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Finite mass corrections for $B \to (\bar{D}^{(*)}, \bar{D}^{**})\ell\nu$ decays in the Bakamjian-Thomas relativistic quark model

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The Bakamjian-Thomas relativistic quark model for hadron current matrix elements, while noncovariant at finite mass, is successful in the heavy quark limit: form factors are covariant and satisfy Isgur-Wise scaling and Bjorken-Uraltsev sum rules. Motivated by the so-called "1/2 vs 3/2 puzzle" in $\bar{B}$ decays to positive parity D^{**}, we examine the implications of the model at finite mass. In the elastic case $\frac{1}{2}^- \to \frac{1}{2}^-$, the heavy quark effective theory (HQET) constraints for the $O(1/m_Q)$ corrections are analytically fulfilled. A number of satisfying regularities is also found for inelastic transitions. We compute the form factors using the wave functions given by the Godfrey-Isgur potential. We find a strong enhancement in the case $\frac{1}{2}^- \to \frac{1}{2}^+$ for $0^- \to 0^+$. This enhancement is linked to a serious difficulty of the model at finite mass for the inelastic transitions, namely a violation of the HQET constraints at zero recoil formulated by Leibovich *et al.* These are nevertheless satisfied in the nonrelativistic limit for the light quark. We conclude that these HQET rigorous constraints are crucial in the construction of a sensible relativistic quark model of inelastic form factors.

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I. INTRODUCTION

The Bakamjian-Thomas (BT) relativistic quark models [1–4] are a class of models with a fixed number of constituents in which the states are covariant under the Poincaré group. The model relies on an appropriate Lorentz boost of the eigenfunctions of a Hamiltonian describing the hadron spectrum at rest.

We have proposed a formulation of this scheme for the meson ground states [5] and demonstrated the important feature that, in the heavy quark limit, the current matrix elements, when the current is coupled to the heavy quark, are *covariant*. We have extended this scheme to *P*-wave excited states [6].

Moreover, these matrix elements in the heavy quark limit exhibit Isgur-Wise (IW) scaling [7]. As demonstrated in [5,6], given a Hamiltonian describing the spectrum, the model provides an unambiguous result for the Isgur-Wise functions, the elastic $\xi(w)$ [7] and the inelastic to P-wave states $\tau_{1/2}(w)$, $\tau_{3/2}(w)$ [8].

On the other hand, the sum rules (SR) in the heavy quark limit of QCD, like Bjorken [8,9] and Uraltsev SR [10] are analytically satisfied in the model [11–13], as well as SR involving higher derivatives of $\xi(w)$ at zero recoil [14–16].

In [17], we have chosen the Godfrey-Isgur (GI) Hamitonian [18], that gives a very complete description of the light $q\bar{q}$ and heavy $Q\bar{q}$ meson spectra in order to predict within the BT scheme the corresponding IW functions for the ground state and the excited states.

Similar work has been been performed for $Q\bar{q}$ meson decay constants [19] and to demonstrate within the BT scheme new heavy quark effective theory (HQET) SR involving Isgur-Wise functions and decay constants [20].

A detailed and very useful account of the BT scheme for the calculation of Isgur-Wise functions and heavy meson decay constants and their numerical calculation within the Godfrey-Isgur Hamiltonian has been given in the Ph.D. thesis of Morénas [21].

As a further test, we have computed in [22], the vector, scalar, and axial charge densities for the ground states 0^- and 1^- ($\frac{1}{2}^-$ doublet) and for the excited states 0^+ and 1^+ ($\frac{1}{2}^+$ doublet). In this case the active quark is the light quark, and one can show that, unlike the case of the active heavy quark, the current matrix elements are not covariant. For the calculation, we have adopted the natural reference frame for this problem, the heavy meson rest frame. As shown in [22], the agreement with lattice data in the unquenched approximation is really striking, and provides both a test of the BT scheme and of the GI Hamiltonian that describes the spectrum.

A main motivation to undertake this work has been the so-called "$\frac{1}{2}$ versus $\frac{3}{2}$ puzzle" that, based on rather old data, states the fact that the semileptonic decay rates $\frac{1}{2}^- \to \frac{1}{2}^+$ are

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much larger than the expectations of the heavy quark limit, while the semileptonic decay rates $\frac{1}{2}^- \to \frac{3}{2}^+$ are roughly consistent with this limit. A precise discussion of this puzzle has been given in Ref. [23]. Updated data by *BABAR* [24] and Belle [25] confirm the problem, although there are significant differences between both experiments.

The $\frac{1}{2}$ vs $\frac{3}{2}$ puzzle is nicely exemplified by the Uraltsev sum rule [10]:

$$\sum_n (|\tau_{3/2}^{(n)}(1)|^2 - |\tau_{1/2}^{(n)}(1)|^2) = \frac{1}{4}. \tag{1}$$

If one neglects completely higher excitations and the ground state ($n = 0$) dominates the sum of the *differences* of the left-hand side of (1), one expects $|\tau_{3/2}^{(0)}(1)|^2 > |\tau_{1/2}^{(0)}(1)|^2$. In addition, the phase space factors make much larger the BR for $\frac{1}{2}^- \to \frac{3}{2}^+$ relatively to the $\frac{1}{2}^- \to \frac{1}{2}^+$ one. The BT model satisfies analytically [13] the SR (1) with, for $n = 0$ [17]

$$\tau_{1/2}(1) = 0.22, \qquad \tau_{3/2}(1) = 0.54. \tag{2}$$

On the other hand, calculations in the lattice in the unquenched approximation [26] point to a similar conclusion

$$\tau_{1/2}(1) = 0.29 \pm 0.03, \qquad \tau_{1/2}(1) = 0.52 \pm 0.03. \tag{3}$$

Let us finally underline that the $\frac{1}{2}$ vs $\frac{3}{2}$ puzzle does not seem to be present, assuming factorization, in the nonleptonic decays $\bar{B} \to D^{**}\pi$, as shown by the Belle results [27], phenomenologically analyzed in Ref. [28]. This feature makes the puzzle even more obscure. Recently, in Ref. [29] a necessary, precise and updated discussion of the situation for both the semileptonic and nonleptonic data was given.

The paper is organized as follows. In Sec. II we give the definitions of the form factors for the transitions on which we are interested, reproducing some needed results at leading and $O(1/m_Q)$ order within HQET. In Sec. III we give the master formulas defining the theoretical framework of BT quark models. Since the current matrix elements in the BT model are only covariant in the heavy quark limit if the current is coupled to the heavy quark, the calculation of the $1/m_Q$ corrections must be done in a particular reference frame. We discuss this problem in Sec. IV and give arguments to adopt the equal velocity frame (EVF), where the moduli of the initial and final three-vector meson velocities are equal. In Sec. V we check that this frame allows one to obtain very reasonable results for the $1/m_Q$ corrections for the elastic transitions $\frac{1}{2}^- \to \frac{1}{2}^-$. In Sec. VI we give the analytical results of the BT model for the $O(1/m_Q)$ of form factors to excited states $\bar{B} \to D^{**}\ell\nu$ at zero recoil, and compare to the results of HQET. Section VII is devoted to the description of the Godfrey-Isgur quark model for spectroscopy. In Sec. VIII we give the results of the BT model for the $\frac{1}{2}^- \to \frac{1}{2}^-$ in the heavy quark limit, at finite mass and at the order $1/m_Q$. Section IX is devoted to the calculation of the different form factors for the inelastic transitions $\frac{1}{2}^- \to \frac{1}{2}^+$ and $\frac{1}{2}^- \to \frac{3}{2}^+$ at infinite and finite mass. In Sec. X we give the numerical results for the branching ratios $\bar{B} \to D^{(*)}\ell\nu, D^{(*)}\pi$ and $\bar{B} \to D^{**}\ell\nu, D^{**}\pi$ in the heavy mass limit and also at finite mass, and in Sec. XI we expose a discussion of the obtained results and problems. We leave a number of technicalities to the appendixes. In Appendix A we write the needed formulas of the different form factors in terms of matrix elements. In Appendixes B and C we give the wave functions in the GI model, respectively in the heavy quark limit and at finite mass. In Appendix D we write some formulas defining a family of collinear frames and in Appendix E we give the formulas for the decay rates in the different cases.

II. MATRIX ELEMENTS FOR $B \to D^{(*)}\ell\nu$ AND $B \to \bar{D}^{**}\ell\nu$

For the ground state mesons $D(0^-)$ and $D^*(1^-)$ we adopt the notation of [30]:

$$\frac{\langle D(v')|V^\mu|B(v)\rangle}{\sqrt{m_B m_D}} = h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu, \tag{4}$$

$$\frac{\langle D^*(v',\epsilon')|V^\mu|B(v)\rangle}{\sqrt{m_B m_{D^*}}} = i h_V(w)\epsilon^\mu{}_{\nu\alpha\beta}\epsilon'^{*\nu}v'^\alpha v^\beta, \tag{5}$$

$$\begin{aligned}\frac{\langle D^*(v',\epsilon')|A^\mu|B(v)\rangle}{\sqrt{m_B m_{D^*}}} &= h_{A_1}(w)(w+1)\epsilon'^{*\mu} - h_{A_2}(w)(\epsilon'^*.v)v^\mu \\ &\quad - h_{A_3}(w)(\epsilon'^*.v)v'^\mu,\end{aligned} \tag{6}$$

while for the excited P-wave mesons, $D_{1/2}(0^+)$, $D_{1/2}(1^+)$, $D_{3/2}(1^+)$ and $D_{3/2}(2^+)$, we adopt the notation of [31] for the form factors:

$$\frac{\langle D_{3/2}(1^+)(v',\epsilon')|A^\mu|B(v)\rangle}{\sqrt{m_B m_{D^{**}}}} = i f_A(w)\epsilon^\mu{}_{\alpha\beta\gamma}\epsilon'^{*\alpha}v'^\beta v'^\gamma, \tag{7}$$

$$\begin{aligned}\frac{\langle D_{3/2}(1^+)(v',\epsilon')|V^\mu|B(v)\rangle}{\sqrt{m_B m_{D^{**}}}} &= f_{V_1}(w)\epsilon'^{*\mu} + (\epsilon'^*.v)[f_{V_2}(w)v^\mu + f_{V_3}(w)v'^\mu],\end{aligned} \tag{8}$$

$$\frac{\langle D_{3/2}(2^+)(v',\epsilon')|V^\mu|B(v)\rangle}{\sqrt{m_B m_{D^{**}}}} = i k_V(w)\epsilon'^\mu{}_{\alpha\beta\gamma}\epsilon'^{*\alpha}_\sigma v^\sigma v^\beta v'^\gamma, \tag{9}$$

$$\frac{\langle D_{3/2}(2^+)(v',\epsilon')|A^\mu|B(v)\rangle}{\sqrt{m_B m_{D^{**}}}} = k_{A_1}(w)\epsilon'^{*\mu}_{\alpha} v^\alpha + \epsilon'^{*}_{\alpha\beta} v^\alpha v^\beta [k_{A_2}(w)v^\mu + k_{A_3}(w)v'^\mu], \tag{10}$$

$$\frac{\langle D_{1/2}(0^+)(v')|A^\mu|B(v)\rangle}{\sqrt{m_B m_{D^{**}}}} = g_+(w)(v+v')^\mu + g_-(w)(v-v')^\mu, \tag{11}$$

$$\frac{\langle D_{1/2}(1^+)(v',\epsilon')|A^\mu|B(v)\rangle}{\sqrt{m_B m_{D^{**}}}} = i g_A(w)\epsilon^\mu{}_{\alpha\beta\gamma}\epsilon^{*\alpha} v^\beta v'^\gamma, \tag{12}$$

$$\frac{\langle D_{1/2}(1^+)(v',\epsilon')|V^\mu|B(v)\rangle}{\sqrt{m_B m_{D^{**}}}} = g_{V_1}(w)\epsilon'^{*\mu} + (\epsilon'^{*}.v)[g_{V_2}(w)v^\mu + g_{V_3}(w)v'^\mu]. \tag{13}$$

In the equations for the excited states D^{**} denotes generically any excited state, but in each equation the physical mass of the corresponding excited meson is understood.

A. Heavy quark expansion of form factors in HQET

1. Elastic form factors $B \to D^{(*)}\ell\nu$ in HQET

To compare with the results of the BT model at finite mass, let us give here the expressions of the form factors in powers of $\frac{1}{m_Q}$ in HQET. Let us set the notation $\epsilon_Q = \frac{1}{2m_Q}$. To first order in the heavy quark expansion one has, for the elastic form factors $B \to D^{(*)}$ [30],

$$h_+(w) = \xi(w) + (\epsilon_c + \epsilon_b)L_1(w) + O^{h_+}_{1/m_Q^2}(w), \tag{14}$$

$$h_-(w) = (\epsilon_c - \epsilon_b)L_4(w) + O^{h_-}_{1/m_Q^2}(w), \tag{15}$$

$$h_V(w) = \xi(w) + \epsilon_c[L_2(w) - L_5(w)] + \epsilon_b[L_1(w) - L_4(w)] + O^{V}_{1/m_Q^2}(w), \tag{16}$$

$$h_{A_1}(w) = \xi(w) + \epsilon_c\left[L_2(w) - \frac{w-1}{w+1}L_5(w)\right] + \epsilon_b\left[L_1(w) - \frac{w-1}{w+1}L_4(w)\right] + O^{A_1}_{1/m_Q^2}(w), \tag{17}$$

$$h_{A_2}(w) = \epsilon_c[L_3(w) + L_6(w)] + O^{A_2}_{1/m_Q^2}(w), \tag{18}$$

$$h_{A_3}(w) = \xi(w) + \epsilon_c[L_2(w) - L_3(w) - L_5(w) + L_6(w)] + \epsilon_b[L_1(w) - L_4(w)] + O^{A_3}_{1/m_Q^2}(w). \tag{19}$$

Luke's theorem [32] states that, at first order in $\frac{1}{m_Q}$, one has

$$L_1(1) = L_2(1) = 0 \tag{20}$$

and therefore follows the important result that at zero recoil ($w = 1$) the subleading corrections to $h_+(1)$ and $h_{A_1}(1)$ begin at order $1/m_Q^2$:

$$h_+(1) = 1 + \delta^{h_+}_{1/m_Q^2}, \qquad h_{A_1}(1) = 1 + \delta^{h_{A_1}}_{1/m_Q^2}. \tag{21}$$

The functions $L_i(w)(i = 4, 5, 6)$, corresponding to the so-called Current perturbations, are not independent according to HQET, and are given in terms of two functions $\bar{\Lambda}\xi(w)$ and $\xi_3(w)$ [30]:

$$L_4(w) = -\bar{\Lambda}\xi(w) + 2\xi_3(w), \tag{22}$$

$$L_5(w) = -\bar{\Lambda}\xi(w), \tag{23}$$

$$L_6(w) = -\frac{2}{w+1}(\bar{\Lambda}\xi(w) + \xi_3(w)), \tag{24}$$

where $\xi(w)$ is the elastic IW function.

