

**Naturally large tensor-to-scalar ratio in inflation**Guo-Chin Liu,<sup>1</sup> Kin-Wang Ng,<sup>2,3</sup> and I-Chin Wang<sup>1</sup><sup>1</sup>*Department of Physics, Tamkang University, Tamsui, New Taipei City 25137, Taiwan*<sup>2</sup>*Institute of Physics, Academia Sinica, Taipei 11529, Taiwan*<sup>3</sup>*Institute of Astronomy and Astrophysics, Academia Sinica, Taipei 11529, Taiwan*

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Recently, BICEP2 measurements of the cosmic microwave background (CMB)  $B$ -mode polarization at degree angular scales has indicated the presence of tensor modes with a high tensor-to-scalar ratio of  $r = 0.2$  when assuming nearly scale-invariant tensor and scalar spectra, although the signal may be contaminated by dust emission, as implied by recent Planck polarization data. This result is in conflict with the Planck best-fit  $\Lambda$  cold dark model with  $r < 0.11$ . Because the inflaton has to interact with other fields to convert its potential energy into radiation to reheat the Universe, the interacting inflaton may result in a suppression of the scalar spectrum at large scales. This suppression has been used to explain the observed low quadrupole in the CMB anisotropy. In this paper, we show that a combination of the tensor modes measured by BICEP2 and the large-scale suppressed scalar modes contributes to the CMB anisotropy in such a way that the resultant CMB anisotropy and polarization power spectra are consistent with both Planck and BICEP2 data. We also project our findings to cases in which  $r$  may become reduced in future CMB polarization measurements.

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**I. INTRODUCTION**

The spatial flatness and homogeneity of the present Universe strongly suggest that a period of de Sitter expansion or inflation occurred in the early Universe [1]. During inflation, quantum fluctuations of the inflaton field may give rise to energy density perturbations (scalar modes) [2], which can serve as the seeds for the formation of large-scale structures of the Universe. In addition, a spectrum of gravitational waves (tensor modes) is produced from the de Sitter vacuum [3].

In the standard big bang cosmology, scalar and tensor modes leave signatures on the cosmic microwave background (CMB) thoroughly determined by the power spectra,  $C_l^{TT}$ ,  $C_l^{TE}$ ,  $C_l^{EE}$ , and  $C_l^{BB}$ , where  $T$ ,  $E$ , and  $B$  denote the temperature anisotropy,  $E$ -mode polarization, and  $B$ -mode polarization, respectively. CMB anisotropy and  $E$ -mode polarization have been well measured by WMAP, Planck, and many other experiments (and references therein) [5]. Unlike scalar modes, tensor modes are very weakly coupled to matter, so once produced they remain as a stochastic background. However, they induce large-scale CMB anisotropy via the Sachs-Wolfe effect and uniquely  $B$ -mode polarization. Detecting these signals would provide a potentially important probe of the inflationary epoch, and the latter is the primary goal of ongoing and future CMB experiments [6]. Recently, WMAP + SPT CMB data has placed an upper limit on the contribution of tensor modes to the CMB anisotropy in terms of the tensor-to-scalar ratio, which is  $r < 0.18$  at 95% confidence level, tightening to  $r < 0.11$  when measurements of the Hubble constant and baryon acoustic oscillations [7] are also

included. Planck Collaboration XVI has quoted  $r < 0.11$  using a combination of Planck, South Pole Telescope (SPT), and Atacama Cosmology Telescope (ACT) anisotropy data, plus WMAP polarization [5]. More recently, a BICEP2 CMB experiment has found an excess of  $B$ -mode power at degree angular scales, indicating the presence of tensor modes, with  $r = 0.20^{+0.07}_{-0.05}$  when assuming nearly scale-invariant tensor and scalar spectra [8]. If this result is confirmed, it would give very strong support to the inflation model and open a new window for probing inflationary dynamics, though it conflicts with the Planck low  $r$  limit.

There have been many attempts to alleviate the tension on the high tensor-to-scalar ratio of BICEP2. Here we will restrict ourselves to those involving inflationary dynamics [9]. The main idea is to accommodate relatively large tensor contribution in the CMB anisotropy by suppressing the large-scale scalar spectrum with a transient fast-roll phase or sound-speed variation of inflaton fluctuations in a slow-roll inflation, or by anticorrelating the tensor and scalar modes for power cancellation. Recently, the authors in Refs. [10,11] considered the effect on density perturbation due to a quantum environment that interacts with the inflaton. It was shown that the quantum environment constitutes a colored noise that induces inflaton fluctuations, resulting in a suppression of the scalar spectrum at large scales. This suppressed scalar power spectrum was then used to explain the observed low quadrupole in the CMB anisotropy. On the other hand, tensor modes generated during inflation are also affected by the quantum environment, but only through gravitational interaction, so the effect is suppressed by the Planck mass and the tensor modes remain nearly intact. In light of this, we will show

that, by the same token, an interacting inflaton could alleviate the tension between Planck and BICEP2 data.

