

## Radiative decay of keV-mass sterile neutrinos in a strongly magnetized plasma

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The radiative decay of sterile neutrinos with typical masses of 10 keV is investigated in the presence of a strong magnetic field and degenerate plasma. A full account is taken of the strongly modified photon dispersion relation relative to vacuum. The limiting cases of relativistic and nonrelativistic plasma are analyzed. The decay rate in a strongly magnetized plasma as a function of the electron number density is compared with the unmagnetized case. We find that a strong magnetic field suppresses the catalyzing influence of the plasma on the decay rate.

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### I. INTRODUCTION

The weak interaction strength of neutrinos as well as their small masses single them out among all elementary particles. While neutrinos play almost no role on Earth, their role in astrophysics and cosmology is important and sometimes dominant. In particular, this pertains to astrophysical cataclysms like core-collapse supernova explosions or coalescence of neutron stars. In these phenomena, a dense and hot plasma interacting with a strong neutrino flux arises. It has become clear that strong magnetic fields of up to  $10^{16}$  G can be generated, exceeding the electron-mass critical field  $B_e = m_e^2/e \approx 4.41 \times 10^{13}$  G. Neutrino processes are also important for the cooling of supernova cores and neutron stars where neutrinos are emitted from the dense central region. Observations of neutron stars lead to a wide spread of magnetic-field values, and very large magnetic fields  $B \gtrsim 10^{15}$  G have been identified in some objects called magnetars [1,2]. Therefore, studying properties and dynamics of such astrophysical phenomena requires a detailed understanding of quantum processes involving neutrinos under the influence of a strong magnetic field and relativistic plasma.

The plasma and magnetic field are optically active media and therefore can significantly influence the photon-neutrino interaction that in vacuum arises at loop level and turns out to be extremely weak. On the other hand, the photon-neutrino interaction within a medium can lead to actually observed effects, notably the neutrino luminosity of a plasma by the  $\gamma \rightarrow \nu\bar{\nu}$  decay [3]. In this process, the plasma has two effects: it provides photons with an effective mass, enabling the decay kinematics, and it provides an effective interaction between neutrinos and

photons. On the other hand, the radiative decay of a massive neutrino is kinematically allowed in vacuum (see, for example, Ref. [4] and references therein). However, an active medium can influence both the decay amplitude and particle kinematics, and hence, the decay rate can change significantly [5,6].

Early studies of the radiative decay of a massless neutrino in a magnetic field were performed in Refs. [7–9]. (Note that the process  $\nu_i \rightarrow \nu_j\gamma$  in the presence of external fields or media has been called “radiative decay,” “Cherenkov effect,” or “bremsstrahlung” in the literature.) The radiative decay of a massive neutrino  $\nu_i \rightarrow \nu_j + \gamma$  with  $i \neq j$  in the framework of the Standard Model with lepton mixing was considered in Ref. [10] for electromagnetic fields of different configurations. In all of these papers, the decay probability was calculated for low-energy neutrinos ( $E_\nu < 2m_e$ ) and under the assumption that the modification of the photon dispersion law can be neglected. In addition, it was shown that the field-induced amplitude of the ultrarelativistic neutrino decay in a magnetic field is not suppressed by the smallness of the neutrino mass, in contrast to vacuum [10].

We recall that with increasing the photon energy, its dispersion in a strong  $B$  field differs from vacuum and each photon polarization has its own dispersion law [11–13]. In particular, the photon four-momentum  $q^\mu$  can be space-like and its square can be sufficiently large,  $|q^2| \gg m_\nu^2$ , to allow the transition  $\nu_i \rightarrow \nu_j + \gamma$  of a lighter neutrino to a heavier one ( $m_i < m_j$ ). In other words, the strongly modified photon dispersion law implies that in practice the radiative decay probability of ultrarelativistic neutrinos in strong magnetic fields does not depend on the neutrino mass spectrum.

For high-energy neutrinos ( $E_\nu \gg m_e$ ) in a strong constant magnetic field, the process  $\nu \rightarrow \nu + \gamma$  was studied in Ref. [14], taking account of the appropriate photon dispersion. The same process in a homogeneous magnetic

\*Nicolay Vladimirovich Mikheev passed away on 19 June 2014 before this manuscript could be completed. The remaining authors hope to have finished this work in his spirit.

field was considered in detail in Ref. [9] for low-energy neutrinos ( $E_\nu < 2m_e$ ) and in the kinematical region where the photon dispersion is similar to vacuum. The neutrino radiative decay was also investigated in plasma [15–21]. In particular, the decay probability of a heavier neutrino to a lighter one and a photon in a thermal medium was calculated in Refs. [17,18] under the assumption that the particle dispersion relations were not affected by the plasma.

Later, the study of the neutrino-photon interaction was extended to high energies in a strongly magnetized electron-positron plasma [22]. In this case, apart from the modified photon dispersion, large radiative corrections exist near the  $e^-e^+$  resonance—otherwise the result is overestimated.

Most recently, the decay of a massive neutrino was analyzed for the conditions of a strongly magnetized, degenerate electron gas [23]. There are no theoretical restrictions on the existence of astrophysical objects where both a strong magnetic field and degenerate plasma can exist. Several objects called magnetars [1,2] have been observed that probably contain such a medium, i.e., 14 Soft Gamma-ray Repeaters of which 10 are confirmed and 4 are candidates as well as 14 Anomalous X-ray Pulsars with 12 being confirmed and candidates [24]. The existence of such objects motivates the study of elementary processes under extreme conditions.