One finds therefore the relation

$$L_4(w) + (1+w)L_6(w) = 3L_5(w) \tag{25}$$

that reduces to the relation at zero recoil

$$L_4(1) + 2L_6(1) = 3L_5(1). \tag{26}$$

2. Inelastic form factors $B \to D^{**}(\mathbf{0^+_{1/2}}, \mathbf{1^+_{1/2}}, \mathbf{1^+_{3/2}}, \mathbf{2^+_{3/2}})\ell\nu$ in HQET

For the inelastic form factors $B \to D^{**}$ we reproduce only the leading order in the heavy quark expansion [8,31]:

$$f_A(w) = -\frac{w+1}{\sqrt{2}}\tau_{3/2}(w) + O^{f_A}_{1/m_Q}(w), \tag{27}$$

$$f_{V_1}(w) = \frac{1-w^2}{\sqrt{2}}\tau_{3/2}(w) + O^{f_{V_1}}_{1/m_Q}(w), \tag{28}$$

$$f_{V_2}(w) = -\frac{3}{\sqrt{2}}\tau_{3/2}(w) + O^{f_{V_2}}_{1/m_Q}(w), \tag{29}$$

$$f_{V_3}(w) = \frac{w-2}{\sqrt{2}}\tau_{3/2}(w) + O^{f_{V_3}}_{1/m_Q}(w), \tag{30}$$

$$k_V(w) = -\sqrt{3}\tau_{3/2}(w) + O^{k_V}_{1/m_Q}(w), \tag{31}$$

$$k_{A_1}(w) = -(w+1)\sqrt{3}\tau_{3/2}(w) + O^{k_{A_1}}_{1/m_Q}(w), \tag{32}$$

$$k_{A_2}(w) = O^{k_{A_2}}_{1/m_Q}(w), \tag{33}$$

$$k_{A_3}(w) = \sqrt{3}\tau_{3/2}(w) + O^{k_{A_3}}_{1/m_Q}(w), \tag{34}$$

$$g_+(w) = O^{g_+}_{1/m_Q}(w), \tag{35}$$

$$g_-(w) = 2\tau_{1/2}(w) + O^{g_-}_{1/m_Q}(w), \tag{36}$$

$$g_A(w) = 2\tau_{1/2}(w) + O^{g_A}_{1/m_Q}(w), \tag{37}$$

$$g_{V_1}(w) = (w-1)2\tau_{1/2}(w) + O^{g_{V_1}}_{1/m_Q}(w), \tag{38}$$

$$g_{V_2}(w) = O^{g_{V_2}}_{1/m_Q}(w), \tag{39}$$

$$g_{V_3}(w) = -2\tau_{1/2}(w) + O^{g_{V_3}}_{1/m_Q}(w), \tag{40}$$

where the different $O_{1/m_Q}(w)$ corrections are given in the detailed and careful paper by Leibovich *et al.* [31]. Among these corrections, we reproduce the ones that do not vanish at zero recoil, and that are very relevant for what follows:

$$g_+(1) = -3(\epsilon_c + \epsilon_b)\Delta E_{1/2}\tau_{1/2}(1), \tag{41}$$

$$g_{V_1}(1) = 2(\epsilon_c - 3\epsilon_b)\Delta E_{1/2}\tau_{1/2}(1), \tag{42}$$

$$f_{V_1}(1) = -4\sqrt{2}\epsilon_c \Delta E_{3/2}\tau_{3/2}(1), \tag{43}$$

where

$$\Delta E_j = m_{D(j^+)} - m_{D(\frac{1}{2}^-)}\left(j = \frac{1}{2}, \frac{3}{2}\right). \tag{44}$$

III. BAKAMJIAN-THOMAS APPROACH TO QUARK MODELS

As explained in [5], the construction of the BT wave function in motion involves a *unitary* transformation that relates the wave function $\Psi^{(P)}_{s_1,\ldots,s_n}(\vec{p}_1,\ldots,\vec{p}_n)$ in terms of one-particle variables, the spin $\vec{S}_i$ and momenta $\vec{p}_i$ to the so-called *internal* wave function $\Psi^{\rm int}_{s_1,\ldots,s_n}(\vec{P},\vec{k}_2,\ldots,\vec{k}_n)$ given in terms of another set of variables, the total momentum $\vec{P}$ and the internal momenta $\vec{k}_1,\vec{k}_2,\ldots,\vec{k}_n$ ($\sum_i \vec{k}_i = 0$). This property ensures that, starting from an orthonormal set of internal wave functions, one gets an orthonormal set of wave functions in any frame. The base $\Psi^{(P)}_{s_1,\ldots,s_n}(\vec{p}_1,\ldots,\vec{p}_n)$ is useful to compute one-particle matrix elements like current one-quark matrix elements, while the second $\Psi^{\rm int}_{s_1,\ldots,s_n}(\vec{P},\vec{k}_2,\ldots,\vec{k}_n)$ allows one to exhibit Poincaré covariance. In order to satisfy the Poincaré commutators, the unique requirement is that the mass operator M, i.e. the Hamiltonian describing the spectrum at rest, should depend only on the internal variables and be rotational invariant, i.e. M must commute with $\vec{P}$, $\frac{\partial}{\partial \vec{P}}$ and $\vec{S}$. The internal wave function at rest $(2\pi)^3\delta(\vec{P})\varphi_{s_1,\ldots,s_n}(\vec{k}_2,\ldots,\vec{k}_n)$ is an eigenstate of M, $\vec{P}$ (with $\vec{P}=0$), $\vec{S}^2$ and $\vec{S}_z$, while the wave function in motion of momentum $\vec{P}$ is obtained by applying the boost $\mathbf{B}_P$, where $P^0 = \sqrt{\vec{P}^2 + M^2}$ involves the dynamical operator M.

The final output of the formalism that gives the total wave function in motion $\Psi^{(P)}_{s_1,\ldots,s_n}(\vec{p}_1,\ldots,\vec{p}_n)$ in terms of the internal wave function at rest $\varphi_{s_1,\ldots,s_n}(\vec{k}_2,\ldots,\vec{k}_n)$ is the formula

$$\begin{aligned}&\Psi^{(P)}_{s_1,\ldots,s_n}(\vec{p}_1,\ldots,\vec{p}_n)\\ &= (2\pi)^3\delta\left(\sum_i \vec{p}_i - \vec{P}\right)\sqrt{\frac{\sum_i p_i^0}{M_0}}\left(\prod_i \frac{\sqrt{k_i^0}}{\sqrt{p_i^0}}\right)\\ &\quad\times \sum_{s'_1,\ldots,s'_n}[D_i(\mathbf{R}_i)]_{s_i,s'_i}\varphi_{s'_1,\ldots,s'_n}(\vec{k}_2,\ldots,\vec{k}_n)\end{aligned} \tag{45}$$

where $p_i^0 = \sqrt{\vec{p}_i^2 + m_i^2}$ and M_0 is the free mass operator, given by

$$M_0 = \sqrt{\left(\sum_i p_i\right)^2}. \tag{46}$$

The internal momenta of the hadron at rest are given in terms of the momenta of the hadron in motion by the *free boost*

$$k_i = \mathbf{B}^{-1}_{\sum_i p_i} p_i \tag{47}$$

where the operator $\mathbf{B}_p$ is the boost $(\sqrt{p^2},\vec{0}) \to p$; the Wigner rotations $\mathbf{R}_i$ in the preceding expression are

$$\mathbf{R}_i = \mathbf{B}^{-1}_{p_i}\mathbf{B}^{-1}_{\sum_i p_i}\mathbf{B}_{k_i} \tag{48}$$

and the states are normalized by

$$\langle \vec{P}', S'_z|\vec{P}, S_z\rangle = (2\pi)^3\delta(\vec{P}' - \vec{P})\delta_{S_z,S'_z}. \tag{49}$$

The one-quark current matrix element acting on quark 1 between two hadrons is then given by the expression

$$\begin{aligned}&\langle \vec{P}', S'_z|J^{(1)}|\vec{P}, S_z\rangle\\ &= \int \frac{d\vec{p'}_1}{(2\pi)^3}\frac{d\vec{p}_1}{(2\pi)^3}\left(\prod_{i=2}^n \frac{d\vec{p}_i}{(2\pi)^3}\right)\Psi^{P'}_{s'_1,\ldots,s'_n}(\vec{p'}_1,\ldots,\vec{p'}_n)^*\\ &\quad\times \langle \vec{p'}_1, s'_1|J^{(1)}|\vec{p}_1, s_1\rangle\Psi^{P}_{s_1,\ldots,s_n}(\vec{p}_1,\ldots,\vec{p}_n)\end{aligned} \tag{50}$$

where $\Psi^P_{s_1,\ldots,s_n}(\vec{p}_1,\ldots,\vec{p}_n)$ is given in terms of the internal wave function by (45) and $\langle p'_1, s'_1|J^{(1)}|\vec{p}_1, s_1\rangle$ is the one-quark current matrix element.

As demonstrated in [5,6], in this formalism, in the heavy quark limit, current matrix elements are covariant and exhibit Isgur-Wise scaling, and one can compute Isgur-Wise functions like $\xi(w)$, $\tau_{1/2}(w)$, $\tau_{3/2}(w)$ [17].

After we have presented the general calculations, we will specify the mass operator M, which will be chosen as the one of the Godfrey and Isgur model in the following section.

We are interested in this paper in transitions between heavy quarks $b \to c$ where the initial meson is a pseudoscalar $\bar{B}$. We particularize the general formula (50) to the meson case $q_1\bar{q}_2$ where $q_1 \to q'_1$ labels the heavy quarks, $\bar{q}_2$ the light antiquark, and the current operator $J^{(1)}$ acts on the heavy quark.

As shown in [5], one can express (50) in a Pauli matrix formalism and then in a Dirac matrix formalism. We reproduce here the needed master formula in the Dirac formalism:

$$\langle \vec{P}', \epsilon'|J^{(1)}|\vec{P}, \epsilon\rangle = \int \frac{d\vec{p}_2}{(2\pi)^3}\frac{1}{p_2^0}F(\vec{p}_2,\vec{P}',\vec{P})\frac{1}{16}\mathrm{Tr}[O(m_1+\not{p}_1)(1+\not{u})(m_2+\not{p}_2)\Gamma_{u'}(1+\not{u}')(m_1+\not{p}'_1)]\varphi'(\vec{k}'_2)^*\varphi(\vec{k}_2) \tag{51}$$

where

$$F(\vec{p}_2,\vec{P}',\vec{P}) = \frac{\sqrt{u^0u'^0}}{p_1^0p'^0_1}\sqrt{\frac{k_1^0k_2^0}{(k_1^0+m_1)(k_2^0+m_2)}} \times \sqrt{\frac{k'^0_1k'^0_2}{(k'^0_1+m_1)(k'^0_2+m_2)}}. \tag{52}$$

In formula (51) the following unit four-vectors are used:

$$u = \frac{p_1+p_2}{M_0}, \qquad u' = \frac{p'_1+p_2}{M'_0} \tag{53}$$

with $M_0 = \sqrt{(p_1+p_2)^2}, M'_0 = \sqrt{(p'_1+p_2)^2}$, as explained above.

In (51) the Dirac matrix O depends on the current, for example $O = \gamma^\mu$ or $O = \gamma^\mu\gamma_5$ for the vector or axial current. On the other hand, the Dirac matrix $\Gamma_{u'}$ depends on the quantum numbers of the final state, namely $0^-, 1^-$ for the ground state and 0^+, two 1^+ states and 2^+ for the excited states. Let us give now these matrices for the different D states [5,12,21]:

$$\begin{aligned}
&D(0^-) \qquad \Gamma_{u'} = 1,\\
&D^*(1^-) \qquad \Gamma_{u'} = \gamma_5\not{\epsilon}'^*_{u'},\\
&D^{**}(0^+_{1/2}) \qquad \Gamma_{u'} = -\frac{[\not{p}_2 - (p_2.u')\not{u}']\gamma_5}{\sqrt{(p_2.u')^2-m_2^2}},\\
&D^{**}(1^+_{1/2}) \qquad \Gamma_{u'} = -\frac{[\epsilon^*_{u'}.p_2 + i\epsilon_{\alpha\beta\rho\sigma}u'^\alpha\epsilon^{*\beta}_{u'}p_2^\rho\gamma^\sigma\gamma_5]}{\sqrt{(p_2.u')^2-m_2^2}},\\
&D^{**}(1^+_{3/2}) \qquad \Gamma_{u'} = -\frac{1}{\sqrt{2}}\frac{[2\epsilon^*_{u'}.p_2 - i\epsilon_{\alpha\beta\rho\sigma}u'^\alpha\epsilon^{*\beta}_{u'}p_2^\rho\gamma^\sigma\gamma_5]}{\sqrt{(p_2.u')^2-m_2^2}},\\
&D^{**}(2^+_{3/2}) \qquad \Gamma_{u'} = -\sqrt{3}\frac{\gamma_\mu p_{2\nu}\epsilon^{*\mu\nu}_{u'}\gamma_5}{\sqrt{(p_2.u')^2-m_2^2}},
\end{aligned} \tag{54}$$

where the convention $\epsilon_{0123} = -1$ is adopted, $\epsilon_{u'}$ are the polarizations relative to the four-vector u', four-vectors for the $J^P = 1^P$ $(P = -,+)$ states, and a tensor for the $J^P = 2^+$ state.

A. Matrix elements in the heavy quark limit

We now consider the heavy mass limit, defined as $m_1, m'_1 \to \infty$ with $v' = P'/M'$ and $v = P/M$ fixed, and $M/m_1 \to 1, M'/m'_1 \to 1$. One has, in this limit

$$\begin{aligned}
&\frac{p_1}{m_1} \to v, \qquad \frac{p'_1}{m'_1} \to v', \qquad \frac{k_1^0}{m_1} \to v, \qquad \frac{k'^0_1}{m'_1} \to 1,\\
&u \to v, \qquad u' \to v', \qquad \epsilon'_{u'} \to \epsilon'_{v'} = \epsilon',\\
&k_2 \to \mathbf{B}_v^{-1}p_2, \qquad k'_2 \to \mathbf{B}_{v'}^{-1}p_2.
\end{aligned} \tag{55}$$

On the other hand, one has, due to the invariance of the scalar product,

$$(\mathbf{B}_v^{-1}p_2)^0 = p_2.v, \qquad (\mathbf{B}_{v'}^{-1}p_2)^0 = p_2.v' \tag{56}$$

and therefore the matrix element (51), (52) is given by the following covariant expression:

$$\begin{aligned}
&\langle \vec{P}', \epsilon'|J^{(1)}|\vec{P}, \epsilon\rangle\\
&= \frac{1}{\sqrt{4v^0v'^0}}\int\frac{d\vec{p}_2}{(2\pi)^3}\frac{1}{p_2^0}\sqrt{\frac{(p_2.v')(p_2.v)}{(p_2.v'+m_2)(p_2.v+m_2)}}\\
&\times\frac{1}{4}\mathrm{Tr}[O(1+\not{v})(m_2+\not{p}_2)\Gamma_{v'}(1+\not{v}')]\varphi'\\
&\times(\overrightarrow{\mathbf{B}_{v'}^{-1}p_2})^*\varphi(\overrightarrow{\mathbf{B}_v^{-1}p_2})
\end{aligned} \tag{57}$$

where the Dirac matrices $\Gamma_{v'}$ are identical to $\Gamma_{u'}$ in (54) with u' replaced by the four-velocity v'.

The radial wave functions $\varphi'(\vec{k})$ and $\varphi(\vec{k})$ depend only on $\vec{k}^2$, and from (56) one has

$$(\overrightarrow{\mathbf{B}_{v'}^{-1}p_2})^2 = (p_2.v')^2 - m_2^2, \qquad (\overrightarrow{\mathbf{B}_{v}^{-1}p_2})^2 = (p_2.v)^2 - m_2^2. \tag{58}$$

B. The Isgur-Wise functions $\xi(w)$, $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$

From the matrix elements (57), the operators (54) and the definitions and $1/m_Q$ expansion of the form factors given in Sec. II, the Isgur-Wise functions $\xi(w), \tau_{1/2}(w), \tau_{3/2}(w)$ are given by the expressions

$$\begin{aligned}\xi(w) = &\frac{1}{w+1}\int \frac{d\vec{p}_2}{(2\pi)^3}\frac{1}{p_2^0}\sqrt{\frac{(p_2.v')(p_2.v)}{(p_2.v'+m_2)(p_2.v+m_2)}}\\ &\times[p_2.(v'+v)+m_2(w+1)]\\ &\times\varphi\left(\sqrt{(p_2.v')^2-m^2}\right)^*\varphi\left(\sqrt{(p_2.v)^2-m_2^2}\right),\end{aligned} \tag{59}$$

$$\begin{aligned}\tau_{1/2}(w) = &\frac{1}{2(1-w)}\int \frac{d\vec{p}_2}{(2\pi)^3}\frac{1}{p_2^0}\sqrt{\frac{(p_2.v')(p_2.v)}{(p_2.v'+m_2)(p_2.v+m_2)}}\\ &\times[(p_2.v)(p_2.v'+m_2)\\ &-(p_2.v')(p_2.v'+wm_2)+(1-w)m_2^2]\\ &\times\frac{\varphi_{\frac{1}{2}^+}(\sqrt{(p_2.v')^2-m_2^2})^*\varphi(\sqrt{(p_2.v)^2-m_2^2})}{\sqrt{(p_2.v')^2-m_2^2}},\end{aligned} \tag{60}$$

$$\begin{aligned}\tau_{3/2}(w) = &\frac{1}{2\sqrt{3}(1-w)(1+w)^2}\int \frac{d\vec{p}_2}{(2\pi)^3}\frac{1}{p_2^0}\\ &\times\sqrt{\frac{(p_2.v')(p_2.v)}{(p_2.v'+m_2)(p_2.v+m_2)}}\{3[p_2.(v+v')]^2\\ &-2(w+1)(p_2.v)(2p_2.v'-m_2)\\ &-2(w+1)(p_2.v')(p_2.v'+wm_2)+(w^2-1)m_2^2\}\\ &\times\frac{\varphi_{\frac{3}{2}^+}(\sqrt{(p_2.v')^2-m_2^2})^*\varphi(\sqrt{(p_2.v)^2-m_2^2})}{\sqrt{(p_2.v')^2-m_2^2}},\end{aligned} \tag{61}$$

where all the radial wave functions for the $\frac{1}{2}^-, \frac{1}{2}^+, \frac{3}{2}^+$ states in the heavy quark limit are normalized by

$$\int \frac{d\vec{p}_2}{(2\pi)^3}|\varphi(\vec{p}_2)|^2 = 1. \tag{62}$$

IV. LIMITATIONS OF THE BT MODEL AT FINITE MASS: CHOICE OF A CONVENIENT REFERENCE FRAME

As we have emphasized above, the BT model provides a Poincaré covariant description of the states in motion, and also a Lorentz invariant formulation of the current matrix elements *in the heavy quark limit*. In the present paper we are interested in studying the $1/m_Q$ corrections to the matrix elements. However, although the current matrix elements can be formulated in the BT model by (51), (52), this expression is not Lorentz covariant.

