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Higher derivative corrections to manifestly supersymmetric nonlinear realizations

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When global symmetries are spontaneously broken in supersymmetric vacua, there appear quasi-Nambu-Goldstone (NG) fermions as superpartners of NG bosons. In addition to these, there can appear quasi-NG bosons in general. The quasi-NG bosons and fermions together with the NG bosons are organized into chiral multiplets. Kähler potentials of low-energy effective theories were constructed some years ago as supersymmetric nonlinear realizations. It is known that higher-derivative terms in the superfield formalism often encounter the auxiliary field problem; the auxiliary fields that accompanied with space-time derivatives and cannot be eliminated. In this paper, we construct higher-derivative corrections to supersymmetric nonlinear realizations in the off-shell superfield formalism free from the auxiliary field problem. As an example, we present the manifestly supersymmetric chiral Lagrangian.

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I. INTRODUCTION

Low-energy field theories can be described by only light fields when one integrates out massive particles above the scale which one considers. In particular, when a global symmetry of a Lagrangian or Hamiltonian is spontaneously broken in the ground state or vacuum, there appear Nambu-Goldstone (NG) bosons as massless scalar fields. The low-energy dynamics of these NG bosons is solely determined from the symmetry argument. When a symmetry group G is spontaneously broken down to its subgroup H, the low-energy dynamics is governed by a nonlinear sigma model whose target space is the coset space G/H [1]. A prime example is the chiral Lagrangian of pions which appear as NG bosons when the chiral symmetry of QCD is spontaneously broken. Low-energy effective theories are usually expanded by the number of space-time derivatives, thereby they inevitably contain higher-derivative corrections. It is known that the chiral perturbation theory includes derivative corrections to the chiral Lagrangian [2].

On the other hand, supersymmetry plays important roles to control quantum corrections in field theories and determines the exact low-energy dynamics [3]. It is also a necessary ingredient to define consistent string theories. It was also proposed as the most promising candidate to solve the naturalness problem in the Standard Model. Among other things, when a global symmetry is spontaneously broken in supersymmetric vacua, there appear quasi-NG fermions [4] in addition to the NG bosons. They are required to form chiral supermultiplets as superpartners of NG bosons. In model building of particle physics, quasi-NG fermions were identified as quarks in supersymmetric preon models [5]. The target spaces of supersymmetric nonlinear sigma models must be Kähler [6] because the lowest components of chiral superfields are complex scalar fields. When a coset space G/H is eventually Kähler, there are no additional massless fields. However, G/H is not Kähler in general, and in that case, there must appear quasi-NG bosons [7] in addition to the NG bosons, to parametrize a Kähler manifold. In this case, target spaces of low-energy effective theories are enlarged from G/H. In general, the problem to construct low-energy effective theories of massless fields reduces to finding G-invariant Kähler potentials. The most general framework to construct G-invariant Kähler potentials was provided as supersymmetric nonlinear realizations [8]. The authors of [8] classified NG supermultiplets into P type, containing two NG bosons, and M type, containing one NG boson and one quasi-NG boson. In one extreme class called a pure realization, all supermultiplets are of P type and there are no quasi-NG bosons, which is possible only when G/H happens to be Kähler. In this case, the most general G-invariant Kähler potential up to Kähler transformations was constructed in Refs. [8,9] (see Ref. [10] as a review), which is unique up to finite number of decay constants (Kähler class). This class was studied extensively in the literature (see, e.g., Refs. [11] and references in Ref. [10]). In the other extreme class called a maximal realization, all supermultiplets are of M type so that there are the same number of quasi-NG bosons with NG bosons. The target manifold in this case is a cotangent bundle $T^*(G/H)$, whose cotangent directions are parametrized by quasi-NG bosons. For instance, the chiral symmetry breaking belongs to this class [12]. If there is at least one quasi-NG boson, the effective Kähler potential is an arbitrary function of strict G

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invariants [8]. Geometrically this arbitrariness corresponds to a degree of freedom to deform noncompact directions of the target space, which cannot be controlled by the isometry G [12–15]. These directions are associated with the quasi-NG bosons. It was proved that there must appear at least one quasi-NG boson in the absence of gauge interactions [16–18]. When there is a gauge symmetry on the other hand, pure realizations without quasi-NG bosons are possible by absorbing M-type superfields by the supersymmetric Higgs mechanism [19].

While the superfield formalism is one of the most powerful off-shell formulations to construct manifestly supersymmetric Lagrangians, it often encounters an auxiliary field problem when higher-derivative terms exist in the Lagrangians. For example, chiral superfields with space-time derivatives (e.g., $\partial_m \Phi$) contain derivatives on the auxiliary fields F so that they cannot be eliminated by their equations of motion. This problem was recognized [20,21] for a supersymmetric extension of Wess-Zumino-Witten (WZW) term [22] in the chiral Lagrangian of supersymmetric QCD. A supersymmetric WZW term proposed in Ref. [23] does not have this problem. Supersymmetric Lagrangians free from the auxiliary field problem were also known before, such as supersymmetric Dirac-Born-Infeld action [24], supersymmetric higher-derivative $\mathbb{C}P^1$ models [25,26], supersymmetric baby Skyrme models [27,28] and supersymmetric k-field theories [29,30]. The most general model of chiral superfields with higher-derivative terms was recently presented in Ref. [31], where it was called a supersymmetric $P(X,\varphi)$ model. The higher-derivative interaction can be written by using a target space tensor with two holomorphic and two antiholomorphic indices which are both symmetric. This term was first found in Ref. [32] as a quantum correction term in a chiral model, and the supersymmetric WZW term in Ref. [23] also contains it [33]. The model in Ref. [31] was extended by the introduction of a superpotential [34] and coupling to supergravity [35,36], and was applied to the supersymmetric Galileon inflation models [37] and the ghost condensation [38]. In our previous paper [39], we have classified 1/2 and 1/4 Bogomol'nyi-Prasad-Sommerfield (BPS) equations for domain walls, lumps, baby Skyrmions and domain wall junctions. See also Ref. [40] for further study on baby Skyrmions.

In this paper, we construct higher-derivative corrections to supersymmetric nonlinear realizations for spontaneous broken global symmetries with keeping supersymmetry. As the leading two derivative terms for pure realizations without quasi-NG bosons, we find that the higher-derivative terms are unique up to constants. On the other hand, higher-derivative terms contain arbitrary functions in the presence of quasi-NG bosons. As one of the most important examples, we discuss chiral symmetry breaking in detail.

This paper is organized as follows. In Sec. II, we give a brief review on supersymmetric nonlinear realizations. In Sec. III we discuss higher-derivative corrections to nonlinear realizations. In Sec. III A, we introduce the supersymmetric higher-derivative chiral model with four supercharges. We write down the equation of motion for the auxiliary fields and analyze the structure of the on-shell Lagrangians. In Sec. III B, we discuss higher-derivative corrections to pure realizations in the absence of quasi-NG bosons, for which each massless chiral superfield contains two NG bosons and there are no quasi-NG bosons. In Secs. III C and III D, we discuss higher-derivative corrections in the presence of quasi-NG bosons. In Sec. IV, we discuss higher-derivative corrections for supersymmetric chiral symmetry breaking, which is a maximal realization where each massless chiral superfield contains one NG boson and one quasi-NG boson. Section V is devoted to conclusion and discussions. We use the notation of the textbook of Wess and Bagger [41].

II. SUPERSYMMETRIC NONLINEAR REALIZATIONS: A REVIEW

In this section, we review supersymmetric nonlinear realizations formulated in Ref. [8].

A. Global symmetry breaking in supersymmetric theories

When a global symmetry group G is spontaneously broken down to its subgroup H, there appear massless Nambu-Goldstone (NG) bosons associated with broken generators of the coset manifold G/H. At low energies, interactions among these massless particles are described by the so-called nonlinear sigma models, whose Lagrangians in the leading order of derivative expansions are completely determined by the geometry of the target manifold G/H parametrized by NG bosons as was found by Callan, Coleman, Wess and Zumino [1].

In four-dimensional $\mathcal{N}=1$ supersymmetric theories, scalar fields belong to chiral superfields Φ^i $(i=1,\ldots,N)$ whose component expansion in the chiral base $y^m = x^m + i\theta\sigma^m\bar{\theta}$ is

$$\Phi^i(y,\theta) = \varphi^i(y) + \theta\psi^i(y) + \theta^2 F^i(y), \qquad (2.1)$$

where φ^i is the complex scalar field, ψ^i is the Weyl fermion and F^i is the complex auxiliary field.

When a global symmetry is spontaneously broken in supersymmetric vacua, there appear massless fermions ψ^i as supersymmetric partners of NG bosons [4]. These massless fermions together with NG bosons are described by chiral superfields. Since chiral superfields are complex, the supersymmetric nonlinear sigma models are closely related to the complex geometry; their target manifolds, where fields variables take their values, must be Kähler manifolds [6]. If the coset manifold G/H itself happens to be a Kähler manifold, both real and imaginary parts of the scalar components of chiral superfields are NG bosons. If G/H is not a Kähler manifold, on the other hand, there is at least one chiral superfield whose real or imaginary part is not a NG boson. This additional massless boson is called the *quasi-NG boson* [7].