The main point of our paper is to extend the analysis of Ref. [23] to include the modified photon dispersion relation. As a motivation we note that in a strongly magnetized plasma, the neutrino-photon interaction is mainly determined by electrons occupying the lowest Landau level. Therefore, the electron chemical potential should satisfy  $\mu_e^2 - m_e^2 < 2eB$ . If the plasma is degenerate ( $\mu_e - m_e \gg T$ ), the plasma frequency is [23,25,26]

$$\omega_0^2 = \frac{2\alpha}{\pi} eB \frac{p_F}{\sqrt{p_F^2 + m_e^2}}, \quad (1)$$

where  $p_F$  is the electron Fermi momentum. The electron number density in a strongly magnetized electron gas is  $n_e = eB p_F / (2\pi^2)$  [27]. This relation allows us to express the plasma frequency of Eq. (1) in the form

$$\omega_0 \approx 37.1 \text{ keV} \left( \frac{n_{30}^2 b^2}{b^2 + 1.3n_{30}^2} \right)^{1/4}, \quad (2)$$

where  $b = B/B_e$  and  $n_{30} = n_e / (10^{30} \text{ cm}^{-3})$ . Our benchmark number density ( $10^{30} \text{ cm}^{-3}$ ), interpreted here as a baryon density, corresponds approximately to a mass density of  $10^6 \text{ g cm}^{-3}$ , where degenerate electrons would still be nonrelativistic.

For the conditions of interest, a typical scale of  $\omega_0$  is therefore 10 keV or larger. Ordinary neutrinos have sub-eV masses so that radiative decays would not be kinematically

possible. Of course, the presence of electrons implies a weak potential for electron neutrinos of  $\sqrt{2}G_F n_e \approx 1.27 \times 10^{-7} \text{ eV} n_{30}$ , which is a very small effect compared with the plasma frequency. Therefore, it is the modification of the photon dispersion relation that tends to be the dominant effect. It is clear that radiative decays would be of interest only for sterile neutrinos  $\nu_s$  with keV masses and above. There has been renewed interest in such particles recently as a possible warm or cold dark matter candidate [28–32]. Moreover, the observation of an unexplained 3.5 keV X-ray line, possibly caused by the  $\nu_s \rightarrow \nu_a \gamma$  decay of dark-matter sterile neutrinos, has recently electrified the community [33–37].

Whatever the final verdict on such speculations, we here go through the exercise of calculating the radiative decay of nonrelativistic sterile neutrinos in an optically active medium that can be identified with both an unmagnetized or a strongly magnetized plasma. Our main new point beyond the previous literature is to include the photon dispersion relation consistently. We limit our discussion to Dirac neutrinos—the Majorana case should only differ by numerical factors. We neglect the modified active neutrino dispersion relation in the final state.

We begin in Sec. II with the simpler case of an unmagnetized degenerate plasma for comparison with our main calculation in Sec. III, the strongly magnetized case. In Sec. IV we summarize our findings.

## II. UNMAGNETIZED PLASMA

A sterile neutrino  $\nu_s$  can mix with an active species and in this way interact with matter, where  $\theta_s$  is the usual mixing angle. It is assumed to be very small so that  $\nu_s$  essentially coincides with a propagation eigenstate of mass  $m_s$ . For the radiative decay  $\nu_s \rightarrow \nu_a \gamma$  in vacuum one finds the probability (or rather decay rate) [4]

$$W_{\text{vac}} = \frac{9\alpha G_F^2}{2048\pi^4} m_s^5 \sin^2(2\theta_s). \quad (3)$$

This result pertains to the Dirac case, whereas for Majorana neutrinos the rate is a factor of 2 larger and then agrees with what is usually stated in the sterile-neutrino literature [28]. We will frequently use this vacuum result to normalize our results.

Turning next to an unmagnetized electron plasma, the contribution to the radiative decay amplitude is defined by the neutrino-photon interaction via real electrons. The neutrino-electron interaction is described by the effective local Lagrangian [9]

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} [\bar{\Psi}_e \gamma^\alpha (C_V - C_A \gamma_5) \Psi_e] j_\alpha, \quad (4)$$

where  $\Psi_e$  is the electron field.  $C_V = \pm 1/2 + 2 \sin^2 \theta_W$  and  $C_A = \pm 1/2$  with the Weinberg angle  $\theta_W$  are the vector and

axial-vector coefficients, respectively, which take into account the  $Z$ - and  $W$ -boson exchange. The plus sign pertains to  $\nu_e$ , and the minus sign to  $\nu_\mu$  and  $\nu_\tau$ .

The neutrino current  $j_\alpha$  in Eq. (4) describes the transition of a heavy neutrino  $\nu_s$  with a mass of several keV to a light neutrino  $\nu_a$  with a sub-eV mass,

$$j_\alpha = \cos \theta_s \sin \theta_s [\bar{\nu}_a \gamma_\alpha (1 - \gamma_5) \nu_s]. \quad (5)$$

The vector current in the Lagrangian (4) has the same structure as the standard electron interaction with a photon,  $\mathcal{L}_{\text{QED}} = e(\bar{\Psi}_e \gamma_\alpha \Psi_e) A^\alpha$ . Therefore, the decay  $\nu_s \rightarrow \nu_a + \gamma$  in plasma corresponds to the Feynman graphs shown in Fig. 1, which is identical to the one shown in Fig. 2 after one of the photon lines has been replaced by the neutrino current.

It is well known that the amplitude of the  $\gamma \rightarrow \gamma$  transition shown in Fig. 2 determines the polarization operator  $\Pi^{\alpha\beta}$  of the photon [38,39]

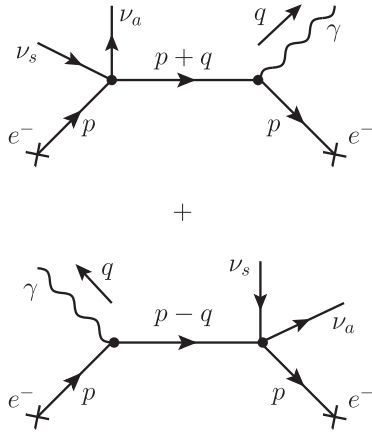


FIG. 1. Feynman graphs for the  $\nu_s \rightarrow \nu_a + \gamma$  decay in plasma. The crosses attached at the ends of the electron lines signify that these particles pertain to the plasma. In the magnetized case, the magnetic field is included on the electron lines.

