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Charmed scalar meson production in B decays

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Charmed scalar meson spectroscopy has become a hot topic on both experimental and theoretical sides. The $B_{(s)}$ decays provide an ideal place to study their properties. We employ the B-meson light-cone sum rules to calculate the $\bar{B}_s^0 \to D_{s0}^{*+}(2317)$ and $B^- \to D_0^{*0}(2400)$ transition form factors at the large recoil
recion assuming $D^{*+}(2317)$ and $D^{*0}(2400)$ are solar quark attiquark states. The results are extrapol region assuming $D_{s0}^{*+}(2317)$ and $D_0^{*0}(2400)$ are scalar quark-antiquark states. The results are extrapolated
to the whole momentum region in heavy quark effective theory. We also estimate the DK continuum state to the whole momentum region in heavy quark effective theory. We also estimate the DK continuum state contribution to $\bar{B}_s^0 \to D_s^{*+}(2317)$ form factors with a phenomenological method. Compared with the recononce contribution it is loss than 15% for f^+ and shout 60% for f^- a differentially the theoretical resonance contribution, it is less than 15% for $f_{BD_{s0}^*}^+$ and about 60% for $f_{BD_{s0}^*}^-$. After including the theoretical uncertainties, our results can be consistent with the previous studies. The branching ratios of semileptonic decays $\bar{B}_s^0 \to D_{s0}^{*+}(2317)l\bar{\nu}_l$ and $B^- \to D_0^{*0}(2400)l\bar{\nu}_l$ are then evaluated. For the light lepton final state,
 $\Pr(\bar{B}_s^0 \to D_s^{*+}(2317)l\bar{\nu}_l)$ is ground 6.0 × 10⁻³ which is decreased to shout 0.8 × 10⁻³ $Br(\bar{B}^0_s \to D_{s0}^{*+}(2317)l\bar{\nu}_l)$ is around 6.0×10^{-3} , which is decreased to about 0.8×10^{-3} for the tau final state
due to the phase space effect. The predicted branching ratio of $B^- \to D^{*0}(2400)l\bar{\nu}_s$ is sli due to the phase space effect. The predicted branching ratio of B^- → $D_0^{*0}(2400)l\bar{\nu}_l$ is slightly larger than that of $\bar{B}^0 \rightarrow D^{*+}/2317 \overline{l}$. that of $\bar{B}_s^0 \to D_{s0}^{*+}(2317)l\bar{\nu}_l$.

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I. INTRODUCTION

Since the observation of $D_{s0}^{*+}(2317)$ by the *BABAR*
Usboration in 2003, the charmed scalar meson spectroscollaboration in 2003, the charmed scalar meson spectroscopy has evoked great theoretical interests. In addition, the signal for the isospin doublet $D_0^*(2400)$ has also been
reported by Belle [11 and Focus [21 in the $D\pi$ final states] reported by Belle [\[1\]](#page-11-0) and Focus [\[2\]](#page-11-1) in the $D\pi$ final states. Recently more measurements on the charmed scalar meson final state in B decays have been performed [\[3\].](#page-11-2) The low mass and narrow width of $D_{s0}^{*+}(2317)$ indicates some hints
on its mysterious inner structure. It is regarded as a scalar on its mysterious inner structure. It is regarded as a scalar meson state in some studies, while it has also been assigned to be a four-quark state or the molecular state. Until now, the structure of $D_{s0}^{*+}(2317)$ is still a controversial problem.
As for $D^{*}(2400)$ there is even less information from As for $D_0^*(2400)$, there is even less information from
experiments, and we have very poor knowledge of its experiments, and we have very poor knowledge of its property. So more phenomenological studies are required to clarify the inner structure of these charmed p-wave states.

A great number of B decay events have been accumulated at B factories which provide a good platform to test the inner structure of the charmed scalar meson. To study the B-to-scalar meson decay modes theoretically, an essential task is to evaluate $\bar{B}_s^0 \to D_{s0}^{*+}(2317)$ and $B^- \to D^{*0}(2400)$ transition form factors In heavy quark $B^{-} \rightarrow D_{0}^{*0}(2400)$ transition form factors. In heavy quark
effective theory (HOFT) [4] the heavy-to-heavy form effective theory (HQET) [\[4\],](#page-11-3) the heavy-to-heavy form factor can be reduced to the universal Isgur-Wise (IW) function $\xi(v \cdot v')$ in the heavy quark limit. In order to estimate the form factors or the IW function, one must employ the nonperturbative methods. There have existed some phenomenological studies using different approaches, including the phenomenological quark model [\[5\]](#page-12-0), the QCD sum rules approach [6–[8\]](#page-12-1), PQCD approach [\[9\]](#page-12-2), lattice QCD [\[10](#page-12-3)–12], as well as the light-cone sum rules (LCSR) [\[13\].](#page-12-4)

LCSR [14–[16\]](#page-12-5) combines the traditional QCD sum rules [\[17\]](#page-12-6) with the theory of hard exclusive processes, and offers a systematic way to compute the hadron transition form factor. The vacuum-to-hadron correlation function is computed in terms of light-cone operator product expansion (OPE). The conventional LCSR for $\bar{B}^0_s \to D_{s0}^{*+}(2317)$ form
factor employ a correlation function which is a nonlocal factor employ a correlation function which is a nonlocal current-current operator sandwiched between vacuum and the $D_{s0}^{*+}(2317)$ state, and the B meson is interpolated by a
local current. The long distance effect of the form factor is local current. The long distance effect of the form factor is then described by the distribution amplitudes (DAs) of $D_{s0}^{*+}(2317)$. As the structure of $D_{s0}^{*+}(2317)$ is not well
understood, the DAs of $D_{s0}^{*+}(2317)$ are rather model understood, the DAs of $D_{s0}^{*+}(2317)$ are rather model
dependent In this paper we employ a different sum rule dependent. In this paper, we employ a different sum rule for the transition form factor following Refs. [\[18\]](#page-12-7) and [\[19\]](#page-12-8), where the correlation function is constructed with the onshell B-meson state and the interpolated current of the charmed scalar meson. As the nonperturbative dynamics is parametrized in terms of the B-meson DAs [\[20,21\]](#page-12-9), the new method is usually called B-meson LCSR and it has been widely applied to the calculation of heavy-to-light matrix elements [\[22,23\].](#page-12-10) The idea of B-meson LCSR is also

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proposed in Refs. [\[24\]](#page-12-11) and [\[25\]](#page-12-12) independently in the framework of soft-collinear effective theory.

In this paper, we will employ the B -meson LCSR approach to evaluate the $\bar{B}^0_s \to D_{s0}^{*+}(2317)$ and $B^- \to D_0^{*0}(2400)$ form factors. In our calculation $D^{*+}(2317)$ and $D^{*0}(2400)$ are factors. In our calculation, $D_{s0}^{*+}(2317)$ and $D_0^{*0}(2400)$ are
regarded as $q\bar{q}$ mesonic states. Compared with the s-wave regarded as $q\bar{q}$ mesonic states. Compared with the s-wave final state, the mass of $D_{s0}^{*+}(2317)$ is very close to the DK continuum state so that the single resonance dominance continuum state, so that the single resonance dominance assumption should not work well. It is meaningful to introduce a phenomenological method to estimate the contribution from continuum states. Inserting the DK state into the correlation function, both the DK timelike form factor and the $B \to DK$ transition form factor will be involved. The former has been obtained in the QCD sum rules to evaluate the mass and decay constant of $D_{s0}^{*+}(2317)$, and the latter can be parametrized by several form factors. After both of the above parametrized by several form factors. After both of the above two parts are evaluated, we can give a rough estimation on the DK continuum state contribution. In the present paper we only assign the scalar D meson state to be a quark-antiquark state for simplicity. If it is regarded as a four-quark state or other structure, the interpolating current is changed which will lead to a different result. For example, in $[26]$, the coupling constant of $D_{s0}^{*+}DK$ is calculated in the four-quark state assumption, and the result is much smaller than that evaluated by supposing D_{s0} being a conventional scalar meson [\[27\]](#page-12-14). In this paper the semileptonic $\bar{B}_s^0 \to D_{s0}^{*+}(2317)l\nu$ and $B^- \to D_0^{*0}(2400)l\nu$
decay modes are also analyzed. The experimental data decay modes are also analyzed. The experimental data accumulated in the B factories and LHC-b can test whether it is reasonable to regard the charmed scalar meson as a $q\bar{q}$ state.