Another important point, also a limitation of the BT model, is that at finite mass, although one has lost Lorentz covariance, one does not even have Galilean covariance. In order to have Galilean covariance one would need to take the full nonrelativistic limit, i.e. to consider the nonrelativistic quark model: the model must be nonrelativistic, not only for the heavy quarks b and c, but also for the light quark.

However, the nonrelativistic quark model is not suited for our purpose, because what we want is to understand the *departures* relatively to the heavy quark limit predictions of the BT model due to the finiteness of the masses m_b and m_c.

Then, we are left to consider the BT model at finite mass in a definite reference frame. How does one choose this frame? Fortunately, there is a theoretical criterium for choosing a convenient frame. Namely, we will adopt the frame that is consistent with known theoretical results in the $1/m_Q$ expansion of HQET.

In Appendix D we have formulated a set of collinear frames, that go from the B meson rest frame to the D meson rest frame, dependent on a single parameter α. The B and D rest frames correspond respectively to $\alpha = 0$ and $\alpha = 1$. There is an intermediate frame, that we call the equal velocity frame (EVF), in which the spatial velocities are equal in modulus ($v^0 = v'^0$, $v^z = -v'^z$), that corresponds to the value $\alpha = \frac{1}{2}$. In this latter frame, the initial and final velocities then read, in terms of the variable $w = v.v'$,

$$\begin{aligned}v &= \left(\sqrt{\frac{2w}{w+1}}, 0, 0, -\sqrt{\frac{w-1}{w+1}}\right),\\ v' &= \left(\sqrt{\frac{2w}{w+1}}, 0, 0, \sqrt{\frac{w-1}{w+1}}\right).\end{aligned} \tag{63}$$

Considering the matrix element at arbitrary masses (51) for the ground state $\bar{B} \to D^{(*)}\ell\nu$ transitions, and making analytically an expansion up to the first power in $1/m_c$ and $1/m_b$, we have realized that the form of the HQET expansion of the form factors as written in (14)–(19) is not fulfilled in any of the considered collinear frames, *except in the EVF*. In this frame, relations (14)–(19), at least up to first order in $1/m_Q$, are exactly satisfied. This seems to us a good enough criterium for choosing the EVF in our calculations. We will compute below all the ground state subleading functions $L_i(w)(i = 1, \ldots 6)$ and verify also that Luke's theorem is satisfied.

A last important point of principle is in order here. Had we adopted the nonrelativistic quark model (including the

light quark), relations (14)–(19) are exactly satisfied in any Galilean frame. However, as pointed out above, we need to consider the b and c quarks as heavy, and the spectator light quark as relativistic. Quantitatively, the results of the nonrelativistic quark model would not make much sense in order to consider departures of the heavy quark limit results of the BT model due to the b and c finite masses.

V. $1/m_Q$ FORM FACTORS FOR THE GROUND STATE TRANSITIONS $B \to D^{(*)}\ell\nu$ IN THE BT MODEL

To make explicit the discussion of the $1/m_Q$ corrections to $\bar{B} \to D^{(*)}\ell\nu$, let us rewrite the basic formulas at finite mass (51) and (52) under the form and new notation

$$\langle D^{(*)}(\vec{P}'), \epsilon' | J^{(1)} | B(\vec{P}) \rangle = \int \frac{d\vec{p}_2}{(2\pi)^3} \frac{1}{p_2^0} G_{D^{(*)}B}(\vec{p}_2, \vec{P}', \vec{P}) \varphi'_{D^{(*)}}(\vec{k}'_2)^* \varphi_B(\vec{k}_2) \quad (64)$$

with

$$G_{D^{(*)}}(\vec{p}_2, \vec{P}', \vec{P}) = \frac{\sqrt{u^0 u'^0}}{p_1^0 p'^0_1} \sqrt{\frac{k_1^0 k_2^0}{(k_1^0 + m_1)(k_2^0 + m_2)}} \sqrt{\frac{k'^0_1 k'^0_2}{(k'^0_1 + m_1)(k'^0_2 + m_2)}} \times \frac{1}{16} \mathrm{Tr}[O(m_1 + \not{p}_1)(1 + \not{u})(m_2 + \not{p}_2)\Gamma^{D^{(*)}}_{u'} \times (1 + \not{u}')(m_1 + \not{p}'_1)] \quad (65)$$

where $\Gamma^D_{u'} = 1$ and $\Gamma^{D^*}_{u'} = \gamma_5 \not{\epsilon}'^*_{u'}$. For the sake of clarity we now adopt the notation

$$\epsilon_b = \frac{1}{2m_1} = \frac{1}{2m_b}, \qquad \epsilon_c = \frac{1}{2m'_1} = \frac{1}{2m_c}. \quad (66)$$

To compute the $1/2m_Q$ subleading functions $L_i(w)(i = 1, \ldots 6)$ (14)–(19), we need to expand the matrix element (64) in powers of ϵ_b, ϵ_c up to the first order. Symbolically we can write, simplifying the notation,

$$\langle D^{(*)}(\vec{P}'), \epsilon' | J^{(1)} | B(\vec{P}) \rangle = \langle D^{(*)}(\vec{P}'), \epsilon' | J^{(1)} | B(\vec{P}) \rangle_0 + \int \frac{d\vec{p}_2}{(2\pi)^3} \frac{1}{p_2^0} [\epsilon_b G_0^{(b)}(\vec{p}_2, \vec{P}', \vec{P}) + \epsilon_c G_0^{(c)}(\vec{p}_2, \vec{P}', \vec{P})] \varphi'_0(\vec{k}'_2)^* \varphi_0(\vec{k}_2) + \int \frac{d\vec{p}_2}{(2\pi)^3} \frac{1}{p_2^0} G_0(\vec{p}_2, \vec{P}', \vec{P}) [\epsilon_b \varphi'_0(\vec{k}'_2)^* \varphi_0^{(b)}(\vec{k}_2) + \epsilon_c \varphi'^{(c)}_0(\vec{k}'_2)^* \varphi_0(\vec{k}_2)]. \quad (67)$$

In the preceding equation, the subindex 0 means $\epsilon_b = \epsilon_c = 0$ (heavy quark limit).

We have separated the perturbation of the kernel G and of the wave functions φ, in an obvious notation. In what follows we will neglect the second term in (67) since we have realized numerically that the perturbation of the wave functions gives a very small contribution.

Using (67), it is convenient to write the matrix elements (14)–(19) using the following notation:

$$h_+(w) = \xi(w) + \epsilon_c H_+^{(c)}(w) + \epsilon_b H_+^{(b)}(w) + O^{h_+}_{1/m_Q^2}(w), \quad (68)$$

$$h_-(w) = \epsilon_c H_-^{(c)}(w) + \epsilon_b H_-^{(b)}(w) + O^{h_-}_{1/m_Q^2}(w), \quad (69)$$

$$h_V(w) = \xi(w) + \epsilon_c H_V^{(c)}(w) + \epsilon_b H_V^{(b)}(w) + O^{V}_{1/m_Q^2}(w), \quad (70)$$

$$h_{A_1}(w) = \xi(w) + \epsilon_c H_{A_1}^{(c)}(w) + \epsilon_b H_{A_1}^{(b)}(w) + O^{A_1}_{1/m_Q^2}(w), \quad (71)$$

$$h_{A_2}(w) = \epsilon_c H_{A_2}^{(c)}(w) + \epsilon_b H_{A_2}^{(b)}(w) + O^{A_2}_{1/m_Q^2}(w), \quad (72)$$

$$h_{A_3}(w) = \xi(w) + \epsilon_c H_{A_3}^{(c)}(w) + \epsilon_b H_{A_3}^{(b)}(w) + O^{A_3}_{1/m_Q^2}(w). \quad (73)$$

Performing analytically an expansion of the matrix elements for the different currents in powers of ϵ_b, ϵ_c, we can compute the different functions $H^{(Q)}(Q = b, c)$, and from them obtain the subleading functions $L_i(w)(i = 1, \ldots 6)$ appearing in (14)–(19), by using the straightforward relations

$$L_1(w) = H_+^{(c)}(w) = H_+^{(b)}(w) = \frac{1}{2}[(w+1)H_{A_1}^{(b)}(w) - (w-1)H_{A_3}^{(b)}(w)], \quad (74)$$

$$L_2(w) = \frac{1}{2}[(w+1)H_{A_1}^{(c)}(w) - (w-1)H_V^{(c)}(w)], \quad (75)$$

$$L_3(w) = \frac{1}{2}[H_{A_2}^{(c)}(w) - H_{A_3}^{(c)}(w) + H_V^{(c)}(w)], \quad (76)$$

$$L_4(w) = H_-^{(c)}(w) = -H_-^{(b)}(w) = \frac{w+1}{2}[H_{A_1}^{(b)}(w) - H_{A_3}^{(b)}(w)], \quad (77)$$

$$L_5(w) = \frac{w+1}{2}[H_{A_1}^{(c)}(w) - H_V^{(c)}(w)], \quad (78)$$

$$L_6(w) = \frac{1}{2}[H_{A_2}^{(c)}(w) + H_{A_3}^{(c)}(w) - H_V^{(c)}(w)]. \quad (79)$$

From these relations, and the expressions for the different functions $H^{(Q)}(Q = b, c)$, we find analytically that Luke's theorem [32] (20) is satisfied

$$L_1(1) = L_2(1) = 0. \quad (80)$$

Moreover we find, for the functions $L_i(w)(i = 1, 2, 3)$, corresponding to the so-called Lagrangian perturbations, the following results, that do not follow from HQET, and are specific to the BT model:

$$L_1(w) = L_2(w), \qquad L_3(w) = 0. \quad (81)$$

In the BT model, for the functions $L_i(w)(i = 4, 5, 6)$ that correspond to the Current perturbations, we find analytically relation (26) that holds in HQET:

$$L_4(1) + 2L_6(1) = 3L_5(1). \quad (82)$$

More explicitly, we find in the limit $m_D = m_{D^*} = m_c + \bar{\Lambda}$, calling from now on the light quark mass $m_2 = m$:

$$L_4(1) = -\bar{\Lambda} + \frac{2}{3}\int \frac{d\vec{p}}{(2\pi)^3}\frac{\vec{p}^2}{m + \sqrt{m^2 + \vec{p}^2}}|\varphi(\vec{p})|^2, \quad (83)$$

$$L_5(1) = -\bar{\Lambda}, \quad (84)$$

$$L_6(1) = -\bar{\Lambda} - \frac{1}{3}\int \frac{d\vec{p}}{(2\pi)^3}\frac{\vec{p}^2}{m + \sqrt{m^2 + \vec{p}^2}}|\varphi(\vec{p})|^2, \quad (85)$$

where the internal wave function normalization

$$\int \frac{d\vec{p}}{(2\pi)^3}|\varphi(\vec{p})|^2 = 1 \quad (86)$$

has been used. Relation (84) is in agreement with (23) at zero recoil.

VI. $1/m_Q$ FORM FACTORS AT ZERO RECOIL FOR TRANSITIONS TO EXCITED STATES $B \to D^{**}\ell\nu$ IN THE BT MODEL

Performing a series expansion of the relevant form factors one finds, in the BT model, at zero recoil:

$$g_+(1) = -3(\epsilon_c + \epsilon_b)\frac{1}{3}\int \frac{d\vec{p}}{(2\pi)^3}|\vec{p}|\varphi_{\frac{1}{2}^+}(|\vec{p}|)^*\varphi(|\vec{p}|), \quad (87)$$

$$g_{V_1}(1) = 2(\epsilon_c - 3\epsilon_b)\frac{1}{3}\int \frac{d\vec{p}}{(2\pi)^3}|\vec{p}|\varphi_{\frac{1}{2}^+}(|\vec{p}|)^*\varphi(|\vec{p}|), \quad (88)$$

$$f_{V_1}(1) = -4\sqrt{2}\epsilon_c\frac{1}{3}\int \frac{d\vec{p}}{(2\pi)^3}|\vec{p}_2|\varphi_{\frac{3}{2}^+}(|\vec{p}|)^*\varphi(|\vec{p}|). \quad (89)$$

These formulas hold for all collinear reference frames considered in Appendix D. We observe that the $1/m_Q$ dependence agrees with the prediction of HQET for all three form factors $g_+(1)$, $g_{V_1}(1)$, and $f_{V_1}(1)$ [formulas (41)–(43)], in particular the BT model predicts for the two states belonging to the same doublet $0^- \to 0^+_{1/2}$, $0^- \to 1^+_{1/2}$

$$\frac{g_+(1)}{g_{V_1}(1)} = -\frac{3}{2}\frac{\epsilon_c + \epsilon_b}{\epsilon_c - 3\epsilon_b}, \quad (90)$$

while the form factor $f_{V_1}(1)$ for $0^- \to 1^+_{3/2}$ is independent because a different radial wave function $\varphi_{\frac{3}{2}^+}(|\vec{p}|)$ appears in formula (89). Formula (90) is consistent with the expectations of HQET (41)–(43).

Another matter is the absolute magnitude of the BT results (87)–(89) as compared with the HQET results by Leibovich *et al.* [31] (41)–(43). In the latter expressions we see that there is factorization between the level spacings and the corresponding inelastic IW functions at zero recoil: $\Delta E_{\frac{1}{2}}\tau_{1/2}(1)$ or $\Delta E_{\frac{1}{2}}\tau_{3/2}(1)$.

The spin-orbit term is small and one can therefore assume that the level spacing is about the same for both j^+ states:

$$\Delta E_{\frac{1}{2}} \simeq \Delta E_{\frac{3}{2}}. \quad (91)$$

Then, the form factors at zero recoil (41)–(43) are in the ratios

$$g_+(1) : g_{V_1}(1) : f_{V_1}(1) = -3(\epsilon_c + \epsilon_b)\tau_{1/2}(1) : 2(\epsilon_c - 3\epsilon_b)\tau_{1/2}(1) : -4\sqrt{2}\epsilon_c\tau_{3/2}(1) \quad (92)$$

while we find, from (87)–(89), in the BT model within the same assumption of small spin-orbit coupling

$$g_+(1) : g_{V_1}(1) : f_{V_1}(1) = -3(\epsilon_c + \epsilon_b) : 2(\epsilon_c - 3\epsilon_b) : -4\sqrt{2}\epsilon_c. \quad (93)$$

The contradiction between the results of HQET (92) and the ones of the BT model (93) is obvious because of the values (2) found in the heavy quark limit in the BT model (for the IG potential): $\tau_{1/2}(1) = 0.22, \tau_{3/2}(1) = 0.54$.

