Tachyon condensation on a nonstationary Dp-brane with background fields in superstring theory

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Using the boundary state formalism we obtain the partition function corresponding to a dynamical (rotating-moving) Dp-brane in the presence of electromagnetic and tachyonic background fields in the superstring theory. The instability of such a Dp-brane due to the tachyon condensation is investigated.

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I. INTRODUCTION

D-branes can be described in terms of closed string states; hence by using the boundary state formalism many interesting properties have been shown [1–9]. By means of the boundary state, all relevant properties of the D-branes could be revealed. The boundary state formalism has been applied to the various D-brane configurations in the presence of different background fields [10–14].

On the other hand, investigating the stability of D-branes is one of the most important subjects that can be studied via the tachyon dynamics of open string and tachyon condensation phenomenon [15]. These concepts have been verified by various methods [16–18] and more recently by the boundary string field theory (BSFT) in different configurations [19–24]. It has been conjectured that the open string tachyon condensation describes the decay of unstable D-branes into the closed string vacuum or to the lower dimensional unstable D-branes as intermediate states. Study of this physical process namely, decaying of unstable objects, is an important phenomenon because of its interpolation between two different vacua and also because it is a way to reach the concept of the background independent formulation of string theory.

Some aspects of the boundary state, accompanied by the tachyon condensation, are as follows. The boundary state is a source for closed strings; therefore, by using this state and tachyon condensation, one can find the time evolution of the source for each closed string mode. Also it has been argued that the boundary state description of the rolling tachyon is valid during the finite time that is determined by string coupling, and the energy could be dissipated into the bulk beyond this time [23]. Moreover, this method shows the decoupling of the open string modes at the nonperturbative minima of the tachyon potential [25].

Previously we have calculated the boundary states associated with a dynamical (rotating and moving) Dp-brane in the presence of the electromagnetic and tachyonic background fields [12,24]. Now, by making use of the same boundary state we shall construct the corresponding partition function, which is obtained by the BSFT method. Then, we shall examine the instability of a Dp-brane. We demonstrate that this process can make such a dynamical brane unstable, and hence reduces the brane's dimension.

II. BOUNDARY STATE OF A DYNAMICAL BRANE

For constructing a boundary state corresponding to a dynamical (rotating-moving) D-brane in the presence of some background fields, we start with the action

$$
\begin{aligned}
S = &-\frac{1}{4\pi\alpha'}\int_{\Sigma} d^2\sigma\Big(\sqrt{-g}g^{ab}G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu \\
&+ \varepsilon^{ab}B_{\mu\nu}\partial_a X^\mu \partial_b X^\nu\Big) \\
&+\frac{1}{2\pi\alpha'}\int_{\partial\Sigma} d\sigma (A_\alpha \partial_\sigma X^\alpha + \omega_{\alpha\beta}J^{\alpha\beta}_\tau + T(X^\alpha)), \quad (1)
\end{aligned}
$$

where Σ and $\partial\Sigma$ are the world sheet of closed string and its boundary, respectively. This action contains the Kalb-Ramond field $B_{\mu\nu}$, a $U(1)$ gauge field A_α, an ω term for rotation and motion of the brane, and a tachyonic field. We shall apply $\{X^\alpha | \alpha = 0, 1, \ldots, p\}$ for the world volume directions of the brane and $\{X^i | i = p+1, \ldots, d-1\}$ for directions perpendicular to it.

The background fields $G_{\mu\nu}$ and $B_{\mu\nu}$ are considered to be constant, and for the $U(1)$ gauge field we use the gauge $A_\alpha = -\frac{1}{2}F_{\alpha\beta}X^\beta$ that possesses a constant field strength. Besides, the tachyon profile $T = \frac{1}{2}U_{\alpha\beta}X^\alpha X^\beta$ will be used, where the symmetric matrix $U_{\alpha\beta}$ is constant. The ω term, which is responsible for the brane's rotation and motion, contains the antisymmetric angular velocity $\omega_{\alpha\beta}$ and angular momentum density $J^{\alpha\beta}_\tau$ that is given by $\omega_{\alpha\beta}J^{\alpha\beta}_\tau = 2\omega_{\alpha\beta}X^\alpha\partial_\tau X^\beta$. In fact, the component $\omega_{0\bar{\alpha}}|_{\bar{\alpha}\neq 0}$ denotes the velocity of the brane along the direction $X^{\bar{\alpha}}$ while $\omega_{\bar{\alpha}\bar{\beta}}$ represents its rotation.

