

Strong $\Lambda_b NB$ and $\Lambda_c ND$ vertices

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We investigate the strong vertices among the Λ_b , nucleon, and B meson as well as the Λ_c , nucleon, and D meson in QCD. In particular, we calculate the strong coupling constants $g_{\Lambda_b NB}$ and $g_{\Lambda_c ND}$ for different Dirac structures entering the calculations. In the case of the $\Lambda_c ND$ vertex, we compare the result with the only existing prediction obtained at $Q^2 = 0$.

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I. INTRODUCTION

The last decade has witnessed significant experimental progresses on the spectrum and decay products of the hadrons containing heavy quarks. This progress has stimulated theoretical interest on the spectroscopy of these baryons via various methods (for some of them see [1–11] and references therein). For a better understanding of the heavy flavor physics, it is also necessary to gain deeper insight into the radiative, strong, and weak decays of the baryons containing a heavy quark. For related studies, see [12–28] and references therein.

The strong coupling constants are the main ingredients of the strong interactions of the heavy baryons. To improve our understanding of the strong interactions among the heavy baryons and other hadrons and gain knowledge about the nature and structure of the participating particles, one needs an accurate determination of these coupling constants. In the present paper, we calculate the strong coupling constants $g_{\Lambda_b NB}$ and $g_{\Lambda_c ND}$ within the framework of the QCD sum rule [29] as one of the most powerful and applicable tools in hadron physics. These coupling constants are relevant in the bottom and charmed meson cloud description of the nucleon which may be used to explain exotic events observed by different collaborations. Besides, in order to exactly determine the modifications in the masses, decay constants and other parameters of the B and D mesons in the nuclear medium, we should immediately consider the contributions of the baryons $\Lambda_{b[c]}$ and $\Sigma_{b[c]}$ in the medium produced by the interactions of B and D mesons with the nucleon, viz.,

$$\begin{aligned} B^-(b\bar{u}) + p(uud) \quad \text{or} \quad n(udd) &\rightarrow \Lambda_b^0(udb) \\ \text{or} \quad \Sigma_b^-(ddb), \\ D^0(c\bar{u}) + p(uud) \quad \text{or} \quad n(udd) &\rightarrow \Lambda_c^+(udc) \\ \text{or} \quad \Sigma_c^+(udc) \quad \text{or} \quad \Sigma_c^0(ddc). \end{aligned} \quad (1)$$

Hence, we need to know the exact values of the strong coupling constants $g_{\Lambda_b NB}$, $g_{\Lambda_c ND}$, $g_{\Sigma_b NB}$, and $g_{\Sigma_c ND}$ entering the Born term in the calculations [30–34]. Note that among these couplings, we have only one approximate prediction for the strong coupling $g_{\Lambda_c ND}$ in the literature calculated at zero transferred momentum square, taking the Borel masses in the initial and final channels as the same [19]. We shall also refer to a pioneering work [35], which estimates the strong coupling constant $g_{N\Lambda}$. Here we should also stress that our work on the calculation of the strong coupling constants $g_{\Sigma_b NB}$ and $g_{\Sigma_c ND}$ is in progress.

The layout of this paper is as follows. The next section presents the details of the calculations of the strong coupling constants under consideration. In Sec. III, we numerically analyze the sum rules obtained and discuss the results.

II. THE STRONG COUPLING FORM FACTORS

The purpose of the present section is to give the details of the calculations of the coupling form factors $g_{\Lambda_b NB}(q^2)$ and $g_{\Lambda_c ND}(q^2)$. The values of these form factors at $Q^2 = -q^2 = -m_{B[D]}^2$ give the strong coupling constants among the participating particles. To fulfill this aim, the starting point is the usage of the following three-point correlation function,

$$\begin{aligned} \Pi(p, p', q) = i^2 \int d^4x \int d^4y e^{-ip \cdot x} e^{ip' \cdot y} \\ \times \langle 0 | \mathcal{T} (J_N(y) J_{B[D]}(0) \bar{J}_{\Lambda_b[\Lambda_c]}(x)) | 0 \rangle, \end{aligned} \quad (2)$$

where \mathcal{T} denotes the time-ordering operator and $q = p - p'$ is transferred momentum. The three-point correlation function contains interpolating currents that can be written in terms of the quark field operators as

$$\begin{aligned} J_{\Lambda_b[\Lambda_c]}(x) &= \varepsilon_{abc} u^{aT}(x) C \gamma_5 d^b(x) b[c]^c(x), \\ J_N(y) &= \varepsilon_{ijk} (u^{iT}(y) C \gamma_\mu u^j(y)) \gamma_5 \gamma_\mu d^k(y), \\ J_{B[D]}(0) &= \bar{u}(0) \gamma_5 b[c](0), \end{aligned} \quad (3)$$

where C is the charge conjugation operator.