The paper is arranged as follows: We first derive the LCSR for the $\bar{B}^0_s \to D_{s0}^{*+}(2317)$ and $B^- \to D_0^{*0}(2400)$ form factors in Sec. II. The contributions from both two-particle factors in Sec. [II.](#page-1-0) The contributions from both two-particle and three-particle wave functions of B meson are calculated. In Sec. [III](#page-3-0), the framework of estimating the DK contribution to $\bar{B}_s^0 \to D_{s0}^{*+}(2317)$ is presented. The numeri-
cal analysis of LCSR for the transition form factors at large cal analysis of LCSR for the transition form factors at large recoil region is displayed in Sec. [IV.](#page-5-0) The HQET is adopted to describe transitions at the small recoil region. In addition, the result for the DK state contribution is also included. Utilizing these form factors, the branching fractions of semileptonic decays are calculated. The last section is devoted to the conclusion.

II. THE LIGHT-CONE SUM RULES FOR FORM FACTORS

The B-to-charmed scalar meson transition form factor induced by an axial vector current is defined as

$$
\langle D_0^*(p)|\bar{c}\gamma_{\mu}\gamma_5 b|\bar{B}(p+q)\rangle = -i\{p_{\mu}f_{BD_0^*}^*(q^2) + q_{\mu}f_{BD_0^*}^*(q^2)\},
$$
 (1)

where \bar{B} denotes \bar{B}^0 , B^+ and \bar{B}_s respectively, D_0^* refers to D_{s0}^{*+} (2317) and D_0^{*0} (2400). To obtain the form factors with B_s -meson I CSR, we consider the following correlation B-meson LCSR, we consider the following correlation function with an on-shell B-meson state:

$$
F_{\mu}(p,q) = i \int d^4x e^{ip\cdot x} \langle 0|T\{\bar{q}(x)c(x),\bar{c}(0)\gamma_{\mu}(1-\gamma_5)b(0)\}\vert \bar{B}(P+q)\rangle, \tag{2}
$$

where $\bar{c}\gamma_{\mu}(1-\gamma_5)b$ is the $b \to c$ weak current and $\bar{q}c$ is the interpolating current for a charmed scalar meson. The hadronic representation of the correlation function can be written as

 $F_\mu(p,q) = \frac{\langle 0 | \bar{q}(0) c(0) | D_0^*(p) \rangle \langle D_0^*(p) | \bar{c}(0) \gamma_\mu \gamma_5 b(0) | \bar{B}(P+q) \rangle}{m_{D_0^*}^2 - p^2} + \sum_h$ $\frac{\langle 0|\bar q(0)c(0)|h(p)\rangle\langle h(p)|\bar c(0)\gamma_\mu\gamma_5b(0)|\bar B(P+q)\rangle}{s-p^2}.$ (3)

The decay constants $f_{D_0^*}$ and $\tilde{f}_{D_0^*}$ are given by

$$
\langle 0|\bar{q}\gamma_{\mu}c|D_0^*(p)\rangle = f_{D_0^*}p_{\mu}, \qquad \langle 0|\bar{q}c|D_0^*(p)\rangle = m_{D_0^*}\tilde{f}_{D_0^*}, \qquad (4)
$$

where $f_{D_0^*}$ $\hat{p}_{\alpha}^{*} = (m_c - m_q) \tilde{f}_{D_0^*}/m_{D_0^*}$ and m_c , m_q are the masses of charm quark and light quark, respectively. Inserting the soft the form factors and decay constants, the correlation function reads definitions of the form factors and decay constants, the correlation function reads

$$
F_{\mu}(p,q) = \frac{-im_{D_0^*}^2 f_{D_0^*}}{(m_c - m_q)(m_{D_0^*}^2 - p^2)} [f_{D_{s0}^*}^+(q^2)p_{\mu} + f_{D_0^*}^-(q^2)q_{\mu}] + \int_{S_0^{p^*}}^{\infty} ds \frac{\rho_+^h(s,q^2)p_{\mu} + \rho_-^h(s,q^2)q_{\mu}}{s - p^2},
$$
(5)

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FIG. 1. Diagrams corresponding to the contributions of (a) twoparticle and (b) three-particle B-meson DA's to the correlation function in Eq. [\(2\).](#page-1-2)

where $s_0^{D_0^*}$ is the threshold parameter corresponding to the D_0^* channel. Similar to the s-wave final state, we adopt a simple hadronic spectrum density (single narrow resonance and continuum states). Note that the invariant mass of the DK continuum state is very close to the mass of $D_{s0}^{*+}(2317)$, thus it cannot be included in the second term
of Eq. (5) Actually this contribution should be mixed with of Eq. [\(5\)](#page-1-1). Actually this contribution should be mixed with the resonance state contribution. In this section we simply neglect the contribution of the DK continuum state to the $\bar{B}_s^0 \rightarrow D_{s0}^{*+}$ (2317), and it will be studied in the next section.
On the other hand in the deep Euclidean region the

On the other hand, in the deep Euclidean region, the correlation function can be calculated in the perturbation theory using the operator production expansion near the light cone:

$$
F_{\mu}(p,q) = F_{+}^{\text{QCD}}(q^{2}, p^{2})p_{\mu} + F_{-}^{\text{QCD}}(q^{2}, p^{2})q_{\mu}
$$

=
$$
\int_{m_{c}^{2}}^{\infty} ds \frac{1}{\pi} \frac{\text{Im}F_{+}^{\text{QCD}}(q^{2}, p^{2})}{s - p^{2}} p_{\mu}
$$

+
$$
\int_{m_{c}^{2}}^{\infty} ds \frac{1}{\pi} \frac{\text{Im}F_{-}^{\text{QCD}}(q^{2}, p^{2})}{s - p^{2}} q_{\mu}.
$$
 (6)

Applying the quark-hadron duality

$$
\rho_i^h(s, q^2) = \frac{1}{\pi} \text{Im} F_i^{\text{QCD}}(q^2, p^2) \Theta(s - s_0^h), \tag{7}
$$

with $i = +, -$ and performing the Borel transformation with respect to the variable p^2 , we can derive the sum rules for the form factors as

$$
f_i(q^2) = -i \frac{m_c - m_q}{\pi f_{D_0^*} m_{D_0^*}^2} \int_{m_c^2}^{s_0^h} ds \text{Im} F_i^{\text{QCD}}(q^2, s) \exp\left(\frac{m_{D_0^*}^2 - s}{M_B^2}\right). \tag{8}
$$

The leading order contribution to the OPE is illustrated in Fig. [1\(a\).](#page-2-0) The correlation function can be calculated by contracting the charm quark fields in Eq. [\(2\)](#page-1-2) and inserting the c quark propagator, then we obtain

$$
F_{\mu}^{(B)}(p) = i \int d^4x e^{ip\cdot x} \int \frac{d^4k}{(2\pi)^4} i e^{-ik\cdot x} \langle 0|T\{\bar{q}(x)S_F(x,0)\gamma_{\nu}(1-\gamma_5)b(0)\}|\bar{B}(P_B)\rangle.
$$
 (9)

The full quark propagator can be written as [\[28\]](#page-12-15)

$$
S_F(x,0)_{ij} = \delta_{ij} \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{i}{k - m_c} - ig \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 d\alpha \left[\frac{1}{2} \frac{k + m_c}{(m_c^2 - k^2)^2} G_{ij}^{\mu\nu}(\alpha x) \sigma_{\mu\nu} + \frac{1}{m_c^2 - k^2} \alpha x_\mu G^{\mu\nu}(\alpha x) \gamma_\nu \right],
$$
\n(10)

where the first term is the free-quark propagator and $G_{ij}^{\mu\nu} = G_{\mu\nu}^a T_{ij}^a$ with $Tr[T^aT^b] = \frac{1}{2}\delta^{ab}$. Inserting this propagator to $F_a(9)$ the long distance contribution to the correlation function can be expres Eq. [\(9\),](#page-2-1) the long distance contribution to the correlation function can be expressed by nonlocal matrix elements, which defines the B-meson light-cone DA. In the leading Fock state approximation

$$
\langle 0|\bar{q}_{2\alpha}(x)[x,0]h_{v\beta}(0)|\bar{B}_v\rangle = -\frac{if_Bm_B}{4}\int_0^\infty d\omega e^{-i\omega v \cdot x} \left[(1+x)\left\{ \phi_+^B(\omega) - \frac{\phi_+^B(\omega) - \phi_-^B(\omega)}{2v \cdot x} x \right\} \gamma_5 \right]_{\beta\alpha},\tag{11}
$$

where $[x, 0]$ is the path-ordered gauge factor. The variable $\omega > 0$ is the plus component of the spectator-quark momentum in the B meson. The three-particle DAs' contribution is shown in Fig. $1(b)$, with the definition