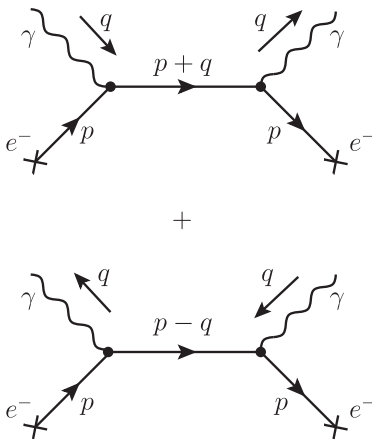


FIG. 2. Diagrams for the photon forward scattering on plasma electrons in analogy to Fig. 1.

$$M_{\gamma \rightarrow \gamma} = -\varepsilon_\alpha^* \Pi^{\alpha\beta} \varepsilon_\beta. \quad (6)$$

Therefore, the vector part of the  $\nu_s \rightarrow \nu_a + \gamma$  amplitude can be expressed in terms of the photon polarization operator  $\Pi^{\alpha\beta}$  in plasma,

$$M_{\text{pl}}^{(V)} = \frac{C_V G_F}{e\sqrt{2}} (j_\alpha \Pi^{\alpha\beta} \varepsilon_\beta^*), \quad (7)$$

where  $\varepsilon_\beta$  is the photon polarization vector.

The corresponding axial-vector contribution is much smaller. In a nonrelativistic plasma one finds explicitly that it is suppressed by a factor  $(C_A/C_V)(m_s/m_e) \ll 1$ . In a relativistic plasma,  $m_e$  is replaced by the chemical potential  $\mu_e$ . We conclude that the axial coupling contributes very little to the process, in analogy to a photon absorption by neutrinos [15] and for a plasmon decay into neutrino pairs [40,41].

As mentioned earlier, photons in plasma acquire an effective mass in the form of the plasma frequency  $\omega_0$ . Under a wide range of conditions,  $\omega_0$  is small enough to fulfill the kinematical conditions for  $\nu_s \rightarrow \nu_a \gamma$  with  $m_s$  of several tens of keV,

$$\omega_0 < m_s \ll m_e. \quad (8)$$

We concentrate on a nonrelativistic plasma where

$$\omega_0^2 = \frac{4\pi a n_e}{m_e}, \quad (9)$$

where  $n_e = p_F^3/(3\pi^2)$  for degenerate electrons. Therefore, the kinematical condition (8) restricts the Fermi velocity to  $V_F^2 < 0.25(m_s/(10 \text{ keV}))^{4/3}$ . This condition provides an upper bound  $m_s \ll 30 \text{ keV}$  for which the nonrelativistic approximation is appropriate.

Photons in plasma have three polarization modes, one longitudinal (polarization vector  $\varepsilon^\ell$ ) and two transverse ( $\varepsilon^t$ ). They are the eigenvectors of the polarization operator  $\Pi_{\alpha\beta}$  and determine the corresponding set of eigenvalues  $\Pi_\lambda$  ( $\lambda = \ell, t$ ). In a nonrelativistic plasma they are  $\Pi_t \approx \omega_0^2$  and  $\Pi_\ell \approx \omega_0^2(1 - k^2/\omega^2)$ , where  $k = |\mathbf{k}|$  is the photon momentum.

The probability for  $\nu_s \rightarrow \nu_a \gamma$  can be written in the form

$$W_{\text{pl}}^\lambda = \frac{1}{32\pi^2 m_s} \int Z_{A\lambda} |M_{\text{pl}}^\lambda|^2 [1 + f_\gamma(\omega)] \times \delta(m_s - k - \omega) \frac{d^3 \mathbf{k}}{k\omega}, \quad (10)$$

where  $f_\gamma(\omega)$  is the photon distribution function. In a cold plasma ( $T \ll \omega_0$ ), the deviation of the photon stimulation factor  $[1 + f_\gamma(\omega)]$  from unity can be neglected. The factor  $Z_{A\lambda}$  accounts for the renormalized wave function of the photon,

$$Z_{A\lambda}^{-1} = 1 - \frac{\partial \Pi_\lambda}{\partial \omega^2}. \quad (11)$$

The matrix element is largely determined by the vector part of Eq. (7). In terms of the eigenvalues and eigenvectors of the photon polarization operator we find

$$|M_{\text{pl}}^\lambda|^2 = \frac{G_F^2 C_V^2}{16\pi\alpha} \sin^2(2\theta_s) [m_s^2 - q^2 + 4(p\epsilon^\lambda)^2] \Pi_\lambda^2. \quad (12)$$

The  $\nu_s$  decay probabilities are then found to be

$$W_{\text{pl}}^t = \frac{(G_F \omega_0^2)^2 C_V^2}{128\pi^2 \alpha} \sin^2(2\theta_s) m_s \left(1 - \frac{\omega_0^2}{m_s^2}\right)^2, \quad (13)$$

$$W_{\text{pl}}^\ell = \frac{(G_F m_s^2)^2 C_V^2}{64\pi^2 \alpha} \sin^2(2\theta_s) \omega_0 \left(1 - \frac{\omega_0}{m_s}\right)^2. \quad (14)$$

The rate with a transverse photon coincides with a well-known result in the limit  $\omega_0 \rightarrow 0$  [18]. Besides the different phase space, the longitudinal case involves a nontrivial wave-function renormalization factor  $Z_{A\ell}$ .