The origin of the difference between $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$ in the BT model is the following. From expressions (60) and (61) one obtains at zero recoil [6,21]

$$\tau_{1/2}(1) = -m\frac{1}{12\pi^2}\int_0^\infty p^2 dp \varphi_{1/2}(p) \times \left[\frac{p}{m+p^0}\left(3+\frac{m}{p^0}\right)+2\frac{d}{dp}\right]\varphi(p), \quad (94)$$

$$\tau_{3/2}(1) = -m\frac{1}{12\pi^2}\int_0^\infty p^2 dp \varphi_{3/2}(p) \times \left[\frac{p}{m+p^0}\frac{m}{p^0}+2\frac{d}{dp}\right]\varphi(p). \quad (95)$$

Therefore, due to the first terms in the right-hand side of (94) and (95) one gets in the BT model $\tau_{1/2}(1) \neq \tau_{3/2}(1)$. As analyzed in detail in [11] the Wigner rotations are at the origin of these terms:

$$\tau_j(1) \sim \left\langle j^+\right| -\frac{p^0 iz + iz p^0}{2} + \frac{i}{2}\frac{(\vec{\sigma}\times\vec{p}_T)_z}{p^0+m}\left|\frac{1}{2}^-\right\rangle \times \left(j=\frac{1}{2},\frac{3}{2}\right). \quad (96)$$

The Wigner rotation, second term in (96), is a relativistic effect dependent on the spin that gives the difference between $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$.

A. BT model $1/m_Q$ form factors at zero recoil for transitions to excited states in the nonrelativistic limit

Let us first observe that expressions (87)–(89) are independent of the light quark mass m. Therefore, the same expressions must be valid *in the nonrelativistic limit of the BT model*, i.e. taking $|\vec{p}| \ll m$. Let us assume this limit and consider the nonrelativistic Hamiltonian for the light quark interacting with the heavy quark:

$$H = \frac{p^2}{2m} + V(r) \quad (97)$$

where $\vec{r}$ is the relative position between the light quark and the heavy quark.

Let us first remark that in the nonrelativistic limit, since the spin-orbit term does not contribute, one has

$$\varphi_{\frac{1}{2}}(p) = \varphi_{\frac{3}{2}}(p), \qquad \Delta E_{\frac{1}{2}} = \Delta E_{\frac{3}{2}}. \quad (98)$$

In the nonrelativistic limit one has $\tau_{1/2}(w) = \tau_{3/2}(w)$, that at zero recoil is given by

$$\tau_j(1) = -m\frac{1}{6\pi^2}\int_0^\infty p^2 dp \varphi_j(p)\frac{d}{dp}\varphi(p) = -\frac{1}{3}m\left\langle 0^+\right|\frac{d}{dp}\left|0^-\right\rangle \qquad \left(j=\frac{1}{2},\frac{3}{2}\right). \quad (99)$$

Using (99) and the nonrelativistic Hamiltonian (97) let us compute

$$\Delta E_j \tau_j(1) = -\frac{1}{3}m\left\langle 0^+\right|\left[H,\frac{d}{dp}\right]\left|0^-\right\rangle = -\frac{1}{3}m\left\langle 0^+\right|\left[\frac{p^2}{2m},\frac{d}{dp}\right]\left|0^-\right\rangle = \frac{1}{6\pi^2}\int_0^\infty p^3 dp \varphi_j(p)\varphi(p) \qquad \left(j=\frac{1}{2},\frac{3}{2}\right) \quad (100)$$

and we obtain therefore the common factor in the right-hand side of Eqs. (87)–(89).

Finally, in the nonrelativistic limit we obtain relations (41)–(43) with $\Delta E_{\frac{1}{2}}\tau_{\frac{1}{2}}(1) = \Delta E_{\frac{3}{2}}\tau_{\frac{3}{2}}(1)$ given by the right-hand side of (100).

The argument has a transparent physical interpretation in configuration space. In the nonrelativistic limit of (96) the Wigner rotations are subleading and one has

$$\tau_j(1) \sim m\left\langle j^+\right| - iz\left|\frac{1}{2}^-\right\rangle \qquad \left(j=\frac{1}{2},\frac{3}{2}\right) \quad (101)$$

Computing the matrix element of the axial current A^0 at zero recoil one has, since the active quark is the heavy quark labelled 1:

$$\langle 0^+|A^0|0^-\rangle \sim \left(\frac{1}{m_c}+\frac{1}{m_b}\right)\langle 0^+|p_{1z}|0^-\rangle \qquad \left(j=\frac{1}{2},\frac{3}{2}\right) \quad (102)$$

then one has, from the nonrelativistic Hamiltonian (97) and $\vec{p}_1 = -\vec{p}_2 = -\vec{p}$, where $\vec{p}$ is the momentum of the light spectator quark:

$$\langle 0^+|p_{1z}|0^-\rangle = m\left\langle 0^+\right| -\frac{p_z}{m}\left|0^-\right\rangle = -im\langle 0^+|[H,z]|0^-\rangle = -im(E_1-E_0)\langle 0^+|z|0^-\rangle \quad (103)$$

where E_0, E_1 are the energies of the ground state and the excited state. Therefore, the dependence on the level spacing of HQET follows in the nonrelativistic limit, as we have already seen from (100).

VII. THE GODFREY-ISGUR QUARK MODEL FOR SPECTROSCOPY

Let us now particularize the above expressions for the choice of the mass operator M given by the Godfrey-Isgur model [18], and perform the numerical calculations.

The GI model for meson spectroscopy describes the whole set of meson spectra $q\bar{q}$ and $Q\bar{q}$, where q is a light quark ($q = u, d, s$) and Q is a heavy quark ($Q = c, b$), with the important exception of the recently discovered narrow states D_{sJ} (0^+ and 1^+), which are too low in mass compared with the predictions of the model. The model contains a relativistic kinetic term of the form

$$K = \sqrt{\vec{k}_1^2 + m_1^2} + \sqrt{\vec{k}_2^2 + m_2^2} \tag{104}$$

that is identical to the operator M_0 at rest, and a complicated interaction term that includes (1) a Coulomb part with a q^2 dependent α_s, (2) a linear confining piece, and (3) terms describing the spin-orbit and spin-spin interactions. All singularities are regularized–e.g. terms of the type $\delta(\vec{r})$ or $1/m_2$, where m_2 is the light quark mass. The hamiltonian H depends on a number of parameters that are fitted to describe all the meson spectra.

VIII. FORM FACTORS FOR THE GROUND STATE $B \to D^{(*)}\ell\nu$

This section contains the numerical results for the ground state form factors $B \to D^{(*)}\ell\nu$ using the Bakamjian-Thomas model exposed above and the internal wave functions provided by the GI spectrocopic potential, given in Appendixes B (heavy quark limit) and C (at finite mass).

In Fig. 1 we give the prediction for the elastic IW function $\xi(w)$ and in Figs. 2–7 we give the results for the different $B \to D^{(*)}\ell\nu$ form factors at finite mass compared with their heavy quark limit. The finite mass effect is rather small in general, even in the case of the form factors that vanish in the heavy quark limit, $h_-(w)$ and $h_{A_2}(w)$.

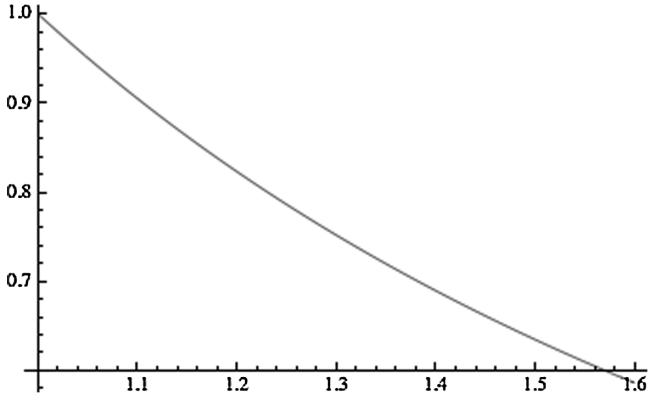

FIG. 1. The elastic Isgur-Wise function $\xi(w) = (\frac{2}{w+1})^{2\rho^2}$ in the BT model ($\rho^2 = 1.023$).

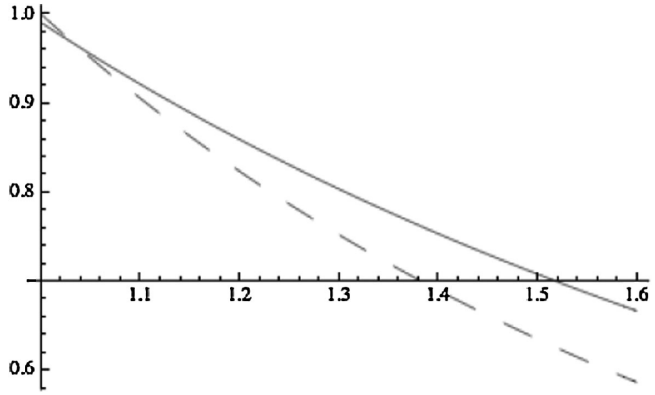

FIG. 2. The form factor $h_+(w)$ in the BT model at finite mass [continuous line, $h_+(1) = 0.99033$] and in the heavy quark limit (dashed line).

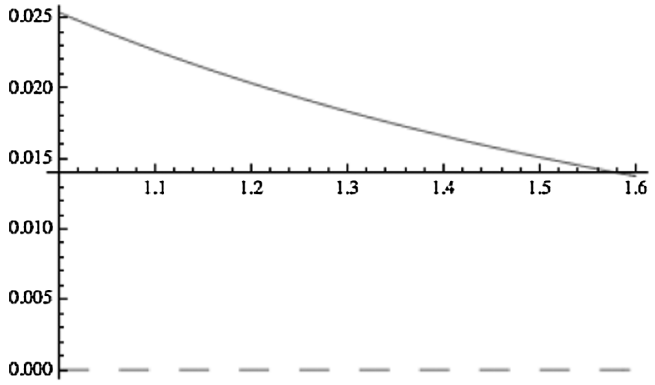

FIG. 3. The form factor $h_-(w)$ in the BT model at finite mass [continuous line, $h_-(1) = 0.02535$] and in the heavy quark limit (dashed line).

A. First order $1/m_Q$ functions and Luke theorem

Here we compute within the BT model with the GI internal wave functions the subleading functions $L_i(w)$ defined in (14)–(19) and given by equations (74)–(79) in

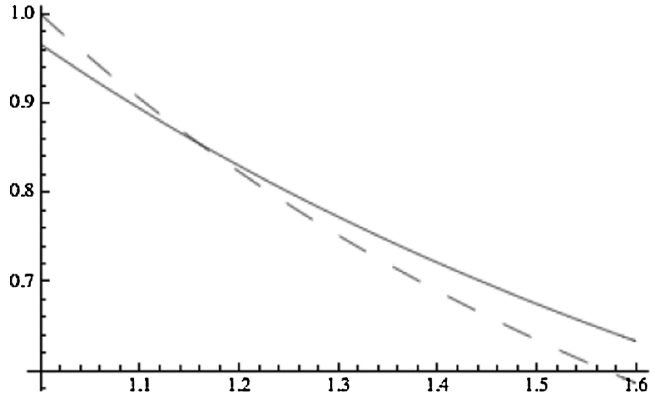

FIG. 4. The form factor $h_{A_1}(w)$ in the BT model at finite mass [continuous line, $h_{A_1}(1) = 0.96606$] and in the heavy quark limit (dashed line).

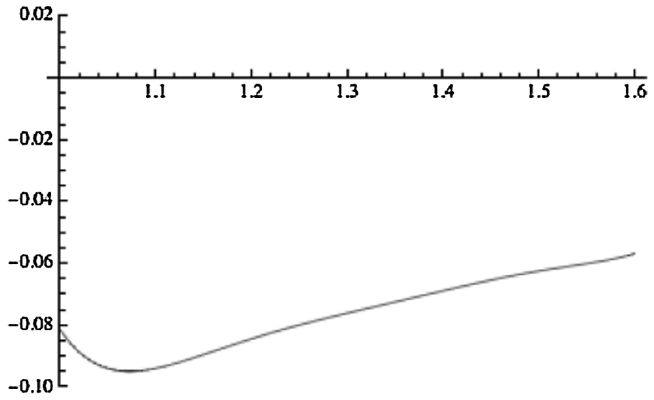

FIG. 5. The form factor $h_{A_2}(w)$ at finite mass in the BT model (it vanishes at infinite mass).

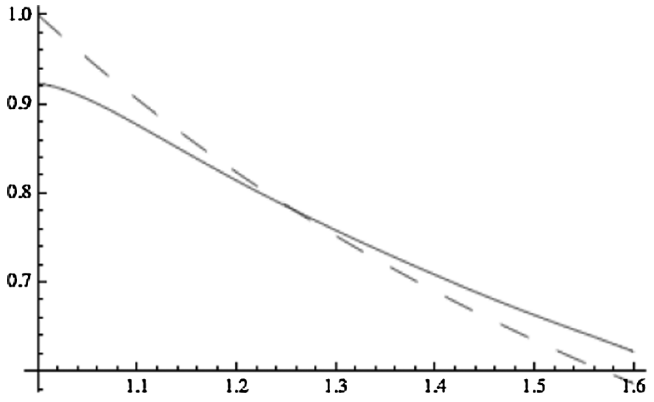

FIG. 6. The form factor $h_{A_3}(w)$ in the BT model at finite mass [continuous line, $h_{A_3}(1) = 0.92299$] and in the heavy quark limit (dashed line).

terms of the functions $H^{(Q)}$. In the results given below we consider only the expansion of the kernel G in (67), since the perturbation of the wave function φ gives a negligible numerical contribution.

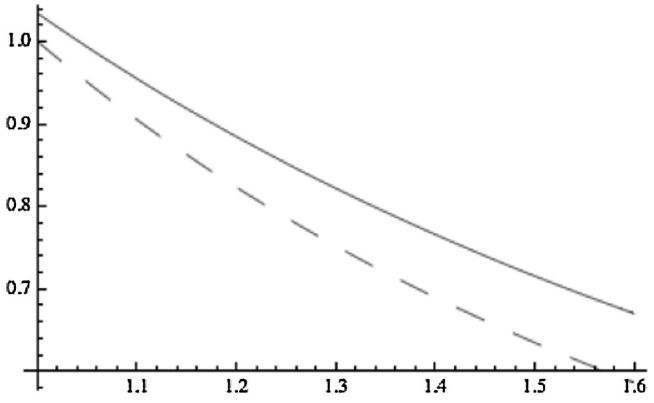

FIG. 7. The form factor $h_V(w)$ in the BT model at finite mass [continuous line, $h_V(1) = 1.03414$] and in the heavy quark limit (dashed line).

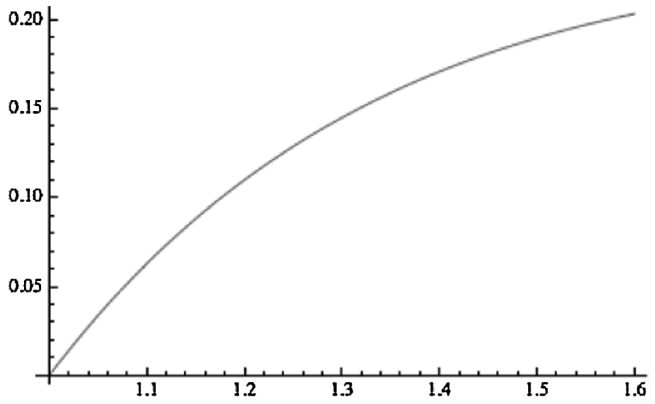

FIG. 8. The subleading functions $L_1(w), L_2(w)$ in the BT model, in which $L_1(w) = L_2(w)$ (in GeV units). Luke's theorem $L_1(1) = L_2(1) = 0$ is satisfied.

Figures 8–11 concern the subleading function $L_i(w)(i = 1, \ldots 6)$. Some comments are in order concerning these figures.

Let us begin with the Lagrangian perturbation functions $L_i(w)(i = 1, 2, 3)$. First, we observe that Luke's theorem [32] (20) is indeed satisfied:

$$L_1(1) = L_2(1) = 0. \tag{105}$$

On the other hand, the result that we find for $L_3(w)$, $L_3(1) = 0$, is not a prediction of HQET.

Considering now the Current perturbation functions $L_i(w)(i = 4, 5, 6)$, these functions are not independent according to HQET, and are given in terms of two independent functions $\bar{\Lambda}\xi(w)$ and $\xi_3(w)$ [30] (22)–(24). We recall here the expression of $L_5(w)$ in terms of the elastic IW function $\xi(w)$:

$$L_5(w) = -\bar{\Lambda}\xi(w) \tag{106}$$

and the linear relation

$$L_4(w) + (1 + w)L_6(w) = 3L_5(w). \tag{107}$$

It is important to emphasize that relation (106) is in analytical agreement with the prediction of the BT model for the elastic IW function (Fig. 1, where $\bar{\Lambda} = 0.3$ GeV). From the explicit formulas for $L_4(w), L_5(w)$, and $L_6(w)$ in the BT model, we have checked that this relation is also analytically exact within the model.

