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Higgs boson-radion similarity in production processes involving off-shell fermions

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The appearance of the radion field and of the radion, the corresponding lowest Kaluza-Klein (KK) mode, is a generic prediction of stabilized brane world models. In such models the radion plays the role of the dilaton, and its mass may be somewhat smaller than that of all the KK modes of other particles propagating in the multidimensional bulk. Because of its origin, the radion couples to the trace of the energy-momentum tensor of the standard model (SM), the interaction Lagrangian of the radion, and the standard model fermions similar to those of the SM Higgs boson-fermion interactions except for additional terms, which come into play only in the case of off-shell fermions. In the present paper it is shown that all the contributions due to these additional terms to perturbative amplitudes of physical processes with emitted single radion and an arbitrary number of gauge bosons are canceled out for both massless and massive off-shell fermions. Thus, in this case the additional fermion-radion terms in the interaction Lagrangian do not alter any production and decay properties of the radion compared to those of the Higgs boson.

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I. INTRODUCTION

There are few generic predictions in brane world models as possible extensions of the standard model (SM). The fields propagating in the multidimensional bulk manifest themselves as Kaluza-Klein (KK) towers of states on the brane, where we are supposed to live. These KK towers of states are an example of such generic predictions. Another phenomenon of this kind is the presence of a new scalar field (or fields) on "our" brane (TeV brane) associated with spin-0 fluctuations of the metric component corresponding to extra space dimension. The size of the extra space dimension should be stabilized in order to get a physically meaningful picture. This leads to the violation of the dilatation invariance, and the spin-0 component of metric fluctuations together with the fluctuations of the additional scalar field introduced for stabilizing the size renders a new scalar field called the radion field, which plays the role of the dilaton in the system at hand.

The radion field is the collection of the spin-0 degrees of freedom, coming from the scalar fluctuations of the metric in extra dimension and of the stabilizing scalar field [1]. For both minimal and nonminimal couplings of the stabilizing scalar field to gravity, this field can be identified with certain combinations of the fluctuations of the metric and of the scalar field in convenient gauges, which connect the fluctuations of the metric with those of the scalar field and coincide with a gauge for isolating the radion field in the unstabilized Randall-Sundrum (RS) model, if the stabilizing scalar field is put equal to zero [2,3].

The Randall-Sundrum setup [4] with stabilizing scalar field as proposed by Goldberger and Wise [5] and worked out by DeWolfe, Freedman, Gubser, and Karch [6], taking into account the backreaction by solving exactly the coupled equations for the metric and the scalar field, provides an example of a concrete realization of a stabilized brane world model. In a number of studies it was argued that the lowest scalar state, usually called the radion [7–9], might be significantly lighter than all the other KK excitations [1,10].

The phenomenology of the radion follows from the simple effective Lagrangian

$$L = -\frac{r(x)}{\Lambda_r} T^{\mu}_{\mu}, \tag{1}$$

where T^{μ}_{μ} is the trace of the SM energy-momentum tensor, $r(x)$ stands for the radion field, and Λ_r[1] is a dimensional scale parameter. Since the radion plays the role of the dilaton, it interacts with the massless photon and gluon fields via the well-known dilatation anomaly in the trace of the energy-momentum tensor. At the lowest order in the SM couplings the trace has the following form, where the fields, as supposed in most of the studies, are taken on the mass shell,

$$T^{\mu}_{\mu} = \frac{\beta(g_s)}{2g_s} G^{ab}_{\rho\sigma} G^{\rho\sigma}_{ab} + \frac{\beta(e)}{2e} F_{\rho\sigma} F^{\rho\sigma} - m_Z^2 Z^{\mu} Z_{\mu} - 2m_W^2 W^{+}_{\mu} W^{-\mu} + \sum_f m_f \bar{f} f, \tag{2}$$

[1]For the parameter Λ_r, the notation Λ_ϕ is also often used. Λ_r and another often used parameter Λ_π are related by $\Lambda_r = 2\sqrt{3}\Lambda_\pi$.

and the sum is taken over all SM fermions; $\beta(g_s)$ and $\beta(e)$ are the well-known QCD and QED β-functions.

The interaction vertices of the radion with the SM fields are similar to those of the SM Higgs boson except for the anomaly enhanced interactions with gluons and photons. This leads to the corresponding relative enhancement in gluon and photon decay modes and the gluon-gluon fusion production channel of the radion. Various aspects of the decay and production properties of the radion have been touched upon in a number of studies, including a possible mixing of the radion with the Higgs boson, which is important for the radion and Higgs phenomenology [1], [11–18].