We explain how quasi-NG bosons appear. The spontaneous symmetry breaking of a global symmetry G in supersymmetric theories is caused by the superpotential W: the chiral superfields acquire the vacuum expectation values $v = \langle\varphi\rangle$ as a result of the F-term condition $\frac{\partial W}{\partial\varphi} = 0$. Since the superpotential W is *holomorphic*—namely, it contains only chiral superfields—this condition is *invariant under the complex extension of G, namely,* $G^{\mathbb{C}}$. Hence, if we define the *complex isotropy group* $\hat{H}(\subset G^{\mathbb{C}})$ by[1]

$$\hat{H}v = v, \qquad \hat{\mathcal{H}}v = 0, \tag{2.2}$$

the target space parametrized by NG and quasi-NG bosons can be written as a complex coset space:

$$M \simeq G^{\mathbb{C}}/\hat{H}. \tag{2.3}$$

In general, $\hat{H}$ is larger than $H^{\mathbb{C}}$, and it is decomposed as

$$\hat{\mathcal{H}} = \mathcal{H}^{\mathbb{C}} \oplus \mathcal{B}, \tag{2.4}$$

where $\mathcal{B}$ consists of non-Hermitian generators $E \in \hat{\mathcal{H}}$ and is called (the subalgebra of) the *Borel subalgebra* in $\hat{\mathcal{H}}$ [8].[2]

(i) As an example, let us consider a doublet $\phi = (\phi_1, \phi_2)^T$ of $G = SU(2)$ and suppose that they acquire the vacuum expectation values $v = (1,0)^T$. Since the raising operator

$$\sigma_+ = \frac{1}{2}(\sigma_1 + i\sigma_2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

satisfies $\sigma_+ v = 0$, it is the complex unbroken generator in $\hat{\mathcal{H}}$. On the other hand, σ_3 and the lowering operator $\sigma_-(= {\sigma_+}^\dagger)$ are the elements of the broken generators in $\mathcal{G}^{\mathbb{C}} - \hat{\mathcal{H}}$.

The coset representative can be written as

$$\xi(\Phi) = \exp(i\Phi \cdot Z) \in G^{\mathbb{C}}/\hat{H}, \qquad Z \in \mathcal{G}^{\mathbb{C}} - \hat{\mathcal{H}}, \tag{2.5}$$

where Z are complex broken generators and Φ are NG chiral superfields generated by them. There are two kinds of broken generators: the hermitian broken generators X and the non-Hermitian broken generators $\bar{E}$:

$$\mathcal{G}^{\mathbb{C}} - \hat{\mathcal{H}} = \{Z\} = \{X, \bar{E}\}. \tag{2.6}$$

The NG superfields Φ corresponding to non-Hermitian and Hermitian generators are called P-type (or nondoubled-type) and M-type (or doubled-type) superfields, respectively [8,16]. Note that there are as many non-Hermitian broken generators $\bar{E}$ as non-Hermitian unbroken generators E, since they are Hermitian conjugate to each other. On a suitable basis, $\bar{E}$ and E can be written as off-diagonal lower and upper half matrices, respectively.

(i) In the previous example where the representative of $G^{\mathbb{C}}/\hat{H}$ is given by $\phi = \exp i(\varphi_3\sigma_3 + \varphi\sigma_-) \cdot v$, φ_3 is an M-type and φ is a P-type superfield. The non-Hermitian broken generator $\bar{E} = \sigma_-$ written as a lower half matrix is hermitian conjugate to the non-Hermitian unbroken generator $E = \sigma_+$ written as a upper half matrix.

The directions parametrized by quasi-NG bosons are noncompact, whereas those of NG bosons are compact.[3] The scalar components of the M-type superfields consist of a quasi-NG boson in addition to a NG boson, whereas those of the P-type superfields consist of two genuine NG bosons. This can be understood as follows: note that, for each non-Hermitian broken generator $\bar{E}$, there is a non-Hermitian unbroken generator E. Since the vacuum is invariant under $\hat{H}$, we can multiply the representative of the coset manifold by an arbitrary element of $\hat{H}$ from the right. Hence, for any P-type superfield Φ generated by a non-Hermitian generator $\bar{E}$, there exists an element $\exp(i\Phi^\dagger E) \in \hat{H}$ such that

$$\begin{aligned} \xi v &= \exp i(\cdots + \Phi\bar{E} + \cdots)v \\ &= \exp i(\cdots + \Phi\bar{E} + \cdots)\exp(i\Phi^\dagger E)v \\ &= \exp i(\cdots + \Re\Phi X_1 + \Im\Phi X_2 + O(\Phi^2) + \cdots)v, \end{aligned} \tag{2.7}$$

where we have used the Baker-Campbell-Hausdorff formula and defined two hermitian broken generators $X_1 = \bar{E} + E, X_2 = i(\bar{E} - E)$. Here $\Re$ and $\Im$ denote real and imaginary parts, respectively. Therefore two scalar components of the P-type superfield parametrize compact directions, and hence are considered NG bosons. On the other hand, since any M-type superfield is generated by an hermitian generator, there is no partner in $\hat{\mathcal{H}}$. Therefore its imaginary part of scalar component parametrizes a

[1]We use the calligraphic font for a Lie algebra corresponding to a Lie group.

[2]In the group level, $\hat{H}$ can be written as a semidirect product of $H^{\mathbb{C}}$ and the Borel subgroup B: $\hat{H} = H^{\mathbb{C}} \wedge B$. Here the symbol $\wedge$ denotes a semidirect product. If there are two elements of $\hat{H}$, hb and $h'b'$, where $h, h' \in H^{\mathbb{C}}$ and $b, b' \in B$, their product is defined as $(hb)(h'b') = hh'(h'^{-1}bh')b' = (hh')(b''b)$, where $b'' = h'^{-1}bh' \in B$ [8]. It is, however, sufficient to consider only the Lie algebra in this paper.

[3]We use the word "compactness" in the sense of topology. The kinetic terms of quasi-NG bosons have the same sign as those of NG bosons.

noncompact direction, and hence is considered to be a quasi-NG boson.

(i) In our previous example, we can rewrite it as $\exp i(\varphi_3\sigma_3 + \Re\varphi\sigma_1 + \Im\varphi\sigma_2)\cdot v$ by multiplying an appropriate factor generated by σ_+ for sufficiently small $|\varphi_3|$ and $|\varphi|$. The NG bosons parametrizing $S^3 \simeq SU(2)$ are $\Re\varphi_3, \Re\varphi, \Im\varphi$, whereas $\Im\varphi_3$ is the quasi-NG boson parametrizing the radius of S^3.

As a notation, we write the number of chiral superfields N_Φ parametrizing the target manifold as

$$N_\Phi = N_\mathrm{M} + N_\mathrm{P}, \tag{2.8}$$

where the numbers of the M-type and P-type superfields are denoted by N_M and N_P, respectively. The number of quasi-NG bosons is[4]

$$\begin{aligned} N_\mathrm{Q} = N_\mathrm{M} &= 2\dim_\mathbf{C}(G^\mathbb{C}/\hat{H}) - \dim(G/H) \\ &= \dim(G/H) - \dim B. \end{aligned} \tag{2.9}$$

Hence if there is as large Borel subalgebra as the number of NG bosons, $\dim B = \dim(G/H)$, there is no quasi-NG boson. This case is called the "pure realization" (total pairing or nondoubling). On the other hand, if there is no Borel subalgebra, $\dim B = 0$, there appear as many quasi-NG bosons as NG bosons. This case is called the "maximal realization" (or full doubling). It is known that a pure realization cannot occur in the model with a linear origin without gauge symmetry [16–18]. It was shown in Ref. [16] that a maximal realization occurs when a field belonging to a real representation obtains a vacuum expectation value or when NG boson part G/H brought by a vacuum expectation value is a symmetric space. In the presence of a gauge symmetry, pure realizations without quasi-NG bosons are possible, since gauge fields absorb M-type superfields as a consequence of the supersymmetric Higgs mechanism [19].