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The three-point correlation function is calculated in the following two ways. In the first way, called the hadronic side, one calculates it in terms of the hadronic degrees of freedom. In the second way, called the operator product expansion (OPE) side, it is calculated in terms of quark and gluon degrees of freedom using the operator product expansion in the deep Euclidean region. These two sides are then matched to obtain the QCD sum rules for the coupling form factors. We apply a double

Borel transformation with respect to the variables p^2 and p'^2 to both sides to suppress the contributions of the higher states and continuum.

The calculation of the hadronic side of the correlation function requires its saturation with complete sets of appropriate $\Lambda_b[\Lambda_c]$, $B[D]$, and N hadronic states having the same quantum numbers as their interpolating currents. This step is followed by performing the four integrals over x and y , which leads to

$$\begin{aligned} \Pi(p, p', q) &= \frac{\langle 0|J_N|N(p')\rangle\langle 0|J_{B[D]}|B[D](q)\rangle\langle\Lambda_b[\Lambda_c](p)|\bar{J}_{\Lambda_b[\Lambda_c]}|0\rangle}{(p^2 - m_{\Lambda_b[\Lambda_c]}^2)(p'^2 - m_N^2)(q^2 - m_{B[D]}^2)} \\ &\times \langle N(p')B[D](q)|\Lambda_b[\Lambda_c](p)\rangle + \dots, \end{aligned} \quad (4)$$

where ... represents the contributions coming from the higher states and continuum. The matrix elements in this equation are parametrized as

$$\begin{aligned} \langle 0|J_N|N(p')\rangle &= \lambda_N u_N(p', s'), \\ \langle\Lambda_b(p)|\bar{J}_{\Lambda_b[\Lambda_c]}|0\rangle &= \lambda_{\Lambda_b[\Lambda_c]} \bar{u}_{\Lambda_b[\Lambda_c]}(p, s), \\ \langle 0|J_{B[D]}|B[D](q)\rangle &= i \frac{m_{B[D]}^2 f_{B[D]}}{m_u + m_{b[c]}}, \\ \langle N(p')B[D](q)|\Lambda_b[\Lambda_c](p)\rangle &= g_{\Lambda_b NB[\Lambda_c ND]} \bar{u}_N(p', s') \\ &\times i\gamma_5 u_{\Lambda_b[\Lambda_c]}(p, s), \end{aligned} \quad (5)$$

where λ_N and $\lambda_{\Lambda_b[\Lambda_c]}$ are the residues and u_N and $u_{\Lambda_b[\Lambda_c]}$ are the spinors for the nucleon and $\Lambda_b[\Lambda_c]$ baryon, respectively. In the above equations, $f_{B[D]}$ is the leptonic decay constant of the $B[D]$ meson and $g_{\Lambda_b NB[\Lambda_c ND]}$ is the strong coupling form factor among $\Lambda_b[\Lambda_c]$, N and $B[D]$ particles. The use of Eqs. (5) in Eq. (4) is followed by summing over the spins of the N and $\Lambda_b[\Lambda_c]$ baryons, i.e.,

$$\begin{aligned} \sum_{s'} u_N(p', s') \bar{u}_N(p', s') &= \not{p}' + m_N, \\ \sum_s u_{\Lambda_b[\Lambda_c]}(p, s) \bar{u}_{\Lambda_b[\Lambda_c]}(p, s) &= \not{p} + m_{\Lambda_b[\Lambda_c]}. \end{aligned} \quad (6)$$

As a result, we have

$$\begin{aligned} \Pi(p, p', q) &= i^2 \frac{m_{B[D]}^2 f_{B[D]}}{m_{b[c]} + m_u} \\ &\times \frac{\lambda_N \lambda_{\Lambda_b[\Lambda_c]} g_{\Lambda_b NB[\Lambda_c ND]}}{(p^2 - m_{\Lambda_b[\Lambda_c]}^2)(p'^2 - m_N^2)(q^2 - m_{B[D]}^2)} \\ &\times \{(m_N m_{\Lambda_b[\Lambda_c]} - m_{\Lambda_b[\Lambda_c]}^2)\gamma_5 \\ &+ (m_{\Lambda_b[\Lambda_c]} - m_N)\not{p}\gamma_5 + \not{q}\gamma_5 - m_{\Lambda_b[\Lambda_c]}\not{q}\gamma_5\} \\ &+ \dots. \end{aligned} \quad (7)$$

