

**Composite  $Z'$** 

Michio Hashimoto\*

*Chubu University, 1200 Matsumoto-cho, Kasugai-shi, Aichi 487-8501, Japan*

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We investigate the possibility of a composite  $Z'$  vector boson. For the compositeness, the required gauge coupling  $g$  in low energy is not so big,  $g^2/(4\pi) \gtrsim 0.015$  in the case of the  $U(1)_{B-L}$  model. We show that the Stückelberg model is effectively induced in low energy via the fermion loop from the Nambu-Jona-Lasinio model having the vectorial four-fermion interaction. In terms of the renormalization group equations, this situation is expressed by the compositeness conditions. We find that the solutions of the renormalization group equations with the compositeness conditions are determined by the infrared fixed points. As a result, the ratio of the masses of the extra electroweak singlet scalar and the right-handed neutrino is fixed. The mass of the composite  $Z'$  boson contains the contribution  $\Delta$  of the Stückelberg mass term. This nonzero  $\Delta$  might be a remnant of a strongly interacting theory in high energy.

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**I. INTRODUCTION**

Recently, the Higgs boson has been discovered at the LHC [1]. The Higgs mass is revealed around 125 GeV, and the nature seems to be consistent with the standard model (SM) [2]. This suggests that the magnitude of the Higgs quartic coupling  $\lambda_H$  is  $\lambda_H \sim 0.1$ . It then turns out that the perturbation works up to a very high energy scale, although  $\lambda_H$  becomes negative at some scale [3]. Recall that the color and weak gauge interactions are asymptotic free within the SM. The SM top-Yukawa coupling  $y_t$  is also a decreasing function with respect to the energy scale  $\mu$ . The hypercharge gauge coupling  $g_Y$  is increasing, but it stays perturbative up to the Planck scale  $\Lambda_{Pl}$ . Therefore it seems that there is no room for strongly interacting theories in high energy.

On the other hand, the gravity is apparently strong around the Planck scale. If the complicated Yukawa structure of the SM is dynamically generated as in low energy QCD, we may expect some strong dynamics in a high energy scale [4,5]. Is there a remnant of such a strong interaction?

Nonzero neutrino mass is one of the evidences of physics beyond the SM. Considering the quark-lepton correspondence [6–9], the existence of the right-handed neutrinos is highly possible. The seesaw mechanism might help to generate tiny neutrino masses [10]. Since the Majorana mass term explicitly breaks the lepton number, we may introduce an extra complex scalar field and a Majorana Yukawa coupling between the scalar and the right-handed neutrino. A simple extension of the SM including the above is the  $B-L$  model [11,12]. The right-handed neutrino is now *inevitably* required in order to cancel the anomaly of the  $U(1)_{B-L}$  gauge current.

In this paper, we explore the possibility of a composite  $Z'$ . Note that we do not need such a strong  $U(1)$  gauge

coupling, if the strong coupling region is around the Planck scale or that of grand unified theories.

The Landau pole  $\Lambda$  of the  $U(1)$  gauge interaction is expressed as  $\Lambda = M \exp(8\pi^2/ag^2)$ , where  $g$  and  $a$  denote the  $U(1)$  gauge coupling and the coefficient of the renormalization group equation (RGE) for  $g$ , respectively. The typical mass scale of the  $U(1)$  gauge boson is  $M$ . In the case of  $U(1)_Y$ , the Landau pole is well above the Planck scale,  $\Lambda \sim 10^{42}$  GeV by taking  $M \sim 100$  GeV,  $a = 41/6$ , and  $g_Y^2/(4\pi) \approx 0.01$ . This is the reason the  $U(1)_Y$  gauge interaction stays perturbative. However, to obtain  $\Lambda \lesssim \Lambda_{Pl}$ , we just need  $g^2/(4\pi) \gtrsim 0.015$  for the  $U(1)_{B-L}$  model, where  $a = 12$  [13]. This lower bound is about half of the weak coupling square,  $g_2^2/(4\pi) \approx 0.03$ . Of course, bigger  $U(1)_{B-L}$  gauge coupling implies an exponentially smaller scale of the Landau pole. In any case, if  $g^2/(4\pi) \gtrsim 0.015$  for the  $U(1)_{B-L}$  model is established in experiments, this suggests the existence of the strong interaction in high energy. If so, we may expect that the  $Z'$  boson associated with the  $U(1)_{B-L}$  model is composite. (For the earlier approach, see, e.g., Ref. [14].)