B. G-invariant Kähler potentials

The kinetic term in the effective Lagrangian is described by the Kähler potential $K(\Phi,\Phi^\dagger)$ of NG chiral superfields

$$\begin{aligned} \mathcal{L} &= \int d^2\theta d^2\bar{\theta} K(\Phi,\Phi^\dagger) \\ &= -g_{i\bar{j}}(\varphi,\bar{\varphi})\partial_\mu\varphi^i\partial^\mu\bar{\varphi}^{\bar{j}} + (\text{fermion terms}), \end{aligned} \tag{2.10}$$

where we have eliminated the auxiliary fields F^i by its equation of motion and $g_{i\bar{j}} \equiv \frac{\partial}{\partial\varphi^i}\frac{\partial}{\partial\bar{\varphi}^{\bar{j}}}K(\varphi,\bar{\varphi})$ is the Kähler metric. Since the Kähler potential includes both chiral and antichiral superfields, the symmetry group of the effective theory is still G, but not its complexification. Hence our goal is to construct G-invariant Kähler potentials of complex coset spaces $G^\mathbb{C}/\hat{H}$. Here the G-invariance means

$$K(\Phi,\Phi^\dagger) \xrightarrow{g} K(\Phi',\Phi'^\dagger) = K(\Phi,\Phi^\dagger) + F(\Phi,g) + F^*(\Phi^\dagger,g), \tag{2.11}$$

where F (F^*) is a (anti)holomorphic function of Φ ($\Phi^\dagger$) which depends on $g \in G$. The latter two terms in Eq. (2.11) disappear in the superspace integral $\int d^4\theta$.[5] Since the redefinition of the Kähler potential by adding holomorphic and antiholomorphic functions is called the Kähler transformation, we denote that it is G invariant under a Kähler transformation or quasi-G invariant if $F(\Phi,g)$ exists in Eq. (2.11).

First of all, we note that the transformation law under G of the representative ξ of the complex coset $G^\mathbb{C}/\hat{H}$ is

$$\xi \xrightarrow{g} \xi' = g\xi\hat{h}^{-1}(g,\xi), \tag{2.12}$$

where $\hat{h}^{-1}(g,\xi)$ is a compensator to project $g\xi$ onto the coset representative (see Fig. 1).

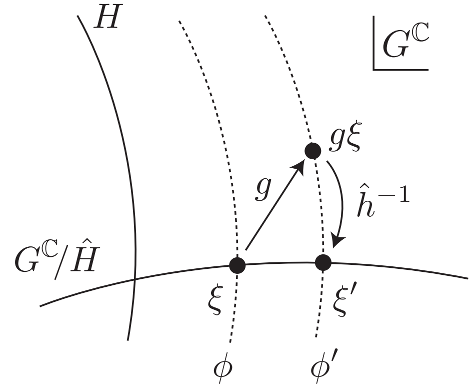

FIG. 1. The G-transformation law for ξ.

Bando *et al.* constructed the following three types of G-invariant Kähler potentials called A, B, and C types [8].

A type. We prepare a representation (ρ, V) of G in the representation space V. If there are $\hat{H}$-invariant vectors v_a,

$$\rho(\hat{H})v_a = v_a, \tag{2.13}$$

the transformation law of the quantity $\rho(\xi)v_a$ under G is

$$\rho(\xi)v_a \xrightarrow{g} \rho(\xi')v_a = \rho(g)\rho(\xi)\rho(\hat{h}^{-1})v_a = \rho(g)\rho(\xi)v_a. \tag{2.14}$$

Then, by using strict G invariants,

$$X_{ab} \equiv v_a^\dagger\rho(\xi^\dagger\xi)v_b, \tag{2.15}$$

[4]We use '$\dim_\mathbf{C}$' for complex dimensions and 'dim' for real dimensions.

[5]Here $F(\Phi,g)$ is called the cocycle function, which satisfies the cocycle condition, $F(\Phi,g_2g_1) = F(g_2\Phi,g_1) + F(\Phi,g_2)$.

we can construct a G-invariant Kähler potential

$$K_{\mathrm{A}}(\Phi,\Phi^\dagger)=f(X_{ab}), \tag{2.16}$$

where f is an *arbitrary* real function of all possible G invariants X_{ab}.

B type. It is sufficient to consider the fundamental representation [8], hence we do not write ρ for simplicity. We need the projection matrices, which project a fundamental representation space onto an $\hat{H}$-invariant subspace. They satisfy the projection conditions,

$$\eta_a^\dagger=\eta_a, \qquad \eta_a\hat{H}\eta_a=\hat{H}\eta_a, \qquad \eta_a^2=\eta_a. \tag{2.17}$$

Define the projected determinant as

$$\det{}_\eta A\equiv\det(\eta A\eta+\mathbf{1}-\eta), \tag{2.18}$$

where $\det_\eta$ stands for the determinant in the projected space. By using these, if we construct[6]

$$K_{\mathrm{B}}(\Phi,\Phi^\dagger)=\sum_a c_a\log\det{}_{\eta_a}\xi^\dagger\xi, \tag{2.19}$$

it is G invariant up to a Kähler transformation:

$$\begin{aligned}\log\det{}_\eta\xi^\dagger\xi &\xrightarrow{g}\log\det{}_\eta\xi'^\dagger\xi'\\ &=\log\det{}_\eta(\eta\xi'^\dagger\xi'\eta)\\ &=\log\det{}_\eta(\eta\hat{h}^{\dagger-1}\xi^\dagger\xi\hat{h}^{-1}\eta)\\ &=\log\det{}_\eta(\eta\hat{h}^{\dagger-1}\eta\xi^\dagger\xi\eta\hat{h}^{-1}\eta)\\ &=\log\det{}_\eta(\eta\hat{h}^{\dagger-1}\eta\eta\xi^\dagger\xi\eta\eta\hat{h}^{-1}\eta)\\ &=\log\det{}_\eta\xi^\dagger\xi+\log\det{}_\eta\hat{h}^{-1}+\log\det{}_\eta\hat{h}^{\dagger-1},\end{aligned} \tag{2.20}$$

where the last two terms include only chiral and antichiral superfields, respectively, and disappear in the superspace integral $\int d^4\theta$. (2.17).[7] Here we have used Eq. (2.17).

C type. Again, the fundamental representation is sufficient [8]. We define $[A]_\eta^{-1}\equiv[\eta A\eta+\mathbf{1}-\eta]^{-1}$, where the inverse is calculated in the projected space. The quantities defined by[8]

$$P_a=\xi\eta_a[\xi^\dagger\xi]_{\eta_a}^{-1}\eta_a\xi^\dagger \tag{2.21}$$

transform under G as

$$\begin{aligned}P\xrightarrow{g}P' &=\xi'\eta[\xi'^\dagger\xi']_\eta^{-1}\eta\xi'^\dagger\\ &=g\xi\hat{h}^{-1}\eta[\eta\hat{h}^{-1\dagger}\xi^\dagger\xi\hat{h}^{-1}\eta]_\eta^{-1}\eta\hat{h}^{-1\dagger}\xi^\dagger g^\dagger\\ &=g\xi\eta(\eta\hat{h}^{-1}\eta)[\eta\hat{h}^{-1\dagger}\eta\eta\xi^\dagger\xi\eta\eta\hat{h}^{-1}\eta]_\eta^{-1}(\eta\hat{h}^{-1\dagger}\eta)\eta\xi^\dagger g^\dagger\\ &=g\xi\eta[\hat{h}^{-1}]_\eta([\hat{h}^{-1\dagger}]_\eta[\xi^\dagger\xi]_\eta[\hat{h}^{-1}]_\eta)_\eta^{-1}[\hat{h}^{-1\dagger}]_\eta\eta\xi^\dagger g^\dagger\\ &=gPg^\dagger.\end{aligned} \tag{2.22}$$

By noting the relations

$$P_a^2=\xi\eta_a[\xi^\dagger\xi]_{\eta_a}^{-1}(\eta_a\xi^\dagger\xi\eta_a)[\xi^\dagger\xi]_{\eta_a}^{-1}\eta_a\xi^\dagger=P_a, \tag{2.23}$$

$$\mathrm{tr}P_a=\mathrm{tr}([\xi^\dagger\xi]_{\eta_a}^{-1}(\eta_a\xi^\dagger\xi\eta_a))=\mathrm{tr}\eta_a=\mathrm{const}, \tag{2.24}$$

a G-invariant Kähler potential can be constructed as

$$K_{\mathrm{C}}(\Phi,\Phi^\dagger)=f(\mathrm{tr}(P_aP_b),\mathrm{tr}(P_aP_bP_c),\ldots), \tag{2.25}$$

where f is again an *arbitrary* real function and all the indices $a,b,c,\ldots$ are different.

III. HIGHER-DERIVATIVE CORRECTIONS

In this section we study higher-derivative corrections to supersymmetric nonlinear realizations. In the first subsection, we present general higher-derivative chiral models with multiple chiral superfields. In the second subsection, we consider pure realizations described by B-type Kähler potentials, for which each massless chiral superfield contains two NG bosons. In the third and fourth subsections, we consider A-and C-type Kähler potentials, respectively, for which some chiral superfields are M-type superfields, consisting of one quasi-NG boson and one genuine NG boson.