The final form of the hadronic side of the correlation function is obtained after the application of the double

Borel transformation with respect to the initial and final momenta squared, viz.,

$$\begin{aligned} \hat{\mathbf{B}}\Pi(q) &= i^2 \frac{m_{B[D]}^2 f_{B[D]} \lambda_N \lambda_{\Lambda_b[\Lambda_c]} g_{\Lambda_b NB[\Lambda_c ND]}}{m_{b[c]} + m_u} \frac{e^{-\frac{m_{\Lambda_b[\Lambda_c]}^2}{M^2}} e^{-\frac{m_N^2}{M'^2}}}{(q^2 - m_{B[D]}^2)} \\ &\times \{(m_N m_{\Lambda_b[\Lambda_c]} - m_{\Lambda_b[\Lambda_c]}^2)\gamma_5 (m_{\Lambda_b[\Lambda_c]} - m_N)\not{p}\gamma_5 \\ &+ \not{q}\gamma_5 - m_{\Lambda_b[\Lambda_c]}\not{q}\gamma_5\} + \dots, \end{aligned} \quad (8)$$

where M^2 and M'^2 are Borel mass parameters.

The OPE side of the correlation function is calculated in the deep Euclidean region, where $p^2 \rightarrow -\infty$ and $p'^2 \rightarrow -\infty$. To proceed, the explicit expressions of the interpolating currents are inserted into the correlation function in Eq. (2). After contracting out all quark pairs via Wick's theorem, we get

$$\begin{aligned} \Pi(p, p', q) &= i^2 \int d^4x \int d^4y e^{-ip \cdot x} e^{ip' \cdot y} \varepsilon_{abc} \varepsilon_{ij\ell} \\ &\times \{\gamma_5 \gamma_\mu S_d^{cj}(y-x) \gamma_5 C S_u^{biT}(y-x) \\ &\times C \gamma_\mu S_u^{ah}(y) \gamma_5 S_{b[c]}^{h\ell}(-x) - \gamma_5 \gamma_\mu S_d^{cj}(y-x) \gamma_5 \\ &\times C S_u^{aiT}(y-x) C \gamma_\mu S_u^{bh}(y) \gamma_5 S_{b[c]}^{h\ell}(-x)\}, \end{aligned} \quad (9)$$

where $S_{b[c]}^{i\ell}(x)$ represents the heavy quark propagator which is given by [36]

$$\begin{aligned} S_{b[c]}^{i\ell}(x) &= \frac{i}{(2\pi)^4} \int d^4k e^{-ik \cdot x} \left\{ \frac{\delta_{i\ell}}{k - m_{b[c]}} - \frac{g_s G_{i\ell}^{\alpha\beta}}{4} \right. \\ &\times \frac{\sigma_{\alpha\beta}(k + m_{b[c]}) + (k + m_{b[c]})\sigma_{\alpha\beta}}{(k^2 - m_{b[c]}^2)^2} \\ &\left. + \frac{\pi^2}{3} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \delta_{i\ell} m_{b[c]} \frac{k^2 + m_{b[c]}k}{(k^2 - m_{b[c]}^2)^4} + \dots \right\}, \end{aligned} \quad (10)$$

and $S_u(x)$ and $S_d(x)$ are the light quark propagators given by

$$\begin{aligned} S_q^{ij}(x) = & i \frac{x}{2\pi^2 x^4} \delta_{ij} - \frac{m_q}{4\pi^2 x^2} \delta_{ij} - \frac{\langle \bar{q}q \rangle}{12} \left(1 - i \frac{m_q}{4} x \right) \delta_{ij} \\ & - \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle \left(1 - i \frac{m_q}{6} x \right) \delta_{ij} \\ & - \frac{ig_s G_{\theta\eta}^{ij}}{32\pi^2 x^2} [x\sigma^{\theta\eta} + \sigma^{\theta\eta}x] + \dots \end{aligned} \quad (11)$$

The substitution of these explicit forms of the heavy and light quark propagators into Eq. (9) is followed by the use of the following Fourier transformations in $D = 4$ dimensions:

$$\begin{aligned} \frac{1}{[(y-x)^2]^n} &= \int \frac{d^D t}{(2\pi)^D} e^{-it \cdot (y-x)} i (-1)^{n+1} 2^{D-2n} \pi^{D/2} \\ &\quad \times \frac{\Gamma(D/2 - n)}{\Gamma(n)} \left(-\frac{1}{t^2} \right)^{D/2-n}, \\ \frac{1}{[y^2]^n} &= \int \frac{d^D t'}{(2\pi)^D} e^{-it' \cdot y} i (-1)^{n+1} 2^{D-2n} \pi^{D/2} \\ &\quad \times \frac{\Gamma(D/2 - n)}{\Gamma(n)} \left(-\frac{1}{t'^2} \right)^{D/2-n}. \end{aligned} \quad (12)$$