We finally express Eqs. (13) and (14) in terms of the vacuum rate of Eq. (3) and find

$$W_{\text{pl}}^t = W_{\text{vac}} \frac{32\pi^2}{18\alpha^2} x_0^4 (1 - x_0^2)^2, \quad (15)$$

$$W_{\text{pl}}^\ell = W_{\text{vac}} \frac{32\pi^2}{9\alpha^2} x_0 (1 - x_0)^2, \quad (16)$$

where we have introduced  $x_0 = \omega_0/m_s$  in terms of the plasma frequency (9). The kinematical constraint  $x_0 < 1$  implies that typically the decay into longitudinal plasmons is much faster than into transverse ones.

In Fig. 3 we show the total rate  $W_{\text{pl}} = W_{\text{pl}}^\ell + W_{\text{pl}}^t$  as a function of the electron density (dashed lines). The strong

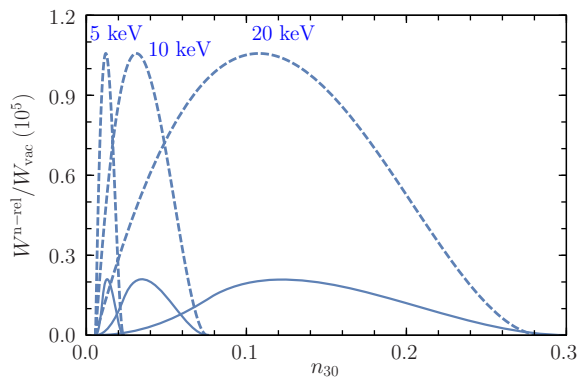


FIG. 3 (color online). Sterile-neutrino radiative decay probability for the indicated mass values as a function of the electron density  $n_{30} = n_e/(10^{30} \text{ cm}^{-3})$ . Dashed lines: unmagnetized plasma. Solid lines: strongly magnetized plasma with  $B = B_e = 4.41 \times 10^{13} \text{ G}$ .

catalyzing effect of the plasma is clearly seen with an enhancement of up to 5 orders of magnitude compared with vacuum. There is also a maximum of these functions for an electron density  $n_e$ , which moves to larger number densities with increasing the neutrino mass.

### III. STRONGLY MAGNETIZED PLASMA

#### A. Analytic calculation

In the strongly magnetized case the neutrino-photon interaction is defined by the same effective Lagrangian (4) as before. However, the electron field now is a superposition of solutions of the Dirac equation in a strong  $B$  field. We assume that the hierarchy of plasma parameters is

$$2eB > \mu_e^2 - m_e^2 \gg T^2, \quad (17)$$

and we take the magnetic field to be oriented along the third axis, i.e.,  $\mathbf{B} = (0, 0, B)$ .

The neutrino-photon interaction is mainly determined by electrons on the lowest Landau level [42]. Therefore, the electron quantum field  $\Psi_e$  is an eigenfunction of the projection operator [6,43]

$$\Pi_- = \frac{1 + i(\gamma\varphi\gamma)}{2} = \frac{1 - i\gamma_1\gamma_2}{2}, \quad (18)$$

where  $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$  is the dimensionless tensor of the external magnetic field. We use the shorthand notation  $(\gamma\varphi\gamma) = \gamma^\alpha \varphi_{\alpha\beta} \gamma^\beta$  for the contraction of Lorentz indices.

The properties of this projection operator reveal an effective equality [6,43]

$$\Pi_- \gamma_\alpha \gamma_5 \Pi_- = (\tilde{\varphi}\gamma)_\alpha \Pi_-, \quad (19)$$

where  $\tilde{\varphi}_{\alpha\beta} = \tilde{F}_{\alpha\beta}/B$  is the dual dimensionless tensor of the external magnetic field and  $(\tilde{\varphi}\gamma)_\alpha = \tilde{\varphi}_{\alpha\beta} \gamma^\beta$ . This equality differs from zero only at  $\alpha = 0$  and 3. Therefore, we may transform the axial-vector electron current in the Lagrangian (4) to a vector current of the form

$$\bar{\Psi}_e \gamma_\alpha \gamma_5 \Psi_e = \bar{\Psi}_e \Pi_- \gamma_\alpha \gamma_5 \Pi_- \Psi_e = \bar{\Psi}_e (\tilde{\varphi}\gamma)_\alpha \Psi_e, \quad (20)$$

where  $\Pi_- \Psi_e = \Psi_e$  was used. Therefore, Eq. (4) becomes

$$\mathcal{L}_{\text{eff}} = e(\bar{\Psi}_e \gamma^\alpha \Psi_e) V_\alpha, \quad (21)$$

where we have introduced the local vector operator

$$V_\alpha = -\frac{G_F}{e\sqrt{2}} [C_V(\tilde{\Lambda}j)_\alpha + C_A(\tilde{\varphi}j)_\alpha]. \quad (22)$$

The Lorentz tensor  $\tilde{\Lambda}_{\mu\nu} = (\tilde{\varphi}\tilde{\varphi})_{\mu\nu}$  determines the metric of the two-dimensional Minkowski subspace of the four-dimensional space-time [6,43]. The direct analogy of the

Lagrangian (21) with the electromagnetic case  $\mathcal{L}_{\text{QED}} = e(\bar{\Psi}_e \gamma_\alpha \Psi_e) A^\alpha$  again allows us to map results from electro-dynamics to neutrino processes.

The diagrams for  $\nu_s \rightarrow \nu_a + \gamma$  and photon forward scattering in a strongly magnetized plasma are the same as before (Figs. 1 and 2), except that the electrons are now states in a strong  $B$  field. The incoming-photon polarization vector  $\mathcal{E}_\alpha^{(\lambda)}$  ( $\lambda = 1, 2, 3$ ) in Fig. 2 is replaced with the effective neutrino current  $V_\alpha$  of Eq. (22) in Fig. 1. We implicitly assume forward scattering of a photon of definite polarization  $\lambda$  and the production of a photon with the same polarization in the sterile-neutrino decay. The basis of photon polarization vectors  $\mathcal{E}_\alpha^{(\lambda)}$  generally differs from the basis  $\varepsilon_\alpha^{(\lambda)}$  of the unmagnetized case. The new polarization operator  $\Pi_{\alpha\beta}$  receives contributions from both the plasma and the external magnetic field. The eigenvalue problem is now rather complicated and has not yet been solved in the general case [44].