A. Higher-derivative chiral models

We consider higher-derivative terms generated by multiple chiral superfields Φ_i in which no dynamical (propagating) auxiliary fields exist. The supersymmetric higher-derivative term can be given by [27,28,31,39]

$$\mathcal{L}_{\mathrm{H.D.}}=\frac{1}{16}\int d^4\theta\Lambda_{ik\bar{j}\bar{l}}(\Phi,\Phi^\dagger)D^\alpha\Phi^iD_\alpha\Phi^k\bar{D}_{\dot{\alpha}}\Phi^{\dagger\bar{j}}\bar{D}^{\dot{\alpha}}\Phi^{\dagger\bar{l}}. \tag{3.1}$$

Here the supercovariant derivatives are defined as

$$D_\alpha=\frac{\partial}{\partial\theta^\alpha}+i(\sigma^m)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_m, \quad \bar{D}_{\dot{\alpha}}=-\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}-i\theta^\alpha(\sigma^m)_{\alpha\dot{\alpha}}\partial_m, \tag{3.2}$$

where the sigma matrices are $\sigma^m=(\mathbf{1},\vec{\tau})$ with the Pauli matrices $\vec{\tau}=(\tau^1,\tau^2,\tau^3)$. Since the term $D_\alpha\Phi^i$ behaves as a vector,

[6]This can be rewritten as [42], $K_{\mathrm{B}}=\sum_a\log\det'(\xi\eta_a\xi^\dagger)$, where $\det'$ is a determinant except zero eigen values.

[7]Here the cocycle function $F(\Phi,g)=\log\det_\eta\hat{h}^{-1}(g,\xi(\Phi))$ satisfies the cocycle condition.

[8]The meaning of P_a can be understood as follows [42]. Since P_a satisfies the properties

$$P_a^\dagger=P_a, \qquad P_a^2=P_a, \qquad \mathrm{tr}P_a=\mathrm{tr}\eta_a, \qquad P_a|_{\Phi=0}=\eta_a,$$

P_a can be considered to be the transformation of η_a from the origin $\Phi=0$ (or $\xi=1$) to $\Phi\neq0$ in the manifold.

$$D_\alpha \Phi'^i = \frac{\partial \Phi'^i}{\partial \Phi^j} D_\alpha \Phi^j, \tag{3.3}$$

under field redefinition $\Phi^i \to \Phi^{i\prime}(\Phi)$, $\Lambda_{ik\bar{j}\bar{l}}$ can be regarded as a (2,2) Kähler tensor symmetric in holomorphic and anti-holomorphic indices, whose components are functions of Φ^i and $\Phi^{\dagger\bar{i}}$ (admitting space-time derivatives acting on them).

We write down the bosonic components of the Lagrangian (3.1). The component expansion of the $\mathcal{N}=1$ chiral superfield in the x basis is

$$\Phi^i(x,\theta,\bar{\theta}) = \varphi^i + i\theta\sigma^m\bar{\theta}\partial_m\varphi^i + \frac{1}{4}\theta^2\bar{\theta}^2\Box\varphi^i + \theta^2 F^i, \tag{3.4}$$

where only the bosonic components are presented. Then, the bosonic component of the supercovariant derivatives of Φ^i can be calculated as

$$\begin{aligned} &D^\alpha\Phi^i D_\alpha\Phi^k \bar{D}_{\dot{\alpha}}\Phi^{\dagger\bar{j}}\bar{D}^{\dot{\alpha}}\Phi^{\dagger\bar{l}} \\ &\quad = 16\theta^2\bar{\theta}^2\Big[(\partial_m\varphi^i\partial^m\varphi^k)(\partial_m\bar{\varphi}^{\bar{j}}\partial^m\bar{\varphi}^{\bar{l}}) \\ &\qquad -\frac{1}{2}(\partial_m\varphi^i F^k + F^i\partial_m\varphi^k)(\partial^n\bar{\varphi}^{\bar{j}}\bar{F}^{\bar{l}} + \bar{F}^{\bar{j}}\partial^n\bar{\varphi}^{\bar{l}}) \\ &\qquad + F^i\bar{F}^{\bar{j}}F^k\bar{F}^{\bar{l}}\Big]. \end{aligned} \tag{3.5}$$

Since the bosonic part of the right-hand side of (3.5) saturates the Grassmann coordinate $\theta^2\bar{\theta}^2$, only the lowest component of the tensor $\Lambda_{ik\bar{j}\bar{l}}$ contributes to the bosonic part of the Lagrangian. Therefore the bosonic part of the Lagrangian (2.10) with the higher-derivative term (3.1) is

$$\begin{aligned} \mathcal{L}_b &= g_{i\bar{j}}(-\partial_m\varphi^i\partial^m\bar{\varphi}^{\bar{j}} + F^i\bar{F}^{\bar{j}}) + \frac{\partial W}{\partial\varphi^i}F^i + \frac{\partial\bar{W}}{\partial\bar{\varphi}^{\bar{j}}}\bar{F}^{\bar{j}} \\ &\quad + \Lambda_{ik\bar{j}\bar{l}}(\varphi,\bar{\varphi})[(\partial_m\varphi^i\partial^m\varphi^k)(\partial_n\bar{\varphi}^{\bar{j}}\partial^n\bar{\varphi}^{\bar{l}}) \\ &\quad - \partial_m\varphi^i F^k\partial^m\bar{\varphi}^{\bar{j}}\bar{F}^{\bar{l}} + F^i\bar{F}^{\bar{j}}F^k\bar{F}^{\bar{l}}], \end{aligned} \tag{3.6}$$

where we have introduced the superpotential W for generality. The model is manifestly (off-shell) supersymmetric and Kähler invariant provided that K and W are scalars and $\Lambda_{ik\bar{j}\bar{l}}$ is a tensor. The auxiliary fields F^i do not have space-time derivatives and consequently can be eliminated by the following algebraic equation of motion,

$$g_{i\bar{j}}F^i - 2\partial_m\varphi^i F^k\Lambda_{ik\bar{j}\bar{l}}\partial^m\bar{\varphi}^{\bar{l}} + 2\Lambda_{ik\bar{j}\bar{l}}F^iF^k\bar{F}^{\bar{l}} + \frac{\partial\bar{W}}{\partial\bar{\varphi}^{\bar{j}}} = 0. \tag{3.7}$$

Since NG fields are all massless, we consider the vanishing superpotential $W=0$.[9] In this case, $F^i=0$ is a solution to this equation, and the on-shell Lagrangian becomes

$$\mathcal{L}_b = -g_{i\bar{j}}\partial_m\varphi^i\partial^m\bar{\varphi}^{\bar{j}} + \Lambda_{ik\bar{j}\bar{l}}(\partial_m\varphi^i\partial^m\varphi^k)(\partial_n\bar{\varphi}^{\bar{j}}\partial^n\bar{\varphi}^{\bar{l}}). \tag{3.8}$$

We call this canonical branch. We note that the second term in (3.8) contains more than the fourth order of space-time derivatives for appropriate functions $\Lambda_{ik\bar{j}\bar{l}}$. We will demonstrate an example of sixth-derivative terms in Sec. IV B.

In general, there are more solutions other than $F^i=0$, although an explicit solution F^i is not easy to find except for one component field. Indeed, for single superfield models, we have other on-shell branches associated with solutions $F^i \neq 0$ [28,39]. We call this noncanonical branch. In the noncanonical branch, the ordinary kinetic term with two space-time derivatives vanishes and the on-shell Lagrangian consists of only four-derivative terms. Although it is interesting, we do not consider this branch because we are considering derivative expansions.

B. *B* type (pure realizations)

When there are no quasi-NG modes, it is called a pure realization. This is possible only when G/H is eventually Kähler. When there is a gauge symmetry, the pure realization without quasi-NG bosons is possible [19]. From the Borel's theorem, compact Kähler coset spaces G/H can be written as

$$G/H = G/[H_{\mathrm{s.s.}} \times U(1)^r] \tag{3.9}$$

with $H_{\mathrm{s.s.}}$ the semisimple subgroup in H and $r \equiv \mathrm{rank}G - \mathrm{rank}H_{\mathrm{s.s.}}$. [43]. In this case, there exists the isomorphism

$$G/H \simeq G^{\mathbb{C}}/\hat{H}. \tag{3.10}$$

The most general G-invariant Kähler potential (up to Kähler transformations) was shown to be written solely by B-type Kähler potentials and A and C types were shown not to give independent Kähler potentials [8–10].

Now we consider higher-derivative terms. In this case, the problem is reduced to find G-invariant (2,2) tensors $\Lambda_{ik\bar{j}\bar{l}}$ on the target manifold G/H. The G-transformation on the fields are

$$\delta\Phi^i_A = k^i_A, \tag{3.11}$$

where $k^i_A(\Phi)$ $(A = 1,2,\ldots,\dim G)$ are holomorphic Killing vectors generated by the isometry G, preserving the metric $\mathcal{L}_k g_{i\bar{j}} = 0$. The (2,2) tensors $\Lambda_{ik\bar{j}\bar{l}}$ for higher-derivative term must be preserved by the isometry G: $\mathcal{L}_k\Lambda_{ik\bar{j}\bar{l}} = 0$. Then, G-invariant four-derivative terms are given by

[9] If we consider the spontaneously breaking of approximate symmetries, a nonzero superpotential W that provides small mass to the pseudo-NG modes is possible.

$$\mathcal{L}^{(4)} = \frac{1}{16}\int d^4\theta \Lambda_{ik\bar{j}\bar{l}}(\Phi, \Phi^\dagger) D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \Phi^{\dagger\bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger\bar{l}},$$
$$\Lambda_{ik\bar{j}\bar{l}} = w_1 g_{(i\bar{j}} g_{k\bar{l})} + w_2 R_{i\bar{j}k\bar{l}} + w_3 R_{(i\bar{j}} R_{k\bar{l})} + w_4 g_{(i\bar{j}} R_{k\bar{l})}, \tag{3.12}$$

where $R_{i\bar{j}k\bar{l}}$ and $R_{i\bar{j}}$ are the Riemann curvature and Ricci form, respectively, brackets $(\ldots)$ imply symmetrization over holomorphic and antiholomorphic indices, and $w_{1,2,3}$ are real constants. The scalar curvature R is also invariant but it is just a constant for G/H. The explicit form of the curvature tensor can be found in Ref. [44]. In some cases, the terms in Eq. (3.12) are not independent. For Einstein manifolds, $R_{i\bar{j}} \sim g_{i\bar{j}}$ holds. For instance, rank one cases ($r = 1$) belong to this class.