It should be noted that the rotation and motion of the brane are considered to be in its volume. In fact, according to the various fields inside the brane, the Lorentz symmetry

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is broken, and hence such a dynamic (rotation and motion) is sensible.

Suppose that the following mixed elements vanish, i.e., $B_{\alpha i} = U_{\alpha i} = 0$. The oscillating part of the bosonic boundary state is given by

$$|B_{\rm Bos}\rangle^{(\rm osc)} = \prod_{n=1}^{\infty} [\det Q_{(n)}]^{-1} \times \exp\left[-\sum_{m=1}^{\infty} \frac{1}{m} \alpha^{\mu}_{-m} S_{(m)\mu\nu} \tilde{\alpha}^{\nu}_{-m}\right] |0\rangle_{\alpha} \otimes |0\rangle_{\tilde{\alpha}}, \tag{2}$$

in which the matrices are as follows:

$$\begin{aligned} Q_{(n)\alpha\beta} &= \eta_{\alpha\beta} - \mathcal{F}_{\alpha\beta} + \frac{i}{2n} U_{\alpha\beta}, \\ S_{(m)\mu\nu} &= (\Delta_{(m)\alpha\beta}, -\delta_{ij}), \\ \Delta_{(m)\alpha\beta} &= (M^{-1}_{(m)} N_{(m)})_{\alpha\beta}, \\ M_{(m)\alpha\beta} &= \eta_{\alpha\beta} + 4\omega_{\alpha\beta} - \mathcal{F}_{\alpha\beta} + \frac{i}{2m} U_{\alpha\beta}, \\ N_{(m)\alpha\beta} &= \eta_{\alpha\beta} + 4\omega_{\alpha\beta} + \mathcal{F}_{\alpha\beta} - \frac{i}{2m} U_{\alpha\beta}, \\ \mathcal{F}_{\alpha\beta} &= \partial_\alpha A_\beta - \partial_\beta A_\alpha - B_{\alpha\beta}. \end{aligned} \tag{3}$$

The normalization factor $\prod_{n=1}^{\infty} [\det Q_{(n)\alpha\beta}]^{-1}$ is an effect of the disk partition function. In addition, the zero-mode part of the bosonic boundary state has the feature

$$|B_{\rm Bos}\rangle^{(0)} = \frac{T_p}{2} \int_{-\infty}^{\infty} \exp\left\{ i\alpha' \left[\sum_{\alpha=0}^{p} (U^{-1}\mathbf{A})_{\alpha\alpha} (p^\alpha)^2 + \sum_{\alpha,\beta=0,\alpha\neq\beta}^{p} (U^{-1}\mathbf{A} + \mathbf{A}^T U^{-1})_{\alpha\beta} p^\alpha p^\beta \right] \right\} \times \left(\prod_{\alpha} |p^\alpha\rangle dp^\alpha \right) \otimes \prod_i \delta(x^i - y^i) |p^i = 0\rangle, \tag{4}$$

where $\mathbf{A}_{\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta}$.

The NS-NS and R-R sectors possess the following fermionic boundary states:

$$|B_{\rm Ferm}\rangle_{\rm NS} = \prod_{r=1/2}^{\infty} [\det Q_{(r)}] \exp\left[i \sum_{r=1/2}^{\infty} (b^{\mu}_{-r} S_{(r)\mu\nu} \tilde{b}^{\nu}_{-r}) \right] |0\rangle, \tag{5}$$

$$|B_{\rm Ferm}\rangle_{\rm R} = \prod_{n=1}^{\infty} [\det Q_{(n)}] \exp\left[i \sum_{m=1}^{\infty} (d^{\mu}_{-m} S_{(m)\mu\nu} \tilde{d}^{\nu}_{-m}) \right] |B\rangle^{(0)}_{\rm R}. \tag{6}$$

The explicit form of the zero-mode state $|B\rangle^{(0)}_{\rm R}$ in the Type IIA and Type IIB theories and its contribution to the spin structure can be found in [24] in complete detail. It is not modified here because for obtaining the partition function it will be projected onto the bra vacuum; hence, the remaining state would be the boundary state built on the vacuum.