Then, the four- x and four- y integrals are performed in the sequel of the replacements $x_\mu \rightarrow i \frac{\partial}{\partial p_\mu}$ and $y_\mu \rightarrow -i \frac{\partial}{\partial p'_\mu}$. As a result, these integrals turn into Dirac delta functions which are used to take the four-integrals over k and t' . Finally the Feynman parametrization and

$$\begin{aligned} & \int d^4 t \frac{(t^2)^\beta}{(t^2 + L)^\alpha} \\ &= \frac{i\pi^2 (-1)^{\beta-\alpha} \Gamma(\beta+2) \Gamma(\alpha-\beta-2)}{\Gamma(2) \Gamma(\alpha) [-L]^{\alpha-\beta-2}} \end{aligned} \quad (13)$$

are used to perform the remaining four-integral over t .

The correlation function in the OPE side is obtained in terms of different structures as

$$\begin{aligned} \Pi(p, p', q) = & \Pi_1(q^2) \gamma_5 + \Pi_2(q^2) \not{p} \gamma_5 + \Pi_3(q^2) \not{q} \not{p} \gamma_5 \\ & + \Pi_4(q^2) \not{q} \gamma_5, \end{aligned} \quad (14)$$

where each $\Pi_i(q^2)$ function includes the contributions coming from both the perturbative and nonperturbative parts and can be written as

$$\Pi_i(q^2) = \int ds \int ds' \frac{\rho_i^{\text{pert}}(s, s', q^2) + \rho_i^{\text{nonpert}}(s, s', q^2)}{(s-p^2)(s'-p'^2)}. \quad (15)$$

The imaginary parts of the Π_i functions give the spectral densities $\rho_i(s, s', q^2)$ appearing in the last equation, viz. $\rho_i(s, s', q^2) = \frac{1}{\pi} \text{Im}[\Pi_i]$. As examples, we present only the explicit forms of the spectral functions $\rho_1^{\text{pert}}(s, s', q^2)$ and $\rho_1^{\text{nonpert}}(s, s', q^2)$ corresponding to the Dirac structure γ_5 , which are obtained as

$$\begin{aligned} \rho_1^{\text{pert}}(s, s', q^2) = & \left\{ -\frac{m_{b[c]} m_u s'^2}{64\pi^4 (q^2 - m_{b[c]}^2)} \Theta[L_1(s, s', q^2)] + \int_0^1 dx \int_0^{1-x} dy \frac{1}{64\pi^4 u^3} \right. \\ & \times [2m_{b[c]}^4 x^2 (1 + 3x^2 - y + 6xy - 4x) + m_{b[c]}^3 x (3m_d u (2x - 1) + m_u (3 + 2x^2 - 3y - 5x - 2xy)) \\ & + 2m_{b[c]}^2 x (s(12x^4 + y^2 - y - 30x^3 + 36x^3 y - 6x + 20xy - 13xy^2 + 24x^2 - 55x^2 y + 24x^2 y^2) \\ & + q^2 xy (18x - 24xy + 7y - 12x^2 - 6) + s'y (12x^3 + 7y - 4y^2 - 27x^2 + 36x^2 y + 18x - 43xy + 24xy^2 - 3)) \\ & + 2s^2 u^2 x (10x^3 + 6x - 15xy + 2y - 16x^2 + 20x^2 y) + 2q^4 x^2 y^2 (10x^2 - 7y - 16x + 20xy + 6) \\ & + 2s^2 y^2 u^2 (10x^2 - 3y - 12x + 20xy) - 4q^2 s' xy^2 (10x^3 + 9y - 5y^2 - 24x^2 + 30x^2 y + 18x - 39xy + 20xy^2 - 4) \\ & + 2suy (q^2 x (32x^2 - 40x^2 y - 20x^3 - 2y - 13x + 22xy + 1) \\ & + s' (20x^4 - 48x^3 + 60x^3 y - y + y^2 - 8x + 27xy - 18xy^2 + 36x^2 - 86x^2 y + 40x^2 y^2)) \\ & + 3m_{b[c]} m_u u (q^2 x (x + 2y - 3xy - 1) + sux (3x - 1) + s'u (3xy - x - y)) - m_{b[c]} m_u (q^2 x (3x^2 y - 3x^2 \\ & + 7y - 4y^2 + 6x - 10xy - 3xy^2 - 3) - sux (3x^2 - y - 6x - 6xy + 3) \\ & \left. - 3s'u (x^2 y - x^2 + y - y + x - 3xy - xy^2)) \right\} \Theta[L_2(s, s', q^2)], \end{aligned} \quad (16)$$