However, limiting cases provide simplifications and allow us to find analytic solutions. A strongly magnetized electron plasma is a beautiful case in point. In particular, the eigenvalues and eigenvectors of the corresponding photon polarization operator were found as a power expansion in the inverse magnetic field strength [45]. In this plasma there are only two physical states of the photon [44] that largely coincide with the photon polarization vectors in the constant uniform magnetic field [6,43]

$$\mathcal{E}_\alpha^{(1)} \approx \frac{(q\varphi)_\alpha}{\sqrt{q_\perp^2}} \quad \text{and} \quad \mathcal{E}_\alpha^{(2)} \approx \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q_\parallel^2}}. \quad (23)$$

The shorthand  $q_\perp^2 = q_\mu \varphi^{\mu\nu} \varphi_{\nu\rho} q^\rho$  and  $q_\parallel^2 = q_\mu \tilde{\varphi}^{\mu\nu} \tilde{\varphi}_{\nu\rho} q^\rho$  were used. The third polarization vector  $\mathcal{E}_\alpha^{(3)}$  is reduced to the photon four-momentum  $q_\mu$  and can be eliminated by a gauge transformation [44,46].

Therefore, the sterile-neutrino decay amplitude also requires the corresponding eigenvalues  $\Pi_\lambda$  of the polarization operator with  $\lambda = 1$  and 2, which are [44,47]

$$\Pi_1 \approx -\frac{2\alpha}{\pi} \omega \mu_e V_F \sqrt{\frac{q_\perp^2}{q_\parallel^2}}, \quad (24)$$

$$\Pi_2 \approx \frac{2\alpha}{\pi} e B V_F \frac{q_\parallel^2}{\omega^2 - V_F^2 k_3^2}. \quad (25)$$

Here,  $\omega$  is the photon energy,  $k_3$  the projection of the photon momentum on the  $B$ -field direction, and  $V_F = p_F/\mu_e$  the Fermi velocity. Equations (24) and (25) apply when the kinematical condition  $\omega \lesssim m_s \ll m_e$  is satisfied.

To go further, it is instructive to compare the above eigenvalues under the plasma conditions of Eq. (17). With the values of the parameters entering Eqs. (24) and (25) close to what is maximally allowed, i. e.,  $\omega \sim m_s$ ,  $k_3 \ll m_s$ , and  $q^2, q_\parallel^2 \sim m_s^2$ , one easily obtains

$$\left| \frac{\Pi_1}{\Pi_2} \right| \approx \frac{\mu_e m_s}{e B} \lesssim \frac{m_s}{\mu_e} \ll 1. \quad (26)$$

This means that if both eigenvalues contribute to the decay amplitude with weights of the same order in  $m_s$ , terms with  $\Pi_1$  can be neglected in the amplitude.

Let us apply the procedure explained above that was successfully worked out in the case of pure plasma. More precisely, after the replacement of the photon polarization vector  $\mathcal{E}_\beta^{(\lambda)} \rightarrow V_\beta$  by the neutrino current in Eq. (6), one can express the sterile-neutrino decay amplitude through the photon polarization operator  $\Pi_{\alpha\beta}$  as

$$M_{\text{pl+f}} = \frac{G_F}{e\sqrt{2}} \mathcal{E}_\alpha^{(\lambda)*} \Pi^{\alpha\beta} [C_V (\tilde{\Lambda} j)_\beta + C_A (\tilde{\varphi} j)_\beta]. \quad (27)$$

Comparison of the amplitude  $M_{\text{pl+f}}$  obtained with the similar one of Eq. (7) calculated in the pure plasma shows that  $C_A$  appears and can no longer be neglected as will be demonstrated later. Taking into account the hierarchy of the polarization operator eigenvalues in Eq. (26), mainly photons with the polarization  $\lambda = 2$  are produced in this decay. So, the photon polarization vector should be identified with  $\mathcal{E}_\alpha^{(2)}$ . As a result, the decay amplitude is

$$M_{\text{pl+f}} = \frac{G_F}{e\sqrt{2}} \Pi_2 [C_V (\mathcal{E}^{(2)*} \tilde{\Lambda} j) + C_A (\mathcal{E}^{(2)*} \tilde{\varphi} j)], \quad (28)$$

where the neutrino current  $j_\alpha$  is given in Eq. (5). The effective neutrino current  $V_\alpha$  in the strongly magnetized plasma, where all electrons are on the lowest Landau level, is the projection of  $j_\alpha$  on the two-dimensional Minkowski subspace and thus is orthogonal to the other polarization vector with  $\lambda = 1$ , i. e.,  $(\mathcal{E}^{(1)} V) = 0$ .

After substituting the polarization vector  $\mathcal{E}_\alpha^{(2)}$  (23) and corresponding eigenvalue  $\Pi_2$  (25) in Eq. (28), we arrive at the final form of the decay amplitude

$$M_{\text{pl+f}} = \frac{G_F \Omega_0^2}{e\sqrt{2}} \sqrt{q_\parallel^2} \frac{C_V (q\tilde{\varphi} j) + C_A (q\tilde{\Lambda} j)}{\omega^2 - V_F^2 k_3^2}. \quad (29)$$

We have introduced the plasma frequency

$$\Omega_0^2 = \frac{2\alpha e B}{\pi} V_F, \quad (30)$$

relevant in the magnetized electron plasma.