## II. COLORED NOISE

Motivated by various theoretical reasons and cosmological observations, there has been a lot of interest in inflation models with an interacting inflaton involving a reheating, preheating, or trapping phenomenon in which the inflaton is coupled to scalar, fermion, or vector fields. Here we will not restrict ourselves to a specific inflation model, but rather we will assume a successful inflation potential that satisfies the standard slow-roll condition. Furthermore, we introduce an interaction between the inflaton and scalar fields as a simple working example to show the occurrence of a colored noise due to the interaction during inflation and to study its effect on inflaton fluctuations. There are many studies on the effects of particle production during inflation, though in different contexts, deriving similar inflaton perturbation equations that exhibit both noise and dissipation effects [11–13].

The colored noise stems from the quantum interaction of the inflaton,  $\phi$ , and other fields such as a scalar,  $\sigma$ , while the inflaton is rolling down the potential  $V(\phi)$ . The Lagrangian of this kind of model usually takes the following form (see, for example, Refs. [11,12]):

$$\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) + \mathcal{L}_\sigma, \quad (1)$$

$$\mathcal{L}_\sigma = \sum_i \frac{1}{2} [\partial^\mu \sigma_i \partial_\mu \sigma_i - m_{\sigma_i}^2 \sigma_i^2 - g_i^2 (\phi - \bar{\phi}_i)^2 \sigma_i^2],$$

where  $g_i$  is a coupling constant and  $\bar{\phi}_i$  is a constant field value. For a single  $\sigma$  field and  $\bar{\phi} = 0$ , it reduces to the simplest case in which a massive scalar couples to the inflaton [11]. In the case with many copies of  $\sigma_i$  fields and  $m_{\sigma_i} = 0$ , when  $\phi$  rolls down to a trapping point  $\bar{\phi}_i$ ,  $\sigma_i$  particles become instantaneously massless and are copiously produced and then backreact on the motion of the inflaton [12]. In either case, it has been shown that particle number density fluctuations (or the noise term) in the  $\sigma_i$  particle production would induce a blue power spectrum of the inflaton fluctuations. Below we will give a brief review of this phenomenon. For the purpose of showing the generation of the blue power spectrum, we consider the simplest interaction in a small-field inflation model that has been investigated in Ref. [11]:

$$\mathcal{L}_\sigma = \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{2} g^2 \phi^2 \sigma^2. \quad (2)$$

We can approximate the background metric to be de Sitter during inflation which is given by

$$ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2), \quad (3)$$

where  $a = -1/(H\eta)$  with the Hubble parameter  $H$  and inflation begins at  $a(\eta_i) = 1$ . Then, using the influence functional method that integrates out the interaction term and the field  $\sigma$  and introduces an auxiliary field  $\xi$ , the effective action becomes [14]

$$S_{\text{eff}}[\phi, \phi_\Delta, \xi] = \int d^4x a^2(\eta) \phi_\Delta(x) \left\{ -\ddot{\phi}(x) - 2aH\dot{\phi}(x) + \nabla^2 \phi(x) - a^2[V'(\phi) + g^2 \langle \sigma^2 \rangle \phi(x)] - g^4 a^2(\eta) \phi(x) \int d^4x' a^4(\eta') \theta(\eta - \eta') \times iG_-(x, x') \phi^2(x') + g^2 a^2(\eta) \phi(x) \xi(x) \right\}, \quad (4)$$

where  $\phi_\Delta$  is a relative field variable, the dot and the prime denote differentiation with respect to  $\eta$  and  $\phi$ , respectively, and the kernels  $G_\pm$  can be obtained from the Green's function of  $\sigma$ :

$$G_\pm(x, x') = \langle \sigma(x) \sigma(x') \rangle^2 \pm \langle \sigma(x') \sigma(x) \rangle^2. \quad (5)$$