and

$$\begin{aligned}
\rho_1^{\text{nonpert}}(s, s', q^2) = & \left\{ \frac{1}{16\pi^2(m_{b[c]}^2 - q^2)} [2m_{b[c]}m_d m_u \langle \bar{d}d \rangle + (m_{b[c]}(3m_u^2 - 3m_d m_u - 2s') + m_d(4m_u^2 + s - s') + 2m_u s') \langle \bar{u}u \rangle] \right. \\
& - \left\langle \alpha_s \frac{G^2}{\pi} \right\rangle \left[\frac{m_{b[c]}m_u q^2 s^2}{192\pi^2(q^2 - m_{b[c]}^2)^4} - \frac{9m_{b[c]}s'(m_d + m_u) + 2s'(q^2 - 2s + 5s')}{1152\pi^2(q^2 - m_{b[c]}^2)^2} - \frac{m_{b[c]}(m_d - 3m_u)}{128\pi^2(q^2 - m_{b[c]}^2)} \right] \\
& - m_0^2 \langle \bar{d}d \rangle \frac{3m_{b[c]} + 4m_d}{96\pi^2(m_{b[c]}^2 - q^2)} + m_0^2 \langle \bar{u}u \rangle \frac{9m_{b[c]} + 3m_d - 7m_u}{96\pi^2(m_{b[c]}^2 - q^2)} \left. \right\} \Theta[L_1(s, s', q^2)] \\
& + \int_0^1 dx \int_0^{1-x} dy \left\{ \frac{1}{8\pi^2 u} [\langle \bar{d}d \rangle (m_{b[c]} - 2m_{b[c]}x - m_u u + m_d(3x - 1)(y + u)) \right. \\
& + \langle \bar{u}u \rangle (m_{b[c]} - 2m_{b[c]}x - 4m_d u - 2m_u(y - 3xy - 3xu))] \\
& \left. + \left\langle \alpha_s \frac{G^2}{\pi} \right\rangle \frac{1}{96\pi^2 u^3} [3u^2(3x - 1)(y + u) + xy(1 - y + x(3x + 6y - 4))] \right\} \Theta[L_2(s, s', q^2)], \quad (17)
\end{aligned}$$

where

$$\begin{aligned}
L_1(s, s', q^2) &= s', \\
L_2(s, s', q^2) &= -m_{b[c]}^2 x + sx - sx^2 + s'y \\
&\quad + q^2 xy - sxy - s'xy - s'y^2, \\
u &= x + y - 1, \quad (18)
\end{aligned}$$

with $\Theta[\dots]$ being the unit-step function.

As we previously mentioned, the QCD sum rules for the strong form factors are obtained by matching the hadronic and OPE sides of the correlation function. As a result, for γ_5 structure, we get

$$\begin{aligned}
& g_{\Lambda_b N B[\Lambda_c N D]}(q^2) \\
&= -e^{\frac{m_{\Lambda_b[\Lambda_c]}^2}{M^2}} e^{\frac{m_N^2}{M'^2}} \frac{(m_{b[c]} + m_u)(q^2 - m_{B[D]}^2)}{m_{B[D]}^2 f_{B[D]} \lambda_{\Lambda_b[\Lambda_c]}^\dagger \lambda_N (m_N m_{\Lambda_b[\Lambda_c]} - m_{\Lambda_b[\Lambda_c]}^2)} \\
&\quad \times \left\{ \int_{(m_{b[c]} + m_u + m_d)^2}^{s_0} ds \int_{(2m_u + m_d)^2}^{s'_0} ds' e^{-\frac{s}{M^2}} e^{-\frac{s'}{M'^2}} \right. \\
&\quad \left. \times [\rho_1^{\text{pert}}(s, s', q^2) + \rho_1^{\text{nonpert}}(s, s', q^2)] \right\}, \quad (19)
\end{aligned}$$

where s_0 and s'_0 are continuum thresholds in $\Lambda_b[\Lambda_c]$ and N channels, respectively.

III. NUMERICAL RESULTS

This section contains the numerical analysis of the obtained sum rules for the strong coupling form factors including their behavior in terms of $Q^2 = -q^2$. For the analysis, we use the input parameters given in Table I.