The probability of  $\nu_s \rightarrow \nu_a + \gamma$  requires a phase-space integration of the amplitude squared (29), including the appropriate dispersion relations. The magnetized plasma does not strongly modify the active-neutrino dispersion properties. To get the modified dispersion relation for a photon with polarization  $\lambda$  one needs to solve

$$q^2 = \Pi_\lambda. \quad (31)$$

For a photon with  $\lambda = 2$  it can be written as

$$\omega^2 = k_3^2 + k_\perp^2 + \Omega_0^2 \frac{\omega^2 - k_3^2}{\omega^2 - V_F^2 k_3^2}. \quad (32)$$

When the photon momentum vanishes,  $k_3^2 = k_\perp^2 = 0$ , the photon energy is  $\omega = \Omega_0$  and means the effective photon mass in the magnetized plasma. Note that the plasma frequency squared (30) differs from the similar quantity (9) defined in the unmagnetized plasma.

The  $\nu_s \rightarrow \nu_a + \gamma$  decay can occur only if  $\Omega_0 < m_s$ . This requirement restricts the Fermi velocity to

$$V_F^2 < 0.01 \left( \frac{B_e}{B} \right)^2 \left( \frac{m_s}{10 \text{ keV}} \right)^4. \quad (33)$$

This expression shows that the radiative decay of a sterile neutrino with the mass 2–20 keV in a highly magnetized plasma requires the latter to be nonrelativistic.

The decay probability has the standard form of an integral over phase space of the final-state particles

$$W_{\text{pl+f}} = \frac{1}{32\pi^2 m_s} \int \frac{d^3 \mathbf{p}_a d^3 \mathbf{k}}{E_a \omega} \delta^{(4)}(p_s - p_a - q) [1 + f_\gamma(\omega)] \times Z_{A2} |M_{\text{pl+f}}|^2, \quad (34)$$

where  $p_s^\mu = (m_s, \mathbf{0})$  is the  $\nu_s$  four-momentum in its rest frame,  $p_a^\mu = (E_a, \mathbf{p}_a)$  is the four-momentum of the active neutrino, and the factor  $Z_{A2}$  defined in Eq. (11) accounts for the photon wave-function renormalization.

After performing the integration over the active neutrino momentum  $\mathbf{p}_a$  and the azimuth angle in the cylindrical momentum frame of the photon, Eq. (34) becomes

$$W_{\text{pl+f}} = \frac{1}{32\pi m_s} \int_{-\infty}^{+\infty} dk_3 \int_0^\infty \frac{dk_\perp^2}{E_a \omega} \times \delta(m_s - E_a - \omega) [1 + f_\gamma(\omega)] Z_{A2} |M_{\text{pl+f}}|^2.$$

The remaining integrations are not simple as one should include the nontrivial photon dispersion relation of Eq. (32) and thus the  $\nu_a$  energy in the form  $E_a = m_s - \omega$ . It is convenient to remove the variable  $k_\perp^2$  in favor of  $\omega$  by  $dk_\perp^2 = 2\omega |\partial k_\perp^2 / \partial \omega| d\omega$ . In the new variables  $k_3$  and  $\omega$ , the integration area is divided into two parts, leading to

$$W_{\text{pl+f}} = \frac{1}{16\pi m_s} \left( \int_{\Omega_0}^{\Omega_F} d\omega \int_0^{k_{3F}} dk_3 F(\omega, k_3) + \int_{\Omega_F}^\infty d\omega \int_0^\omega dk_3 F(\omega, k_3) \right), \quad (35)$$

where  $\Omega_F = \Omega_0 / \sqrt{1 - V_F^2}$  and  $k_{3F} = \sqrt{\omega^2 - \Omega_0^2} / V_F$ . The integrand  $F(\omega, k_3)$  in Eq. (35) can be represented as

$$F(\omega, k_3) = \left| \frac{\partial k_\perp^2}{\partial \omega^2} \right| \frac{\delta(m_s - E_a - \omega)}{E_a} [1 + f_\gamma(\omega)] \times (|M_{\text{pl+f}}|^2 + |M_{\text{pl+f}}|_{k_3 \rightarrow -k_3}^2). \quad (36)$$

The calculation of the squared matrix element is not complicated, and one finds

$$\begin{aligned} & |M_{\text{pl+f}}|^2 + |M_{\text{pl+f}}|_{k_3 \rightarrow -k_3}^2 \\ &= \frac{(G_F \Omega_0^2)^2}{8\alpha\pi} \sin^2(2\theta_s) \frac{q_\parallel^2}{(\omega^2 - V_F^2 k_3^2)^2} \\ &\times \{4m_s^2 [C_A^2 \omega^2 + C_V^2 k_3^2] \\ &+ q_\parallel^2 [(C_V^2 - C_A^2)(m_s^2 - q^2) - 4C_A^2 m_s \omega]\}. \end{aligned} \quad (37)$$

This is our final analytical result for the probability of the sterile-neutrino radiative decay.

## B. Approximations and limiting cases

In applications it may be more useful to have a simple approximate formula valid in certain parameter ranges. We adopt  $m_s = 2\text{--}20$  keV as before and  $B = 1\text{--}10B_e$  to guarantee strong magnetization. In particular, for  $m_s = 10$  keV and  $B = 10B_e$  we find

$$\begin{aligned} \frac{W_{\text{pl+f}}}{W_{\text{vac}}} &\approx \frac{\pi^2}{\alpha^2} \left[ 15.93 \frac{(1-x_0)^{0.65}}{x_0^{18.09}} \exp\left(-11.79 \frac{(1-x_0)}{x_0}\right) \right. \\ &\left. + 1168.96(1-x_0)^{1.46} x_0^{3.88} \exp\left(-0.089 \frac{x_0}{1-x_0}\right) \right], \end{aligned} \quad (38)$$

where  $x_0 = \Omega_0 / m_s$ . The first function within the square brackets mainly determines the behavior at large  $x_0$  values, while the second one is for small  $x_0$ . The variation of  $m_s$  and  $B$  in our chosen parameter range causes only very small changes in the approximation formula. Also, the impact of the stimulating statistical factor  $[1 + f_\gamma(\omega)]$  is numerically small as in the unmagnetized plasma.