An important fact is that there are no strict G invariants, unlike the case with quasi-NG bosons which we discuss in the next subsections. This is the reason why higher-derivative terms are uniquely determined up to constants.

As for derivative terms higher than four derivatives, one uses the covariant derivatives of tensors such as $D_g \bar{D}_{\bar{h}} R_{i\bar{j}k\bar{l}}$. For instance, a six-derivative term can be constructed as

$$\mathcal{L}^{(6)} = \frac{1}{16}\int d^4\theta D_g \bar{D}_{\bar{h}} R_{i\bar{j}k\bar{l}} \partial_m \Phi^g \partial^m \Phi^{\dagger\bar{h}} D^\alpha \Phi^i D_\alpha \Phi^k \times \bar{D}_{\dot{\alpha}} \Phi^{\dagger\bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger\bar{l}} + \cdots. \tag{3.13}$$

C. A type

The Kähler potential of A type is given in Eq. (2.16). There are two ways to construct G-invariant four-derivative terms using the A-type invariants. The first way is a geometrical method which is the same with pure realizations, and the second way is a group theoretical method.

In the first method, G-invariant four-derivative terms are given by

$$\mathcal{L}^{(4)} = \frac{1}{16}\int d^4\theta \Lambda_{ik\bar{j}\bar{l}}(\Phi, \Phi^\dagger) D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \Phi^{\dagger\bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger\bar{l}},$$
$$\Lambda_{ik\bar{j}\bar{l}} = w_1(X_{ab}) g_{(i\bar{j}} g_{k\bar{l})} + w_2(X_{ab}) R_{i\bar{j}k\bar{l}} + w_3(X_{ab}) R_{(i\bar{j}} R_{k\bar{l})} + w_4(X_{ab}) g_{(i\bar{j}} R_{k\bar{l})}. \tag{3.14}$$

Unlike the B-type case, $w_{1,2,3,4}$ are *arbitrary functions* of the strict G invariants X_{ab}. The scalar curvature R is a function of X_{ab} and is not included.

Now we introduce the second method to construct G-invariant four-derivative terms. Here we do not write the representation ρ for simplicity. First, the Maurer-Cartan one form on $G^{\mathbb{C}}/\hat{H}$ is given by

$$i\xi^{-1} d\xi = (E_i^I(\Phi) X_I + \omega_i^a(\Phi) H_a) d\Phi^i \tag{3.15}$$

with the holomorphic vielbein $E_i^I(\Phi)$ and the holomorphic connection $\omega_i^a(\Phi)$. By using this expression, we calculate

$$D_\alpha \xi = D_\alpha \Phi^i \partial_i \xi = i\xi(E_i^I(\Phi) X_I + \omega_i^a(\Phi) H_a) D_\alpha \Phi^i, \tag{3.16}$$

$$D_\alpha \xi v_a = i(\xi X_I v_a) E_i^I(\Phi) D_\alpha \Phi^i. \tag{3.17}$$

Then, the supercovariant derivatives of the G invariants X_{ab} given in Eq. (2.15) can be calculated to be

$$D_\alpha X_{ab} = (v_a \xi^\dagger \xi X_I v_b) E_i^I(\Phi) D_\alpha \Phi^i \tag{3.18}$$

$$D^\alpha D_\alpha X_{ab} = (v_a \xi^\dagger \xi X_J X_I v_b) E_i^I(\Phi) E_j^J(\Phi) D^\alpha \Phi^i D_\alpha \Phi^j \tag{3.19}$$

$$\bar{D}^{\dot{\alpha}} D_\alpha X_{ab} = (v_a X_J^\dagger \xi^\dagger \xi X_I v_b) E_i^I(\Phi) E_j^{*J}(\Phi^\dagger) D_\alpha \Phi^i \bar{D}^{\dot{\alpha}} \Phi^{\dagger\bar{j}}, \tag{3.20}$$

$$\bar{D}_{\dot{\alpha}} D^\alpha X_{ab} \bar{D}^{\dot{\alpha}} D_\alpha X_{cd} = (v_a X_J^\dagger \xi^\dagger \xi X_I v_b)(v_c X_L^\dagger \xi^\dagger \xi X_K v_d) \times E_i^I(\Phi) E_k^K(\Phi) E_j^{*J}(\Phi^\dagger) E_l^{*L}(\Phi^\dagger) \times D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \Phi^{\dagger\bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger\bar{l}}, \tag{3.21}$$

$$\bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} D^\alpha D_\alpha X_{ab} = (v_a X_J^\dagger X_L^\dagger \xi^\dagger \xi X_K X_I v_b) E_i^I(\Phi) E_k^K(\Phi) E_j^{*J}(\Phi^\dagger) E_l^{*L}(\Phi^\dagger) \times D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \Phi^{\dagger\bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger\bar{l}}, \tag{3.22}$$

$$D^\alpha D_\alpha X_{ab} \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} X_{cd} = (v_a \xi^\dagger \xi X_K X_I v_b)(v_c X_J^\dagger X_L^\dagger \xi^\dagger \xi v_d) \times E_i^I(\Phi) E_k^K(\Phi) E_j^{*J}(\Phi^\dagger) E_l^{*L}(\Phi^\dagger) \times D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \Phi^{\dagger\bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger\bar{l}}. \tag{3.23}$$

By using these relations, four-derivative terms can be given by

$$\begin{aligned}\mathcal{L}^{(4)} = \frac{1}{16}\int d^4\theta [&g_1^{ab}(X_{mn}) \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} D^\alpha D_\alpha X_{ab} + g_2^{abcd}(X_{mn}) \bar{D}_{\dot{\alpha}} D^\alpha X_{ab} \bar{D}^{\dot{\alpha}} D_\alpha X_{cd} + g_3^{abcd}(X_{mn}) D^\alpha D_\alpha X_{ab} \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} X_{cd} \\ &+ g_4^{abcdef}(X_{mn}) D^\alpha X_{ab} D_\alpha X_{cd} \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} X_{ef} + g_5^{abcdef}(X_{mn}) D^\alpha D_\alpha X_{ab} \bar{D}_{\dot{\alpha}} X_{cd} \bar{D}^{\dot{\alpha}} X_{ef} \\ &+ g_6^{abcdef}(X_{mn}) D^\alpha X_{ab} D_\alpha \bar{D}_{\dot{\alpha}} X_{cd} \bar{D}^{\dot{\alpha}} X_{ef} + g_7^{abcdefgh}(X_{mn}) D^\alpha X_{ab} D_\alpha X_{cd} \bar{D}_{\dot{\alpha}} X_{ef} \bar{D}^{\dot{\alpha}} X_{gh}]\end{aligned} \tag{3.24}$$

with arbitrary real functions $g_{\#}^{ab\cdots}$ of the G invariants X_{mn}. From this equation, the components of $\Lambda_{ik\bar{j}\bar{l}}$ can be read as

$$\begin{aligned}\Lambda_{ik\bar{j}\bar{l}} = {}& [g_1^{ab}(X_{mn})(v_a X_J^\dagger X_L^\dagger \xi^\dagger \xi X_K X_I v_b) + g_2^{abcd}(X_{mn})(v_a X_J^\dagger \xi^\dagger \xi X_I v_b)(v_c X_L^\dagger \xi^\dagger \xi X_K v_d) \\ & + g_3^{abcd}(X_{mn})(v_a \xi^\dagger \xi X_K X_I v_b)(v_c X_J^\dagger X_L^\dagger \xi^\dagger \xi v_d) + g_4^{abcdef}(X_{mn})(v_a \xi^\dagger \xi X_I v_b)(v_c \xi^\dagger \xi X_K v_d)(v_e X_J^\dagger X_L^\dagger \xi^\dagger \xi v_f) \\ & + g_5^{abcdef}(X_{mn})(v_a \xi^\dagger \xi X_K X_I v_b)(v_c X_J^\dagger \xi^\dagger \xi v_d)(v_e X_L^\dagger \xi^\dagger \xi v_f) + g_6^{abcdef}(X_{mn})(v_a \xi^\dagger \xi X_I v_b)(v_c X_J^\dagger \xi^\dagger \xi X_K v_d)(v_e X_L^\dagger \xi^\dagger \xi v_f) \\ & + g_7^{abcdefgh}(X_{mn})(v_a \xi^\dagger \xi X_I v_b)(v_c \xi^\dagger \xi X_K v_d)(v_e X_J^\dagger \xi^\dagger \xi v_f)(v_g X_L^\dagger \xi^\dagger \xi v_h)] \times E_i^I(\Phi) E_k^K(\Phi) E_j^{*J}(\Phi^\dagger) E_l^{*L}(\Phi^\dagger). \end{aligned} \quad (3.25)$$

Note that Eq. (3.25) contains the multiple functions labeled by $ab\ldots$, implying more general than Eq. (3.14).