First, we show that the strong gauge coupling limit of the Stückelberg model [15,16] corresponds to the Nambu-Jona-Lasinio (NJL) model with the vectorial four-fermion interaction. Next, we argue that the Stückelberg model is effectively induced in low energy via the bubble diagram. For the conceptual diagrams, see Figs. 1 and 2. The compositeness at the scale  $\Lambda$  can be described by the compositeness conditions,

$$\frac{1}{g^2(\Lambda)} = \frac{1}{y^2(\Lambda)} = 0, \quad \frac{\lambda(\Lambda)}{y^4(\Lambda)} = 0, \quad (1)$$

where  $y$  and  $\lambda$  are the Majorana Yukawa interaction and the quartic coupling of the extra complex scalar field, respectively, as in the Bardeen, Hill, and Lindner (BHL)

\*michioh@isc.chubu.ac.jp



FIG. 1. Conceptual diagram of the induced propagator for  $\chi$  by the scalar four-fermion interaction represented by the black dots.



FIG. 2. Conceptual diagram of the induced propagator for  $A_\mu$  by the vector four-fermion interaction represented by the black dots.

approach [17] of the top condensate model [18–20]. (See also the earlier attempt [21].) The behavior of  $y^2/g^2$  is controlled by the Pendleton-Ross-type infrared fixed point (PR-IRFP) [22]. Also, we find that  $\lambda/g^2$  is determined by an infrared fixed point. In our approach, the ratio of the masses of the extra scalar  $\chi$  and the right-handed neutrino  $\nu_R$  is fixed to  $M_\chi/M_{\nu_R} \approx 1.2$ . In sharp contrast to the conventional  $U(1)_{B-L}$  model, the  $Z'$  mass has the contribution of the Stückelberg mass term, so that  $\Delta \equiv M_{Z'}^2/g^2 - 4v_\chi^2 > 0$ , where  $\langle \chi \rangle = v_\chi/\sqrt{2}$ . This nonzero  $\Delta$  might be the remnant of the strong dynamics in high energy.

Although we study only a fine-tuning scenario with a light composite  $Z'$  in this paper, it might be more natural that the masses of  $Z'$ ,  $\chi$ , and  $\nu_R$  are around the compositeness scale  $\Lambda$ . In the latter case, we will use the seesaw mechanism [10]. We here mention that the Higgs potential can be stabilized owing to the tree level threshold corrections for the Higgs quartic coupling, which is generated by the  $Z'$  loop effect [23]. We will investigate such a possibility elsewhere.

## II. STÜCKELBERG FORMALISM AND COMPOSITE VECTOR FIELD

### A. Strong coupling limit of the Stückelberg model

Let us first revisit the Stückelberg formalism for the massive photon [11,16]. Introducing the Stückelberg scalar field  $B$ , the Lagrangian density of a massive vector field  $A_\mu$  is

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_g + \mathcal{L}_{\text{gf}}, \quad (2)$$

with

$$\mathcal{L}_\psi = \bar{\psi} i \partial \psi + g \bar{\psi} A \psi, \quad (3)$$

$$\mathcal{L}_g = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 f^2 \left( A_\mu - \frac{1}{gf} \partial_\mu B \right)^2, \quad (4)$$

and

$$\mathcal{L}_{\text{gf}} = -\frac{1}{2} (\partial_\mu A^\mu + gfB)^2, \quad (5)$$

where  $\psi$  is a four-component fermion,  $F_{\mu\nu}$  denotes the field strength,