The effects from the quantum field  $\sigma$  on the inflaton are given by the dissipation via the kernel  $G_-$  as well as a stochastic force induced by the multiplicative colored noise  $\xi$ , with

$$\langle \xi(x) \xi(x') \rangle = G_+(x, x'). \quad (6)$$

Next, we extremize the effective action  $\delta S_{\text{eff}}/\delta \phi_\Delta$  and obtain the semiclassical Langevin equation for  $\phi$ :

$$\ddot{\phi} + 2aH\dot{\phi} - \nabla^2 \phi + a^2[V'(\phi) + g^2 \langle \sigma^2 \rangle \phi] - g^4 a^2 \phi \int d^4x' a^4(\eta') \theta(\eta - \eta') iG_-(x, x') \phi^2(x') = g^2 a^2 \phi \xi + \xi_w, \quad (7)$$

where we have included the white noise  $\xi_w$  in the free-field stochastic inflation [4], with

$$\langle \xi_w(x) \xi_w(x') \rangle \propto \delta(x - x'), \quad (8)$$

which can be produced by integrating out the high-frequency modes of the inflaton (see, for example, Ref. [10]). The white noise reproduces the inflaton vacuum quantum fluctuations  $\langle \phi_q^2 \rangle$  with a scale-invariant power spectrum given by  $\Delta_k^q = H^2/(4\pi^2)$  [15]. Note that the dissipation in the equation is not important at the beginning of inflation because it is a time accumulated term. We may safely neglect it if we only consider the first few  $e$ -folds of inflation. Let us drop the dissipation for the moment and consider the colored noise only. Then, after decomposing  $\phi$  into a mean field and a classical

perturbation,  $\phi(\eta, \mathbf{x}) = \bar{\phi}(\eta) + \varphi(\eta, \mathbf{x})$ , we obtain the linearized Langevin equation,

$$\ddot{\phi} + 2aH\dot{\phi} - \nabla^2\phi + a^2 m_{\text{eff}}^2 \phi = g^2 a^2 \bar{\phi} \xi, \quad (9)$$

where the effective mass is  $m_{\text{eff}}^2 = V''(\bar{\phi}) + g^2 \langle \sigma^2 \rangle$  and the time evolution of  $\bar{\phi}$  is governed by  $V(\bar{\phi})$ . The equation of motion for  $\sigma$  from which we construct its Green's function can be read off its quadratic terms in the Lagrangian as

$$\ddot{\sigma} + 2aH\dot{\sigma} - \nabla^2\sigma + a^2 m_\sigma^2 \sigma = 0. \quad (10)$$

Let us decompose

$$Y(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} Y_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad \text{where } Y = \varphi, \xi, \\ \sigma(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} [b_{\mathbf{k}} \sigma_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.c.}], \quad (11)$$

where  $b_{\mathbf{k}}^\dagger$  and  $b_{\mathbf{k}}$  are creation and annihilation operators satisfying  $[b_{\mathbf{k}}, b_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}')$ . Then, the solution to Eq. (9) is obtained as

$$\varphi_{\mathbf{k}}(\eta) = -ig^2 \int_{\eta_i}^{\eta} d\eta' a^4(\eta') \bar{\phi}(\eta') \xi_{\mathbf{k}}(\eta') \\ \times [\varphi_{\mathbf{k}}^1(\eta') \varphi_{\mathbf{k}}^2(\eta) - \varphi_{\mathbf{k}}^2(\eta') \varphi_{\mathbf{k}}^1(\eta)], \quad (12)$$

where the homogeneous solutions  $\varphi_{\mathbf{k}}^{1,2}$  are given by

$$\varphi_{\mathbf{k}}^{1,2} = \frac{1}{2a} (\pi|\eta|)^{\frac{1}{2}} H_\nu^{(1),(2)}(k\eta). \quad (13)$$

Here  $H_\nu^{(1)}$  and  $H_\nu^{(2)}$  are Hankel functions of the first and second kinds, respectively, and  $\nu^2 = 9/4 - m_{\text{eff}}^2/H^2$ . In addition, we have from Eq. (10) that

$$\sigma_{\mathbf{k}}(\eta) = \frac{1}{2a} (\pi|\eta|)^{\frac{1}{2}} [c_1 H_\mu^{(1)}(k\eta) + c_2 H_\mu^{(2)}(k\eta)], \quad (14)$$

where the constants  $c_1$  and  $c_2$  are subject to the normalization condition,  $|c_2|^2 - |c_1|^2 = 1$ , and  $\mu^2 = 9/4 - m_\sigma^2/H^2$ .