In the same parameter range we can get another approximate representation for the decay probability. Equation (33) reveals that the Fermi velocity is always small. In the  $V_F \ll 1$  limit the integrand in Eq. (36) becomes a relatively simple function and can be integrated analytically,

$$\begin{aligned} W_{\text{pl+f}}^{\text{n-rel}} &= W_{\text{vac}} \frac{256\pi^2}{25515\alpha^2} (C_V^2 + C_A^2) \\ &\times [\theta(2x_0 - 1) F_1^{\text{n-rel}}(x_0) + \theta(1 - 2x_0) F_2^{\text{n-rel}}(x_0)], \end{aligned} \quad (39)$$

where the functions  $F_{1,2}^{\text{n-rel}}(x_0)$  are

$$\begin{aligned}
F_1^{n\text{-rel}}(x_0) &= \frac{2835x_0^4}{32} \int_0^{1/x_0-1} dx(1-x^2)[1+x_0^2(1-x^2)] \\
&\quad \times [1+3x^2-x_0^2(1-x^2)^2] \\
&= -\frac{11}{x_0} + 129x_0 - 210x_0^2 + 168x_0^3 - 84x_0^4 \\
&\quad - 24x_0^6 + 32x_0^8, \tag{40}
\end{aligned}$$

$$\begin{aligned}
F_2^{n\text{-rel}}(x_0) &= \frac{2835x_0^4}{32} \int_0^1 dx(1-x^2)[1+x_0^2(1-x^2)] \\
&\quad \times [1+3x^2-x_0^2(1-x^2)^2] \\
&= 4x_0^4(21+6x_0^2-8x_0^4). \tag{41}
\end{aligned}$$

The integration variable is  $x = \omega/m_s$ . The reduced plasma frequency  $x_0 = \Omega_0/m_s$  is restricted to the interval  $0 < x_0 < 1$  because of the decay kinematics. The variation of  $W_{\text{pl+f}}^{n\text{-rel}}$  with  $x_0$  is shown in Fig. 4 where both Eqs. (38) and (39) coincide numerically.

In Fig. 3 we compare the decay rate for the unmagnetized (dashed lines) and strongly magnetized (solid lines) plasma as a function of the electron density. For the chosen field strength  $B = B_e$  the decay rate is strongly suppressed, but, of course, it is still much faster than in vacuum. The maximum decay rate is shifted to somewhat larger electron densities, reflecting the different dependence of the plasma frequency on  $n_e$ .

At  $C_V = C_A = 1$  and  $\Omega_0 \ll m_s$  ( $x_0 \ll 1$ ) we reproduce the result of Ref. [23],

$$W_{\text{pl+f}}^{n\text{-rel}} = \frac{256\pi^2}{135\alpha^2} x_0^4 W_{\text{vac}}. \tag{42}$$

For relativistic and strongly magnetized conditions, the plasma frequency is  $\Omega_0 \approx 34.7 \text{ keV} \sqrt{B/B_e}$  and  $\nu_s \rightarrow \nu_a \gamma$  with  $m_s < 20 \text{ keV}$  requires  $B < B_e/3$ . At larger  $B$  values the decay mode  $\gamma \rightarrow \nu_a + \nu_s$  opens. Therefore, we now implicitly consider sufficiently heavy sterile neutrinos.

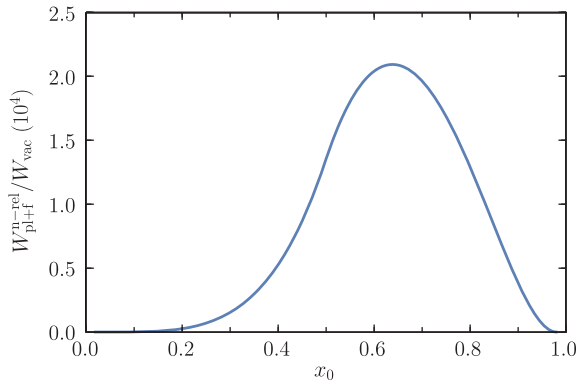


FIG. 4 (color online). The radiative decay probability of sterile neutrinos in a nonrelativistic strongly magnetized plasma as a function of the reduced plasma frequency  $x_0 = \Omega_0/m_s$ .

In the relativistic limit  $V_F \approx \sqrt{1 - m_e^2/\mu_e^2} \rightarrow 1$  and Eq. (35) simplifies to

$$\begin{aligned}
W_{\text{pl+f}}^{\text{rel}} &= \frac{(G_F m_s^2)^2}{64\pi^2 \alpha} m_s \sin^2(2\theta_s) (C_V^2 + C_A^2) \frac{x_0^4 (1+x_0^2)}{1 - e^{-m_s(1+x_0^2)/(2T)}} \\
&\quad \times \left[ F(x_0, V_F) + \theta \left( 1 - x_0 \sqrt{\frac{1+V_F}{1-V_F}} \right) \tilde{F}(x_0, V_F) \right]. \tag{43}
\end{aligned}$$

Analytical expressions for the functions  $F(x_0, V_F)$  and  $\tilde{F}(x_0, V_F)$  are given in the Appendix. This result further simplifies in the limiting case of a very small plasma frequency,  $x_0 \ll m_e/\mu_e$ ,

$$W_{\text{pl+f}}^{\text{rel}} \approx \frac{(G_F \Omega_0^2)^2}{64\pi^2 \alpha} m_s \sin^2(2\theta_s) (C_V^2 + C_A^2) \frac{\ln(2\mu_e/m_e) - 5/4}{1 - e^{-m_s/(2T)}}. \tag{44}$$

A simplification also obtains in the opposite limit  $x_0 \gg m_e/\mu_e$ ,

$$\begin{aligned}
W_{\text{pl+f}}^{\text{rel}} &\approx \frac{(G_F m_s^2)^2}{64\pi^2 \alpha} m_s \sin^2(2\theta_s) (C_V^2 + C_A^2) \frac{x_0^4}{1 - e^{-m_s(1+x_0^2)/(2T)}} \\
&\quad \times \left[ (1+x_0^2) \ln \frac{1}{x_0} - \frac{1}{8} (1-x_0^2)(3+x_0^2) \right]. \tag{45}
\end{aligned}$$

Notice that this result applies close to the kinematical limit  $m_s$ , i.e., for  $x_0 \rightarrow 1$ .