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (6)$$

and we represented the mass of the vector field  $A_\mu$  by a gauge coupling constant  $g$  times a parameter  $f$  having mass-dimension one. Although  $\psi$  may have a Dirac mass, it is irrelevant in the following discussions. The parts of  $\mathcal{L}_\psi$  and  $\mathcal{L}_g$  are invariant under the (infinitesimal) local  $U(1)$  transformation,

$$\delta A_\mu = \partial_\mu \omega(x), \quad \delta B = gf\omega(x), \quad (7)$$

$$\delta \psi = ig\omega\psi, \quad \delta \bar{\psi} = -ig\omega\bar{\psi}. \quad (8)$$

Unlike the Proca field, this system is power-counting renormalizable. We also note that we can rewrite the gauge fixing term by using the Becchi-Rouet-Stora-Tyutin (BRST) construction.<sup>1</sup>

We may normalize  $A_\mu$  and  $B$  as  $gA_\mu \rightarrow A_\mu$  and  $gB \rightarrow B$ , and then we read the gauge sector,

$$\mathcal{L}_g \rightarrow -\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} f^2 \left( A_\mu - \frac{1}{gf} \partial_\mu B \right)^2; \quad (9)$$

also the gauge fixing term becomes

$$\mathcal{L}_{\text{gf}} \rightarrow -\frac{1}{2g^2} (\partial_\mu A^\mu + gfB)^2. \quad (10)$$

Formally, taking the limit of  $g \rightarrow \infty$ , we obtain

$$\mathcal{L} = \bar{\psi} i \partial \psi + \bar{\psi} A \psi + \frac{1}{2} f^2 A_\mu^2 - \frac{1}{2} f^2 B^2, \quad (11)$$

where we dropped the total derivative term. The Stückelberg fields  $A_\mu$  and  $B$  are now the auxiliary ones, and thus, by performing the Gauss integral, we find

$$\mathcal{L} \rightarrow \bar{\psi} i \partial \psi - G_V (\bar{\psi} \gamma^\mu \psi)^2, \quad (12)$$

with  $G_V = 1/(2f^2)$  and  $A^\mu \sim \bar{\psi} \gamma^\mu \psi$ . That is, the NJL model with the vectorial four-fermion interaction is equivalent to the strong coupling limit of the Stückelberg model.

### B. Stückelberg model as a low energy effective theory and compositeness conditions

We now start from a model with a Majorana-type scalar four-fermion coupling and a vector one,

<sup>1</sup>The Stückelberg scalar field  $B$  has a positive metric, in sharp contrast to the Nakanishi-Lautrup field, which is introduced in a general BRST invariant construction of the gauge fixing term.

$$\mathcal{L} = \bar{\eta}i\partial\eta + G_S(\bar{\eta}^c\eta)(\bar{\eta}\eta^c) - G_V(\bar{\eta}\gamma^\mu\eta)^2, \quad (13)$$

where  $\eta$  is a two-component fermion, for example, a right-handed neutrino, and  $\eta^c$  is the charge conjugation. This system classically has a global  $U(1)$  symmetry,

$$\eta \rightarrow e^{i\theta}\eta, \quad \bar{\eta} \rightarrow e^{-i\theta}\bar{\eta}. \quad (14)$$

By introducing auxiliary fields,  $\phi \sim \bar{\eta}\eta^c$ ,  $\phi^\dagger \sim \bar{\eta}^c\eta$ , and  $A_\mu \sim \bar{\eta}\gamma^\mu\eta$ , we can rewrite the model as follows:

$$\mathcal{L} \rightarrow \bar{\eta}i\partial\eta - \bar{\eta}^c\eta\phi - \bar{\eta}\eta^c\phi^\dagger - M_{\phi,0}^2\phi^\dagger\phi \quad (15)$$

$$+ \bar{\eta}\gamma^\mu\eta A_\mu + \frac{1}{2}f_0^2 A_\mu^2, \quad (16)$$

with  $M_{\phi,0}^2 = 1/G_S$  and  $f_0^2 = 1/(2G_V)$ .