To maintain the slow-roll condition,  $m_{\text{eff}}^2 = m_{\text{eff}}^2 \ll H^2$  (i.e.,  $\nu = 3/2$ ), requires that  $g^2 < 1$  and  $m_\sigma^2 > H^2$ . The latter condition limits the growth of  $\langle \sigma^2 \rangle$  during inflation to be less than about  $10^{-2} H^2$  [16,17]. Equation (10) does not include mass corrections to  $m_\sigma^2$  from the mean inflaton field,  $g^2 \bar{\phi}^2$ , and the mass renormalization due to quantum fluctuations of the inflaton,  $g^2 \langle \varphi_q^2 \rangle$ . Under the slow-roll condition,  $\langle \varphi_q^2 \rangle$  grows linearly as  $H^3 t / 4\pi^2$  [16,17] and thus  $\langle \varphi_q^2 \rangle \approx H^2$  after about 60  $e$ -foldings (i.e.,  $Ht \approx 60$ ). Therefore, as long as  $g^2 \bar{\phi}^2 \leq 2H^2$  for the period during which those  $k$  modes of cosmologically relevant scales cross out the horizon, we can conveniently choose

$m_\sigma^2 = 2H^2$  (i.e.,  $\mu = 1/2$ ) for which  $\sigma$  takes a very simple form. After that,  $g^2 \bar{\phi}^2$  may grow to a value much bigger than  $H^2$  and thus the effective mass of  $\sigma$  becomes much larger than  $H^2$ . If so, this large mass will suppress the growth of  $\langle \sigma^2 \rangle$  [17] and may diminish the effect of the noise term. From now on, let us consider only the relevant period with  $g^2 \bar{\phi}^2 \leq 2H^2$ . It was shown that when  $\mu = 1/2$  one can select the Bunch-Davies vacuum (i.e.,  $c_2 = 1$  and  $c_1 = 0$ ) [17]. Hence, using Eqs. (6) and (12), we obtain [11]

$$\langle \varphi_{\mathbf{k}}(\eta) \varphi_{\mathbf{k}'}^*(\eta) \rangle = \frac{2\pi^2}{k^3} \Delta_k^\xi(\eta) \delta(\mathbf{k} - \mathbf{k}'), \quad (15)$$

where the noise-driven power spectrum is given by

$$\Delta_k^\xi(\eta) = \frac{g^4 z_-^2}{8\pi^4} \int_{z_i}^z dz_1 \int_{z_i}^z dz_2 \bar{\phi}(\eta_1) \bar{\phi}(\eta_2) \frac{\sin z_-}{z_1 z_2 z_-} \\ \times [\sin(2\Lambda z_-/k)/z_- - 1] F(z_1) F(z_2), \quad (16)$$

where  $z_- = z_2 - z_1$ ,  $z = k\eta$ ,  $z_i = k\eta_i = -k/H$ ,  $\Lambda$  is the momentum cutoff introduced in the evaluation of the ultraviolet divergent  $k$  integration of  $\sigma_{\mathbf{k}}$  in the Green's function (5), and

$$F(y) = \left(1 + \frac{1}{yz}\right) \sin(y - z) + \left(\frac{1}{y} - \frac{1}{z}\right) \cos(y - z). \quad (17)$$

Note that the term  $\sin(2\Lambda z_-/k)/z_- \approx \pi \delta(z_-)$  when  $\Lambda \gg k$ , so  $\Delta_k^\xi(\eta)$  is insensitive to  $\Lambda$ . Both  $\bar{\phi}(\eta_1)$  and  $\bar{\phi}(\eta_2)$  in Eq. (16) can be approximated as a constant mean field  $\bar{\phi}_0$ .  $\Delta_k^\xi(\eta)$  at the horizon-crossing time given by  $z = -2\pi$  was found in Ref. [11], and here we show the value versus  $k/H$  in Fig. 1. The figure shows that the noise-driven fluctuations depend on the onset time of inflation and approach asymptotically a scale-invariant power spectrum  $\Delta_k^\xi \approx 0.2 g^4 \bar{\phi}_0^2 / (4\pi^2)$  at large  $k$ . Here, the mean field value  $\bar{\phi}_0$  should naturally be of order  $H$  in small-field inflation. In the large-field inflation model discussed in Ref. [12], the obtained noise-driven power spectrum is also blue and there  $\bar{\phi}_0 \sim H$  can be the spacing between the trapping points along the inflaton trajectory.

Let us go back to examining the dissipation term in the Langevin equation (7). As mentioned above, the dissipation is a time accumulated effect. Near the beginning of inflation, by doing integration by parts of this term, it was found [11] that this term only contributes a mass correction of about  $10^{-2} g^4 \bar{\phi}_0^2$  to  $m_{\text{eff}}^2$ , a small friction term of order  $10^{-2} g^4 \bar{\phi}_0^2 a \dot{\phi} / H$  to Eq. (7), and a correction to the slope of the inflaton potential  $V'(\phi)$  of order  $10^{-2} g^4 \bar{\phi}_0^3$ . All of these corrections can be neglected as long as  $g^2 \bar{\phi}_0^2 \leq 2H^2$ .