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$$
\langle 0|\bar{q}_{2\alpha}(x)G_{\lambda\rho}(ux)h_{\nu\beta}(0)|\bar{B}^{0}(v)\rangle = \frac{f_{B}m_{B}}{4}\int_{0}^{\infty}d\omega\int_{0}^{\infty}d\xi e^{-i(\omega+u\xi)v\cdot x} \times \left[(1+x)\left\{ (v_{\lambda}\gamma_{\rho}-v_{\rho}\gamma_{\lambda})(\Psi_{A}(\omega,\xi)-\Psi_{V}(\omega,\xi)) - i\sigma_{\lambda\rho}\Psi_{V}(\omega,\xi) \right. \right. \\ \left. - \left(\frac{x_{\lambda}v_{\rho}-x_{\rho}v_{\lambda}}{v\cdot x} \right)X_{A}(\omega,\xi) + \left(\frac{x_{\lambda}\gamma_{\rho}-x_{\rho}\gamma_{\lambda}}{v\cdot x} \right)Y_{A}(\omega,\xi) \right\}\gamma_{5}\right]_{\beta\alpha}, \tag{12}
$$

where the gauge link factors are omitted for brevity. The DAs Ψ_V, Ψ_A , X_A and Y_A depend on the two variables ω and ξ , corresponding to the plus components of the light-quark and gluon momenta in the B meson.

 θ

Substituting the B-meson distribution function into the correlation function and employing the quark hadron duality [\(7\)](#page-2-2), we can get the sum rules for transition form factors:

$$
f_{BD_0^*}^{+rd} = \frac{f_B m_B (m_c - m_s)}{f_{D_0^*} m_{D_0^*}^2} \int_0^{\sigma_0} d\sigma e^{-(s - m_{D_0^*2})/M^2} \left\{ m_B (\bar{\sigma} - r_c) \left(\frac{1}{\bar{\sigma}} + \frac{m_B m_c}{\bar{\sigma}^2 m_B^2 + m_c^2 - q^2} \right) \phi_+ - \frac{m_B^2 m_c (\bar{\sigma} - r_c)}{\bar{\sigma}^2 m_B^2 + m_c^2 - q^2} \phi_- \right. \right. \\ \left. + \left[-\frac{1}{\bar{\sigma}} - \frac{m_B m_c}{\bar{\sigma}^2 m_B^2 + m_c^2 - q^2} + \frac{2m_B^3 m_c \bar{\sigma} (\bar{\sigma} - r_c)}{(\bar{\sigma}^2 m_B^2 + m_c^2 - q^2)^2} \right] \Phi_{\pm} \right\} + f_{BD_0^*}^{+3p}, \tag{13}
$$

$$
f_{BD_0^*}^{-rd} = -\frac{f_B m_B (m_c - m_s)}{f_{D_0^*} m_{D_0^*}^2} \int_0^{\sigma_0} d\sigma e^{-(s - m_{D_0^*}^2)/M^2} \left\{ m_B (\sigma + r_c) \left(\frac{1}{\bar{\sigma}} + \frac{m_B m_c}{\bar{\sigma}^2 m_B^2 + m_c^2 - q^2} \right) \phi_+ - \frac{m_B^2 m_c (\sigma + r_c)}{\bar{\sigma}^2 m_B^2 + m_c^2 - q^2} \phi_- \right. \right. \\ \left. + \left[\frac{1}{\bar{\sigma}} + \frac{m_B m_c}{\bar{\sigma}^2 m_B^2 + m_c^2 - q^2} + \frac{2m_B^3 m_c \bar{\sigma} (\sigma + r_c)}{(\bar{\sigma}^2 m_B^2 + m_c^2 - q^2)^2} \right] \Phi_{\pm} \right\} + f_{BD_0^*}^{-3p}, \tag{14}
$$

where the superscript "rd" means resonance dominance. The argument of the wave functions is $m_B\sigma$, and $r_c = m_c/m_B$. In addition, $\bar{\sigma} = 1 - \sigma$ and σ_0 is the root of the equation $\bar{\sigma} s_0 - (\sigma \bar{\sigma} + r_c^2) m_B^2 + \sigma m_B^2 q^2 = 0$. The modified wave function $\bar{\sigma} s_0$ (c) $-\int_{-\infty}^{\infty} d\tau (s) d\tau (s)$. The contributions from three pertials *B* mes $\Phi_{\pm}(\omega) = \int_0^{\omega} d\tau [\phi_{+}(\tau) - \phi_{-}(\tau)].$ The contributions from three-particle B-meson DAs are denoted by $f_{BD_0^*}^{+3p}$ and $f_{BD_0^*}^{-3p}$, which are given in the Appendix.

III. THE CONTRIBUTION OF DK CONTINUUM STATE

In general, the $B \to S$ transition form factors are hard to be extracted in the LCSR framework because the continuum states may play a very important role, such as in $B \to f_0$ form factors, large $\pi\pi$ continuum states below the resonance should be included in the hadronic representation in a systematic way. In our case, the mass of $D_{s0}^{*+}(2317)$ is slightly below the continuum DK states, thus it is necessary to estimate the DK contribution if we want to get a continuum DK states, thus it is necessary to estimate the DK contribution if we want to get a clean result of scalar resonance contribution. The contribution of the DK continuum state in the correlation function is written by

$$
2\mathrm{Im}F_{\mu}^{DK\,\text{continuum}} = \int \frac{d^3k_1}{(2\pi)^3} \frac{1}{2E_1} \frac{d^3k_2}{(2\pi)^3} \frac{1}{2E_2} \langle 0|\bar{s}(0)c(0)|K(k_1)D(k_2)\rangle \langle K(k_1)D(k_2)|\bar{c}(0)\gamma_{\mu}(1-\gamma_5)b(0|\bar{B}(P_B)\rangle
$$

× $(2\pi)^4\delta^4(p-k_1-k_2).$ (15)

The timelike DK form factor $F(s)$ is defined by

$$
F(s) = \langle 0|\bar{c}s|DK\rangle. \tag{16}
$$

The energy region in $F(s)$ is below the mass of the first excited state of $D_{s0}^{*+}(2317)$. As pointed out in [\[29\]](#page-12-16), $F(s)$ is dominant by elastic DK scattering because the DK dominant by elastic DK scattering, because the DK inelastic scattering is suppressed by the phase space. The form factor can be represented by a series of bubble diagrams [\[29\],](#page-12-16) and the result is written as the following form:

$$
F(s) = \frac{\lambda}{s - M_0^2 - \Delta(s)},\tag{17}
$$

where the constant λ can be determined by soft-pion theorem,

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$$
\lambda = \frac{f_D m_D^2}{f_K (m_c + m_u)} [m_D^2 - M_0^2 - \Delta(m_D^2)],\tag{18}
$$

and $\Delta(s)$ can be found in Ref. [\[29\]](#page-12-16).