From this section we conclude that the BT model gives a description of the corrections of $O(1/m_Q)$ to the elastic form factors that is consistent with the predictions of HQET, even for their w dependence.

B. $1/m_Q^2$ corrections at zero recoil for $h_+(w)$ and $h_{A_1}(w)$

In the BT model we find indeed that the results satisfy Luke's theorem (20), and therefore the corrections at zero recoil to $h_+(1)$ and $h_{A_1}(1)$ begin at order $1/m_Q^2$, Eq. (21).

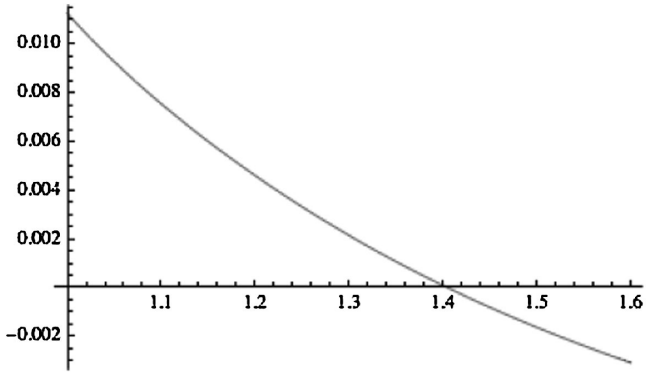

FIG. 9. The subleading function $L_4(w)$ in the BT model [$L_4(1) = 0.011250$ GeV].

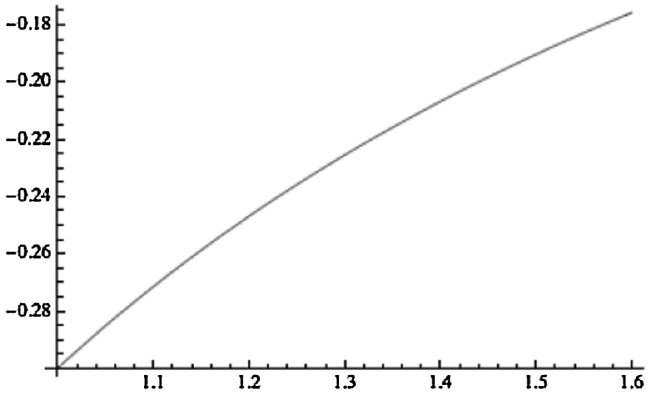

FIG. 10. The subleading function $L_5(w)$ in the BT model [$L_5(1) = -\bar{\Lambda} = -0.3$ GeV].

We get for the sum of all orders $1/m_Q^n (n \geq 2)$ that contribute at zero recoil

$$-\sum_{n\geq 2} \delta^{h_+}_{1/m_Q^n} = 0.0097, \tag{108}$$

$$-\sum_{n\geq 2} \delta^{h_{A_1}}_{1/m_Q^n} = 0.0339. \tag{109}$$

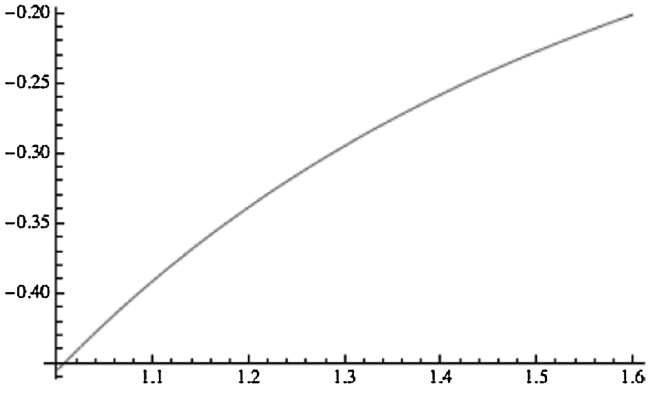

FIG. 11. The subleading function $L_6(w)$ in the BT model [$L_6(1) = -0.455474$ GeV].

These results can be compared with the $O(1/m_Q^2)$ power corrections obtained in HQET [33]. To do that we must switch off the hard gluon radiative corrections in the HQET approach. For the current masses $m_c = 1.25$ GeV, $m_b = 4.75$ GeV and $\mu_G^2 = 0.35\,\mathrm{GeV}^2$, $\mu_\pi^2 = 0.40\ \mathrm{GeV}^2$, the second order HQET power corrections are roughly $-\delta^{h_+}_{1/m_Q^2} \simeq 0.0022$, $-\delta^{h_{A_1}}_{1/m_Q^2} \simeq 0.042$, to be compared with the precedent results of the BT model for the power corrections to all orders with the constituent masses of the model.

IX. FORM FACTORS FOR THE EXCITED STATES $B \to D^{**}\ell\nu$

This section contains the numerical results for the inelastic form factors $\bar{B} \to D^{**}\ell\nu$, using the BT model and the internal wave functions provided by the GI potential tabulated in Appendix B (heavy quark limit) and Appendix C (finite mass).

In Figs. 12 and 13 we give the predictions for the inelastic IW functions $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$.

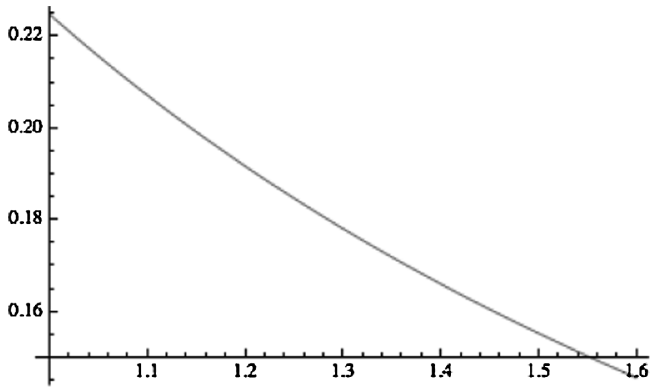

FIG. 12. The IW function $\tau_{1/2}(w) = \tau_{1/2}(1)(\frac{2}{w+1})^{2\sigma^2_{1/2}}$ for the transitions $0^- \to 0^+_{1/2}, 1^+_{1/2}$ in the BT model with the GI hamiltonian ($\tau_{1/2}(1) = 0.2248$, $\sigma^2_{1/2} = 0.84$).

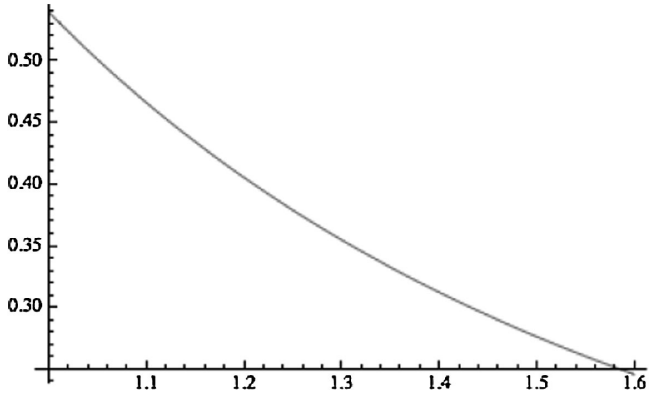

FIG. 13. The IW function $\tau_{3/2}(w) = \tau_{3/2}(1)(\frac{2}{w+1})^{2\sigma^2_{3/2}}$ for the transitions $0^- \to 1^+_{3/2}, 2^+_{3/2}$ in the BT model with the GI hamiltonian ($\tau_{3/2}(1) = 0.5394$, $\sigma^2_{3/2} = 1.50$).

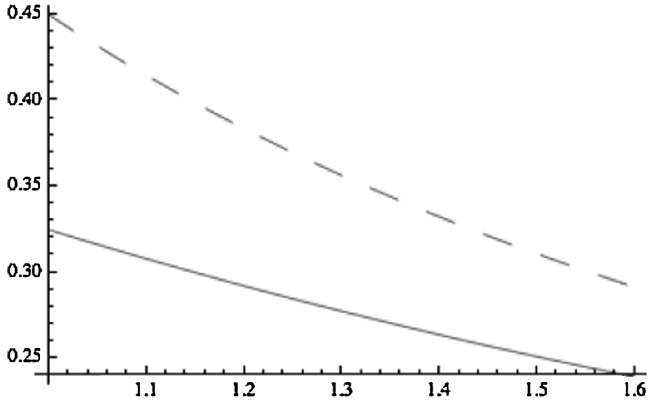

FIG. 14. The form factor $g_-(w)$ for the transition $0^- \to 0^+$ in the BT model at finite mass (full line, $g_-(1) = 0.3241$) and in the heavy quark limit (dashed line).

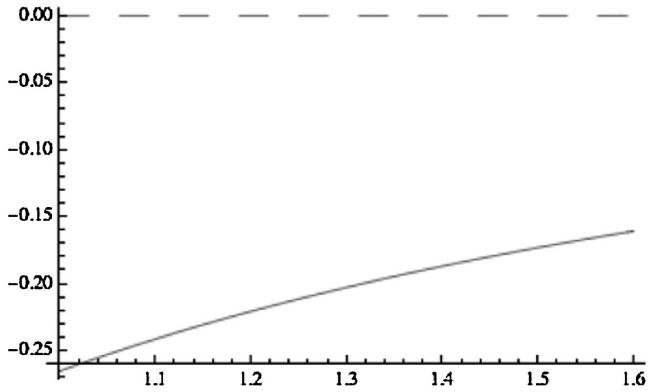

FIG. 15. The form factor $g_+(w)$ for the transition $0^- \to 0^+$ in the BT model at finite mass ($g_+(1) = -0.2657$) and in the heavy quark limit (dashed line).

In Figs. 14–27 we give the results for the different form factors contributing to the transitions $\bar{B} \to D^{**}$ $(0^+_{1/2}, 1^+_{1/2}, 1^+_{3/2}, 2^+_{3/2})$. In the figures we compare the results at finite mass with the corresponding heavy quark limit.

Unlike the elastic case, the finite mass effects for these inelastic form factors are not small, even for some form factors that vanish in the heavy quark limit. This is particularly true for the transition $0^- \to 0^+$. In this case, the leading form factor $g_-(w)$ is reduced by about a factor 1.5, while the absolute magnitude of the form factor $g_+(w)$, that vanishes in the heavy quark limit, becomes of the same order as the leading one.

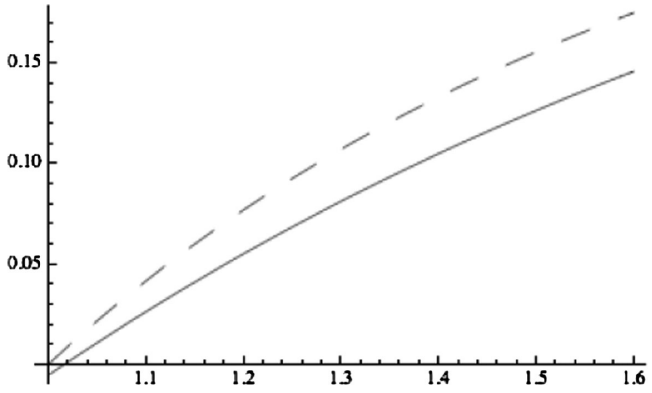

FIG. 16. $g_{V_1}(w)$ for the transition $0^- \to 1^+_{1/2}$ in the BT model at finite mass [$g_{V_1}(1) = -0.0022$] and in the heavy quark limit (dashed line).

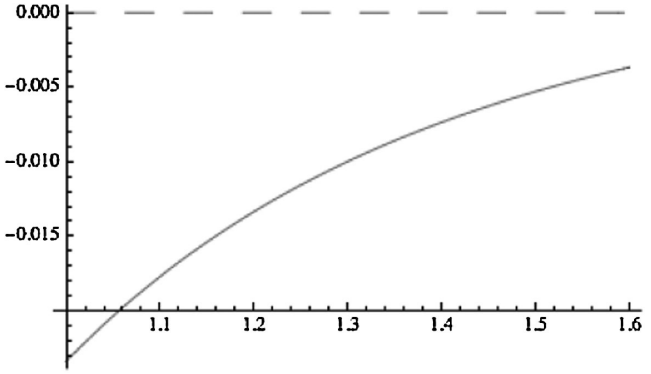

FIG. 17. $g_{V_2}(w)$ for the transition $0^- \to 1^+_{1/2}$ in the BT model at finite mass [$g_{V_2}(1) = -0.0159$] and in the heavy quark limit (dashed line).

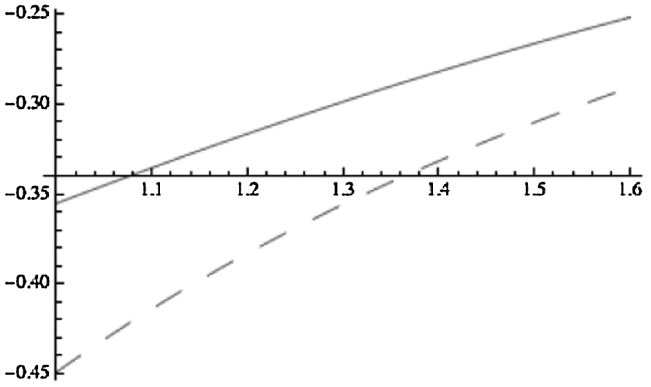

FIG. 18. $g_{V_3}(w)$ for the transition $0^- \to 1^+_{1/2}$ in the BT model at finite mass [$g_{V_3}(1) = -0.3534$] and in the heavy quark limit (dashed line).

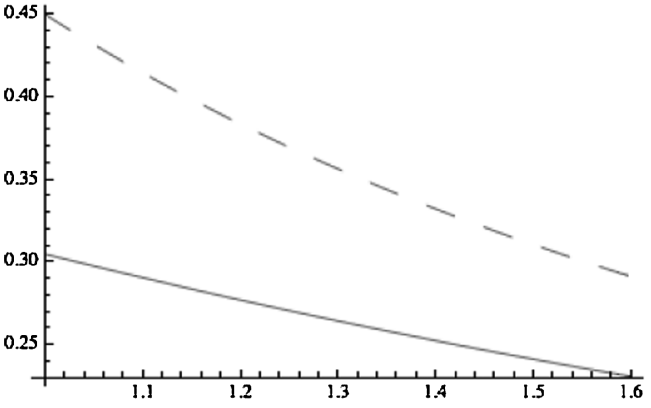

FIG. 19. $g_A(w)$ for the transition $0^- \to 1^+_{1/2}$ in the BT model at finite mass [$g_A(1) = 0.3030$] and in the heavy quark limit (dashed line).

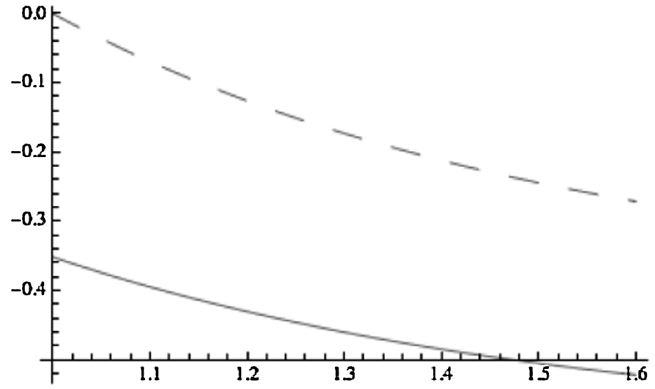

FIG. 20. $f_{V_1}(w)$ for the transition $0^- \to 1^+_{3/2}$ in the BT model at finite mass $[f_{V_1}(1) = -0.3567]$ and in the heavy quark limit (dashed line).

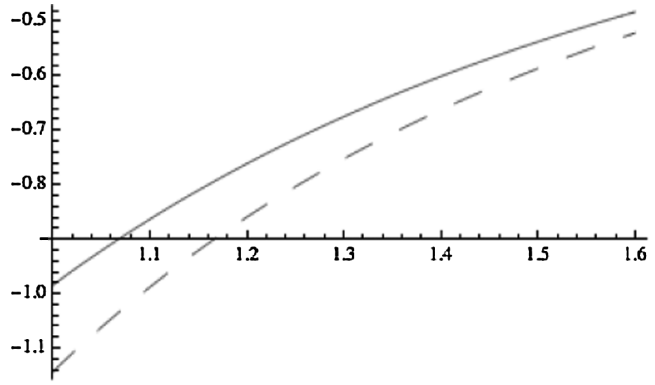

FIG. 21. $f_{V_2}(w)$ for the transition $0^- \to 1^+_{3/2}$ in the BT model at finite mass $[f_{V_2}(1) = -0.9720]$ and in the heavy quark limit (dashed line).