In low energy, the composite scalar and vector fields acquire the kinetic terms via the bubble diagrams,<sup>2</sup> similar to the top condensate model *à la* BHL [17]. See also Figs. 1 and 2. The calculation for the vector-vector correlator is rather tricky, however. As is well known, a sharp cutoff regularization, which is usually used in the calculation of the two point function of the scalar sector, is not appropriate. Instead, we employ the proper time regularization. The vacuum polarization diagram then yields

$$\Pi^{\mu\nu}(p) = -\frac{1}{24\pi^2}(p^2 g^{\mu\nu} - p^\mu p^\nu) \log \Lambda^2, \quad (17)$$

where we introduced the cutoff  $1/\Lambda^2$  in the infrared part of the proper time integral. Thus the induced effective theory in a low energy scale  $\mu$  is

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \bar{\eta}iD\eta + Z_\phi|D_\mu\phi|^2 - M_\phi^2\phi^\dagger\phi - \lambda_\phi(\phi^\dagger\phi)^2 \\ & - \bar{\eta}^c\eta\phi - \bar{\eta}\eta^c\phi^\dagger - \frac{Z_A}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}f^2 A_\mu^2, \end{aligned} \quad (18)$$

where  $D_\mu\eta = \partial_\mu\eta - iA_\mu\eta$ ,  $D_\mu\phi = \partial_\mu\phi + 2iA_\mu\phi$ , and the scalar quartic coupling  $\lambda_\phi$  is induced by the bubble diagram. The wave function renormalization constants are

$$Z_\phi = \frac{1}{16\pi^2} \log \Lambda^2/\mu^2, \quad Z_A = \frac{1}{24\pi^2} \log \Lambda^2/\mu^2. \quad (19)$$

Let us introduce

$$g \equiv \frac{1}{Z_A^2}, \quad y \equiv \frac{1}{Z_\phi^2}, \quad (20)$$

<sup>2</sup>Although we here introduced only a two-component fermion just for a pedagogical explanation, in a realistic  $Z'$  model discussed later, there are many fermions: Three  $\nu_R$ 's contribute to both Figs. 1 and 2, and also the SM fermions contribute to Fig. 2, because they have the  $B-L$  charges. Thus the  $1/N$  approximation is applicable.

and rescale  $A_\mu$  and  $\phi$  as  $A_\mu \rightarrow gA_\mu$  and  $\phi \rightarrow y\phi$ , respectively. The effective theory thereby has the canonical kinetic terms,

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \bar{\eta}iD\eta + |D_\mu\phi|^2 - \tilde{M}_\phi^2\phi^\dagger\phi - \tilde{\lambda}_\phi(\phi^\dagger\phi)^2 \\ & - y\bar{\eta}^c\eta\phi - y\bar{\eta}\eta^c\phi^\dagger - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2f^2A_\mu^2. \end{aligned} \quad (21)$$

We further carry out field-dependent rotations for the fermion and scalar variables,

$$\varphi \equiv e^{i\frac{B(x)}{gf}}\eta, \quad \bar{\varphi} \equiv e^{-i\frac{B(x)}{gf}}\bar{\eta}, \quad (22)$$

and

$$\chi \equiv e^{-2i\frac{B(x)}{gf}}\phi, \quad \chi^\dagger \equiv e^{2i\frac{B(x)}{gf}}\phi^\dagger, \quad (23)$$

also redefine the gauge field  $\tilde{A}_\mu \equiv A_\mu + \frac{1}{gf}\partial_\mu B$ . Because we introduced a redundant field  $B(x)$ , we should add a delta function  $\delta(\xi_B - 1)$  with  $\xi_B \equiv e^{i\frac{B(x)}{gf}}$  in the path integral. Although the fermion path integral measure yields an anomaly from these rotations, this is because we considered a simple model just for a schematic explanation. In the next section, we study an anomaly-free theory. Any vectorial four-fermion couplings consistent with the symmetry would be possible at the compositeness scale  $\Lambda$ , but, only the anomaly-free combinations among the composite gauge interactions should be left in low energy.