The analysis starts by the determination of the working regions for the auxiliary parameters M^2 , M'^2 , s_0 and s'_0 . These parameters, which arise due to the double Borel

transformation and continuum subtraction, are not physical parameters so the strong coupling form factors should be almost independent of these parameters. Being related to the energy of the first excited states in the initial and final channels, the continuum thresholds are not completely arbitrary. The continuum thresholds s_0 and s'_0 are the energy squares which characterize the beginning of the continuum. If we denote the ground states masses in the initial and final channels respectively by m and m' , the quantities $\sqrt{s_0} - m$ and $\sqrt{s'_0} - m'$ are the energies needed to excite the particles to their first excited states with the same quantum numbers. The $\sqrt{s_0} - m$ and $\sqrt{s'_0} - m'$ are well known for the states under consideration [37], where they lie roughly between 0.1 GeV and 0.3 GeV.

TABLE I. Input parameters used in calculations.

Parameters	Values
m_b	(4.18 ± 0.03) GeV [37]
m_c	(1.275 ± 0.025) GeV [37]
m_d	$4.8_{-0.3}^{+0.5}$ MeV [37]
m_u	$2.3_{-0.5}^{+0.7}$ MeV [37]
m_B	(5279.26 ± 0.17) MeV [37]
m_D	(1864.84 ± 0.07) MeV [37]
m_N	$(938.272046 \pm 0.000021)$ MeV [37]
m_{Λ_b}	(5619.5 ± 0.4) MeV [37]
m_{Λ_c}	(2286.46 ± 0.14) MeV [37]
f_B	$(248 \pm 23_{\text{exp}} \pm 25_{Vub})$ MeV [38]
f_D	$(205.8 \pm 8.5 \pm 2.5)$ MeV [39]
λ_N^2	0.0011 ± 0.0005 GeV ⁶ [40]
λ_{Λ_b}	$(3.85 \pm 0.56)10^{-2}$ GeV ³ [22]
λ_{Λ_c}	$(3.34 \pm 0.47)10^{-2}$ GeV ³ [22]
$\langle \bar{u}u \rangle (1 \text{ GeV})$	$-(0.24 \pm 0.01)^3$ GeV ³ [41]
$\langle \bar{d}d \rangle (1 \text{ GeV})$	
$\langle \frac{\alpha_s G^2}{\pi} \rangle$	(0.012 ± 0.004) GeV ⁴ [42]
$m_0^2 (1 \text{ GeV})$	(0.8 ± 0.2) GeV ² [42]

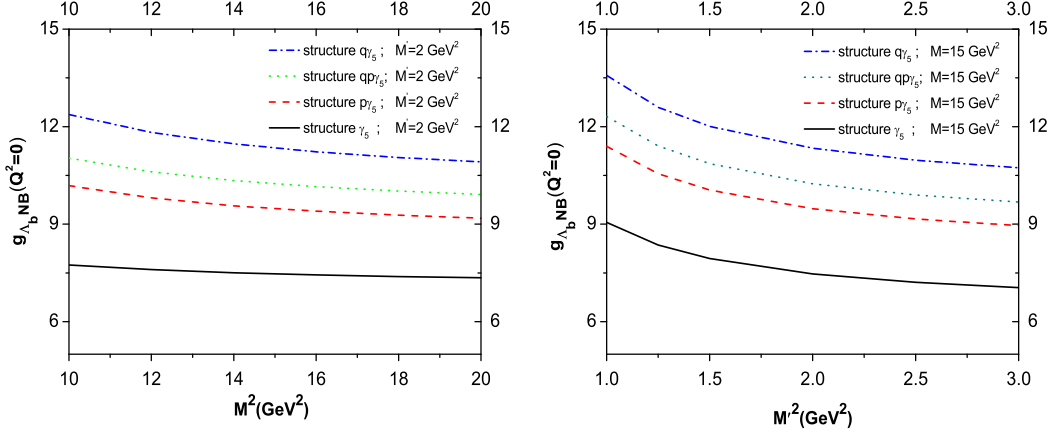


FIG. 1 (color online). *Left:* $g_{\Lambda_b NB}(Q^2 = 0)$ as a function of the Borel mass M^2 at average values of continuum thresholds. *Right:* $g_{\Lambda_b NB}(Q^2 = 0)$ as a function of the Borel mass M^2 at average values of continuum thresholds.