X. BRANCHING RATIOS OF $\bar{B} \to D^{(*)}\ell\nu$, $D^{**}\ell\nu$, $D^{(*)}\pi$, $D^{**}\pi$

We now use formulas (E1)–(E7) to compute the semileptonic branching ratios, and formula (E8) to compute the pionic ones.

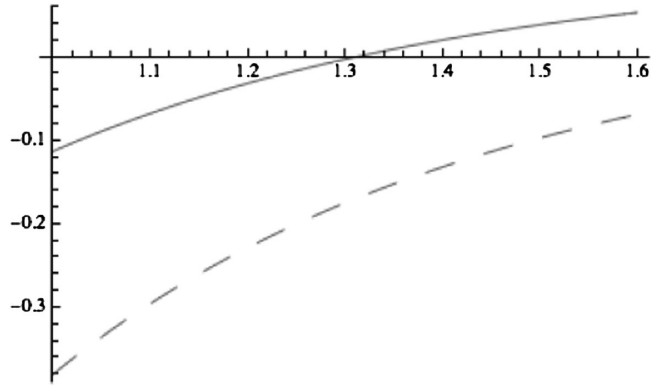

FIG. 22. $f_{V_3}(w)$ for the transition $0^- \to 1^+_{3/2}$ in the BT model at finite mass $[f_{V_3}(1) = -0.1090]$ and in the heavy quark limit (dashed line).

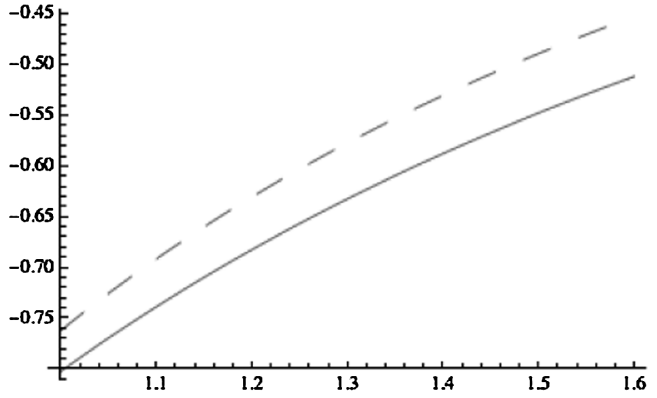

FIG. 23. $f_A(w)$ for the transition $0^- \to 1^+_{3/2}$ in the BT model at finite mass $[f_A(1) = -0.7964]$ and in the heavy quark limit (dashed line).

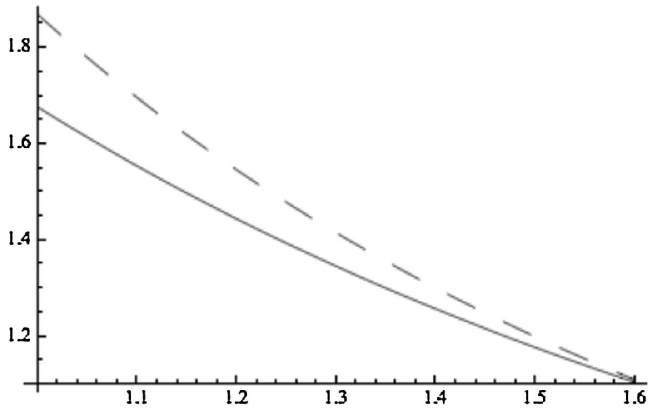

FIG. 24. $k_{A_1}(w)$ for the transition $0^- \to 2^+_{3/2}$ in the BT model at finite mass $[k_{A_1}(1) = 1.6756]$ and in the heavy quark limit (dashed line).

At infinite mass, only the form factors are computed in the heavy quark limit, while the kinematics contains the physical masses. One obtains, for the semileptonic modes,

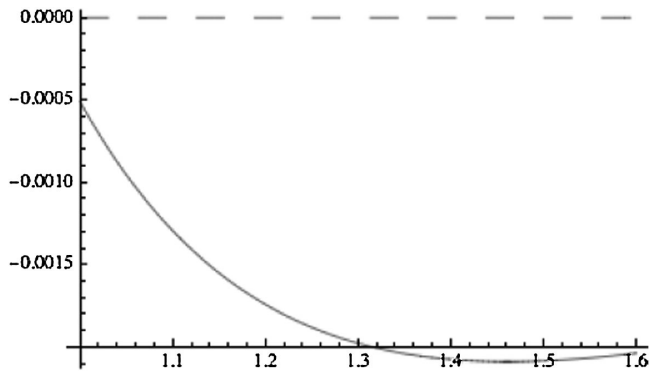

FIG. 25. $k_{A_2}(w)$ for the transition $0^- \to 2^+_{3/2}$ in the BT model at finite mass $[k_{A_2}(1) = -0.00311]$ and in the heavy quark limit (dashed line).

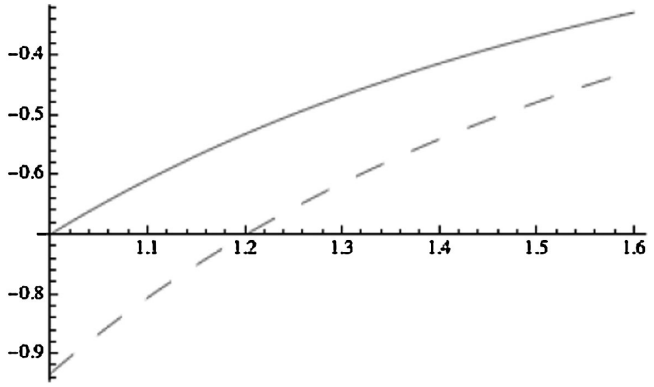

FIG. 26. $k_{A_3}(w)$ for the transition $0^- \to 2^+_{3/2}$ in the BT model at finite mass [$k_{A_3}(1) = -0.69823$] and in the heavy quark limit (dashed line).

$$\begin{aligned}
&\mathrm{BR}(\bar{B} \to D\ell\nu) = 2.022\%,\\
&\mathrm{BR}(\bar{B} \to D^*\ell\nu) = 5.894\%,\\
&\mathrm{BR}(\bar{B} \to D^{**}(0^+_{1/2})\ell\nu) = 5.4 \times 10^{-4},\\
&\mathrm{BR}(\bar{B} \to D^{**}(1^+_{1/2})\ell\nu) = 5.6 \times 10^{-4,}\\
&\mathrm{BR}(\bar{B} \to D^{**}(1^+_{3/2})\ell\nu) = 3.89 \times 10^{-3},\\
&\mathrm{BR}(\bar{B} \to D^{**}(2^+_{3/2})\ell\nu) = 6.04 \times 10^{-3},
\end{aligned} \tag{110}$$

and for the corresponding pionic decays,

$$\begin{aligned}
&\mathrm{BR}(\bar{B} \to D\pi) = 3.73 \times 10^{-3},\\
&\mathrm{BR}(\bar{B} \to D^*\pi) = 3.86 \times 10^{-3},\\
&\mathrm{BR}(\bar{B} \to D^{**}(0^+_{1/2})\pi) = 1.5 \times 10^{-4},\\
&\mathrm{BR}(\bar{B} \to D^{**}(1^+_{1/2})\pi) = 1.2 \times 10^{-4},\\
&\mathrm{BR}(\bar{B} \to D^{**}(1^+_{3/2})\pi) = 1.25 \times 10^{-3},\\
&\mathrm{BR}(\bar{B} \to D^{**}(2^+_{3/2})\pi) = 1.19 \times 10^{-3}.
\end{aligned} \tag{111}$$

The pionic decays with form factors in the heavy quark limit have been compared to the Belle data [27] in Ref. [28].

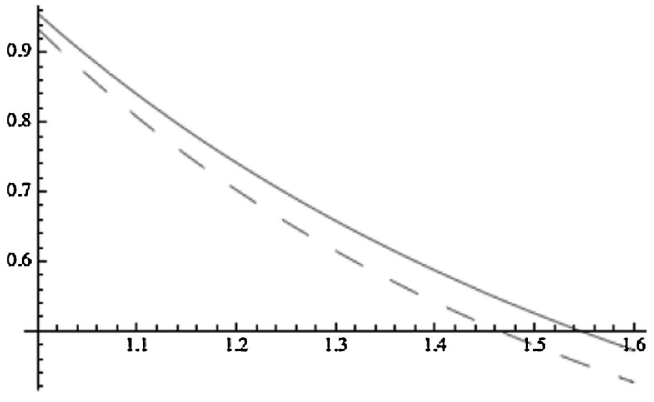

FIG. 27. $k_V(w)$ for the transition $0^- \to 2^+_{3/2}$ in the BT model at finite mass [$k_V(1) = 0.95574$] and in the heavy quark limit (dashed line).

On the other hand, *at finite mass* one has the following semileptonic BR:

$$\begin{aligned}
&\mathrm{BR}(\bar{B} \to D\ell\nu) = 2.354\%,\\
&\mathrm{BR}(\bar{B} \to D^*\ell\nu) = 6.312\%,\\
&\mathrm{BR}(\bar{B} \to D^{**}(0^+_{1/2})\ell\nu) = 2.77 \times 10^{-3},\\
&\mathrm{BR}(\bar{B} \to D^{**}(1^+_{1/2})\ell\nu) = 4.5 \times 10^{-4},\\
&\mathrm{BR}(\bar{B} \to D^{**}(1^+_{3/2})\ell\nu) = 7.04 \times 10^{-3},\\
&\mathrm{BR}(\bar{B} \to D^{**}(2^+_{3/2})\ell\nu) = 5.86 \times 10^{-3},
\end{aligned} \tag{112}$$

and the BR for pionic decays:

$$\begin{aligned}
&\mathrm{BR}(\bar{B} \to D\pi) = 0.469\%,\\
&\mathrm{BR}(\bar{B} \to D^*\pi) = 0.476\%,\\
&\mathrm{BR}(\bar{B} \to D^{**}(0^+_{1/2})\pi) = 7.7 \times 10^{-4},\\
&\mathrm{BR}(\bar{B} \to D^{**}(1^+_{1/2})\pi) = 1.1 \times 10^{-4},\\
&\mathrm{BR}(\bar{B} \to D^{**}(1^+_{3/2})\pi) = 1.74 \times 10^{-3},\\
&\mathrm{BR}(\bar{B} \to D^{**}(2^+_{3/2})\pi) = 1.34 \times 10^{-3}.
\end{aligned} \tag{113}$$

Comparing the finite mass results with those in the heavy quark limit, we observe an enhancement in the case of the 0^+ modes in both the semileptonic and pionic cases (about a factor 5), while the difference is moderate for the other decay modes. The enhancement for the $0^- \to 0^+$ transitions is due to a constructive interference in the decay rates between the two form factors $g_+(w)$ and $g_-(w)$. Of course, the magnitude of the enhancement is not trustable, since in this particular mode it is clearly related to the violation of the relation of Leibovich *et al.* In this case only two form factors contribute, and the subleading one should satisfy this relation.

In such a situation, it is not sensible to compare with the data of *BABAR* and Belle. A detailed discussion was given recently of the experimental situation, compared with the BT model in the heavy quark limit and with the lattice results, in Ref. [29].

XI. DISCUSSION

There cannot be a clear-cut conclusion for this work.

The Bakamjian-Thomas relativistic scheme was originally formulated to build states covariant under the Poincaré group. As shown in a number of papers, the BT relativistic quark model for hadron transitions is very satisfactory in the heavy quark limit. Indeed, in this limit current matrix elements are covariant, form factors exhibit Isgur-Wise scaling, and the Bjorken-Uraltsev sum rules are analytically satisfied.

This model provides also a physical, phenomenological interpretation of a number of features of the heavy quark limit. One notorious example is the inequality $|\tau_{3/2}(1)| > |\tau_{1/2}(1)|$, that in the BT model is a spin

effect due the Wigner rotation of the spin of the spectator light quark.

In the present paper we have tried to extend the BT model to finite mass, for the ground state transitions and for inelastic decays of the ground state to $L^P = 1^+$ excited states. However, at finite mass matrix elements are not covariant anymore and some unwanted results are not unexpected.

As exposed above, a convenient frame is the equal-velocity frame, that we have adopted. On the theoretical side, to test the validity of the model at finite mass, at least the corrections at $O(1/m_Q)$ have to be compared with the rigorous results of HQET for these corrections.

Among the latter, there are the consequences from HQET for the ground state case $0^- \to 0^-, 1^-$, i.e. Luke's theorem for the Lagrangian perturbations at zero recoil, and relations between the different Current perturbations for all w, established by Falk and Neubert. We have checked that these rigorous results of HQET are perfectly satisfied in the BT model at finite mass, even for all w in the case of Current perturbations. In particular, the interesting relation between leading and subleading quantities $L_5(w) = -\bar{\Lambda}\xi(w)$ is analytically fulfilled.

Other rigorous results of HQET at $O(1/m_Q)$ concern the values of the subleading form factors at zero recoil for transitions of the ground state to positive parity mesons $0^- \to 0^+, 1^+_{1/2}, 1^+_{3/2}$. These constraints on the subleading form factors, formulated by Leibovich, Ligeti, Stewart, and Wise, exhibit a certain pattern in $1/m_Q (Q = b, c)$ and are proportional to the level spacings $\Delta E_j (j = 1/2, 3/2)$. In the model, the pattern in $1/m_Q (Q = b, c)$ is obtained, but the proportionality to ΔE_j does not hold. This feature has an important numerical impact on the subleading form factor for the decays $\bar{B}(0^-) \to D^{**}(0^+)\ell\nu$, $\bar{B}(0^-) \to D^{**}(0^+)\pi$ resulting in a spurious enhancement of these decay rates.

As our analysis shows, in a formulation of relativistic quark models for such meson form factors, it is crucial to ensure that the relations of Leibovich *et al.* are satisfied in the heavy quark expansion. It seems to us that to implement these relations is not obvious, and one should investigate whether they hold in other formulations of relativistic quark models.

The BT scheme is not a particular model, but a very general framework. In fact, a framework quite similar to the one of BT is at the basis of the light front relativistic quark models [2–4]. The same inelastic transitions $L = 0$ to $L = 1$ have been studied in the light front models of Cheng *et al.* [34]. But, to our knowledge, the problem of the identities of Leibovich *et al.* has not been evoked in this study.

A similar approach, but based on the point form of BT, has been developed by Gómez-Rocha and Schweiger [35]. These authors compute the form factors for the ground state transitions $\bar{B} \to D^{(*)}\ell\nu$. It would be very interesting to know if within their formalism they could confirm or not our results for the transitions $L = 0 \to L = 1$.

On the other hand, this problem has been clearly raised by Ebert *et al.* [36]. In their relativistic quark model the identities are not automatically fulfilled, but imposed by a choice of the parameters of the potential. In our BT scheme, this latter possibility is clearly excluded.

For our part, one would wish to solve the problem of inelastic form factors in a general way through a fully covariant approach. This approach exists in the Bakamjian-Thomas framework in the heavy quark limit, but is lacking for the moment at finite mass.

