pQCD [22]
$\bar{B}_s^0 \to \pi^0\eta$	$13.7^{+6.6}_{-8.3}$	11.3	…	…	$8.0^{+1.8}_{-2.7}$	17^{+18}_{-13}
	$99.0^{+0.5}_{-1.4}$	91.0	…	…	$91.2^{+3.0}_{-2.4}$	99 ± 1
$\bar{B}_s^0 \to \pi^0\eta'$	$-22.2^{+10.0}_{-7.3}$	-24.9	…	…	$-24.9^{+9.5}_{-6.1}$	-17^{+8}_{-9}
	$94.3^{+2.0}_{-1.8}$	81.3	…	…	$81.8^{+0.5}_{-0.1}$	96^{+2}_{-2}
$\bar{B}_s^0 \to \eta\eta$	$-0.6^{+0.4}_{-0.3}$	2.7	-1.8	-2.2	$-2.2^{+0.6}_{-0.5}$	3.0^{+1}_{-1}
	100.0	99.9	100.0	100.0	99.9	100.0
$\bar{B}_s^0 \to \eta\eta'$	-0.1 ± 0.1	-0.4	0.1	0.1	0.1 ± 0.2	4.0
	100.0	100.0	100.0	100.0	100.0	100.0
$\bar{B}_s^0 \to \eta'\eta'$	0.8 ± 0.1	1.8	2.0	2.5	$2.5^{+0.2}_{-0.4}$	$4.0^{+1.0}_{-1.0}$
	100.0	100.0	99.9	99.9	99.9	100.0

predictions, and the labels "+VC," "+QL," "+MP," and "NLO" mean that the contributions from the vertex corrections, the quark loops, the magnetic penguins, and all known NLO contributions are added to the LO results, respectively. As a comparison, the LO pQCD predictions as given in Ref. [22] and the QCDF predictions in Ref. [16] are also listed in Tables II and III. The errors here are defined in the same way as for the branching ratios.

From the pQCD predictions for the CP-violating asymmetries of the five considered $\bar{B}_s$ decays as listed in Tables II and III, one can see the following points:

(i) For $\bar{B}_s^0 \to (\eta\eta, \eta\eta', \eta'\eta')$ decays, the pQCD predictions for $\mathcal{A}_f^{\rm dir}$ and $\mathcal{S}_f$ are very small: less than 3% in magnitude. The NLO effects are in fact also negligibly small.

(ii) For $\bar{B}_s^0 \to (\pi^0\eta, \pi^0\eta')$ decays, however, the NLO pQCD predictions for $\mathcal{A}_f^{\rm dir}$ can be as large as 40%–52%. The NLO contributions can provide large enhancements to the LO pQCD predictions for $\mathcal{A}_f^{\rm dir}$. Since the branching ratios of $\bar{B}_s^0 \to (\pi^0\eta, \pi^0\eta')$ decays are at the 10^{-8} level, unfortunately, there is no hope to observe their CP violation even at Super-B factory experiments.

IV. SUMMARY

In short, we calculated the branching ratios and CP-violating asymmetries of the five $\bar{B}_s^0 \to (\pi^0, \eta^{(\prime)})\eta^{(\prime)}$ decays by employing the pQCD factorization approach. All currently known NLO contributions, specifically those NLO twist-2 and twist-3 contributions to the relevant form factors, are taken into account. From our studies, we found the following results:

(i) For $\bar{B}_s^0 \to (\eta\eta', \eta'\eta')$ decays, the NLO enhancements to their branching ratios can be as large as 100%. For the other three decay modes, however, the NLO enhancements are less than 30%. The newly known NLO twist-2 and twist-3 contributions to the form factors alone can provide ~10% enhancements to the branching ratios.

(ii) For the $\bar{B}_s \to \pi^0\eta^{(\prime)}$ decays, the LO pQCD predictions for $\mathcal{A}_f^{\rm dir}$ can be enhanced significantly by the inclusion of the NLO contributions. For the other three decays, the NLO contributions are small in size.

(iii) For $\bar{B}_s \to (\eta\eta^{(\prime)}, \eta'\eta')$ decays, their branching ratios are at the order of 4×10^{-5}, which may be measurable at LHCb or Super-B factory experiments.

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