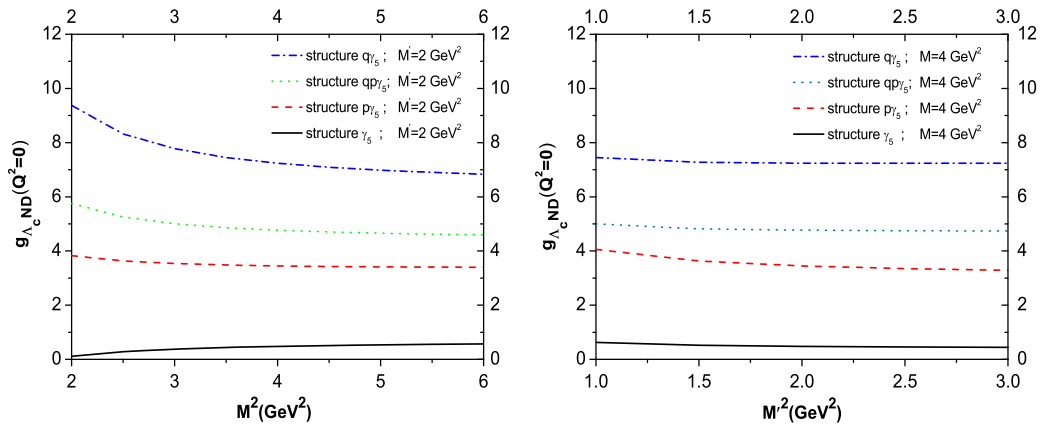


FIG. 2 (color online). The same as Fig. 1 but for $g_{\Lambda_c ND}(Q^2 = 0)$.

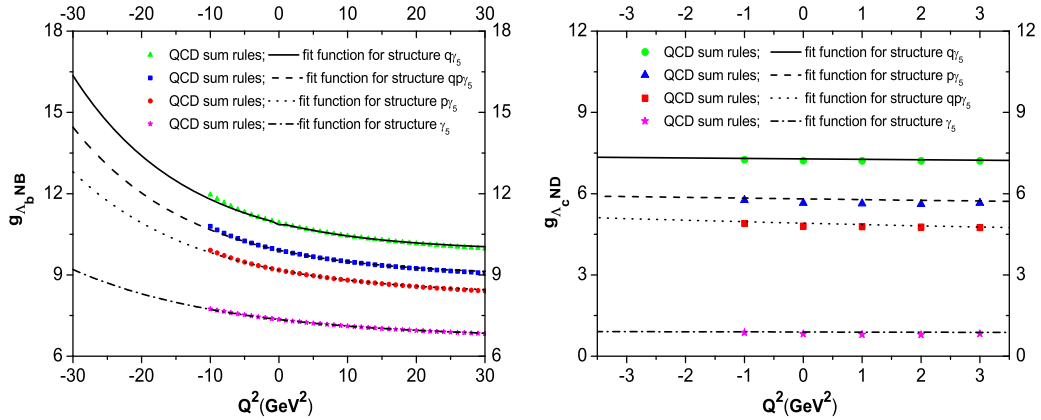


FIG. 3 (color online). *Left:* $g_{\Lambda_b NB}(Q^2)$ as a function of Q^2 at average values of the continuum thresholds and Borel mass parameters. *Right:* $g_{\Lambda_c ND}(Q^2)$ as a function of Q^2 at average values of the continuum thresholds and Borel mass parameters.

These values lead to the working intervals of the continuum thresholds as $32.7[5.7] \text{ GeV}^2 \leq s_0 \leq 34.5[6.7] \text{ GeV}^2$ and $1.08 \text{ GeV}^2 \leq s'_0 \leq 1.56 \text{ GeV}^2$ for the strong vertex $\Lambda_b NB$ [$\Lambda_c ND$].

In the determination of the working regions of Borel parameters M^2 and M'^2 , one considers the pole dominance as well as the convergence of the OPE. In technique language, the upper bounds on these parameters are obtained

TABLE II. Parameters appearing in the fit function of the coupling form factor for $\Lambda_b NB$ vertex.

Structure	c_1	$c_2(\text{GeV}^2)$	c_3
γ_5	0.69 ± 0.21	22.96 ± 6.66	6.67 ± 2.00
$p\gamma_5$	0.90 ± 0.27	18.60 ± 5.58	8.28 ± 2.32
$q\gamma_5$	1.04 ± 0.31	16.40 ± 4.92	9.87 ± 2.67
$qp\gamma_5$	0.95 ± 0.28	17.10 ± 4.79	8.96 ± 2.69

by requiring that the pole contribution exceeds the contributions of the higher states and continuum; i.e., the condition

$$\frac{\int_{s_0}^{\infty} ds \int_{s'_0}^{\infty} ds' e^{-\frac{s}{M^2}} e^{-\frac{s'}{M'^2}} \rho_i(s, s', Q^2)}{\int_{s_{\min}}^{\infty} ds \int_{s'_{\min}}^{\infty} ds' e^{-\frac{s}{M^2}} e^{-\frac{s'}{M'^2}} \rho_i(s, s', Q^2)} < 1/3 \quad (20)$$