In any case, the theory is then

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \bar{\varphi}(i\partial + g\tilde{A})\varphi + |(\partial_\mu + 2ig\tilde{A}_\mu)\chi|^2 - \tilde{M}_\chi^2\chi^\dagger\chi \\ & - \lambda(\chi^\dagger\chi)^2 - y\bar{\varphi}^c\varphi\chi - y\bar{\varphi}\varphi^c\chi^\dagger \\ & - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2f^2\left(\tilde{A}_\mu - \frac{1}{gf}\partial_\mu B\right)^2, \end{aligned} \quad (24)$$

which is essentially equivalent to the Stückelberg model with the complex scalar field. The gauge fixing term comes from the delta function  $\delta(\xi_B - 1)$  added in the partition function. In the NJL picture, we cannot avoid quadratically fine-tuning in order to obtain wanted values of the mass terms for the composite scalar and vector fields in (24) from (13).

In summary, we introduced the redundant field  $B(x)$ , which corresponds to the Stückelberg scalar field, and thereby the global  $U(1)$  symmetry is upgraded to the local one,

$$\varphi \rightarrow e^{ig\omega(x)}\varphi, \quad \bar{\varphi} \rightarrow e^{-ig\omega(x)}\bar{\varphi}, \quad (25)$$

$$\chi \rightarrow e^{-2ig\omega(x)}\chi, \quad \chi^\dagger \rightarrow e^{2ig\omega(x)}\chi^\dagger, \quad (26)$$

$$\tilde{A}_\mu \rightarrow \tilde{A}_\mu + \partial_\mu\omega, \quad B \rightarrow B + gf\omega(x). \quad (27)$$

In the hidden local symmetry approach for the chiral symmetry breaking, the polar decomposition of the complex scalar field is used [24,25]. The hidden symmetry is essentially connected with the ambiguity of the polar decomposition. If we apply the same manner in the above model, the  $U(1)$  symmetry is *explicitly* broken down owing to the Majorana-type interaction. In our approach, when the  $\chi$  field develops a nonzero vacuum expectation value (VEV) in some low energy scale, the local  $U(1)$  symmetry is *spontaneously* broken down. In this case, the gauge field acquires the mass from both the nonzero VEV of  $\chi$  and the Stückelberg mass term built in the model from the very beginning.

Furthermore, it turns out that the Stückelberg model as a low energy effective theory corresponds to the composite model in a high energy scale  $\Lambda$ , when we impose the compositeness conditions,

$$\frac{1}{g^2(\Lambda)} = \frac{1}{y^2(\Lambda)} = 0, \quad \frac{\lambda(\Lambda)}{y^4(\Lambda)} = 0. \quad (28)$$

In the next section, we apply this formulation to a realistic  $Z'$  model.

### III. COMPOSITE $Z'$ MODEL

Let us study the  $U(1)_{B-L}$  extension of the SM. The Lagrangian density is

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_\nu + \mathcal{L}_\chi + \mathcal{L}_{Z'} + \mathcal{L}_{\text{gf}}, \quad (29)$$

where  $\mathcal{L}_{\text{SM}}$  represents the SM part, and

$$\mathcal{L}_\nu = \sum_{f=1,2,3} \bar{\nu}_R^f i \mathcal{D} \nu_R^f, \quad (30)$$

$$\begin{aligned} \mathcal{L}_\chi = & |D_\mu \chi|^2 - M_\chi^2 \chi^\dagger \chi - \lambda_\chi (\chi^\dagger \chi)^2 - \lambda_{\chi H} |H|^2 |\chi|^2 \\ & - Y_{jk} \bar{\nu}_R^{jc} \nu_R^k \chi - Y_{jk} \bar{\nu}_R^{jk} \nu_R^{kc} \chi^\dagger, \end{aligned} \quad (31)$$

$$\mathcal{L}_{Z'} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 f^2 \left( A_\mu - \frac{1}{gf} \partial_\mu B \right)^2. \quad (32)$$