One should emphasize that the white noise (8) causes a delta-function response and thus produces a scale-invariant spectrum of inflaton quantum fluctuations, whereas the

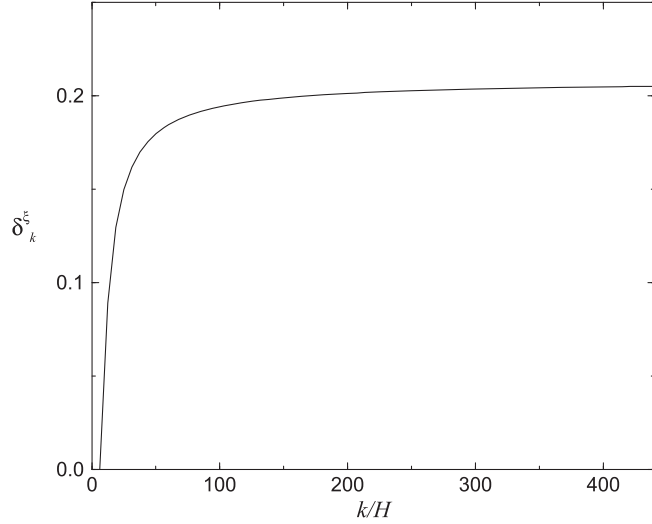


FIG. 1. Power spectrum of the noise-driven inflaton fluctuations  $\delta_k^\xi \equiv 4\pi^2 \Delta_k^\xi / g^4 \bar{\phi}_0^2$ , where  $\bar{\phi}_0$  is the mean field near the beginning of inflation. The starting point,  $k/H = 2\pi$ , corresponds to the  $k$  mode that leaves the horizon at the start of inflation.

colored noise (6) has a causal response due to an interaction between the inflaton and the quantum environment. The latter gives rise to a suppressed power spectrum at cosmologically relevant scales as long as inflation lasts for just about 60  $e$ -foldings. Although the exact form of the colored noise varies with the type of interaction, the blue-tilted power spectrum is just a consequence of the causal response. This characteristic feature in the power spectrum should be quite general; it exists whenever there is interaction between the inflaton and any other quantum fields.

### III. EFFECTS ON LARGE-SCALE CMB

Here, assuming a slow-roll inflation, the scalar power spectrum contains both inflaton quantum fluctuations and those driven by the colored noise, given by

$$P^S = P_{\Lambda\text{CDM}}^S \frac{1 + \Delta_k^\xi / \Delta_k^q}{1 + r'}, \quad (18)$$

where the noise contribution is quantified by a parameter

$$r' \equiv \lim_{k \rightarrow \infty} \Delta_k^\xi / \Delta_k^q \approx 0.2 g^4 \bar{\phi}_0^2 / H^2, \quad (19)$$

the power spectrum of the inflaton quantum fluctuations is given by the scale-invariant  $\Delta_k^q = H^2 / (4\pi^2)$ , and  $P_{\Lambda\text{CDM}}^S$  is the scalar power spectrum in the Planck best-fit lambda cold dark model ( $\Lambda\text{CDM}$ ) [5]. In Fig. 1, we will need to specify the duration of inflation in order to determine the value of  $k/H$  that corresponds to a given comoving scale. In the following, instead of fixing the duration of inflation,

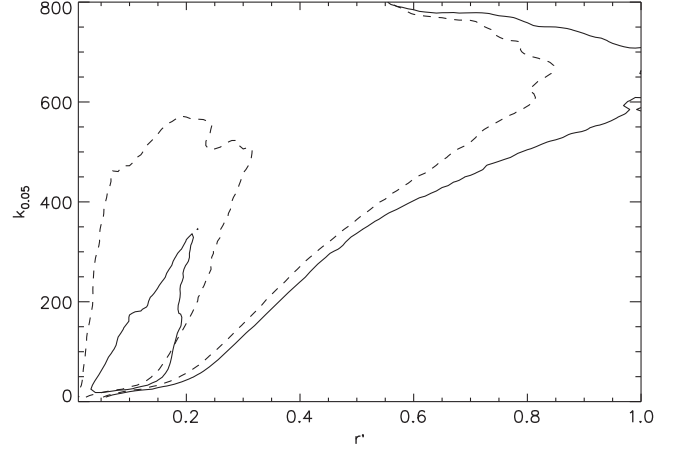


FIG. 2. Solid lines are the likelihood plot of the parameters  $r'$  and  $k_{0.05}$  for the case with  $r = 0.2$ , with one-sigma (the loop in the lower left-hand corner) and two-sigma (unbound) contours. The maximum likelihood values are  $r' = 0.1$  and  $k_{0.05} = 33$ . The dashed lines are for the case with  $r = 0.1$ , with the maximum likelihood values given by  $r' = 0.05$  and  $k_{0.05} = 33$ .

we will treat the value of  $k/H$  that corresponds to  $0.05 \text{ Mpc}^{-1}$  as a free parameter denoted by  $k_{0.05}$ .