Derivative terms higher than four derivatives can be constructed by using space-time derivative on X_{ab}. For instance, six-derivative terms can be constructed as

$$\begin{aligned}\mathcal{L}^{(6)} = {}& \frac{1}{16}\int d^4\theta \sum_{p=1,2} Y_p [h_{1,p}^{ab}(X_{mn}) \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} D^\alpha D_\alpha X_{ab} + h_{2,p}^{abcd}(X_{mn}) \bar{D}_{\dot{\alpha}} D^\alpha X_{ab} \bar{D}^{\dot{\alpha}} D_\alpha X_{cd} + h_{3,p}^{abcd}(X_{mn}) D^\alpha D_\alpha X_{ab} \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} X_{cd} \\ & + h_{4,p}^{abcdef}(X_{mn}) D^\alpha X_{ab} D_\alpha X_{cd} \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} X_{ef} + h_{5,p}^{abcdef}(X_{mn}) D^\alpha D_\alpha X_{ab} \bar{D}_{\dot{\alpha}} X_{cd} \bar{D}^{\dot{\alpha}} X_{ef} \\ & + h_{6,p}^{abcdef}(X_{mn}) D^\alpha X_{ab} D_\alpha \bar{D}_{\dot{\alpha}} X_{cd} \bar{D}^{\dot{\alpha}} X_{ef}] \end{aligned} \quad (3.26)$$

with arbitrary functions $h_{\#,p}^{ab\cdots}$ of the G invariants X_{mn} and the extra derivative terms Y_p $(p = 1, 2)$ defined by

$$Y_1 = \partial_m \partial^m X_{a'b'}, \qquad Y_2 = \partial_m X_{a'b'} \partial^m X_{c'd'}. \quad (3.27)$$

D. C type

Here, we discuss the construction of higher-derivative terms from the C-type invariants. In the geometrical method, G-invariant four-derivative terms are given by

$$\begin{aligned}\mathcal{L}^{(4)} &= \frac{1}{16}\int d^4\theta \Lambda_{ik\bar{j}\bar{l}}(\Phi, \Phi^\dagger) D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \Phi^{\dagger \bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger \bar{l}}, \\ \Lambda_{ik\bar{j}\bar{l}} &= w_1(\mathrm{tr}(P_a P_b), \ldots) g_{(i\bar{j}} g_{k\bar{l})} + w_2(\mathrm{tr}(P_a P_b), \ldots) R_{i\bar{j}k\bar{l}} \\ & \quad + w_3(\mathrm{tr}(P_a P_b), \cdots) R_{(i\bar{j}} R_{k\bar{l})} \\ & \quad + w_4(\mathrm{tr}(P_a P_b), \cdots) g_{(i\bar{j}} R_{k\bar{l})}. \end{aligned} \quad (3.28)$$

As the A-type case, $w_{1,2,3,4}$ are arbitrary functions of the strict G invariants $\mathrm{tr}(P_a P_b)$, $\mathrm{tr}(P_a P_b P_c)$ and so on.

In the group theoretical method, four-derivative terms can be constructed from the C-type projectors P_a and the supercovariant derivatives D_α and $\bar{D}^{\dot{\alpha}}$. All possible G-invariant terms $X_A(D, \bar{D}; P_a, P_b, \ldots)$ including P_a and two D's and two $\bar{D}$'s are summarized in Table I. These terms are classified by the number of traces and the number of P_a, where each trace should contain more than two P_a's with different $a, b, c\ldots$. Then, the four-derivative term constructed from the C type can be written as

$$\begin{aligned}\mathcal{L}^{(4)} = {}& \frac{1}{16}\int d^4\theta \sum_{A;a,b,\ldots} g_{ab\ldots}^A(\mathrm{tr}(P_c P_d), \ldots) \\ & \times X_A(D, \bar{D}; P_a, P_b, \ldots), \end{aligned} \quad (3.29)$$

where $X_A(D, \bar{D}; P_a, P_b, \ldots)$ are the G-invariant four-derivative terms given in Table I and $g_{ab\ldots}^A$ are *arbitrary* functions of the C-type G invariants $\mathrm{tr}(P_c P_d)$, $\mathrm{tr}(P_c P_d P_e)$ and so on.

This method can be generalized to derivative terms higher than four derivatives. It can be achieved by allowing $g_{ab\ldots}^A$ to contain linear terms including space-time derivatives or allowing X_A to contain space-time derivatives. For instance, six-derivative terms can be constructed as

TABLE I. Four-derivative terms $X_A(D, \bar{D}; P_a, P_b, \ldots)$ constructed from the C-type invariants. The columns denote the number of traces, and the lows denote the number of P_a. Each trace contains more than two P_a's with different $a, b, c\ldots$.

$\#P \backslash \#\mathrm{tr}$	1	2	3
2	$\mathrm{tr}(D^\alpha D_\alpha P_a \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} P_b)$	non	non
3	$\mathrm{tr}(P_a D^\alpha D_\alpha P_b \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} P_c)$	non	non
4	$\mathrm{tr}(D^\alpha P_a D_\alpha P_b \bar{D}_{\dot{\alpha}} P_c \bar{D}^{\dot{\alpha}} P_d)$	$\mathrm{tr}(D^\alpha P_a D_\alpha P_b)\mathrm{tr}(\bar{D}_{\dot{\alpha}} P_c \bar{D}^{\dot{\alpha}} P_d)$	non
	$\mathrm{tr}(P_a D^\alpha D_\alpha P_b \cdot P_c \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} P_d)$	$\mathrm{tr}(D^\alpha P_a \bar{D}_{\dot{\alpha}} P_b)\mathrm{tr}(D_\alpha P_c \bar{D}^{\dot{\alpha}} P_d)$	
5	$\cdots$		$\cdots$

$$\mathcal{L}^{(6)} = \frac{1}{16}\int d^4\theta\Bigg[\sum_{p=1,2;A;a,b,\ldots} h^{ab\cdots}_{A,p}(\mathrm{tr}(P_eP_f),\ldots)\mathrm{tr}(Y_p)X_A(D,\bar{D};P_a,P_b,\ldots)$$
$$+\sum_{p=1,2;a,b,\ldots}\{H^{ab\cdots}_{1,p}(\mathrm{tr}(P_eP_f),\ldots)\mathrm{tr}(Y_pD^\alpha D_\alpha P_a\bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha}P_b)+H^{ab\cdots}_{2,p}(\mathrm{tr}(P_eP_f),\ldots)\mathrm{tr}(Y_pP_aD^\alpha D_\alpha P_b\bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha}P_c)$$
$$+H^{ab\cdots}_{3,p}(\mathrm{tr}(P_eP_f),\ldots)\mathrm{tr}(Y_pD^\alpha P_aD_\alpha P_b\bar{D}_{\dot\alpha}P_c\bar{D}^{\dot\alpha}P_d)+H^{ab\cdots}_{4,p}(\mathrm{tr}(P_eP_f),\ldots)\mathrm{tr}(Y_pP_aD^\alpha D_\alpha P_b\cdot P_c\bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha}P_d)\}+\cdots\Bigg] \quad (3.30)$$

with arbitrary functions $h^{ab\cdots}_{A,p}$ and $H^{ab\cdots}_{A,p}$ of the C-type G invariants, and the extra two-derivative terms Y_p $(p=1,2)$ given by

$$Y_1 = \partial_m\partial^m P_{a'}, \qquad Y_2 = \partial_m P_{a'}\partial^m P_{b'}. \quad (3.31)$$

The dots in Eq. (3.30) imply multi-trace terms such as $\mathrm{tr}(\partial_m P_{a'}D^\alpha D_\alpha P_a)\mathrm{tr}(\partial^m P_{b'}\bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha}P_b)$ and so on.

IV. SUPERSYMMETRIC CHIRAL SYMMETRY BREAKING

In this section, we show an explicit example of higher-derivative interactions of quasi-NG bosons. We consider higher-derivative corrections for supersymmetric chiral symmetry breaking, which is a maximal realization with each massless chiral superfield containing one NG boson and one quasi-NG boson.