The total boundary state in the NS-NS and R-R sectors is given by

$$|B\rangle_{\rm NS,R} = |B_{\rm Bos}\rangle^{(\rm osc)} \otimes |B_{\rm Bos}\rangle^{(0)} \otimes |B_{\rm Ferm}\rangle_{\rm NS,R}. \tag{7}$$

In fact, the total boundary state also has the ghosts and superghosts boundary states. Since these parts are free of the background fields, and especially free of the characteristic matrix of the tachyon, we put them away. Note that the boundary state (7) contains significant information about the nature of the brane.

III. TACHYON CONDENSATION AND COLLAPSE OF A Dp-BRANE

The structure of the configuration space for the BSFT can be described as follows: the space of two-dimensional world-sheet theories on the disk with arbitrary boundary interactions deals with the disk partition function of the open string theory and a fixed conformal world-sheet action in the bulk. It has been demonstrated that, at the tree level, the disk partition function in the BSFT appears as the normalization factor of the boundary state. In other words, the partition function can be acquired by the vacuum amplitude of the boundary state

$$Z^{\rm Disk} = \langle {\rm vacuum} | B \rangle. \tag{8}$$

Thus, in our setup the partition function possesses the following feature:

$$Z^{\rm Disk}_{\rm Bos} = \frac{T_p}{2} \int_{-\infty}^{\infty} \prod_{\alpha} dp^\alpha \exp\left\{ i\alpha' \left[\sum_{\alpha=0}^{p} (U^{-1}\mathbf{A})_{\alpha\alpha} (p^\alpha)^2 + \sum_{\alpha,\beta=0,\alpha\neq\beta}^{p} (U^{-1}\mathbf{A} + \mathbf{A}^T U^{-1})_{\alpha\beta} p^\alpha p^\beta \right] \right\} \times \prod_{n=1}^{\infty} [\det Q_{(n)}]^{-1} \tag{9}$$

for the bosonic part of the partition function, and

$$Z^{\rm Disk}_{\rm Ferm} = \prod_{k>0}^{\infty} [\det Q_{(k)}] \tag{10}$$

for the fermionic part, where k is a half-integer (integer) for the NS-NS (R-R) sector. Therefore, after integrating on the momenta and considering both fermionic and bosonic parts, the total partition function in superstring theory is given by

$$Z^{\rm Disk}_{\rm total} = \frac{T_p}{2}\left(\frac{i\pi}{\alpha'}\right)^{(p+1)/2} \frac{1}{\sqrt{\det(D+H)}} \frac{\prod_{k>0}^{\infty}[\det Q_{(k)}]}{\prod_{n=1}^{\infty}[\det Q_{(n)}]}, \tag{11}$$

where the diagonal matrix possesses the elements $D_{\alpha\beta} = (U^{-1}A)_{\alpha\alpha}\delta_{\alpha\beta}$, and the matrix $H_{\alpha\beta}$ is defined by

$$H_{\alpha\beta} = \begin{cases} (U^{-1}A + A^T U^{-1})_{\alpha\beta}, & \alpha \neq \beta, \\ 0, & \alpha = \beta. \end{cases} \tag{12}$$

The partition function enables us to investigate the effect of the tachyon condensation on the instability of the Dp-brane. According to the conventional literature, the tachyonic mode of the open string spectrum makes the D-branes instable. This phenomenon is called tachyon condensation. As the tachyon condenses, the dimension of the brane decreases and in the final stage, one receives a closed string vacuum. Using the boundary sigma model, the tachyon condensation usually starts with a conformal theory with d Neumann boundary conditions in the UV, and then adding a relevant tachyon field will cause the theory to roll toward an IR fixed point as a closed string vacuum with a Dp-brane, which corresponds to a new vacuum with $(d-p-1)$ Dirichlet boundary conditions.

According to the characteristic matrix of our tachyon, investigating the tachyon condensation in this work is more general than the conventional studies that usually consider a single parameter for the tachyon field. Now let us check the stability or instability of the Dp-brane in our setup. The tachyon condensation can occur by taking at least one of the tachyon's elements to infinity, i.e., $U_{pp} \to \infty$. At first, look at the R-R sector. By making use of the $\lim_{U_{pp}\to\infty}(U^{-1})_{p\alpha} = \lim_{U_{pp}\to\infty}(U^{-1})_{\alpha p} = 0$, the dimensional reduction of the matrices $U^{-1}\mathbf{A}$, $\mathbf{A}^T U^{-1}$, and D is obvious. Therefore, according to Eq. (11), in the R-R sector we observe that the direction x^p has been omitted from the resulting brane.