The matrix element $\langle DK|\bar{c}(0)\gamma_{\mu}\gamma_5b(0)|\bar{B}_s\rangle$ can be parametrized as [\[30](#page-12-17)–33]

$$
\langle D(p_2)K(p_1)|\bar{c}(0)\gamma_{\mu}\gamma_5 b(0)|\bar{B}_s(p_B)\rangle = ir(p_B - p_1 - p_2)_{\mu} + i\omega_+(p_2 + p_1)_{\mu} + i\omega_-(p_2 - p_1)_{\mu} + h\epsilon_{\mu\nu\alpha\beta}p_B^{\nu}(p_1 + p_2)^{\alpha}(p_2 - p_1)^{\beta},
$$
\n(19)

the parameters r, ω_+, ω_- of the B-to-two pseudoscalar transition have been calculated using heavy meson chiral perturbation theory (HMChPT) and we quote the results below [\[34\]:](#page-12-18)

$$
\omega_{+} = -\frac{g}{f_{1}f_{2}} \frac{f_{B^{*}}m_{B^{*}}^{3/2}m_{B_{s}}^{1/2}}{p_{B} - p_{1})^{2} - m_{B^{*}}^{2}} \left[1 - \frac{p_{1} \cdot (p_{B} - p_{1})}{m_{B^{*}}^{2}}\right] + \frac{f_{B_{s}}}{2f_{1}f_{2}},
$$
\n
$$
\omega_{-} = \frac{g}{f_{1}f_{2}} \frac{f_{B^{*}}m_{B^{*}}^{3/2}m_{B}^{1/2}}{(p_{B} - p_{1})^{2} - m_{B^{*}}^{2}} \left[1 + \frac{p_{1} \cdot (p_{B} - p_{1})}{m_{B^{*}}^{2}}\right],
$$
\n
$$
r = -\sqrt{\frac{m_{B_{c}}}{m_{B}} \frac{4g^{2}f_{B_{c}}f_{B}^{*}m_{B^{*}}}{[(p_{B} - p_{1} - p_{2})^{2} - m_{B_{c}}^{2}]f_{1}f_{2}} \frac{p_{1} \cdot p_{2} - p_{2} \cdot (p_{B} - p_{1})p_{1} \cdot (p_{B} - p_{1})/m_{B^{*}}^{2}}{(p_{B} - p_{1})^{2} - m_{B^{*}}^{2}} + \frac{2g}{f_{1}f_{2}} \frac{f_{B^{*}}m_{B^{*}}}{(p_{B} - p_{1})^{2} - m_{B^{*}}^{2}} \frac{p_{1} \cdot (p_{B} - p_{1})}{m_{B^{*}}^{2}} + \frac{f_{B}}{f_{1}f_{2}},
$$
\n(20)

with $f_1 = f_K$, $f_2 = f_D$. The form factors ω_{\pm} , r evaluated from HMChPT are applicable only when the final state meson is soft. If it is generalized to the region beyond its validity, the predicted decay rates in three body B decays will be too large. To give a rough estimation on the DK continuum states contrition in LCSR, we neglect the 3-momentum of D meson, which also favors the condition of HMChPT. With the approximation mentioned above, the form factors ω_{+} , r are reduced to

$$
\omega_{+} = -\frac{g}{f_D f_K} \frac{f_{B^*} m_{B^*}^{3/2} m_{B_s}^{1/2}}{(m_{B_s}^2 + m_K^2 - 2m_{B_s} E_K) - m_{B^*}^2} \left[1 - \frac{m_{B_s} E_K - m_K^2}{m_{B^*}^2} \right] + \frac{f_{B_s}}{2f_D f_K},
$$

\n
$$
\omega_{-} = \frac{g}{f_D f_K} \frac{f_{B^*} m_{B^*}^{3/2} m_{B_s}^{1/2}}{(m_{B_s}^2 + m_K^2 - 2m_{B_s} E_K) - m_{B^*}^2} \left[1 + \frac{m_{B_s} E_K - m_K^2}{m_{B^*}^2} \right],
$$

\n
$$
r = \frac{2g}{f_D f_K} \frac{f_{B^*} m_{B^*}}{(m_{B_s}^2 + m_K^2 - 2m_{B_s} E_K) - m_{B^*}^2} \frac{m_{B_s} E_K - m_K^2}{m_{B^*}^2} + \frac{f_{B_s}}{f_D f_K} - \sqrt{\frac{m_{B_c}}{m_{B_s}}} \frac{4g^2 f_{B_c} f_{B^*} m_{B^*}}{[(p_B - p)^2 - m_{B_c}^2] f_D f_K}
$$

\n
$$
\times \frac{(p^2 - m_D^2 - m_K^2) - (m_{B_s} E_K - m_K^2)(m_B^2 - m_D^2 - m_K^2 - p^2 - q^2)/m_{B^*}^2}{(m_{B_s}^2 + m_K^2 - 2m_{B_s} E_K) - m_{B^*}^2},
$$
\n(21)

where $E_K = [(m_{B_s} - m_D)^2 + m_K^2 - q^2]/[2(m_{B_s} - m_D)].$
Employing the above parametrization in the correlation function, the corresponding spectral function for DK continuum state is given by

$$
\rho_{+}^{(DK)}(p,q) = \frac{i}{8\pi^2} F(p^2) \sqrt{\left(1 - \frac{s_+}{p^2}\right) \left(1 - \frac{s_-}{p^2}\right)} \left(\omega_+ + \frac{m_D^2 - m_K^2}{p^2}\omega_-\right) \theta(s_+) \theta(s_0 - p^2),
$$
\n
$$
\rho_{-}^{(DK)}(p,q) = \frac{i}{8\pi^2} F(p^2) \sqrt{\left(1 - \frac{s_+}{p^2}\right) \left(1 - \frac{s_-}{p^2}\right)} r \theta(s_+) \theta(s_0 - p^2),
$$
\n(22)

where $s_{\pm} = p^2 - (m_K \pm m_D)^2$. Including the DK continuum state hadronic spectral function, an additional term will be added in the sum rules of the $B_s \to D_{s0}^{*+}(2317)$ form factor:

$$
f_{BD_0^*}^+ = f_{BD_0^*}^{+rd} - \frac{1}{8\pi^2} \frac{(m_c - m_q)}{m_{D_0^*}^2 f_{D_0^*}} \int_{(m_D + m_K)^2}^{s_0} ds F(s) \sqrt{\left(1 - \frac{s_+}{s}\right) \left(1 - \frac{s_-}{s}\right)} \left(\omega_+ + \frac{m_D^2 - m_K^2}{p^2} \omega_- \right) e^{-(s - m_{D_0^*}^2)/M^2},
$$

$$
f_{BD_0^*}^- = f_{BD_0^*}^{-rd} - \frac{1}{8\pi^2} \frac{(m_c - m_q)}{m_{D_0^*}^2 f_{D_0^*}} \int_{(m_D + m_K)^2}^{s_0} ds F(s) \sqrt{\left(1 - \frac{s_+}{s}\right) \left(1 - \frac{s_-}{s}\right)} r e^{-(s - m_{D_0^*}^2)/M^2},
$$
(23)

where D_0^* denotes $D_{s0}^{*+}(2317)$.

IV. NUMERICAL ANALYSIS

A. Form factors without continuum states

Now we are going to calculate the form factors $f_{D_0^*}(q^2)$
d₂ $\left(\frac{q^2}{q^2}\right)$ municipality in the following and list that and $f_{D_{s0}^*}(q^2)$ numerically. In the following, we list the relevant input parameters for $D_{s0}^{*+}(2317)$ and $D_0^{*}(2400)$.
Their mass can be taken from PDG [35]: $m_{\text{rel}}(2400)$. Their mass can be taken from PDG [\[35\]](#page-12-19): $m_{D_{s0}^{++}(2317)} =$
2.218 GeV and we are 2.400 GeV. The deeps son 2.318 GeV and $m_{D_0^*(2400)} = 2.400$ GeV. The decay constant $\tilde{f}_{D_{s0}^{*+}(2317)} = (250 \pm 25)$ MeV [\[36\].](#page-12-20) For the $D_0^{*}(2400)$ state, we expect $\tilde{f}_{D_0^*}$ $\int_{0}^{*}(2400) / \tilde{f}_{D_{s0}^{*+}}(2317) = f_{D}^{i} / f_{D_{s}}^{i}$ in the SU(3) limit. We adopt the values $f_D = (223 \pm 18)$ MeV
and $f_{D_s} = (274 \pm 20)$ MeV, then $\tilde{f}_{D_s^*(2400)} =$ $f_{D_s} = (274 \pm 20)$ MeV, $D_0^*(2400) =$
 $\sum_{r=1}^{6}$ (203 ± 30) MeV. As for the decay constant of the B_s meson, we use the results $f_B = 130$ MeV [\[37\]](#page-12-21) and $f_{B_s}/f_B = 1.16 \pm 0.09$ [\[38\]](#page-12-22) determined from QCD sum
rules (QCDSR). The threshold parameter s, can be fixed by rules (QCDSR). The threshold parameter s_0 can be fixed by fitting the LCSR of the charmed meson masses to the experimental data. Numerically, the threshold value in the X channel would be $s_X^0 = (m_X + \Delta_X)^2$, where Δ_X is about 0.6 GeV [39-411] and it is taken as $(0.6 + 0.1)$ GeV in the 0.6 GeV [\[39](#page-12-23)–41], and it is taken as (0.6 ± 0.1) GeV in the error analysis. The two-particle DAs of B-meson inspired from QCD sum rule analysis read [\[20\]](#page-12-9)