After the discovery of the Higgs-like boson at the LHC [19] few studies have been carried out investigating a possible impact of the radion as the second scalar particle on the interpretation of the observed boson [20–26]. The details depend on particular assumptions of the scenario under consideration, such as which fields propagate in the bulk and which are localized on the brane, what is the Higgs boson-radion mixing mechanism and what is the mixing parameter, etc. But generically the light radion with mass below or above the observed 126 GeV boson is still not fully excluded by all the electroweak precision constrains and the LHC data.

The fermion part of Lagrangian (1) for on-shell fermions has a very simple form,

$$L = -\sum_f \frac{r}{\Lambda_r} m_f \bar{f} f, \tag{3}$$

being the same as for the Higgs boson with the replacement of the scale Λ_r by the Higgs vacuum expectation value, $\Lambda_r \to v$. However, for the case of off-shell fermions Lagrangian (3) needs to be modified. The corresponding Lagrangian can be written symbolically in the following form (the explicit form of the Lagrangian is given in Appendix A) [27]:

$$L = -\sum_f \frac{r}{\Lambda_r}\left[\frac{3i}{2}((D_\mu \bar{f})\gamma^\mu f - \bar{f}\gamma^\mu(D_\mu f)) + 4m_f \bar{f} f\right] + \cdots, \tag{4}$$

where D_μ are the SM covariant derivatives. Note that Lagrangian (4) is nontrivial even for massless fermions.

The aim of this short article is to investigate a possible influence of these additional terms on the processes with tree-level diagrams involving the radion and off-shell fermion lines. The extra terms in the Lagrangian lead to a momentum dependence in the vertices potentially giving additional contributions if virtual fermions participate in the process. For the main radion decay processes the fermions are on shell, and these additional terms in the vertices vanish, but this might not be the case for the radion production.

However, it will be explicitly demonstrated that due to the gauge invariance all the additional contributions to the amplitudes of physical processes are canceled out, and the result for the matrix element is the same as computed with only Higgs-like Lagrangian (3). One should stress that in order to get the mentioned cancellations the trace T^μ_μ of the SM energy-momentum tensor should be computed including the SM covariant derivatives as given in Appendix A. The corresponding Feynman rules containing new four-point vertices are presented in Appendix B.

We begin with a simple example of the radion production in the radion-strahlung process. Then we give a more general explanation demonstrating how the cancellation takes place and show a few more examples for the main radion production processes at the LHC and at a linear collider.

II. THE RADION STRAHLUNG AS A SIMPLE EXAMPLE

We begin with a very simple example, the radion production in association with Z boson in e^+e^- collisions called the radion strahlung by analogy with the well-known Higgs production process, the Higgs strahlung. Even neglecting the very small electron mass there are four contributing Feynman diagrams at the lowest order shown in Fig. 1.

Note that the presence of the last four-point diagram is a consequence of the gauge invariance. The Feynman rules for the vertices involved are given in Appendix B. Correspondingly the diagrams give the following contributions to the process amplitude:

$$M_1 = -2iC\bar{e}^+(p_2)\Gamma_\mu e^-(p_1)\frac{1}{p^2 - M_Z^2} M_Z^2 \varepsilon^\mu(p_Z) r(p_r) \tag{5}$$

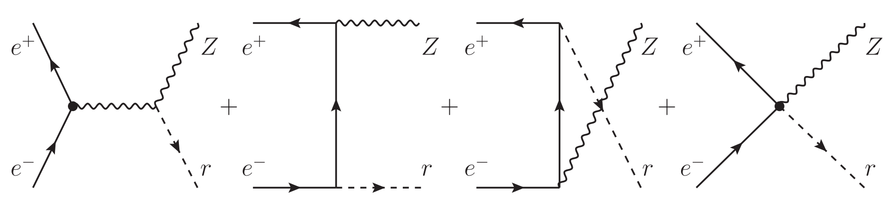

FIG. 1. Feynman diagrams contributing to the radion production in association with the Z boson in e^+e^- collisions (the radion-strahlung process).