The fields  $H$  and  $\mathcal{L}_{\text{gf}}$  denote the SM Higgs doublet and the gauge fixing term, respectively. The  $U(1)$  part of the covariant derivative is

$$D_\mu = \partial_\mu - iQ_Y (g_Y Y_\mu + \tilde{g} A_\mu) - ig Q_{B-L} A_\mu, \quad (33)$$

where  $Q_Y$  and  $Q_{B-L}$  denote the hypercharge and the  $B-L$  charge, respectively. The gauge couplings of  $U(1)_Y$  and  $U(1)_{B-L}$  are  $g_Y$  and  $g$ , respectively. In general, the gauge mixing coupling  $\tilde{g}$  appears. It is natural to set  $\tilde{g}(\Lambda) = 0$ , because of no gauge kinetic mixing term at the compositeness scale  $\Lambda$ . As for the scalar quartic mixing  $\lambda_{\chi H}$ , the operator  $|H|^2 |\chi|^2$  has a higher dimension than six at the

compositeness scale  $\Lambda$ . Thus we may safely neglect it, i.e.,  $\lambda_{\chi H}(\Lambda) = 0$ .

Unlike the conventional  $Z'$  model, the Stückelberg mass term is incorporated in this composite model as in Eq. (32). This is essential in our formalism of the composite vector field.

The full set of the RGEs for the  $U(1)_{B-L}$  model is shown in Refs. [13,26,27]. Essentially, the RGEs for the gauge and Yukawa couplings are

$$\beta_g \equiv \mu \frac{\partial}{\partial \mu} g = \frac{a}{16\pi^2} g^3, \quad (34)$$

$$\beta_y \equiv \mu \frac{\partial}{\partial \mu} y = \frac{y}{16\pi^2} [by^2 - cg^2], \quad (35)$$

where we took  $Y_{jk} = \text{diag}(y, y, y)$ . The coefficients of the RGEs are  $a = 12$ ,  $b = 10$ , and  $c = 6$ . We may take the number of the right-handed neutrinos having relevant Majorana yukawa couplings to  $N_\nu$  in general, and then the coefficient  $b$  is  $b = 4 + 2N_\nu$ . For details, see Refs. [27,28]. We fix  $N_\nu = 3$  hereafter. Imposing the compositeness conditions,  $1/g^2(\Lambda) = 1/y^2(\Lambda) = 0$ , we analytically find the solutions,

$$\frac{1}{g^2(\mu)} = \frac{a}{8\pi^2} \ln \frac{\Lambda}{\mu}, \quad \frac{1}{y^2(\mu)} = \frac{b}{a+c} \frac{1}{g^2(\mu)}, \quad (36)$$

where  $\Lambda$  is the compositeness scale. This solution corresponds to the PR-IRFP [22].<sup>3</sup> Actually, we can easily rewrite the RGEs as follows:

$$(8\pi^2)\mu \frac{\partial}{\partial \mu} \left( \frac{y^2}{g^2} \right) = bg^2 \cdot \frac{y^2}{g^2} \left( \frac{y^2}{g^2} - \frac{a+c}{b} \right), \quad (37)$$

and hence the solution (36) is the PR-IRFP. Owing to this nature of the PR-IRFP, if we relax the compositeness conditions to  $1/g^2(\Lambda), 1/y^2(\Lambda) \ll 1$  (nonvanishing), the RG flows are not changed so much. The RGE for  $\lambda_\chi$  is a bit complicated,

$$\begin{aligned} \beta_{\lambda_\chi} \equiv \mu \frac{\partial}{\partial \mu} \lambda_\chi = & \frac{1}{16\pi^2} [20\lambda_\chi^2 + \lambda_\chi(24y^2 - 48g^2) \\ & - 48y^4 + 96g^4], \end{aligned} \quad (38)$$

where we ignored the numerically irrelevant  $\lambda_{\chi H}^2$  term. Imposing the compositeness condition for  $\lambda_\chi$ ,  $\lambda_\chi(\Lambda)/y^4(\Lambda) = 0$ , and positivity of  $\lambda_\chi$  in any scale, we obtain the analytical solution,