$$
\phi_{+}^{B}(\omega) = \frac{\omega}{\omega_{0}^{2}} e^{-\frac{\omega}{\omega_{0}}},
$$

$$
\phi_{-}^{B}(\omega) = \frac{1}{\omega_{0}} e^{-\frac{\omega}{\omega_{0}}},
$$
 (24)

and the 3-particle DAs are given by

$$
\Psi_A(\omega, \xi) = \Psi_V(\omega, \xi) = \frac{\lambda_E^2}{6\omega_0^4} \xi^2 e^{-(\omega + \xi)/\omega_0},
$$
\n
$$
X_A(\omega, \xi) = \frac{\lambda_E^2}{6\omega_0^4} \xi(2\omega - \xi) e^{-(\omega + \xi)/\omega_0},
$$
\n
$$
Y_A(\omega, \xi) = -\frac{\lambda_E^2}{24\omega_0^4} \xi(7\omega_0 - 13\omega + 3\xi) e^{-(\omega + \xi)/\omega_0}.
$$
\n(25)

The parameters ω_0 , λ_H and λ_E satisfy the conditions adopted in [\[20\]](#page-12-9)

$$
\omega_0 = \frac{2}{3}\bar{\Lambda}, \qquad \lambda_E^2 = \lambda_H^2 = \frac{3}{2}\omega_0^2 = \frac{2}{3}\bar{\Lambda}^2. \tag{26}
$$

The parameter ω_0 is actually the inverse moment of ϕ_B^+ , and plays an important role in nonleptonic B decays. Experimentally there are two analyses by BABAR collabo-ration [\[42,43\];](#page-12-24) the first reports $\omega_0 > 0.669$ GeV and in the second $\omega_0 > 0.3$ GeV. The nonleptonic B decays phenomenology in QCD factorization prefers a small value $\omega_0 \sim 0.2$ GeV [44–[46\].](#page-12-25) A theoretical study in QCD sum rules gives $\omega_0 = 0.46 \pm 0.11$ GeV [\[47\].](#page-12-26) To cover the range mentioned above, we employ the values $\omega_0^B = 0.45 \pm 0.20$ GeV and $\omega_0^{B_s} = 0.50 \pm 0.25$ GeV,
where a slight SU(3) breaking effect has been taken into where a slight SU(3) breaking effect has been taken into account. Note that for the B-meson LCDAs, we only consider the tree level contribution in this paper. If the radiative corrections and the RG effect is included, the parameters ω_0 and λ_H are independent [\[48,49\],](#page-12-27) then the uncertainties coming from of hadronic physics parameters should be significantly underestimated. The RG effect is very important in the next-to-leading-order contribution to form factors, and such discussion is beyond the scope of the present paper.

After fixing the corresponding parameters, the numerical values of the form factors can be obtained. In principle, the form factors should not depend on the unphysical Borel mass M_B^2 . However, the OPE series are truncated up to the next to leading Fock state of the B meson and the QCD corrections are not considered, a manifest dependence of the form factors on the Borel parameter M^2 would emerge. Therefore, we should search for the so-called "Borel window," where Borel mass dependence is mild to ensure the truncation is acceptable.

We first focus on the form factors at zero momentum transfer. To extract the form factor $f_{D_0^*}^i(0)$, the value of $f_{D_0^*}^i(0)$ should not be sensitive to the Borel mass. In view of these considerations, the Borel parameter M_B^2 should not be either too large or too small. To make sure that the contributions from the higher states are exponentially damped [see Eq. [\(14\)](#page-3-1)] and the global quark-hadron duality

FIG. 2. The dependence of form factors $f_{D_{s0}^{++}(2317)}^+(0)$ (left) and $f_{D_{s0}^{++}(2317)}^-(0)$ (right) on the Borel mass M^2 , the contribution from
2 pertials B mason DA is depended by the dephabilities and the dephabiti 2-particle B-meson DA is denoted by the dashed line, and the dash-dotted line represents the 3-particle DA contribution. The solid line gives the total results.

FIG. 3. The dependence form factor $f_{D_{s0}^{*+}(2317)}^{+}(0)$ (left) and $f_{D_{s0}^{*+}(2317)}^{-}(0)$ (right) on the Borel mass M^2 , the contribution of higher exited states and the continuum states in the whole sum rules is sho exited states and the continuum states in the whole sum rules is shown by the dashed line.

is satisfied, we need a smaller Borel mass. On the other hand, the Borel mass could not be too small for the validity of light-cone OPE for the correlation function, since the contributions of higher twist distribution amplitudes amount to the higher power of $1/M_B^2$ relative to the perturbative part. In this way, we find a Borel platform $M_B^2 \in [3.5, 5]$ GeV². The Borel mass dependence of the form factors is plotted in Figs. 2 and 3: the former includes form factors is plotted in Figs. [2](#page-6-0) and [3](#page-6-1); the former includes the contribution from the three-particle B-meson distribution amplitudes and the higher states contribution is shown in the latter one. From these diagrams we can easily see that the higher Fock state is highly suppressed in the Borel window, and higher exited states and the continuum states contribution is within 15% for $f_{D_0^*}^+(0)$ [30% for $f_{D_0^*}^-(0)$]. The numerical values for these form factors are collected in Table [I](#page-7-0), where the uncertainties are from the variation of shape parameter ω_0 , the fluctuation of threshold value, the uncertainties of quark masses and the errors of decay constants for the involved mesons. The results in the other studies are listed for comparison. We can see that for $f_{D_{s0}}^{+}(0)$ our result is sightly larger than that of D-meson LCSR, however the results are consistent with each other within the errors. For $f_{D_{s0}^*}^-(0)$, the sign of our result is consistent with the result obtained from the QCDSR, but it is different from that derived in the light meson LCSR. This discrepancy is expected to be smeared by power corrections.

We can also investigate the q^2 dependence of the form factors $f_{D_0^*}(q^2)$. It is known that the OPE for the correlation
function is valid only at small momentum transfer region function is valid only at small momentum transfer region $0 < q^2 < (m_b - m_c)^2 - 2\Lambda_{\text{QCD}}(m_b - m_c)$. At the large momentum transfer region, we need to parametrize them in terms of phenomenological models. To achieve this goal we first analyze the form factors within the HQET framework, which works well for the $b \rightarrow c$ transition. The matrix elements responsible for $B \to D_0^*$ transition can be parametrized as [\[50\]](#page-12-28)

TABLE I. Numbers of $f^{\pm}_t(0)$ and $\eta^{\pm}_t(w)$ determined from the LCSR approach with single resonance dominance assumption, where the uncertainties from the ω , threshold value, quark masses and decay constants are in the uncertainties from the ω_0 , threshold value, quark masses and decay constants are included. For comparison, the results in the QCDSR approach are also collected here.