On the other hand, in order to fit BICEP2 data of the  $BB$  power spectrum, we prepare a scale-invariant tensor power spectrum in the  $\Lambda\text{CDM}$  model,  $P^T = P_{\Lambda\text{CDM}}^T$ , such that the  $r$  ratio  $P^T / P_{\Lambda\text{CDM}}^S = 0.2$  at  $k = 0.002 \text{ Mpc}^{-1}$ . Recently, Planck polarization data [18] has implied that the BICEP2  $B$ -mode signal may contain contributions from polarized dust emission, so the ratio  $r$  may be reduced significantly. In light of this, we also study the case where  $r = 0.1$ , in which we repeat every step of the case with  $r = 0.2$  except for the use of the  $r = 0.1$  tensor power spectrum.

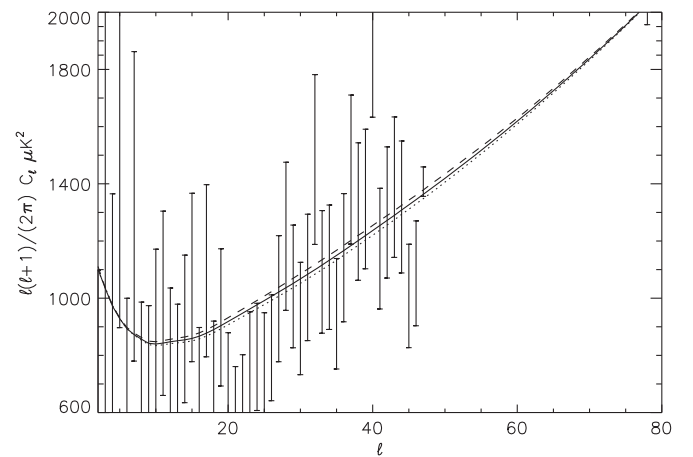


FIG. 3. CMB temperature anisotropy power spectrum of the colored noise model is denoted by the dashed (solid) line for the case with  $r = 0.2$  ( $r = 0.1$ ). The dotted line is the power spectrum predicted in the  $\Lambda\text{CDM}$  model. Overlaid are the Planck measurements. For  $\ell > 70$ , all three spectra overlap.



Then, using  $P^S$  and  $P^T$ , we compute the CMB  $TT$  and  $BB$  power spectra based on the CMBFAST code [19]. We tune the values of  $r'$  and  $k_{0.05}$ , by fixing the other cosmological parameters to the best-fit values of the Planck  $\Lambda$ CDM model [5], to best fit the Planck and BICEP2 data. For the case with  $r = 0.2$ , the likelihood plot in Fig. 2 shows the maximum likelihood values of  $r' = 0.1$  and  $k_{0.05} = 33$ . Figure 3 shows that the  $TT$  power spectrum with  $r' = 0.1$  and  $k_{0.05} = 33$ , which is induced by a combination of both  $P^S$  and  $P^T$ , is indeed the same as that induced only by  $P^S_{\Lambda\text{CDM}}$  within measurement errors. The large-scale  $TT$  power suppression due to the colored noise can really be made up for by the tensor contribution. If  $\bar{\phi}_0 \sim H$  is assumed, then  $r' = 0.1$  would imply that  $g \sim 0.84$ . For the case with  $r = 0.1$ , we find that the results are very similar to those for  $r = 0.2$  except that the value of  $r'$  is now reduced to about  $r' = 0.05$ . Interestingly, recent studies based on Bayesian statistics have indicated that the Planck and BICEP2 joint likelihood analysis strongly favors a scalar power spectrum like that one in Fig. 1 [20].