<sup>3</sup>Strictly speaking, the asymptotic free theory, i.e.,  $a < 0$ , is used in Ref. [22]. Thus the situation  $1/g^2(\Lambda) \rightarrow 0$  occurs in low energy unlike in the asymptotic nonfree theory.

$$\lambda_\chi(\mu) = \frac{2}{25}(9 + \sqrt{546})g^2(\mu). \quad (39)$$

This is a new infrared fixed point (IRFP) solution. Substituting the solutions (36) for  $g$  and  $y$ , we find

$$(16\pi^2)\mu \frac{\partial}{\partial \mu} \left( \frac{\lambda_\chi}{g^2} \right) = 20g^2 \left( \frac{\lambda_\chi}{g^2} - k_+ \right) \left( \frac{\lambda_\chi}{g^2} - k_- \right), \quad (40)$$

where  $k_+ \equiv \frac{2}{25}(9 + \sqrt{546}) \approx 2.589$  and  $k_- \equiv \frac{2}{25}(9 - \sqrt{546}) \approx -1.149$ . Thus the analytical solution (39) corresponds to the IRFP in fact.

We depict the RG flows in Fig. 3, by using the full set of the RGEs. We took  $m_t = 173.5$  GeV,  $m_h = 126.8$  GeV, and  $\Lambda = 1/\sqrt{8\pi G} = 2.435 \times 10^{18}$  GeV. Also, it is reasonable to put  $\tilde{g}(\Lambda) = \lambda_{\chi H}(\Lambda) = 0$ , as we mentioned before. Admittedly, the values are nonperturbative in the UV region, as shown in Fig. 3, but we confirmed that the flows in the IR region are almost unchanged, even if we vary the compositeness conditions to  $g^2(\Lambda), y^2(\Lambda), \lambda_\chi(\Lambda) \sim 3-10$ .

How about the nature of the IRFP? We show  $y/g$  and  $\sqrt{\lambda_\chi}/g$  in Fig. 4. These are slightly running, but almost constants. Numerically, we find the infrared values as  $y/g = 1.27, 1.28, 1.31, 1.32, 1.33$  and  $\sqrt{\lambda_\chi}/g = 1.47, 1.51, 1.55, 1.57, 1.59$  for  $\Lambda = 2.435 \times 10^{18}, 10^{15}, 10^{10}, 10^8, 10^6$  GeV, respectively. Analytical expressions suggest  $y/g = 3/\sqrt{5} = 1.342$  and  $\sqrt{\lambda_\chi}/g = \sqrt{2(9 + \sqrt{546})}/5 = 1.609$ .

Taking the VEV of  $\chi$  as  $\langle v_\chi \rangle = v_\chi/\sqrt{2}$ , the square of the masses of  $\nu_R, \chi$ , and  $Z'$  are

$$\begin{aligned} M_{\nu_R}^2 &\approx 2y^2 v_\chi^2, & M_\chi^2 &\approx 2\lambda_\chi v_\chi^2, \\ M_{Z'}^2 &\approx 4g^2 v_\chi^2 + g^2 f^2. \end{aligned} \quad (41)$$

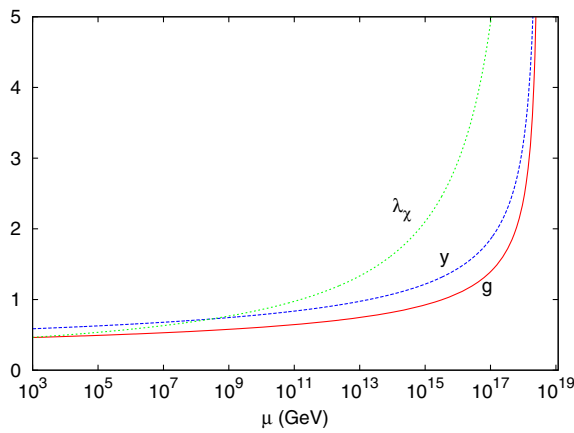


FIG. 3 (color online). The RG flows of  $g, y$ , and  $\lambda_\chi$ . We took  $\Lambda = 1/\sqrt{8\pi G} = 2.435 \times 10^{18}$  GeV, and imposed the compositeness conditions,  $1/g^2(\Lambda) = 1/y^2(\Lambda) = 0$  and  $\lambda(\Lambda)/y^4(\Lambda) = 0$ .