	This work	D-meson LQSR	OCDSR		$\eta_i^{\pm}(1)$	a_{\cdot}^{\perp}	b^{\pm}
$f_{D_{s0}^{*+}(2317)}^+(q^2)$	$0.80^{+0.64}_{-0.32}$	$0.53_{-0.11}^{+0.12}$	0.40 ± 0.10 [7]	$\eta_{D_{s0}^{*+}(2317)}^{+}(w)$	$0.29_{-0.05}^{+0.19}$	$-0.49^{+0.57}_{-1.38}$	$0.53_{-0.80}^{+2.19}$
$f^-_{D_{s0}^{*+}(2317)}(q^2)$	$-0.20^{+0.17}_{-0.28}$	$0.18^{+0.06}_{-0.04}$	-0.12 ± 0.13 [7]	$\eta_{D_{s0}^{*+}(2317)}^-(w)$	$-0.86^{+0.42}_{-0.45}$	$1.59_{-0.51}^{+0.80}$	$-1.61^{+0.94}_{-1.34}$
$f_{D_0^*}^+(q^2)$	$0.94_{-0.41}^{+0.56}$			$\eta_{D_0^*}^+(w)$	$0.28_{-0.07}^{+0.13}$	$-0.34_{-0.65}^{+1.47}$	$0.32_{-1.21}^{+1.22}$
$f^-_{D_0^*}(q^2)$	$-0.27^{+0.16}_{-0.25}$			$\eta_{D_0^*}^-(w)$	$-1.01_{-0.42}^{+0.53}$	$1.86^{+1.14}_{-1.16}$	$-2.00^{+1.69}_{-1.71}$

$$
\langle D_0^{*+}(P)|\bar{c}\gamma_{\mu}\gamma_5 b|\bar{B}(P+q)\rangle = -i\sqrt{m_B m_{D_0^*}}[\eta_{D_0^*}^+(w)(v+v')_{\mu} + \eta_{D_0^*}^-(w)(v-v')_{\mu}],
$$
\n(27)

where $v = (P+q)/m_B$ and $v' = P/m_{D_0^*}$ are the four-
velocity vectors of B and D_0^* mesons and $w = v \cdot v'$ velocity vectors of B and D_0^* mesons, and $w = v \cdot v' =$
 $\frac{m^2 + m^2}{2} = a^2/2m_0m_{\text{B}}$ Combining Eqs. (1) and (27) $\left(m_B^2 + m_{D_0^*}^2 - q^2\right)/2m_B m_{D_0^*}$ $_{0}^{*}$. Combining Eqs. [\(1\)](#page-1-3) and [\(27\)](#page-6-2), we have

$$
f_i^+(q^2) = \frac{1}{\sqrt{m_{B_i}m_{D_i}}} [(m_{B_i} + m_{D_i})\eta_i^+(w) - (m_{B_i} - m_{D_i})\eta_i^-(w)],
$$

$$
f_i^-(q^2) = \sqrt{\frac{m_{D_i}}{m_{B_i}}} [\eta_i^+(w) + \eta_i^-(w)],
$$
 (28)

where $i = 1, 2$ denotes strange and strangeless charmed scalar meson respectively. Similarly to the Isgur-Wise function $\xi(v \cdot v')$ for the s-wave transitions, heavy quark
symmetry allows one to relate the form factors $n^{\pm}(w)$ and symmetry allows one to relate the form factors $\eta_i^+(w)$ and $n(\omega)$ to a universal function $\tau_i \circ (w)$ [4] $\eta_i^-(w)$ to a universal function $\tau_{1/2}(w)$ [\[4\]](#page-11-3),

$$
\eta_i^+(w) + \eta_i^-(w) = -2\tau_{1/2}(w), \n\eta_i^+(w) - \eta_i^-(w) = 2\tau_{1/2}(w).
$$
\n(29)

Different from the Isgur-Wise function $\xi(w)$, one cannot employ the heavy quark symmetry to predict the normalization of $\tau_{1/2}(w)$ [\[51\]](#page-12-29).

Phenomenologically, one can parametrize the $B \to D_0^*$ form factors in the small recoil region as

$$
\eta_i^{\pm}(w) = \eta_i^{\pm}(1) + a_i^{\pm}(w-1) + b_i^{\pm}(w-1)^2, \quad (30)
$$

where the parameters $\eta_{\pm}^{\pm}(1)$, α_{\pm}^{\pm} and b_{\pm}^{\pm} can be determined
by connecting the form factors derived from the LCSR and by connecting the form factors derived from the LCSR and HQET approaches in the vicinity of the region with $q^2 \sim (m_b - m_c)^2 - 2\Lambda_{\text{OCD}}(m_b - m_c)$. In this way, we can derive the results of form factors in the whole kinematical region, in Fig. [4](#page-7-1) we plot the dependence of $f_{D_{s0}^{*+}(2317)}^+$ on q^2 . The parameters related to all the form factors are tabulated in Table [I](#page-7-0).

Another uncertainty of the form factors is from the scale dependence of the quark masses and B_s -meson decay

constant. The decay constant $f_{B_s}(\mu)$ defined in HQET is
related to the OCD f_{B} by related to the QCD f_{B_s} by

$$
f_B = \left[1 + \frac{\alpha_s C_F}{4\pi} \left(\frac{3}{2} \ln \frac{m_b^2}{\mu^2} - 2\right)\right] f_B(\mu),\qquad(31)
$$

where the running coupling constant

$$
\alpha_s(\mu) = \frac{12\pi}{(33 - 2N_f) \ln(\mu^2/\Lambda_{QCD}^2)},
$$
\n(32)

and the running mass of c and s quarks reads

$$
m_q(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_s(m_q)}\right]^{4/(11-2/3N_f)} m_q(m_q), \quad (33)
$$

here we take $N_f = 4$. To show the scale dependence of the form factors manifestly, we plot the $f_{D_{s0}^{*+}(2317)}^+ \sim \mu$ curve in Fig. 5. in Fig. [5](#page-8-0).

As discussed before, the power-suppressed form factors f_i^- in Table [I](#page-7-0) suffer from sizable power corrections, which can even change the sign. Generally speaking, the corrections can be picked up by performing the heavy quark expansion of the current

FIG. 4. The dependence of form factor $f_{D_{s0}^{*+}(2317)}^+$ on q^2 .

FIG. 5. The dependence of form factor $f_{D_{s0}^{++}(2317)}^{+}$ on the renormalization scale renormalization scale.

$$
\bar{c}\Gamma_i b = \bar{c}_{v_2}\Gamma_i b_{v_1} - \frac{1}{2m_c}\bar{c}_{v_2}\Gamma_i i\cancel{D}_{\perp 2} b_{v_1} + \frac{1}{2m_b}\bar{c}_{v_2}\Gamma_i i\cancel{D}_{\perp 1} b_{v_1} + \cdots.
$$
\n(34)

The last two terms in the above equation might give an important contribution for finite quark mass, which could help to reduce the discrepancy among different approaches. In addition, the radiative correction may also help.

B. Contribution from the DK continuum state in the $\bar{B}_{s}^{0} \to D_{s0}^{*+}(2317)$ form factor

The analytic form of the contribution from the DK continuum state to $\bar{B}_s^0 \to D_{s0}^{*+}(2317)$ transition has been
obtained in the previous section and now we evaluate the obtained in the previous section, and now we evaluate the numerical results of this part. In the form factor $F(s)$, the coupling constant which describes the $D_{s0}^{*+}(2317)DK$
interaction is taken to be the same as [29] i.e. interaction is taken to be the same as [\[29\],](#page-12-16) i.e., $g_0 = 5.5 \pm 1.5$ GeV, and the mass parameter M_0 equals the mass of $D_{s0}^{*+}(2317)$. In $B_s \to DK$ transition, the form factors ω_s and r include B^* and R, poles, and their masses factors ω_{\pm} and r include B^* and B_c poles, and their masses
and decay constants are given by $m_{\pm} = 5.33$ GeV $m_{\pm} =$ and decay constants are given by $m_{B^*} = 5.33$ GeV, $m_{B_c} = 6.27$ GeV; $f_{\text{net}} = 0.130$ GeV, $f_{\text{net}} = 0.489$ GeV. In addi-6.27 GeV; $f_{B^*} = 0.130$ GeV, $f_{B_c} = 0.489$ GeV. In addition, the flavor independent coupling constant which can tion, the flavor independent coupling constant which can be extracted from D^{*+} decay width is fixed to be $q = -0.59 \pm 0.07$.