Now concentrate on the factor $\prod_{r=\frac{1}{2}}^{\infty}[\det Q_{(r)}]/\prod_{n=1}^{\infty}[\det Q_{(n)}]$ in the NS-NS sector of the superstring partition function. Using the limit

$$\lim_{U_{pp}\to\infty} \prod_{n=1}^{\infty}\left[\det\left(\eta - \mathcal{F} + \frac{iU}{2n}\right)_{(p+1)\times(p+1)}\right]^{-1} = \prod_{n=1}^{\infty} \frac{2n}{iU_{pp}}\left[\det\left(\eta - \mathcal{F} + \frac{iU}{2n}\right)_{p\times p}\right]^{-1}, \tag{13}$$

the effect of tachyon condensation on this factor is given by

$$\begin{aligned} &\lim_{U_{pp}\to\infty} \frac{\prod_{r=\frac{1}{2}}^{\infty}[\det(Q_{(r)})_{(p+1)\times(p+1)}]}{\prod_{n=1}^{\infty}[\det(Q_{(n)})_{(p+1)\times(p+1)}]} \\ &\longrightarrow \sqrt{\frac{i\pi U_{pp}}{2}} \frac{\prod_{r=\frac{1}{2}}^{\infty}[\det(Q_{(r)})_{p\times p}]}{\prod_{n=1}^{\infty}[\det(Q_{(n)})_{p\times p}]} \\ &\longrightarrow \sqrt{\frac{i\pi U_{pp}}{2}} \det(\eta - \mathcal{F}) \det\left[\frac{\sqrt{\pi}\Gamma(1+\frac{i}{2}(\eta-\mathcal{F})^{-1}U)}{\Gamma(\frac{1}{2}+\frac{i}{2}(\eta-\mathcal{F})^{-1}U)}\right]_{p\times p}. \end{aligned} \tag{14}$$

The $p\times p$ matrices are similar to the initial $(p+1)\times(p+1)$ matrices in which the last rows and last columns have been omitted. To avoid divergent quantities due to the existence of infinite product, in the second and third factors we used the ζ-function regularization. For this reason we used the arrow sign instead of equality. However, it is evident that in this sector the dimensional reduction also occurs.

Let us check this factor after successive tachyon condensation, i.e.,

$$\begin{aligned} &\lim_{U\to\infty} \frac{\prod_{r=\frac{1}{2}}^{\infty}[\det(Q_{(r)})_{(p+1)\times(p+1)}]}{\prod_{n=1}^{\infty}[\det(Q_{(n)})_{(p+1)\times(p+1)}]} \\ &= \lim_{U\to\infty} \frac{\prod_{r=\frac{1}{2}}^{\infty}[\det(\frac{iU}{2r})]}{\prod_{n=1}^{\infty}[\det(\frac{iU}{2n})]} \longrightarrow \lim_{U\to\infty}\left(\frac{i\pi}{2}\right)^{(p+1)/2}\sqrt{\det U}, \end{aligned} \tag{15}$$

where in the last term again, ζ-function regularizations for infinite products have been used. Therefore, the total partition function finds the feature

$$Z^{\rm Disk}_{\rm total} = \frac{T_p}{2}\left(-\frac{\pi^2}{2\alpha'}\right)^{(p+1)/2}\sqrt{\frac{\det U}{\det(D+H)}}. \tag{16}$$

In this limit, condensation would take place for all directions of the brane's world volume. As can be seen, the dimensional reduction followed by the sequential condensation process could not make the tachyon disappear.

For completing the discussion, let us see the tachyon condensation effect via the boundary state approach, directly. According to Eqs. (5) and (6), apart from the normalization factors, i.e., the partition functions, look at the $\Delta_{(m)}$ matrix in which the tachyon has been entered. After applying the limit $U_{pp}\to\infty$, this matrix possesses an eigenvalue "-1"; i.e., we deduce that the Neumann direction x^p has been omitted, and instead it has been added to the Dirichlet directions. This process would be the same as in the bosonic case.

According to the above condensation processes, via the boundary state and the BSFT approaches, the result is that in our setup the dimensional reduction has taken place in both NS-NS and R-R sectors of the superstring theory. That is, after tachyon condensation, such a rotating-moving Dp-brane with photonic and tachyonic background fields reduces to an unstable D$(p-1)$-brane with its own background fields, rotation, and motion. Thus, imposing rotation and motion to an unstable D-brane does not preserve it against collapse during the process of tachyon condensation.