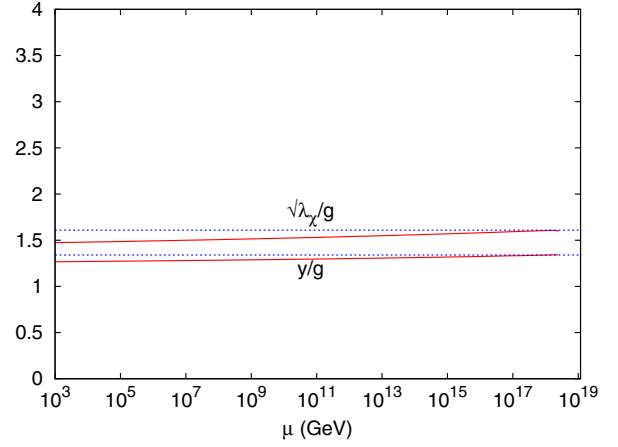


FIG. 4 (color online). The IRFP nature of  $\sqrt{\lambda_\chi}/g$  (upper solid line) and  $y/g$  (lower solid line). We took  $\Lambda = 1/\sqrt{8\pi G} = 2.435 \times 10^{18}$  GeV. The upper and lower dashed lines correspond to the approximate values  $\sqrt{2(9 + \sqrt{546})}/5 \approx 1.609$  and  $3/\sqrt{5} \approx 1.342$ , respectively.

The IRFP solutions yield the mass relation between  $\nu_R$  and  $\chi$ ,

$$\frac{M_\chi}{M_{\nu_R}} = \frac{\sqrt{\lambda_\chi}}{y} \approx 1.2. \quad (42)$$

In sharp contrast to the conventional approach for  $Z'$ , we have the contribution of the Stückelberg mass to  $M_{Z'}$ ,

$$\Delta \equiv \frac{M_{Z'}^2}{g^2} - 4v_\chi^2 = f^2 > 0. \quad (43)$$

If the experiments such as LHC and ILC observe  $\Delta > 0$  and confirm  $g^2/(4\pi) \gtrsim 0.015$ , it implies the compositeness of  $Z'$ .

#### IV. SUMMARY AND DISCUSSIONS

We studied the composite  $Z'$  vector boson. We found that the strong coupling limit of the Stückelberg model corresponds to the NJL model and that the Stückelberg model is effectively induced in low energy via the fermion loop from the NJL model. This correspondence can be encoded in terms of the RGEs with the compositeness conditions. We showed that the RG flows are determined by the IRFP. The nature of the IRFP yields the mass ratio,  $M_\chi/M_{\nu_R} = \sqrt{\lambda_\chi}/y \approx 1.2$ . The  $Z'$  mass is composed of the VEV of  $\chi$  and the Stückelberg mass. Owing to this extra mass term, the constraints of the  $Z'$  model from the precision measurements, the bounds of the direct searches at Tevatron, LHC, etc. [29–31], should be relaxed. If  $\Delta \equiv M_{Z'}^2/g^2 - 4v_\chi^2 > 0$  is established in experiments [32], this might be evidence of the strong dynamics in high energy.

We here considered a scenario that the composite  $Z'$  boson survives in low energy. From the viewpoint of the naturalness, however, the masses of  $Z'$ ,  $\chi$ , and  $\nu_R$  might not be so far below the compositeness scale  $\Lambda$ . In this case, the seesaw mechanism may work. We also note that the Higgs potential can be stabilized by the tree level shift of the Higgs quartic coupling essentially

generated by the  $Z'$  loop contribution [23]. Our approach is unlikely to be applicable to the Stückelberg extension of the SM [33], because the weak and hypercharge gauge couplings are perturbative up to the Planck scale. However, the dark matter might be connected with the composite  $Z'$  [34]. We will study several such scenarios elsewhere.

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