#### IV. CONCLUSIONS

Because an inflaton has to be interacting with other fields to convert its potential energy into radiation to reheat the

Universe, the interaction may induce a large-scale power suppression in the CMB temperature anisotropy power spectrum. We have shown that the presence of a significant amount of tensor modes, as indicated in the BICEP2 measurement of the CMB  $B$ -mode polarization, would lift up this temperature suppression. This implies that the measured temperature anisotropy power spectrum made by the Planck team can be a combination of scalar and  $r = 0.2$  tensor contributions. If BICEP2 results are confirmed to be genuine tensor  $B$  modes, our work would indicate that we may have a one-sigma detection of the interacting inflation model. On the other hand, if the BICEP2  $B$ -mode signal is mostly polarized dust contamination and  $r$  is thus reduced significantly, this would put a tighter upper bound on the interaction strength. Lastly, we admit that the duration of inflation in the present work must be assumed to be about 60  $e$ -foldings, such that the power suppression takes place only at the large scales of the Universe.

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- [1] For reviews, see K. A. Olive, *Phys. Rep.* **190**, 307 (1990); D. H. Lyth and A. Riotto, *Phys. Rep.* **314**, 1 (1999).
  - [2] A. H. Guth and S.-Y. Pi, *Phys. Rev. Lett.* **49**, 1110 (1982); S. W. Hawking, *Phys. Lett.* **115B**, 295 (1982); A. A. Starobinsky, *Phys. Lett.* **117B**, 175 (1982).
  - [3] A. A. Starobinsky, *Pis'ma Zh. Eksp. Teor. Fiz.* **30**, 719 (1979) [*JETP Lett.* **30**, 682 (1979)].
  - [4] A. A. Starobinsky, in *Field Theory, Quantum Gravity and Strings*, edited by H. J. de Vega and N. Sanchez (Springer, Berlin, 1986); A. S. Goncharov, A. D. Linde, and V. F. Mukhanov, *Int. J. Mod. Phys. A* **02**, 561 (1987).
  - [5] P. A. R. Ade *et al.* (Planck Collaboration), *Astron. Astrophys.* **571**, A16 (2014).
  - [6] See, for example, R. Weiss *et al.*, *arXiv:astro-ph/0604101*.
  - [7] K. T. Story *et al.*, *Astrophys. J.* **779**, 86 (2013).
  - [8] P. A. R. Ade *et al.* (BICEP2 Collaboration), *Phys. Rev. Lett.* **112**, 241101 (2014).
  - [9] C. R. Contaldi, M. Peloso, and L. Sorbo, *J. Cosmol. Astropart. Phys.* **07** (2014) 014; V. Miranda, W. Hu, and P. Adshead, *Phys. Rev. D* **89**, 101302 (2014); S. Choudhury and A. Mazumdar, *arXiv:1403.5549*; M. Kawasaki and S. Yokoyama, *J. Cosmol. Astropart. Phys.* **05** (2014) 046; A. Ashoorioon, K. Dimopoulos, M. M. Sheikh-Jabbari, and G. Shiu, *Phys. Lett. B* **737**, 98 (2014); J.-Q. Xia, Y.-F. Cai, H. Li, and X. Zhang, *Phys. Rev. Lett.* **112**, 251301 (2014); D. K. Hazra, A. Shafieloo, G. F. Smoot, and A. A. Starobinsky, *J. Cosmol. Astropart. Phys.* **06** (2014) 061; *Phys. Rev. Lett.* **113**, 071301 (2014); *J. Cosmol. Astropart. Phys.* **08** (2014) 048; M. Kawasaki, T. Sekiguchi, T. Takahashi, and S. Yokoyama, *J. Cosmol. Astropart. Phys.* **08** (2014) 043; R. Bouso, D. Harlow, and L. Senatore, *arXiv:1404.2278*; H. Firouzjahi and M. H. Namjoo, *Phys. Rev. D* **90**, 063525 (2014); S. Choudhury and A. Mazumdar, *arXiv:1404.3398*; J. P. Zibin, *Phys. Rev. D* **89**, 121301 (2014); M. Bastero-Gil, A. Berera, R. O. Ramos, and J. G. Rosa, *J. Cosmol. Astropart. Phys.* **10** (2014) 053; R. Emami, H. Firouzjahi, and Y. Wang, *Phys. Rev. D* **90**, 021301 (2014); T. Higaki and F. Takahashi, *J. High Energy Phys.* **07** (2014) 074; R. Kallosh, A. Linde, and A. Westphal, *Phys. Rev. D* **90**, 023534 (2014); Y. Wan, S. Li, M. Li, T. Qiu, Y. Cai, and X. Zhang, *Phys. Rev. D* **90**, 023537 (2014); I. P. Neupane, *arXiv:1408.6613*.
  - [10] S. Matarrese, M. A. Musso, and A. Riotto, *J. Cosmol. Astropart. Phys.* **05** (2004) 008; M. Liguori, S. Matarrese, M. A. Musso, and A. Riotto, *J. Cosmol. Astropart. Phys.* **08** (2004) 011.
  - [11] C.-H. Wu, K.-W. Ng, W. Lee, D.-S. Lee, and Y.-Y. Charng, *J. Cosmol. Astropart. Phys.* **02** (2007) 006.
  - [12] W. Lee, K.-W. Ng, I.-C. Wang, and C.-H. Wu, *Phys. Rev. D* **84**, 063527 (2011).
  - [13] A. Berera, M. Gleiser, and R. O. Ramos, *Phys. Rev. D* **58**, 123508 (1998); *Phys. Rev. Lett.* **83**, 264 (1999); J. Yokoyama and A. Linde, *Phys. Rev. D* **60**, 083509 (1999).