[1] M. Frau, I. Pesando, S. Sciuto, A. Lerda, and R. Russo, Phys. Lett. B **400**, 52 (1997).

[2] C. G. Callan and I. R. Klebanov, Nucl. Phys. **B465**, 473 (1996); C. G. Callan, C. Lovelace, C. R. Nappi, and S. A. Yost, Nucl. Phys. **B308**, 221 (1988).

[3] S. Gukov, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **423**, 64 (1998).

[4] M. Li, Nucl. Phys. **B460**, 351 (1996).

[5] T. Kitao, N. Ohta, and J. G. Zhou, Phys. Lett. B **428**, 68 (1998).

[6] M. B. Green and P. Wai, Nucl. Phys. **B431**, 131 (1994); C. Schmidhuber, Nucl. Phys. **B467**, 146 (1996); M. B. Green and M. Gutperle, Nucl. Phys. **B476**, 484 (1996).

[7] M. Billo, P. Di Vecchia, and D. Cangemi, Phys. Lett. B **400**, 63 (1997).

[8] F. Hussain, R. Iengo, and C. Nunez, Nucl. Phys. **B497**, 205 (1997).

[9] O. Bergman, M. Gaberdiel, and G. Lifschytz, Nucl. Phys. **B509**, 194 (1998).

[10] P. Di Vecchia, M. Frau, I. Pesando, S. Sciuto, A. Lerda, and R. Russo, Nucl. Phys. **B507**, 259 (1997); M. Billo, P. Di Vecchia, M. Frau, A. Lerda, I. Pesando, R. Russo, and S. Sciuto, Nucl. Phys. **B526**, 199 (1998); P. Di Vecchia and A. Liccardo, arXiv:hep-th/9912275.

[11] H. Arfaei and D. Kamani, Phys. Lett. B **452**, 54 (1999); Nucl. Phys. **B561**, 57 (1999); Phys. Lett. B **475**, 39 (2000).

[12] F. Safarzadeh-Maleki and D. Kamani, Phys. Rev. D **89**, 026006 (2014).

[13] M. M. Sheikh-Jabbari, Phys. Lett. B **425**, 48 (1998).

[14] C. Bachas, Phys. Lett. B **374**, 37 (1996).

[15] A. Sen, Int. J. Mod. Phys. A **14**, 4061 (1999); **20**, 5513 (2005); J. High Energy Phys. 08 (1998) 010; 08 (1998) 012; 10 (1999) 008; 12 (1999) 027.

[16] P. Mukhopadhyay and A. Sen, J. High Energy Phys. 11 (2002) 047; G. Arutyunov, S. Frolov, S. Theisen, and A. A. Tseytlin, J. High Energy Phys. 02 (2001) 002.

[17] E. Witten, J. High Energy Phys. 12 (1998) 019; Nucl. Phys. **B268**, 253 (1986).

[18] O. Bergman and M. R. Gaberdiel, Phys. Lett. B **441**, 133 (1998).

[19] D. Kutasov, M. Marino, and G. Moore, J. High Energy Phys. 10 (2000) 045.

[20] P. Kraus and F. Larsen, Phys. Rev. D **63**, 106004 (2001); T. Takayanagi, S. Terashima, and T. Uesugi, J. High Energy Phys. 03 (2001) 019; M. Naka, T. Takayanagi, and T. Uesugi, J. High Energy Phys. 06 (2000) 007.

[21] E. Witten, Phys. Rev. D **46**, 5467 (1992); **47**, 3405 (1993).

[22] S. P. de Alwis, Phys. Lett. B **505**, 215 (2001); K. Okuyama, Phys. Lett. B **499**, 305 (2001); T. Lee, K. S. Viswanathan, and Y. Yang, J. Korean Phys. Soc. **42**, 34 (2003); Z. Rezaei, Phys. Rev. D **85**, 086011 (2012).

[23] T. Okuda and S. Sugimoto, Nucl. Phys. **B647**, 101 (2002); S. J. Rey and S. Sugimoto, Phys. Rev. D **67**, 086008 (2003).

[24] F. Safarzadeh-Maleki and D. Kamani, Zh. Eksp. Theor. Phys. **146**, 769 (2014) [J. Exp. Theor. Phys. **119**, 04 (2014)].

[25] G. Chalmers, J. High Energy Phys. 06 (2001) 012.