A. Supersymmetric chiral Lagrangian

Let us consider the chiral symmetry breaking

$$G = SU(N)_\mathrm{L}\times SU(N)_\mathrm{R} \to H = SU(N)_{\mathrm{L+R}}. \quad (4.1)$$

The corresponding NG modes span the coset space

$$G/H = \frac{SU(N)_\mathrm{L}\times SU(N)_\mathrm{R}}{SU(N)_{\mathrm{L+R}}} \simeq SU(N). \quad (4.2)$$

We denote generators of the coset by $T_A \in \mathcal{SU}(N)$. It was shown in Ref. [16] that a vacuum expectation value belonging to a real representation gives rise to the same numbers of quasi-NG bosons and NG bosons, which is a maximal realization. The chiral symmetry breaking belongs to this class, and the total target space is

$$G^\mathbb{C}/\hat{H} \simeq SU(N)^\mathbb{C} = G^\mathbb{C}/H^\mathbb{C} \simeq SL(N,\mathbb{C}) \simeq T^*SU(N). \quad (4.3)$$

The coset representative is written as

$$M = \exp(i\Phi^A T_A) \in G^\mathbb{C}/\hat{H}, \quad (4.4)$$

where the NG superfields are in the form of

$$\Phi^A(y,\theta) = \pi^A(y) + i\sigma^A(y) + \theta\psi^A(y) + \theta\theta F^A(y), \quad (4.5)$$

with NG bosons π^A, quasi-NG bosons σ^A, and quasi-NG fermions ψ^A.

The nonlinear transformation law of the NG supermultiplets is

$$M \to M' = g_\mathrm{L} M g_\mathrm{R}, \qquad (g_\mathrm{L}, g_\mathrm{R}) \in SU(N)_\mathrm{L}\times SU(N)_\mathrm{R}. \quad (4.6)$$

From the transformation

$$MM^\dagger \to g_\mathrm{L} MM^\dagger g_\mathrm{L}^\dagger, \quad (4.7)$$

the simplest Kähler potential is found to be

$$K_0 = f_\pi^2\mathrm{tr}(MM^\dagger), \quad (4.8)$$

where f_π is a constant. Therefore, the leading order of the bosonic part of the Lagrangian in the derivative expansion reads

$$\mathcal{L}_0 = -f_\pi^2\mathrm{tr}(\partial_m M\partial^m M^\dagger), \quad (4.9)$$

where M is the lowest component of the NG superfield (4.4). However, the Kähler potential in Eq. (4.8) is not general. In fact, the most general Kähler potential can be written as [12,15]

$$K = f(\mathrm{tr}(MM^\dagger), \mathrm{tr}[(MM^\dagger)^2], \ldots, \mathrm{tr}[(MM^\dagger)^{N-1}]) \quad (4.10)$$

with an *arbitrary* function of $N-1$ variables. The physical reason why we have an arbitrary function is the existence of the quasi-NG bosons. Since the isometry of the target manifold is G but not $G^\mathbb{C}$, the target manifold is not homogeneous. One can deform the shape of the target manifold along the directions of the quasi-NG bosons, with keeping the isometry G.[10]

If we set all quasi-NG bosons to be zero [12,13]

$$U = M|_{\sigma^A=0} \in SU(N), \quad (4.11)$$

[10]If one requires the Ricci-flat condition on the target manifold, the arbitrary function is fixed. That is known as the Stenzel metric. This is not the scope of this paper.

we have usual chiral Lagrangian

$$\mathcal{L} = -f_\pi^2 \mathrm{tr}(\partial_m U \partial^m U^\dagger) = -f_\pi^2 \mathrm{tr}(U^\dagger \partial_m U)^2 \tag{4.12}$$

with the decay constant f_π determined from f.

One interesting feature of chiral symmetry breaking in supersymmetric vacua is that the unbroken group $H = SU(N)_{\mathrm{L+R}}$ can be further broken to its subgroup due to the vacuum expectation value of the quasi-NG bosons [12]. Some of quasi-NG bosons change to NG bosons at less symmetric vacua [12,15]

B. Higher-derivative terms: Supersymmetric chiral perturbation

Let us discuss possible higher-derivative terms for the supersymmetric chiral Lagrangian. The simplest candidate of a four-derivative term is

$$\begin{aligned}\mathcal{L}_0^{(4)} &= \frac{1}{16}\int d^4\theta \Lambda_{ik\bar{j}\bar{l}}(\Phi,\Phi^\dagger) D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \Phi^{\dagger\bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger\bar{l}} \\ &= \int d^4\theta \mathrm{tr}(D^\alpha M \bar{D}_{\dot{\alpha}} M^\dagger D_\alpha M \bar{D}^{\dot{\alpha}} M^\dagger),\end{aligned} \tag{4.13}$$

where components of $\Lambda_{ik\bar{j}\bar{l}}$ are determined from the right-hand side. The bosonic part of this term is

$$\mathcal{L}_{0,b}^{(4)} = \mathrm{tr}(\partial^m M \partial^n M^\dagger \partial_m M \partial_n M^\dagger) \tag{4.14}$$

in the canonical branch with $F^A = 0$.

However, Eq. (4.13) is not general. As in the leading term, we have a freedom to deform the tensor along the directions of the quasi-NG bosons. The most general Lagrangian can be written as

$$\begin{aligned}\mathcal{L}^{(4)} &= \frac{1}{16}\int d^4\theta \Lambda_{ik\bar{j}\bar{l}}(\Phi,\Phi^\dagger) D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \Phi^{\dagger\bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger\bar{l}} \\ &= \int d^4\theta \Bigg[\sum_{k=1}^{N-1} g_1^k(\mathrm{tr}(MM^\dagger),\ldots,\mathrm{tr}[(MM^\dagger)^{N-1}])\mathrm{tr}(D^\alpha M \bar{D}_{\dot{\alpha}} M^\dagger D_\alpha M \bar{D}^{\dot{\alpha}} M^\dagger (MM^\dagger)^k) \\ &\quad + \sum_{k,l=1}^{N-1} g_2^{kl}(\mathrm{tr}(MM^\dagger),\ldots,\mathrm{tr}[(MM^\dagger)^{N-1}])\mathrm{tr}(D^\alpha M \bar{D}_{\dot{\alpha}} M^\dagger (MM^\dagger)^k)\mathrm{tr}(D_\alpha M \bar{D}^{\dot{\alpha}} M^\dagger (MM^\dagger)^l)\Bigg]\end{aligned} \tag{4.15}$$

with an arbitrary functions g_1^k and g_2^{kl} of $N-1$ G invariants $\mathrm{tr}(MM^\dagger),\ldots,\mathrm{tr}(MM^\dagger)^{N-1}$. The bosonic part of this term is

$$\begin{aligned}\mathcal{L}_b^{(4)} &= \sum_{k=1}^{N-1} g_1^k(\mathrm{tr}(MM^\dagger),\ldots,\mathrm{tr}[(MM^\dagger)^{N-1}])\mathrm{tr}(\partial^m M \partial^n M^\dagger \partial_m M \partial_n M^\dagger (MM^\dagger)^k) \\ &\quad + \sum_{k,l=1}^{N-1} g_2^{kl}(\mathrm{tr}(MM^\dagger),\ldots,\mathrm{tr}[(MM^\dagger)^{N-1}])\mathrm{tr}(\partial^m M \partial^n M^\dagger (MM^\dagger)^k)\mathrm{tr}(\partial_m M \partial_n M^\dagger (MM^\dagger)^l).\end{aligned} \tag{4.16}$$

If we set all quasi-NG bosons to be zero as in Eq. (4.11),

$$MM^\dagger|_{\sigma^A=0} = UU^\dagger = \mathbf{1}_N \quad (\mathrm{tr}[(MM^\dagger)^k]|_{\sigma^A=0} = N), \tag{4.17}$$

and the bosonic part in the canonical branch with $F^A = 0$ becomes

$$\mathcal{L}_b^{(4)}|_{\sigma=0} = g_{1,0}\mathrm{tr}(\partial^m U \partial^n U^\dagger \partial_m U \partial_n U^\dagger) + g_{2,0}\mathrm{tr}(\partial^m U \partial^n U^\dagger)\mathrm{tr}(\partial_m U \partial_n U^\dagger) \tag{4.18}$$

with $g_{1,0} = \sum_{k=1}^{N-1} g_1^k(N,\ldots,N)$ and $g_{2,0} = \sum_{k,l=1}^{N-1} g_2^{k,l}(N,\ldots,N)$. One notes that the term $\mathrm{tr}(\partial^m U \partial_m U^\dagger \partial^n U \partial_n U^\dagger)$ or $\mathrm{tr}(\partial^m U \partial_m U^\dagger)\mathrm{tr}(\partial^n U \partial_n U^\dagger)$ is not allowed as a bosonic part of the supersymmetric Lagrangian.