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APPENDIX A: FORM FACTORS IN TERMS OF MATRIX ELEMENTS

From the definitions of Sec. II, one can isolate the different form factors by introducing convenient four-vectors. The form factors for the $0^- \to 0^-$ transitions are simply given by

$$\sqrt{m_B m_D}\, h_+(w) = \frac{\langle D(v')|(v+v').V|B(v)\rangle}{2(1+w)}, \tag{A1}$$

$$\sqrt{m_B m_D}\, h_-(w) = \frac{\langle D(v')|(v-v').V|B(v)\rangle}{2(1-w)}. \tag{A2}$$

To isolate the $B \to D^*$ form factors we need to consider the longitudinal and transverse polarization four-vectors. Assuming the motion along the Oz axis, we can adopt the following four-vectors:

$$v = (v^0, 0, 0, v^z), \qquad v' = (v'^0, 0, 0, v'^z), \tag{A3}$$

$$\epsilon'^{(L)} = (v'^z, 0, 0, v'^0), \qquad \epsilon'^{(T)} = (0, 1, 0, 0). \tag{A4}$$

Then, the different form factors for the $0^- \to 1^-$ transitions are given by the expressions

$$\sqrt{m_B m_{D^*}}\, h_V(w) = -\frac{\langle D^{*(T)}(v')|i\epsilon_{\mu\nu\rho\sigma}\epsilon'^{(T)\nu}v'^{\rho}v^{\sigma}V^{\mu}|B(v)\rangle}{w^2-1}, \tag{A5}$$

$$\sqrt{m_B m_{D^*}}\, h_{A_1}(w) = -\frac{\langle D^{*(T)}(v')|\epsilon'^{(T)}.A|B(v)\rangle}{w+1}, \tag{A6}$$

$$\sqrt{m_B m_{D^*}}\, h_{A_2}(w) = -\frac{\langle D^{*(L)}(v')|\epsilon'^{(L)}.A|B(v)\rangle - \langle D^{*(T)}(v')|\epsilon'^{(T)}.A|B(v)\rangle}{(\epsilon'^{(L)}.v)^2}, \tag{A7}$$

$$\sqrt{m_B m_{D^*}}\,h_{A_3}(w) = -\frac{\langle D^{*(L)}(v')|v'.A|B(v)\rangle}{(\epsilon'^{(L)}.v)} + \frac{w(\langle D^{*(L)}(v')|\epsilon'^{(L)}.A|B(v)\rangle - \langle D^{*(T)}(v')|\epsilon'^{(T)}.A|B(v)\rangle)}{(\epsilon'^{(L)}.v)^2}. \tag{A8}$$

Similar relations for the form factors of the transitions to excited states can be obtained from the definitions (7)–(13)

$$\sqrt{m_B m_D}\;g_+(w) = \frac{\langle D^{(1/2)}(0^+)(v')|(v+v').A|B(v)\rangle}{2(1+w)}, \tag{A9}$$

$$\sqrt{m_B m_D}\;g_-(w) = \frac{\langle D^{(1/2)}(0^+)(v')|(v-v').A|B(v)\rangle}{2(1-w)}, \tag{A10}$$

$$\sqrt{m_B m_{D^{**}}}\;g_A(w) = -\frac{\langle D^{(1/2)}(1^+)^{(T)}(v')|i\epsilon_{\mu\nu\rho\sigma}\epsilon'^{(T)\nu}v'^{\rho}v^{\sigma}A^{\mu}|B(v)\rangle}{w^2-1}, \tag{A11}$$

$$\sqrt{m_B m_{D^{**}}}\;g_{V_1}(w) = -\langle D^{(1/2)}(1^+)^{(T)}(v')|\epsilon'^{(T)}.V|B(v)\rangle \tag{A12}$$

$$\sqrt{m_B m_{D^{**}}}\;g_{V_2}(w) = \frac{\langle D^{(1/2)}(1^+)^{(L)}(v')|\epsilon'^{(L)}.V|B(v)\rangle - \langle D^{(1/2)}(1^+)^{(T)}(v')|\epsilon'^{(T)}.V|B(v)\rangle}{(\epsilon'^{(L)}.v)^2}, \tag{A13}$$

$$\begin{aligned}\sqrt{m_B m_{D^{**}}}\;g_{V_3}(w) = {}& \frac{\langle D^{(1/2)}(1^+)^{(L)}(v')|v'.V|B(v)\rangle}{(\epsilon'^{(L)}.v)} \\ & - \frac{w(\langle D^{(1/2)}(1^+)^{(L)}(v')|\epsilon'^{(L)}.V|B(v)\rangle - \langle D^{(1/2)}(1^+)^{(T)}(v')|\epsilon'^{(T)}.V|B(v)\rangle)}{(\epsilon'^{(L)}.v)^2}\end{aligned} \tag{A14}$$

and similar formulas for the form factors $f_A(w)$, $f_{V_1}(w)$, $f_{V_2}(w)$ and $f_{V_3}(w)$ for the $D^{(3/2)}(1^+)$ state. Notice also that in the definition of the axial current matrix element for the ground state D^*, the form factor $h_{A_1}(w)$ is affected by a factor $(w+1)$, that does not appear in the corresponding definition of the vector form factors $g_{V_1}(w)$, $f_{V_1}(w)$ for the 1^+ states.

To isolate the different form factors for the 2^+ states, let us first write the corresponding tensor polarizations $\epsilon'^{(\lambda)}_{\mu\nu}$, that are symmetric $\epsilon'^{(\lambda)}_{\mu\nu} = \epsilon'^{(\lambda)}_{\nu\mu}$, traceless $g^{\mu\nu}\epsilon'^{(\lambda)}_{\mu\nu} = 0$ and transverse $v'^{\mu}\epsilon'^{(\lambda)}_{\mu\nu} = v'^{\nu}\epsilon'^{(\lambda)}_{\mu\nu} = 0$. The polarization tensors we are interested in (the currents are vectors) can be written as

$$\begin{aligned}\epsilon'^{(0)}_{\mu\nu} &= \frac{1}{\sqrt{6}}[\epsilon'^{(+1)}_{\mu}\epsilon'^{(-1)}_{\nu} + 2\epsilon'^{(0)}_{\mu}\epsilon'^{(0)}_{\nu} + \epsilon'^{(-1)}_{\mu}\epsilon'^{(+1)}_{\nu}], \\ \epsilon'^{(T)}_{\mu\nu} &= \frac{1}{\sqrt{2}}[\epsilon'^{(T)}_{\mu}\epsilon'^{(0)}_{\nu} + \epsilon'^{(0)}_{\mu}\epsilon'^{(T)}_{\nu}],\end{aligned} \tag{A15}$$

where $\epsilon'^{(T)}$ is the linear polarization vector (A4), and $\epsilon'^{(\lambda)}_{\mu}$ are the usual circular polarizations vectors ($\epsilon'^{(\lambda)}.\epsilon'^{(\lambda)} = -1$, $\epsilon'^{(\lambda)}.v' = 0$). In consistency with the motion along Oz (A3) we have

$$\epsilon'^{(0)} = (v'^{z}, 0, 0, v'^{0}), \qquad \epsilon'^{(\pm1)} = \left(0, \mp\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right). \tag{A16}$$

The different 2^+ form factors will read, with the notation $\epsilon'^{(0)} = \epsilon'^{(L)}$,

$$\sqrt{m_B m_{D^{**}}}\,k_{A_1}(w) = -\sqrt{2}\frac{\langle D^{(3/2)}(2^+)^{(T)}(v')|\epsilon'^{(T)}.A|B(v)\rangle}{\epsilon'^{(L)}.v}, \tag{A17}$$

$$\sqrt{m_B m_{D^{**}}}k_{A_2}(w) = \frac{\sqrt{\frac{3}{2}}\langle D^{(3/2)}(2^+)^{(0)}(v')|\epsilon'^{(L)}.A|B(v)\rangle - \sqrt{2}\langle D^{(3/2)}(2^+)^{(T)}(v')|\epsilon'^{(T)}.A|B(v)\rangle}{(\epsilon'^{(L)}.v)^3}, \tag{A18}$$

$$\sqrt{m_B m_{D^{**}}}k_{A_3}(w) = \sqrt{\frac{3}{2}}\frac{\langle D^{(3/2)}(2^+)^{(0)}(v')|v'.A|B(v)\rangle}{(\epsilon'^{(L)}.v)^2} - w\frac{\sqrt{\frac{3}{2}}\langle D^{(3/2)}(2^+)^{(0)}(v')|\epsilon'^{(L)}.A|B(v)\rangle - \sqrt{2}\langle D^{(3/2)}(2^+)^{(T)}(v')|\epsilon'^{(T)}.A|B(v)\rangle}{(\epsilon'^{(L)}.v)^3}, \tag{A19}$$

$$\sqrt{m_B m_{D^{**}}}k_V(w) = -\sqrt{2}\frac{\langle D^{*(T)}(v')|i\epsilon_{\mu\nu\rho\sigma}V^\mu\epsilon'^{(T)\nu}v'^\rho v^\sigma|B(v)\rangle}{(w^2-1)(\epsilon'^{(L)}.v)}. \tag{A20}$$

APPENDIX B: WAVE FUNCTIONS IN THE HEAVY QUARK LIMIT IN THE GI MODEL

We have computed the ground state wave function $j^P = \frac{1}{2}^-$ by expanding it in a truncated harmonic oscillator basis

$$\varphi_{\frac{1}{2}^-}(\vec{k}) = \sum_{n=0}^{n=15} C_n^{\frac{1}{2}^-}(-1)^n(4\pi)^{3/4}2^n\sqrt{\frac{(n!)^2}{(2n+1)!}}\frac{1}{\beta^{3/2}}L_n^{1/2}\left(\frac{\vec{k}^2}{\beta^2}\right)\exp\left(-\frac{\vec{k}^2}{2\beta^2}\right). \tag{B1}$$

With the parameters

$$m_1 = 10^4\ \text{GeV}, \qquad m_2 = 0.220\ \text{GeV}, \qquad \beta = 0.5\ \text{GeV} \tag{B2}$$

one gets the coefficients

$$\begin{aligned} C_{0,\ldots 15}^{\frac{1}{2}^-} = (&0.9793537, 0.1176603, 0.1468293, 4.3721687\times10^{-2},\\ &4.8045449\times10^{-2}, 2.0475958\times10^{-2}, 2.1334046\times10^{-2},\\ &1.0961787\times10^{-2}, 1.1114890\times10^{-2}, 6.3780537\times10^{-3},\\ &6.3600712\times10^{-3}, 3.9184764\times10^{-3}, 3.8404907\times10^{-3},\\ &2.4935019\times10^{-3}, 2.3138365\times10^{-3}, 1.6319989\times10^{-3}). \end{aligned} \tag{B3}$$

Similarly, one gets the following wave function for the lowest $\frac{1}{2}^+$ state:

$$\varphi_{\frac{1}{2}^+}(\vec{k}) = \sum_{n=0}^{n=15} C_n^{\frac{1}{2}^+}(-1)^n(4\pi)^{3/4}2^{n+1}\sqrt{\frac{n!(n+1)!}{(2n+3)!}}\frac{|\vec{k}|}{\beta^{5/2}}L_n^{3/2}\left(\frac{\vec{k}^2}{\beta^2}\right)\exp\left(-\frac{\vec{k}^2}{2\beta^2}\right) \tag{B4}$$

with the following coefficients:

$$\begin{aligned} C_{0,\ldots 15}^{\frac{1}{2}^+} = (&0.9797808, 0.1129152, 0.1477815, 4.7028150\times10^{-2},\\ &4.4749252\times10^{-2}, 2.2688832\times10^{-2}, 1.8693443\times10^{-2},\\ &1.2282215\times10^{-2}, 9.3433624\times10^{-3}, 7.2159977\times10^{-3},\\ &5.1802760\times10^{-3}, 4.5010597\times10^{-3}, 3.0235867\times10^{-3},\\ &2.9367937\times10^{-3}, 1.7230053\times10^{-3}, 1.9955065\times10^{-3}). \end{aligned} \tag{B5}$$

And the wave function for the lowest $\frac{3}{2}^+$ state

$$\varphi_{\frac{3}{2}^+}(\vec{k}) = \sum_{n=0}^{n=15} C_n^{\frac{3}{2}^+} (-1)^n (4\pi)^{3/4} 2^{n+1} \sqrt{\frac{n!(n+1)!}{(2n+3)!}} \frac{|\vec{k}|}{\beta^{5/2}} L_n^{3/2}\left(\frac{\vec{k}^2}{\beta^2}\right) \exp\left(-\frac{\vec{k}^2}{2\beta^2}\right) \tag{B6}$$

with the coefficients

$$\begin{aligned} C_{0,\ldots 15}^{\frac{3}{2}^+} = (&0.9878460, 1.0599474 \times 10^{-2}, 0.1471102, 9.8141907 \times 10^{-3}, \\ &4.3046847 \times 10^{-2}, 5.8332058 \times 10^{-3}, 1.7356267 \times 10^{-2}, \\ &3.4403985 \times 10^{-3}, 8.4537473 \times 10^{-3}, 2.0915067 \times 10^{-3}, \\ &4.6376493 \times 10^{-3}, 1.3029705 \times 10^{-3}, 2.7383780 \times 10^{-3}, \\ &8.2387996 \times 10^{-4}, 1.6385724 \times 10^{-3}, 5.3599390 \times 10^{-4}). \end{aligned} \tag{B7}$$

The set of wave functions (B1)–(B7) are all normalized according to

$$\int \frac{d\vec{k}}{(2\pi)^3} |\varphi(\vec{k})|^2 = 1. \tag{B8}$$

APPENDIX C: WAVE FUNCTIONS IN THE GI MODEL AT FINITE MASS

At finite mass, the wave functions are parametrized by

$$\varphi_{J^-}(\vec{k}) = \sum_{n=0}^{n=15} C_n^{J^-} (-1)^n (4\pi)^{3/4} 2^n \sqrt{\frac{(n!)^2}{(2n+1)!}} \frac{1}{\beta^{3/2}} L_n^{1/2}\left(\frac{\vec{k}^2}{\beta^2}\right) \exp\left(-\frac{\vec{k}^2}{2\beta^2}\right) \tag{C1}$$

for the ground states $J = 0, 1$, and by

$$\varphi_{J_j^+}(\vec{k}) = \sum_{n=0}^{n=15} C_n^{J_j^+} (-1)^n (4\pi)^{3/4} 2^{n+1} \sqrt{\frac{n!(n+1)!}{(2n+3)!}} \frac{|\vec{k}|}{\beta^{5/2}} L_n^{3/2}\left(\frac{\vec{k}^2}{\beta^2}\right) \exp\left(-\frac{\vec{k}^2}{2\beta^2}\right) \tag{C2}$$

with $J_j = 0_{1/2}, 1_{1/2}, 1_{3/2}, 2_{3/2}$.

The pseudoscalar B meson wave function is common to all initial states that we are considering. In the GI model, the mass parameters that fit the data for B mesons are

$$m_1 = 4.977 \text{ GeV}, \qquad m_2 = 0.220 \text{ GeV}, \qquad \beta = 0.5 \text{ GeV}, \tag{C3}$$

and the wave function coefficients are

$$\begin{aligned} C_{0,\ldots 15}^{B(0^-)} = (&0.9690171, 0.1531175, 0.1649211, 6.2490419 \times 10^{-2}, \\ &6.0360532 \times 10^{-2}, 3.1558599 \times 10^{-2}, 2.9348362 \times 10^{-2}, \\ &1.7991375 \times 10^{-2}, 1.6438706 \times 10^{-2}, 1.1053351 \times 10^{-2}, \\ &9.9637937 \times 10^{-3}, 7.1392222 \times 10^{-3}, 6.2874621 \times 10^{-3}, \\ &4.7953418 \times 10^{-3}, 3.8834463 \times 10^{-3}, 3.5072465 \times 10^{-3}). \end{aligned} \tag{C4}$$

For the different charmed D mesons, the spectrum is described using the parameters

$$m_1 = 1.628 \text{ GeV}, \qquad m_2 = 0.220 \text{ GeV}, \qquad \beta = 0.5 \text{ GeV} \tag{C5}$$

and the coefficients of the expansions (C1) and (C2) for the various quantum numbers are given by

$$C^{D(0^-)}_{0,\ldots 15} = (0.9600527, 0.1799335, 0.1767118, 7.6031193 \times 10^{-2}, 6.8335488 \times 10^{-2}, 3.9312087 \times 10^{-2}, 3.4507290 \times 10^{-2}, 2.2833729 \times 10^{-2}, 1.9844856 \times 10^{-2}, 1.4270671 \times 10^{-2}, 1.2243154 \times 10^{-2}, 9.4011556 \times 10^{-3}, 7.7781440 \times 10^{-3}, 6.5341271 \times 10^{-3}, 4.6821525 \times 10^{-3}, 5.1816395 \times 10^{-3}), \tag{C6}$$

$$C^{D(1^-)}_{0,\ldots 15} = (0.9894823, 4.9004469 \times 10^{-2}, 0.1262952, 2.2102771 \times 10^{-2}, 3.5959065 \times 10^{-2}, 1.0480723 \times 10^{-2}, 1.4237838 \times 10^{-2}, 5.380643 \times 10^{-3}, 6.7386944 \times 10^{-3}, 2.9314966 \times 10^{-3}, 3.5624162 \times 10^{-3}, 1.6525286 \times 10^{-3}, 2.0363566 \times 10^{-3}, 9.2892419 \times 10^{-4}, 1.2249013 \times 10^{-3}, 5.2336301 \times 10^{-4}), \tag{C7}$$

$$C^{D(0^+_{1/2})}_{0,\ldots 15} = (0.9848158, 5.2615825 \times 10^{-2}, 0.1519192, 3.4893338 \times 10^{-2}, 4.5274679 \times 10^{-2}, 1.9408170 \times 10^{-2}, 1.8440058 \times 10^{-2}, 1.1052819 \times 10^{-2}, 8.9459708 \times 10^{-3}, 6.5899095 \times 10^{-3}, 4.7909911 \times 10^{-3}, 4.1152863 \times 10^{-3}, 2.6809596 \times 10^{-3}, 2.7001044 \times 10^{-3}, 1.4315639 \times 10^{-3}, 1.9437365 \times 10^{-3}), \tag{C8}$$

$$C^{D(2^+_{3/2})}_{0,\ldots 15} = (0.9766909, -0.1460503, 0.1472010, -3.2608863 \times 10^{-2}, 3.9174896 \times 10^{-2}, -9.5297368 \times 10^{-3}, 1.4054954 \times 10^{-2}, -3.5175697 \times 10^{-3}, 6.1494103 \times 10^{-3}, -1.5897267 \times 10^{-3}, 3.0960441 \times 10^{-3}, -8.5987244 \times 10^{-4}, 1.7280004 \times 10^{-3}, -5.3811091 \times 10^{-4}, 1.0204369 \times 10^{-3}, -3.2012642 \times 10^{-4}). \tag{C9}$$

For the two states $D_1(1^+)$ and $D_2(1^+)$ the situation is more complicated because at finite mass they are not pure $j = \frac{1}{2}$ or $j = \frac{3}{2}$. From the GI model we find that each of these states has two components with $j = \frac{1}{2}$ and $j = \frac{3}{2}$.