#### IV. CONCLUSIONS

We have studied the radiative decay  $\nu_s \rightarrow \nu_a + \gamma$  with cosmologically interesting masses of some 10 keV in a dense magnetized and unmagnetized electron plasma. Our work goes beyond the previous literature in that for the first time we have consistently included the modified photon dispersion relation. The kinematical requirement that the photon effective mass must be smaller than  $m_s$  implies that we should typically restrict the plasma parameters to nonrelativistic conditions.

The decay rate in plasma is much larger than in vacuum because the neutrino-photon interaction is mediated by plasma electrons instead of virtual states. In the unmagnetized case, the enhancement is some 5 orders of magnitude, in detail depending on the electron density. In a strongly magnetized plasma the enhancement is significantly smaller. A strong  $B$  field slows the rate down because the contributing electrons are restricted to the lowest Landau level. It is also noteworthy that here the electron axial-current interaction  $C_A$  contributes on the same level as the vector-current  $C_V$ , in contrast to the unmagnetized case where the vector current dominates by far. This difference would be especially important if the final state active flavor

is not  $\nu_e$  because for  $\nu_\mu$  and  $\nu_\tau$  the vector-coupling constant  $C_V$  nearly vanishes.

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### APPENDIX: PROBABILITY IN THE LIMIT OF RELATIVISTIC PLASMA

The probability of the sterile-neutrino radiative decay in a relativistic magnetized electron plasma has the form

$$W_{\text{pl+nf}}^{\text{rel}} = \frac{(G_F m_s^2)^2}{64\pi^2 \alpha} m_s \sin^2(2\theta_s) (C_V^2 + C_A^2) \times [1 - e^{-m_s(1+x_0^2)/(2T)}]^{-1} x_0^4 (1+x_0^2) \times \left[ \int_0^a dx f(x, x_0) + \Theta\left(1 - x_0 \sqrt{\frac{1+V_F}{1-V_F}}\right) \int_a^1 dx f(x, x_0) \right], \quad (\text{A1})$$

where  $x = 2k_3/[m_s(1+x_0^2)]$ ,  $x_0 = \Omega_0/m_s$ ,  $a = (1-x_0^2)/(1+x_0^2)$ ,  $\Theta(x)$  is the unit-step function, and the integrand is

$$f(x, x_0) = \frac{1-x^2}{(1-V_F^2 x^2)^2} - \frac{3+x_0^2}{4} \frac{(1-x^2)^2}{(1-V_F^2 x^2)^2}. \quad (\text{A2})$$

So, there are two simple integrals:

$$F_1(y, V_F) = \int_0^y \frac{(1-x^2)dx}{(1-V_F^2 x^2)^2} = -\frac{y}{2V_F^2} \left( \frac{1-V_F^2}{1-V_F^2 y^2} + \frac{1+V_F^2}{2V_F y} \ln \frac{1-V_F y}{1+V_F y} \right), \quad (\text{A3})$$

$$F_2(y, V_F) = \int_0^y \frac{(1-x^2)^2 dx}{(1-V_F^2 x^2)^2} = \frac{y}{2V_F^4} \left( 2 + \frac{(1-V_F^2)^2}{1-V_F^2 y^2} + \frac{(3+V_F^2)(1-V_F^2)}{2V_F y} \ln \frac{1-V_F y}{1+V_F y} \right). \quad (\text{A4})$$

The two integrals in Eq. (A1) are

$$F(x_0, V_F) \equiv \int_0^a dx f(x, x_0) = F_{12}(a, V_F) + \frac{1-x_0^2}{4} F_2(a, V_F), \quad (\text{A5})$$

$$\tilde{F}(x_0, V_F) \equiv \int_a^1 dx f(x, x_0) = F_{12}(1, V_F) - F_{12}(a, V_F) + \frac{1-x_0^2}{4} [F_2(1, V_F) - F_2(a, V_F)], \quad (\text{A6})$$

where it is convenient to use the difference of the integrals (A3) and (A4),

$$F_{12}(y, V_F) \equiv F_1(y, V_F) - F_2(y, V_F) = -\frac{y}{2V_F^4} \left[ 2 + \frac{1-V_F^2}{1-V_F^2 y^2} + \frac{3-V_F^2}{2V_F y} \ln \frac{1-V_F y}{1+V_F y} \right]. \quad (\text{A7})$$

We substitute  $x_0^2 = (1-a)/(1+a)$  in Eqs. (A5) and (A6) and use the specific values of the functions (A4) and (A7)

$$F_2(1, V_F) = \frac{1}{2V_F^4} \left[ 3 - V_F^2 + \frac{(3+V_F^2)(1-V_F^2)}{2V_F} \ln \frac{1-V_F}{1+V_F} \right], \quad (\text{A8})$$

$$F_{12}(1, V_F) = -\frac{1}{2V_F^4} \left[ 3 + \frac{3-V_F^2}{2V_F} \ln \frac{1-V_F}{1+V_F} \right] \quad (\text{A9})$$

in Eq. (A6). We thus arrive at the final analytical result Eq. (43) for the decay probability of the sterile neutrino.



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