$$M_2 = -iC\bar{e}^+(p_2)\left[\frac{3}{2}(\not{k}+\not{p}_2)\right]\frac{\not{k}}{k^2}\Gamma_\mu e^-(p_1)\varepsilon^\mu(p_Z)r(p_r) \quad (6)$$

$$M_3 = -iC\bar{e}^+(p_2)\Gamma_\mu\frac{\not{q}}{q^2}\left[\frac{3}{2}(\not{q}-\not{p}_1)\right]e^-(p_1)\varepsilon^\mu(p_Z)r(p_r) \quad (7)$$

$$M_4 = +3iC\bar{e}^+(p_2)\Gamma_\mu e^-(p_1)\varepsilon^\mu(p_Z)r(p_r), \quad (8)$$

where $C = \frac{1}{\Lambda_r}\frac{e}{2\sin\theta_W\cos\theta_W}$, $p = p_1 + p_2 = p_r + p_Z$, $\Gamma_\mu = \gamma_\mu(2\sin^2\theta_W - \frac{1-\gamma_5}{2})$, $k = p_2 - p_r$, $q = p_r - p_1$.

Keeping in mind the Dirac equation $\not{p}_j e(p_j) = 0$ and the equality $\not{q}\frac{\not{q}}{q^2} = 1$ it is easy to see that the sum of two diagrams D2 and D3 is exactly canceled out by the last diagram D4, so

$$M_2 + M_3 + M_4 = 0. \quad (9)$$

Therefore, the matrix element squared $|M|^2 = |M_1|^2$ for the radion production is exactly equal to that for the Higgs boson, and the cross section takes the well-known form with the replacement $\Lambda_r \to v$ and $M_r \to M_h$,

$$\sigma(e^+e^- \to rZ) = \frac{M_Z^2}{\Lambda_r^2}\frac{\alpha(8\sin^4\theta_W - 4\sin^2\theta_W + 1}{24\sin^2\theta_W\cos^2\theta_W}\frac{\sqrt{\lambda_r}}{4s^2}\frac{\lambda_r + 12M_Z^2 s}{s - M_Z^2}, \quad (10)$$

where $\lambda_r = (M_Z^2 + M_r^2 - s)^2 - 4M_Z^2 M_r^2$.

As one can see all the additional contributions are canceled out. The same property of the cancellation of additional to the Higgs-like contributions takes place for the associated radion and $W^\pm$ boson production, for example,

$$u\bar{d} \to rW^+, \quad (11)$$

where there are also four contributing diagrams similar to those in Fig. 1.

III. CANCELLATIONS OF ADDITIONAL TO THE HIGGS-LIKE CONTRIBUTIONS IN TREE-LEVEL AMPLITUDES

The observed cancellation follows from the structure of the fermion current with the emission of the radion and a number of gauge bosons.

Let us begin with a simple case with just one gauge boson emission as shown in Fig. 2. It corresponds to the radion-strahlung process considered in the previous section with Z boson emission. However, the cancellation property is valid for any emitted SM gauge boson and for any massive SM fermion.

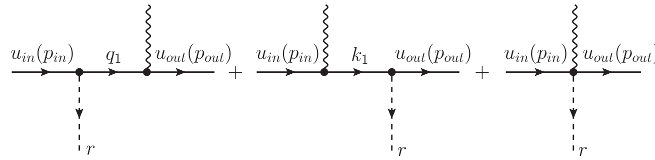

FIG. 2. Fermion current radiating the radion and a SM vector gauge boson.

In order to see that it is instructive to rewrite the fermion-radion vertex in the following way:

$$\begin{aligned}\frac{i}{\Lambda_r}\left[\frac{3}{2}(\not{p}_{\text{out}} + \not{p}_{\text{in}}) - 4m_f\right] &= \frac{i}{\Lambda_r}\left[\frac{3}{2}(\not{p}_{\text{out}} - m_f) + \frac{3}{2}(\not{p}_{\text{in}} - m_f) - m_f\right] \\ &= \frac{i}{\Lambda_r}\left[\frac{3}{2}S^{-1}(p_{\text{in}}) + \frac{3}{2}S^{-1}(p_{\text{out}}) - m_f\right], \end{aligned} \quad (12)$$

where $S^{-1}(p)$ is the inverse function to the propagator[2]

$$S(p) = \frac{\not{p} + m_f}{p^2 - m_f^2}.$$

Note that the last term proportional to m_f in vertex (12) is exactly the same as for the Higgs boson with the obvious replacement $\Lambda_r \to v$.