The two $j = \frac{1}{2}$ and $j = \frac{3}{2}$ components of the $D_1(1^+)$ state, that is dominantly $j = \frac{1}{2}$, are the following:

$$C^{D_1(1^+)_{1/2}}_{0,\ldots 15} = (0.9750784, 4.2226720 \times 10^{-4}, 0.1388684, 1.9337032 \times 10^{-2}, 3.8017304 \times 10^{-2}, 1.3402707 \times 10^{-2}, 1.4626256 \times 10^{-2}, 8.3450762 \times 10^{-3}, 6.9152913 \times 10^{-3}, 5.2573497 \times 10^{-3}, 3.6921142 \times 10^{-3}, 3.424697 \times 10^{-3}, 2.0902278 \times 10^{-3}, 2.3228918 \times 10^{-3}, 1.1427986 \times 10^{-3}, 2.3432890 \times 10^{-3}), \tag{C10}$$

$$C^{D_1(1^+)_{3/2}}_{0,\ldots 15} = (0.1630617, -1.7655547 \times 10^{-2}, 2.4015685 \times 10^{-2}, -4.2665031 \times 10^{-3}, 6.0282979 \times 10^{-3}, -1.6391013 \times 10^{-3}, 1.8471151 \times 10^{-3}, -9.1877274 \times 10^{-4}, 5.8052647 \times 10^{-4}, -6.3214857 \times 10^{-4}, 1.4512054 \times 10^{-4}, -4.7771402 \times 10^{-4}, -2.3710345 \times 10^{-6}, -3.8210297 \times 10^{-4}, -2.8517570 \times 10^{-5}, 1.2153198 \times 10^{-4}). \tag{C11}$$

Of course, the sum of the squared norms of the vectors (C10) and (C11) is normalized, $\sum_i [|C_i^{D_1(1^+)_{1/2}}|^2 + |C_i^{D_1(1^+)_{1/2}}|^2] = 1$.

On the other hand, the two $j=\frac{1}{2}$ and $j=\frac{3}{2}$ components of the $D_2(1^+)$ state, that is dominantly $j=\frac{3}{2}$, are the following:

$$\begin{aligned} C_{0,\ldots 15}^{D_2(1^+)_{1/2}} = (&-0.1633738, -4.8339165\times10^{-3}, -2.5603601\times10^{-2}, -5.8475351\times10^{-3},\\ &-8.1983565\times10^{-3}, -3.9683113\times10^{-3}, -3.8034132\times10^{-3},\\ &-2.6259074\times10^{-3}, -2.1547486\times10^{-3}, -1.7860712\times10^{-3},\\ &-1.3476588\times10^{-3}, -1.2581809\times10^{-3}, -8.6974999\times10^{-4},\\ &-9.28100700\times10^{-4}, -5.1979437\times10^{-4}, -7.6447322\times10^{-4}), \end{aligned} \tag{C12}$$

$$\begin{aligned} C_{0,\ldots 15}^{D_2(1^+)_{3/2}} = (&0.9697097, 6.9651324\times10^{-2}, 0.1564430, -6.7026414\times10^{-3},\\ &4.7112839\times10^{-2}, 2.2269301\times10^{-3}, 1.9536760\times10^{-2},\\ &2.9314493\times10^{-3}, 9.8407984\times10^{-3}, 2.3789247\times10^{-3},\\ &5.5826771\times10^{-3}, 1.7778778\times10^{-3}, 3.3748336\times10^{-3},\\ &1.3486697\times10^{-3}, 1.9880311\times10^{-3}, 1.2300351\times10^{-3}). \end{aligned} \tag{C13}$$

The sum of the squared norms of the vectors (C12) and (C13) is normalized as expected, $\sum_i[|C_i^{D_2(1^+)_{1/2}}|^2+|C_i^{D_2(1^+)_{1/2}}|^2]=1$.

The wave functions of $D_1(1^+)$ and $D_2(1^+)$ must be orthogonal. The spin and orbital angular momentum parts of the wave functions $D_i(1^+)_{1/2}$ and $D_i(1^+)_{3/2}$ $(i=1,2)$ are orthogonal. For the scalar product between $|D_1(1^+)\rangle$ and $|D_2(1^+)\rangle$ we are then left with the sum of products of the radial functions for given $j=\frac{1}{2}$ and $j=\frac{3}{2}$ that, from (C10)–(C13), indeed vanishes:

$$\begin{aligned} \langle D_1(1^+)|D_2(1^+)\rangle \propto \sum_i [&C_i^{D_1(1^+)_{1/2}} C_i^{D_2(1^+)_{1/2}}\\ &+ C_i^{D_1(1^+)_{3/2}} C_i^{D_2(1^+)_{3/2}}] = 0. \end{aligned} \tag{C14}$$

APPENDIX D: A SET OF COLLINEAR FRAMES

We have seen above that the current matrix elements in the BT model are covariant in the heavy quark limit. However, the subleading corrections in $1/m_Q$ are dependent on the frame. We consider a family of collinear frames, with the mesons moving along the Oz axis,

$$v=(v^0,0,0,v^z), \qquad v'=(v'^0,0,0,v'^z), \tag{D1}$$

going continuously between the B meson rest frame through the final D meson rest frame. These frames can be labeled by a parameter α, with $0\le\alpha\le1$:

$$(1-\alpha)v^z+\alpha v'^z=0. \tag{D2}$$

The B and the D meson rest frames correspond respectively to $\alpha=0$ and $\alpha=1$, while the intermediate equal velocity frame (EVF), in which the spatial velocities are equal in modulus $(v^0=v'^0, v^z=-v'^z)$ corresponds to the value $\alpha=\frac{1}{2}$.

In terms of this parameter and of the variable $w=v.v'$, the four-vectors (C1) then read

$$\begin{aligned} v &= \left(\sqrt{1+\frac{\alpha^2(w^2-1)}{\alpha^2+2\alpha(1-\alpha)w+(1-\alpha)^2}},0,0,-\sqrt{\frac{\alpha^2(w^2-1)}{\alpha^2+2\alpha(1-\alpha)w+(1-\alpha)^2}}\right),\\ v' &= \left(\sqrt{1+\frac{(1-\alpha)^2(w^2-1)}{\alpha^2+2\alpha(1-\alpha)w+(1-\alpha)^2}},0,0,\sqrt{\frac{(1-\alpha)^2(w^2-1)}{\alpha^2+2\alpha(1-\alpha)w+(1-\alpha)^2}}\right). \end{aligned} \tag{D3}$$

APPENDIX E: FORMULAS FOR THE DECAY RATES

The differential rates can be expressed in terms of the helicity amplitudes under the form

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2 r^3\sqrt{w^2-1} \times (|H_+(w)|^2 + |H_-(w)|^2 + |H_0(w)|^2), \quad \text{(E1)}$$

where $r = \frac{m_D}{m_B}$ (m_D being the mass of the corresponding charmed meson) and the helicity amplitudes squared read, in the different cases:

(i) $\bar{B} \to D\ell\nu$

$$H_\pm = 0,$$
$$|H_0(w)|^2 = (w^2-1)[(1+r)h_+(w) - (1-r)h_-(w)]^2. \quad \text{(E2)}$$

(ii) $\bar{B} \to D^*\ell\nu$

$$|H_\pm(w)|^2 = (1+r^2-2rw) \times [(w+1)h_{A_1}(w) \mp \sqrt{w^2-1}h_V(w)]^2,$$
$$|H_0(w)|^2 = (w+1)^2\{(w-r)h_{A_1}(w) - (w-1) \times [rh_{A_2}(w) + h_{A_3}(w)]\}^2. \quad \text{(E3)}$$

(iii) $\bar{B} \to D^{**}(0^+_{1/2})\ell\nu$

$$H_\pm = 0,$$
$$|H_0(w)|^2 = (w^2-1)[(1+r)g_+(w) - (1-r)g_-(w)]^2. \quad \text{(E4)}$$

(iv) $\bar{B} \to D^{**}(1^+_{1/2})\ell\nu$

$$|H_\pm(w)|^2 = (1+r^2-2rw)[g_{V_1}(w) \mp \sqrt{w^2-1}g_A(w)]^2,$$
$$|H_0(w)|^2 = \{(w-r)g_{V_1}(w) + (w^2-1)[rg_{V_2}(w) + g_{V_3}(w)]\}^2, \quad \text{(E5)}$$

(v) $\bar{B} \to D^{**}(1^+_{3/2})\ell\nu$

$$|H_\pm(w)|^2 = (1+r^2-2rw)[f_{V_1}(w) \mp \sqrt{w^2-1}f_A(w)]^2,$$
$$|H_0(w)|^2 = \{(w-r)f_{V_1}(w) + (w^2-1)[rf_{V_2}(w) + f_{V_3}(w)]\}^2. \quad \text{(E6)}$$

(vi) $\bar{B} \to D^{**}(2^+_{3/2})\ell\nu$

$$|H_\pm(w)|^2 = \frac{1}{2}(1+r^2-2rw)(w^2-1) \times [k_{A_1}(w) \mp \sqrt{w^2-1}k_V(w)]^2,$$
$$|H_0(w)|^2 = \frac{2}{3}(w^2-1)\{(w-r)k_{A_1}(w) + (w^2-1)[rk_{A_2}(w) + k_{A_3}(w)]\}^2. \quad \text{(E7)}$$

Of course, in the preceding formulas the masses of the charmed mesons, and hence the parameter r, vary according to the considered state $D, D^*, D^{**}(0^+_{1/2})$, $D^{**}(1^+_{1/2}), D^{**}(1^+_{3/2})$ or $D^{**}(2^+_{3/2})$. Remember also that the form factor $h_{A_1}(w)$ is affected by a factor $(w+1)$, that does not appear in the corresponding definition of the form factors $g_{V_1}(w)$, $f_{V_1}(w)$ for the 1^+ states and also the form factors $h_{A_2}(w)$ and $h_{A_3}(w)$ are affected by a minus sign, contrary to the definitions of $g_{V_2}(w)$, $f_{V_2}(w)$ and $g_{V_3}(w)$, $f_{V_3}(w)$ for the 1^+ states, as we see in the definitions (6)–(13).

The decays rates for pionic decays read:

$$\Gamma_\pi = \frac{3\pi^2|V_{ub}|^2 a_1^2 f_\pi^2}{m_B m_D}\left(\frac{d\Gamma_{sl}}{dw}\right)_{w_{\max}}, \quad \left(w_{\max} = \frac{m_B^2+m_D^2}{2m_B m_D}\right), \quad \text{(E8)}$$

where $a_1 \simeq 1$ is a combination of Wilson coefficients, and m_D is the mass of the corresponding charmed meson.

[1] B. Bakamjian and L. H. Thomas, Phys. Rev. **92**, 1300 (1953).
[2] B. D. Keister and W. N. Polyzou, Adv. Nucl. Phys. **20**, 225 (1991).
[3] M. Terent'ev, Sov. J. Nucl. Phys. **24**, 106 (1976).
[4] F. Cardarelli, I. L. Grach, I. M. Nadoretskii, E. Pace, G. Sale, and S. Simula, Phys. Lett. B **332**, 1 (1994).
[5] A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Lett. B **365**, 319 (1996).
[6] V. Morénas, A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Lett. B **386**, 315 (1996).
[7] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
[8] N. Isgur and M. B. Wise, Phys. Rev. D **43**, 819 (1991).
[9] J. D. Bjorken, Report No. SLAC-PUB-5278.
[10] N. Uraltsev, Phys. Lett. B **501**, 86 (2001); N. Uraltsev J. Phys. G **27**, 1081 (2001).

[11] A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Lett. B **386**, 304 (1996).
[12] V. Morénas, A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Lett. B **408**, 357 (1997).
[13] A. Le Yaouanc, L. Oliver, O. Pène, J.-C. Raynal, and V. Morénas, Phys. Lett. B **520**, 25 (2001).
[14] A. Le Yaouanc, L. Oliver, and J.-C. Raynal, Phys. Rev. D **67**, 114009 (2003).
[15] A. Le Yaouanc, L. Oliver, and J.-C. Raynal, Phys. Lett. B **557**, 207 (2003).
[16] A. Le Yaouanc, L. Oliver, and J.-C. Raynal, Phys. Rev. D **69**, 094022 (2004).
[17] V. Morénas, A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Rev. D **56**, 5668 (1997).
[18] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[19] V. Morénas, A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Rev. D **58**, 114019 (1998).
[20] A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Lett. B **387**, 582 (1996).
[21] V. Morénas, Thèse, Université Blaise Pascal, Clermont-Ferrand, 1997.
[22] D. Bećirević, E. Chang, A. Le Yaouanc, L. Oliver, and J.-C. Raynal, Phys. Rev. D **84**, 054507 (2011).
[23] I. I. Bigi, B. Blossier, A. Le Yaouanc, L. Oliver, O. Pène, J.-C. Raynal, A. Oyanguren, and P. Roudeau, Eur. Phys. J. C **52**, 975 (2007).
[24] B. Aubert *et al.*, Phys. Rev. Lett. **101**, 261802 (2008).
[25] D. Liventsev *et al.*, Phys. Rev. D **77**, 091503 (2008).
[26] B. Blossier, M. Wagner, and O. Pène, J. High Energy Phys. 06 (2009) 022.
[27] K. Abe *et al.* (BELLE Collaboration), arXiv:hep-ex/0412072.
[28] F. Jugeau, A. Le Yaouanc, L. Oliver, and J.-C. Raynal, Phys. Rev. D **72**, 094010 (2005).
[29] A. Le Yaouanc and O. Pène, arXiv:1408.5104.
[30] A. F. Falk and M. Neubert, Phys. Rev. D **47**, 2965 (1993).
[31] A. Leibovich, Z. Ligeti, I. Stewart, and M. Wise, Phys. Rev. D **57**, 308 (1998).
[32] M. E. Luke, Phys. Lett. B **252**, 447 (1990).
[33] I. Bigi, M. Shifman, N. Uraltsev, and A. Vainshtein, Phys. Rev. D **52**, 196 (1995); Annu. Rev. Nucl. Part. Sci. **47**, 591 (1997).
[34] H.-Y. Cheng, C.-K. Chua, and C.-W. Hwang, Phys. Rev. D **69**, 074025 (2004).
[35] M. Gómez-Rocha and W. Schweiger, Phys. Rev. D **86**, 053010 (2012).
[36] D. Ebert, R. Faustov, and V. Galkin, Phys. Rev. D **61**, 014016 (1999); **62**, 014032 (2000).