- (1999); C.-H. Wu, K.-W. Ng, and L. H. Ford, *Phys. Rev. D* **75**, 103502 (2007); N. Barnaby, Z. Huang, L. Kofman, and D. Pogosyan, *Phys. Rev. D* **80**, 043501 (2009); D. Green, B. Horn, L. Senatore, and E. Silverstein, *Phys. Rev. D* **80**, 063533 (2009); A. Berera, I. G. Moss, and R. O. Ramos, *Rep. Prog. Phys.* **72**, 026901 (2009); M. M. Anber and L. Sorbo, *Phys. Rev. D* **81**, 043534 (2010); L. H. Ford, S. P. Miao, K.-W. Ng, R. P. Woodard, and C.-H. Wu, *Phys. Rev. D* **82**, 043501 (2010); N. Barnaby and M. Peloso, *Phys. Rev. Lett.* **106**, 181301 (2011); N. Barnaby, R. Namba, and M. Peloso, *J. Cosmol. Astropart. Phys.* **04** (2011) 009; J. L. Cook and L. Sorbo, *Phys. Rev. D* **85**, 023534 (2012); N. Barnaby, E. Pajer, and M. Peloso, *Phys. Rev. D* **85**, 023525 (2012); G. J. Turiaci and M. Zaldarriaga, [arXiv:1310.4531](https://arxiv.org/abs/1310.4531); R. O. Ramos and L. A. da Silva, *J. Cosmol. Astropart. Phys.* **03** (2013) 032; M. Bastero-Gil, A. Berera, I. G. Moss, and R. O. Ramos, *J. Cosmol. Astropart. Phys.* **05** (2014) 004.
- [14] M. Morikawa, *Phys. Rev. D* **33**, 3607 (1986); B. L. Hu, J. P. Paz, and Y. Zhang, in *The Origin of Structure in the Universe*, edited by E. Gunzig and P. Nardone (Kluwer, Dordrecht, Netherlands, 1993); E. Calzetta and B. L. Hu, *Phys. Rev. D* **49**, 6636 (1994); M. Gleiser and R. O. Ramos, *Phys. Rev. D* **50**, 2441 (1994); E. Calzetta and B. L. Hu, *Phys. Rev. D* **52**, 6770 (1995); E. A. Calzetta and S. Gonorazky, *Phys. Rev. D* **55**, 1812 (1997); H. Kubotani, T. Uesugi, M. Morikawa, and A. Sugamoto, *Prog. Theor. Phys.* **98**, 1063 (1997); W. Lee, Y.-Y. Charng, D.-S. Lee, and L.-Z. Fang, *Phys. Rev. D* **69**, 123522 (2004); F. C. Lombardo and D. L. Nacir, *Phys. Rev. D* **72**, 063506 (2005).
- [15] S. W. Hawking, *Phys. Lett.* **115B**, 295 (1982); A. A. Starobinsky, *Phys. Lett.* **117B**, 175 (1982); A. H. Guth and S. Y. Pi, *Phys. Rev. Lett.* **49**, 1110 (1982).
- [16] T. S. Bunch and P. C. W. Davies, *Proc. R. Soc. A* **360**, 117 (1978); A. Vilenkin and L. H. Ford, *Phys. Rev. D* **26**, 1231 (1982).
- [17] K. Enqvist, K.-W. Ng, and K. A. Olive, *Nucl. Phys.* **B303**, 713 (1988).
- [18] R. Adam *et al.* (Planck Collaboration), [arXiv:1409.5738](https://arxiv.org/abs/1409.5738).
- [19] U. Seljak and M. Zaldarriaga, *Astrophys. J.* **469**, 437 (1996).
- [20] G. Aslanyan, L. C. Price, K. N. Abazajian, and R. Easther, *J. Cosmol. Astropart. Phys.* **08** (2014) 052; K. N. Abazajian, G. Aslanyan, R. Easther, and L. C. Price, *J. Cosmol. Astropart. Phys.* **08** (2014) 053.