should be satisfied, where for each structure $\rho_i(s, s', Q^2) = \rho_i^{\text{pert}}(s, s', Q^2) + \rho_i^{\text{nonpert}}(s, s', Q^2)$, $s_{\min} = (m_{b[c]} + m_u + m_d)^2$ and $s'_{\min} = (2m_u + m_d)^2$. The lower bounds on M^2 and M'^2 are obtained by demanding that the contribution of the perturbative part exceeds the nonperturbative contributions. These considerations lead to the windows $10[2] \text{ GeV}^2 \leq M^2 \leq 20[6] \text{ GeV}^2$ and $1 \text{ GeV}^2 \leq M'^2 \leq 3 \text{ GeV}^2$ for the Borel mass parameters corresponding to the strong vertex $\Lambda_b NB[\Lambda_c ND]$ in which our results have weak dependencies on the Borel mass parameters (see Figs. 1–2).

Now, we use the working regions of auxiliary parameters as well as values of other input parameters to find out the dependency of the strong coupling form factors on Q^2 . Our numerical calculations reveal that the following fit function well describes the strong coupling form factors in terms of Q^2 ,

$$g_{\Lambda_b NB[\Lambda_c ND]}(Q^2) = c_1 \exp\left[-\frac{Q^2}{c_2}\right] + c_3, \quad (21)$$

where the values of the parameters c_1 , c_2 , and c_3 for different structures are presented in Tables II and III for $\Lambda_b NB$ and $\Lambda_c ND$, respectively. In Fig. 3, we depict the dependence of the strong coupling form factors on Q^2 at average values of the continuum thresholds and Borel mass parameters for both the QCD sum rules and fitting results. From this figure, we see that the QCD sum rules are truncated at some points at negative values of Q^2 and the fitting results coincide well with the sum rules predictions up to these points. The values of the strong coupling constants obtained from the fit function at $Q^2 = -m_{B[D]}^2$ for all structures are given in Table IV. The errors appearing in the results are due to the uncertainties of the input

TABLE III. Parameters appearing in the fit function of the coupling form factor for the $\Lambda_c ND$ vertex.

Structure	c_1	$c_2(\text{GeV}^2)$	c_3
γ_5	-0.08 ± 0.02	-17.74 ± 4.79	0.97 ± 0.29
$p\gamma_5$	-9.01 ± 2.70	-328.82 ± 98.65	14.81 ± 4.00
$q\gamma_5$	-20.04 ± 5.81	-1221.76 ± 366.53	27.32 ± 8.20
$qp\gamma_5$	0.86 ± 0.26	16.63 ± 4.82	4.05 ± 1.22

TABLE IV. Values of the $g_{\Lambda_b NB}$ and $g_{\Lambda_c ND}$ coupling constants for different structures.

Structure	$g_{\Lambda_b NB}(Q^2 = -m_B^2)$	$g_{\Lambda_c ND}(Q^2 = -m_D^2)$
γ_5	8.97 ± 2.69	0.91 ± 0.27
$p\gamma_5$	12.31 ± 3.57	5.90 ± 1.77
$q\gamma_5$	15.57 ± 4.67	7.34 ± 2.20
$qp\gamma_5$	13.81 ± 4.14	5.11 ± 1.53
Average	12.67 ± 3.76	4.82 ± 1.44

parameters and those coming from the calculations of the working regions for the auxiliary parameters. From Table IV, we see that all structures except that γ_5 lead to very close results. We also depict the average of the coupling constants under consideration, obtained from all the structures used, in Table IV.

At this stage, we compare our result of the coupling constant $g_{\Lambda_c ND}$ obtained at $Q^2 = 0$ with that of Ref. [19] for the Dirac structure $q\gamma_5$. At $Q^2 = 0$, we get the result $g_{\Lambda_c ND} = 7.28 \pm 2.18$ for this structure, which is consistent with the prediction of [19], i.e., $g_{\Lambda_c ND} = \sqrt{4\pi}(1.9 \pm 0.6) = 6.74 \pm 2.12$ within the errors.

To summarize, we have calculated the strong coupling constants $g_{\Lambda_b NB}$ and $g_{\Lambda_c ND}$ in the framework of the three-point QCD sum rules. Our results can be used in the bottom and charmed meson cloud description of the nucleon, which may be used to explain exotic events observed by different experiments. The obtained results can also be used in analysis of the results of heavy ion collision experiments like *PANDA* at FAIR. These results may also be used in exact determinations of the modifications in the masses, decay constants, and other parameters of the B and D mesons in nuclear medium.

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