In Fig. [6](#page-8-1), the Borel parameter dependence of the DK continuum state to the form factors $f_{BD_{s0}^{*+}(2317)}^{+}$ and $f_{BD_{10}^{*+}(2317)}^{-}$ are given, where the coupling constants g_0 and g are fixed at their central value. To keep the DK continuum state contribution no less than 50%, the Borel parameter is constrained to be lower than 5.0 GeV. From this diagram we can see that there exists a platform for M_B at [3.5 GeV,5.0 GeV], which is the same as the $D_{s0}^{*+}(2317)$
pole case. We fix the Borel parameter $M_2 = 4.0$ GeV, and pole case. We fix the Borel parameter $M_B = 4.0 \text{ GeV}$, and plot the dependence of $f_{DK}^{+,-}$ on q^2 in Fig. [7](#page-8-2). The solid and the dashed line are corresponding to f_{DK}^+ , while f_{DK}^- is

FIG. 6. Dependence of the DK continuum state contribution to form factor $f_{D_{s0}^{*+}(2317)}^{+}$ (the dashed line) and $f_{D_{s0}^{*+}(2317)}^{-}$ (the solid line) on the Borel parameter ($q^2 = 0$).

described by the dotted and the dash-dotted line. It is clear that f_{DK}^- is almost independent of q^2 , because the q^2 dependence is almost canceled between the denominator and numerator in the form factor r . To get the upper and lower limit of these form factors, we consider the uncertainty from the coupling constant g_0 and g. Actually the dominant one is from g_0 , and the other uncertainty is negligible. The numerical results indicate that in $f_{BD_{s0}^{++}(2317)}^+$, the contribution of the DK continuum state is within 10%, while it can reach about one half in $f_{BD_{s0}^{*+}(2317)}^-$

C. Semileptonic decays

The semileptonic decays $\bar{B}_{(s)} \to D_{(s)}^* l \nu$ are important
exergences in the *B* factories which can be connected measurements in the B factories which can be connected with the form factors directly. The differential decay width is given by

FIG. 7. Dependence of the DK continuum state contribution to form factor $f_{D_{s0}^{s+}(2317)}^{+}$ and $f_{D_{s0}^{s+}(2317)}^{-}$ on q^2 , the Borel parameter is fixed to be 4.0 GeV.

$$
\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cb}|^2 (q^2 - m_l^2)^2}{768\pi^3 m_B^3 (q^2)^3} \sqrt{\lambda} [(2m_l^2(\lambda + 3q^2m_{D_0^*}^2) + q^2\lambda)]f_i^+(q^2)|^2
$$

+ $6q^2m_l^2(m_B^2 - m_{D_0^*}^2 - q^2)f_i^+(q^2)f_i^-(q^2) + 6q^4m_l^2|f_i^-(q^2)|^2],$ (35)

with $\lambda = (m_B^2 - m_{D_0^*}^2 - q^2)^2 - 4q^2 m_{D_0^*}^2$.
The a^2 dependence of $\bar{R}^0 \rightarrow D^{*+}/2^2$.

The q^2 dependence of $\bar{B}_s^0 \to D_{s0}^{*+}(\bar{2}317)l^-\bar{\nu}_l(l = e(\mu), \tau)$
trial decay rates are plotted in Figs. 8 and 9. In these two partial decay rates are plotted in Figs. [8](#page-9-0) and [9](#page-9-1). In these two figures, we include both the contribution from the D_{s0}^{*+} (2317) resonance and the DK continuum state. The DK continuum state contribution to form factor f^{+} DK continuum state contribution to form factor $f_{D_{s0}^{*+}}^{+}$ S_{A} considered in Fig. [8,](#page-9-0) while in Fig. [9](#page-9-1) only $f_{D_{s0}^{*+}(2317)}^{-1}$ contains this contribution. From these diagrams we can see that the ratio of the continuum state contribution in the decay width becomes smaller as q^2 increased. For the light lepton final state, this ratio reaches its maximum when $q^2 = 0$, and numerically it is about 20%. If the lepton $e(\mu)$ is replaced by τ , the maximum position is shifted and the ratio is also larger. This difference is due to large τ mass which manifestly changes the phase space.

The branching fractions of $\overline{B}_{(s)} \to D_{0(s)}^* l \nu$ are grouped in Table [II.](#page-10-0) Here the error for the $\bar{B}_s^0 \to D_{s0}^{*+} l^- \bar{\nu}_l (l = e(\mu), \tau)$
comes from two sources, the first one is from the resonance comes from two sources, the first one is from the resonance

dominant result of the form factors which are listed in Table [I,](#page-7-0) and the DK continuum state contribution is regarded as the second uncertainty. From this result we can see the DK continuum contribution is about one third for $e(\mu)$ final states, which implies that the continuum state contribution is less important and single resonance dominates this process. For the τ lepton final state, the continuum state contribution is relatively larger. The decay rates for the final state with τ lepton are generally 3–4 times smaller than those for the muon case due to the suppression of phase spaces. The results from the constituent quark model, the QCD sum rules and the D-meson LCSR are also listed here. Our result is slightly larger than the D-meson LCSR due to larger form factors. Note that the theoretical error is very large, which makes all the results actually consistent. The branching fractions for $\overline{B^0} \to D_0^{*0}(2400)l\overline{\nu}_l$
are also available. Here we do not consider the $D\pi$ are also available. Here we do not consider the $D\pi$ continuum state, which is left for future study. We hope the future experiments can check our results.

FIG. 8 (color online). The q² dependence of differential decay width $\frac{d}{dq} \Gamma(\bar{B}^0_s \to D_{s0}^{*+}(2317)l^-\bar{\nu}_l)$ for the final states with $l = e, \mu$ (left) and $l = e, \mu$ (left) and dependented line stands for the result and $l = \tau$ (right). The dash-dotted line stands for the result without $D\ddot{K}$ continuum state, while the region between the solid and dashed line includes the contribution of DK continuum states to f^+ line includes the contribution of DK continuum states to $f_{D_{s0}^{*+}(2317)}^{+}$.

FIG. 9 (color online). The q² dependence of differential decay width $\frac{d}{dq^2}\Gamma(\bar{B}_s^0 \to D_{s0}^{*+}(2317)/\bar{\nu}_l)$ for the final states with $l = e, \mu$ (left) and $l - \tau$ (right). The meaning of each line is the same with Fi and $l = \tau$ (right). The meaning of each line is the same with Fig. [8](#page-9-0) except that the DK continuum states contribute to $f_{D_{s0}^{*+}(2317)}^{-1}$.

TABLE II. Branching ratios for the semileptonic decays $\bar{B}_s^0 \to D_{s0}^{*+}(2317)l\bar{\nu}_l$ and $B^- \to D_0^{*0}(2400)l\bar{\nu}_l$ with the form factors estimated in *R*-meson I CSR, where the results calculated in the conventional l form factors estimated in B-meson LCSR, where the results calculated in the conventional light meson LCSR, the constituent quark model and QCDSR are also displayed for comparison.

$\bar{B}_s^0 \rightarrow D_{s0}^{*+}(2317)l^{-} \bar{\nu}_l$ This work	$l = e, \mu$ $(6.0^{+8.6+2.1}_{-3.8-1.6}) \times 10^{-3}$	$l=\tau$ $(8.2^{+8.1+4.3}_{-3.0-5.0}) \times 10^{-4}$
D-meson LCSR	$(2.3^{+1.2}_{-1.0}) \times 10^{-3}$	$(5.7^{+2.8}_{-2.3}) \times 10^{-4}$
OCDSR [7]	$\sim 10^{-3}$	$\sim 10^{-4}$
Constituent quark model [5]	$(4.90-5.71)\times 10^{-3}$	
QCDSR in HQET [6]	$(0.9 - 2.0) \times 10^{-3}$	
$\bar{B}^0 \to D_0^{*0} l^- \bar{\nu}_l$	$l = e, \mu$	$l=\tau$
This work	$(6.8^{+6.8}_{-2.0}) \times 10^{-3}$	$(6.3^{+4.9}_{-1.4}) \times 10^{-4}$

V. DISCUSSION AND CONCLUSION

In this paper, we employ the B -meson light-cone sum rules to calculate the $\bar{B}_s^0 \to D_{s0}^{*+}(2317)$ and $B^- \to D_{s0}^{*0}(2400)$ transition form factors at the large recall region. D_0^{*0} (2400) transition form factors at the large recoil region,
which can beln to probe the structure of the charmed scalar which can help to probe the structure of the charmed scalar meson. We assume $D_{s0}^{*+}(2317)$ and $D_0^{*0}(2400)$ are scalar
quark-antiquark states. In HOET we extrapolate the result quark-antiquark states. In HQET, we extrapolate the result to the whole momentum region, the q^2 dependence has been studied. Our results are compared with the studies using the other nonperturbative methods, such as the D-meson LCSR, the QCD sum rules and the quark models. Considering large uncertainties, our results are consistent with these studies. Nevertheless, we have to recognize that our study is only a rough estimate, and the results might be dramatically changed by some unknown factors. First, the quark-hadron duality assumption is less solid due to large continuum state contributions. We have introduced a phenomenological method to estimate the DK continuum state contribution to the $\bar{B}_{s}^{0} \to D_{s0}^{*+}(2317)$ form factor and
the numerical result indicates that this kind of contribution the numerical result indicates that this kind of contribution is less than 50%, while a systematical method is still unknown. Another important factor is power corrections, such as the $1/m_b$ term, higher "twist" wave functions, etc., may be large because the energy release is not extraordinarily large in the $\bar{B}_s^0 \to D_{s0}^{*+}(2317)$ transition. But power
corrections are still an open question so far, and it is very corrections are still an open question so far, and it is very difficult to perform a reliable calculation. For the form factor $f_{D_{s0}^{s+}(2317)}^{-1}$, the results from different methods have rather large discrepancy, which may indicate large contribution from higher states or the power corrections.