Next, let us construct six-derivative terms. They can be written as

$$\begin{aligned}\mathcal{L}^{(6)} = \int d^4\theta \Bigg[&\sum_{k=1}^{N-1} h_1^k(\mathrm{tr}(MM^\dagger),\ldots)\mathrm{tr}(\partial_m M \partial^m M^\dagger D^\alpha M \bar{D}_{\dot{\alpha}} M^\dagger D_\alpha M \bar{D}^{\dot{\alpha}} M^\dagger (MM^\dagger)^k) \\ &+ \sum_{k,l=1}^{N-1} h_2^{kl}(\mathrm{tr}(MM^\dagger),\ldots)\mathrm{tr}(\partial_m M \partial^m M^\dagger D^\alpha M \bar{D}_{\dot{\alpha}} M^\dagger (MM^\dagger)^k)\mathrm{tr}(D_\alpha M \bar{D}^{\dot{\alpha}} M^\dagger (MM^\dagger)^l) \\ &+ \sum_{k,l,j=1}^{N-1} h_3^{klj}(\mathrm{tr}(MM^\dagger),\ldots)\mathrm{tr}(\partial_m M \partial^m M^\dagger (MM^\dagger)^k)\mathrm{tr}(D^\alpha M \bar{D}_{\dot{\alpha}} M^\dagger (MM^\dagger)^l)\mathrm{tr}(D_\alpha M \bar{D}^{\dot{\alpha}} M^\dagger (MM^\dagger)^j) + \cdots\Bigg]\end{aligned} \tag{4.19}$$

with arbitrary functions $h_1^k, h_2^{kl}, h_3^{klj}$ of $N-1$ G invariants $\mathrm{tr}(MM^\dagger), \ldots, \mathrm{tr}(MM^\dagger)^{N-1}$. The dots in Eq. (4.19) imply multi-trace terms such as $\mathrm{tr}(D^\alpha M\partial^m M^\dagger D_\alpha M\bar{D}_{\dot\alpha}M^\dagger(MM^\dagger)^k)\mathrm{tr}(\partial_m M\bar{D}^{\dot\alpha}M^\dagger(MM^\dagger)^l)$ and $\mathrm{tr}(\partial_m M\bar{D}_{\dot\alpha}M^\dagger D^\alpha M\bar{D}^{\dot\alpha}M^\dagger(MM^\dagger)^k)\times \mathrm{tr}(D_\alpha M\partial^m M^\dagger(MM^\dagger)^l)$. The bosonic part of this term is

$$\begin{aligned}\mathcal{L}_b^{(6)} = \sum_{k=1}^{N-1}\Bigg[& h_1^k(\mathrm{tr}(MM^\dagger),\ldots)\mathrm{tr}(\partial_m M\partial^m M^\dagger\partial^n M\partial^o M^\dagger\partial_n M\partial_o M^\dagger(MM^\dagger)^k)\\ &+\sum_{k,l=1}^{N-1} h_2^{kl}(\mathrm{tr}(MM^\dagger),\ldots)\mathrm{tr}(\partial_m M\partial^m M^\dagger\partial^n M\partial_o M^\dagger(MM^\dagger)^k)\mathrm{tr}(\partial_n M\partial^o M^\dagger(MM^\dagger)^l)\\ &+\sum_{k,l,j=1}^{N-1} h_3^{klj}(\mathrm{tr}(MM^\dagger),\ldots)\mathrm{tr}(\partial_m M\partial^m M^\dagger(MM^\dagger)^k)\mathrm{tr}(\partial^n M\partial_o M^\dagger(MM^\dagger)^l)\mathrm{tr}(\partial_n M\partial^o M^\dagger(MM^\dagger)^j)\Bigg] + \cdots. \end{aligned}\tag{4.20}$$

If we set all quasi-NG bosons to be zero, these terms reduce to

$$\begin{aligned}\mathcal{L}_b^{(6)}|_{\sigma=0} =& h_{1,0}\mathrm{tr}(\partial_m U\partial^m U^\dagger\partial^n U\partial^o U^\dagger\partial_n U\partial_o U^\dagger)\\ &+h_{2,0}\mathrm{tr}(\partial_m U\partial^m U^\dagger\partial^n U\partial_o U^\dagger)\mathrm{tr}(\partial_n U\partial^o U^\dagger)\\ &+h_{3,0}\mathrm{tr}(\partial_m U\partial^m U^\dagger)\mathrm{tr}(\partial^n U\partial_o U^\dagger)\mathrm{tr}(\partial_n U\partial^o U^\dagger)\\ &+\cdots.\end{aligned}\tag{4.21}$$

with $h_{1,0}=\sum_{k=1}^{N-1}h_1^k(N,\cdots,N)$, $h_{2,0}=\sum_{k,l=1}^{N-1}h_2^{k,l}(N,\cdots,N)$ and $h_{3,0}=\sum_{k,l,j=1}^{N-1}h_3^{k,l,j}(N,\cdots,N)$.

We can construct the eight- or higher-derivative terms in the same way.

V. CONCLUSION AND DISCUSSIONS

In this paper we have constructed higher-derivative correction terms for massless NG and quasi-NG bosons and fermions in the manifestly supersymmetric off-shell formalism. In general, when a global symmetry is broken in supersymmetric vacua, massless quasi-NG bosons and fermions appear. Low-energy effective theories are governed by supersymmetric nonlinear sigma models of the NG and quasi-NG fields. The number of the quasi-NG fields is determined by the structure of the coset group $G^{\mathbb{C}}/\hat{H}$. The G-invariant Kähler potentials of the nonlinear sigma models are classified into A, B, and C types. We have shown the G-invariant quantities and examples of Kähler potentials.

In superfield formalism, the higher-derivative term in the chiral model is given by a (2,2) Kähler tensor $\Lambda_{ij\bar{k}\bar{l}}$ symmetric in holomorphic and antiholomorphic indices, whose components are functions of the chiral superfields Φ^i. By using this formalism we have constructed higher-derivative corrections to supersymmetric nonlinear realizations. The tensors $\Lambda_{ij\bar{k}\bar{l}}$ are constructed by the G-invariant Kähler metrics in the A, B, and C types. Remarkably, in the A and C types, the tensors $\Lambda_{ij\bar{k}\bar{l}}$ include degrees of freedom for the strict G-invariant quantities X_{ab} and $\mathrm{tr}(P_a P_b\cdots)$. For the B type, this is the pure realization, and there are no quasi-NG modes. We have found that the higher-derivative terms are unique up to constants. For the A and C types, there are quasi-NG modes and higher-derivative terms contain arbitrary functions which depends on the strict G invariants. We have also constructed the higher-derivative terms in purely group theoretical manners. As a practical example, we have further studied the case of chiral symmetry breaking in more detail.

Several discussions are addressed here.

In this paper, we have studied spontaneous breaking of exact symmetry leading to exactly massless NG bosons and quasi-NG bosons (fermions). For approximate symmetry, an explicit breaking term should be introduced which give NG bosons masses. Consequently, they become pseudo NG bosons, such as pions for the chiral symmetry breaking. In supersymmetric theory, a symmetry breaking potential term can be introduced by the superpotential W. The introduction of the superpotential can be treated perturbatively, which was done at least for single component cases [34,39].

As for another future work, the inclusion of the supersymmetric WZW term [23,33] should be discussed for supersymmetric chiral perturbation theory. For supersymmetric chiral perturbation theory with general target spaces, Kähler normal coordinates [45] should be useful as in Ref. [33].

A BPS Skyrme model was discovered some years back [46], which consists of only the sixth-order higher-derivative term as well as appropriate potentials. Our result should be useful to investigate supersymmetric version of this model.

In this paper, we have considered the canonical branch with $F=0$ for solutions to the auxiliary field equations. It is known for the $\mathbb{C}P^1$ model that there is also a noncanonical branch with $F\neq 0$ [28,39]. While the usual kinetic term disappears in this case, the theory admits a baby Skyrmion [28], which was shown to be a $1/4$ BPS state [39]. Investigating noncanonical branches and $1/4$ BPS baby Skyrmions for general Kähler G/H are one of interesting future directions.

The supersymmetric $\mathbb{C}P^{N-1}$ model with four supercharges also appears as the world-volume effective action of a BPS non-Abelian vortex in $\mathcal{N}=2$ supersymmetric

$U(N)$ gauge theory with N hypermultiplets in the fundamental representation [47]. Higher-derivative corrections to the effective action were calculated in Ref. [48]. It was shown that $1/2$ BPS lumps (sigma model instantons) are not modified in the presence of higher-derivative terms [39,48]. This should be so because a composite state of lumps inside a non-Abelian vortex is a $1/4$ BPS state and it is nothing but a Yang-Mills instanton in the bulk point of view [49]. See Refs. [50–52] for a review of BPS composite solitons.

As this regards, some other Kähler G/H manifolds are realized on a vortex in supersymmetric gauge theories with gauge groups G [53]. In particular, the cases of $G = SO(N), USp(N)$ were studied in detail [54]. Therefore, $1/2$ BPS lumps in sigma models on Kähler G/H with higher-derivative terms describe instantons in gauge theories with gauge group G. It should be checked whether higher-derivative corrections for lumps in these cases are canceled out.

The supersymmetric chiral Lagrangian studied in Sec. IV also appears as the effective theory on BPS non-Abelian domain walls in $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theories with $2N$ hypermultiplets in the fundamental representation with mass $\pm m$ [55]. A four-derivative correction was partly derived in Ref. [56].

A general framework of a superfield formulation of the effective theories on BPS soliton world-volumes with four supercharges was formulated in Ref. [57]. This should be generalized to the case with higher-derivative corrections.

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