Subsequently, we utilize the form factors obtained using B-meson LCSR to estimate the semileptonic decays $\bar{B}_s^0 \to D_{s0}^{*+} (2317) l \bar{\nu}_l$ and $B^- \to D_0^{*0} (2400) l \bar{\nu}_l$. It has been
shown in this paper that the branching fraction of the shown in this paper that the branching fraction of the semileptonic $\bar{B}_s^0 \to D_{s0}^{*+}(2317)l\bar{\nu}_l$ decay is around 6.0×10^{-3} for light leptons and 0.8×10^{-3} for the tau 6.0×10^{-3} for light leptons and 0.8×10^{-3} for the tau final state. The difference is due to the phase space suppression. The contribution from the continuum state is about 30%–50% in $\bar{B}_s^0 \to D_s^{*+}(2317)l\bar{\nu}_l$, which means
that the LCSR result should be reliable for this channel that the LCSR result should be reliable for this channel. The predicted branching ratio of $B^- \to D_0^{*0}(2400)l\bar{\nu}_l$ is
close to $\bar{R}^0 \to D^{*+}(2317)l\bar{\nu}_l$ and may suffer from larger close to $\bar{B}_s^0 \to D_{s0}^{*+}(2317)l\bar{\nu}_l$, and may suffer from larger
uncertainties. All these results can be tested at both I HCb uncertainties. All these results can be tested at both LHCb and improved B factories.

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APPENDIX: CONTRIBUTION FROM 3-PARTICLE DISTRIBUTION AMPLITUDE OF B MESON

In the following we show the form factors from the 3 point B-meson DA:

$$
f_{\pm}^{3p} = \frac{f_B(m_c - m_q)}{2f_{D_0^*}m_{D_0^*}} \left\{ \int_0^{\eta_0 m_B} d\omega \int_{\eta_0 m_B - \omega}^{\infty} \frac{d\xi}{\xi} e^{-(s_0 - m_{D_0^*})/M_B^2} f(\eta_0) \left(m_B A_1^{\pm} + m_c A_2^{\pm} + A_3^{\pm} - A_4^{\pm} + A_5^{\pm} - \frac{A_6^{\pm} + A_7^{\pm} + m_B A_8^{\pm}}{M_B^2} \right) \right\}
$$

+
$$
\int_0^{\eta_0} \frac{d\eta}{\eta^2} \int_0^{\eta m_B} d\omega \int_{\eta m_B - \omega}^{\infty} \frac{d\xi}{\xi} \frac{1}{M_B^2} e^{-(s - m_{D_0^*})/M_B^2} \left(m_B B_1^{\pm} + m_c B_2^{\pm} + B_3^{\pm} - B_4^{\pm} + B_5^{\pm} - \frac{B_6^{\pm} + B_7^{\pm} + m_B B_8^{\pm}}{2M_B^2} \right)
$$

-
$$
\frac{f(\eta_0) e^{-(s_0 - m_{D_0^*})/M_B^2}}{2m_B^3} \int_0^{\eta_0 m_B} d\omega \int_{\eta_0 m_B - \omega}^{\infty} \frac{d\xi}{\xi} (C_4^{\pm} + C_6^{\pm} + C_7^{\pm} + C_8^{\pm}) \right\}, \tag{A1}
$$

where the functions $A_i^{\pm}(i = 1, 2, ..., 8)$, $B_i^{\pm}(i = 1, 2, ..., 8)$ and $C_i^{\pm}(i = 1, 2, ..., 8)$ entering the integration are given below. below:

$$
A_{1}^{+} = \frac{2\alpha_{0}[2+g_{1}(\eta_{0},s_{0})] - [1+g_{2}(\eta_{0},s_{0})]}{m_{B}\bar{\eta_{0}}^{2}}(\psi_{A}-\psi_{V}); \t A_{2}^{+} = \frac{6\alpha_{0}\bar{\eta_{0}}-6r_{c}}{m_{B}\bar{\eta_{0}}^{2}}\psi_{V};
$$

\n
$$
A_{3}^{+} = \frac{-2\alpha_{0}}{m_{B}\bar{\eta_{0}}^{2}}X_{A}; \t A_{4}^{+} = \frac{\alpha_{0}g_{3}(\bar{\eta}_{0})}{m_{B}\bar{\eta_{0}}^{3}}X_{A}; \t A_{5}^{+} = \frac{\bar{\eta}_{0}+2r_{c}}{m_{B}\bar{\eta_{0}}^{2}}X_{A};
$$

\n
$$
A_{6}^{+} = \frac{2g_{4}(\bar{\eta}_{0},\bar{\eta}_{0})(\bar{\eta}_{0}+r_{c})}{m_{B}\bar{\eta_{0}}^{3}}X_{A}; \t A_{7}^{+} = \frac{-24\alpha_{0}m_{c}^{2}}{m_{B}\bar{\eta_{0}}^{3}}X_{A}; \t A_{8}^{+} = \frac{(-24m_{c})g_{4}(\bar{\eta}_{0},\bar{\eta}_{0})(\bar{\eta}_{0}+r_{c})}{m_{B}\bar{\eta_{0}}^{3}}Y_{A};
$$

\n
$$
A_{1}^{-} = \frac{2\alpha_{0}[1+g_{1}(g_{0},s_{0})] - [2+g_{2}(g_{0},s_{0})]}{m_{B}\bar{\eta_{0}}^{2}}W_{A}; \t A_{4}^{-} = \frac{\alpha_{0}g_{5}(\eta_{0})}{m_{B}\bar{\eta_{0}}^{3}}X_{A}; \t A_{5}^{-} = \frac{1+g_{0}-2r_{c}}{m_{B}\bar{\eta_{0}}^{3}}X_{A};
$$

\n
$$
A_{6}^{-} = \frac{2g_{4}(\eta_{0},\bar{\eta}_{0})(-\eta_{0}+r_{c})}{m_{B}\bar{\eta_{0}}^{3}}X_{A}; \t A_{7}^{-} = \frac{-24\alpha_{0}m_{c}^{2}}{m_{B}\bar{\eta_{0}}^{3}}X_{A}; \t A_{8}^{-} = \frac{(-24m_{c})g_{4}(\eta_{0},\bar{\eta}_{0})(-\eta_{0}
$$

where the notations $\eta = \omega + \xi \alpha$, $f(\eta) = (1 + \frac{m_c^2 - q^2}{\bar{\eta}^2 m_B^2})$ $g_1(x, y) = -3x + (y - q^2)/m_B^2$, $g_2(x, y) = 3(r_c - x)2(y - q^2)/m_B^2$, $g_3(x) = -m_B^2 x + m_c^2 + q^2$, $g_4(x, y) = m_B^2(x - 2r_c) + (m_c^2 - q^2)/y$, $g_5(x) = m_B^2 x \bar{x} + (2 - x)m_c^2/\bar{x} + xq^2/\bar{x}$, $\bar{X}_A(\omega, \xi) =$ $\int_0^{\omega} d\tau X_A(\tau,\xi), \bar{Y}_A(\eta,\xi) = \int_0^{\omega} d\tau Y_A(\tau,\xi)$, and η_0 satisfies the equation $\bar{\eta} s_0 - (\eta \bar{\eta} + r_c^2) m_B^2 + \eta m_B^2 = 0$.

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