The diagrams in Fig. 2 can be expressed as follows[3]:

$$D_1 = -iC\bar{u}_{\text{out}}(p_{\text{out}})\Gamma_\mu S(k_1) \times \left[\frac{3}{2}(S^{-1}(k_1) + S^{-1}(p_{\text{in}})) - m_f\right]u_{\text{in}}(p_{\text{in}}) \quad (13)$$

$$D_2 = -iC\bar{u}_{\text{out}}(p_{\text{out}})\left[\frac{3}{2}(S^{-1}(p_{\text{out}}) + S^{-1}(q_1)) - m_f\right] \times S(q_1)\Gamma_\mu u_{\text{in}}(p_{\text{in}}) \quad (14)$$

$$D_3 = +i3C\bar{u}_{\text{out}}(p_{\text{out}})\Gamma_\mu u_{\text{in}}(p_{\text{in}}), \quad (15)$$

where the constant C includes the product of all the factors of the vertices involved, Γ_μ is the Lorentz part of the fermion boson, and fermion-boson-radion vertices, which is the same in two vertices coming from the same covariant derivative. The initial and the final spinors correspond to the same fermion for the neutral fermion current and they are different for the charged current. Once more one can easily see that using the Dirac equation for in and out fermion states and summing up $D_1 + D_2 + D_3$ the only

[2]All the relevant constants from propagators and vertices are included in the common factors of the amplitudes.

[3]To make expressions shorter in all formulas below for the fermion currents, obvious Lorentz indices are omitted in the left-hand sides.

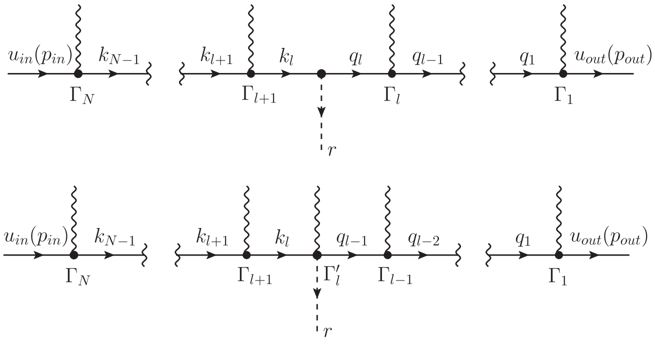

FIG. 3. Fermion current radiating the radion and N SM vector gauge bosons in a three-point vertex (upper graph) and in a four-point vertex (lower graph) corresponding to the contributions M^l and M'_l, respectively.

remaining contribution is proportional to the fermion mass and is the same as that for the Higgs boson.

This property of the cancellation of all the contributions additional to the Higgs-like part can be generalized to an arbitrary number, say N, of gauge bosons emitted from the fermion line in association with the radion as shown in Fig. 3.

The sum of all diagrams can be split into parts containing the fermion-fermion-radion vertex and the boson-fermion-fermion-radion vertex

$$M_{N\text{vector bosons}} = M_0 + \sum_{l=1}^{N}(M_l + M'_l), \tag{16}$$

where

$$M_l \sim i^{2N+1}\bar{f}_{\rm out}(p_{\rm out})\left[\prod_{j=1}^{l}\Gamma^j_{\mu_j}S(q_j)\right] \times \left[-\frac{3}{2}(S^{-1}(q_l)+S^{-1}(k_l))+m_{f_l}\right] \times \left[\prod_{j=1+1}^{N}S(k_{j-1})\Gamma^j_{\mu_j}\right]f_{\rm in}(p_{\rm in}), \tag{17}$$

for $l = 1, \ldots, N-1$

$$M_0 \sim i^{2N+1}\bar{f}_{\rm out}(p_{\rm out})\left[-\frac{3}{2}(S^{-1}(p_{\rm out})+S^{-1}(k_0))+m_{f_{\rm out}}\right] \times \left[\prod_{j=1}^{N}S(k_{j-1})\Gamma^j_{\mu_j}\right]f_{\rm in}(p_{\rm in}), \tag{18}$$

$$M_N \sim i^{2N+1}\bar{f}_{\rm out}(p_{\rm out})\left[\prod_{j=1}^{N}\Gamma^j_{\mu_j}S(q_j)\right] \times \left[-\frac{3}{2}(S^{-1}(q_N)+S^{-1}(p_{\rm in}))+m_{f_{\rm in}}\right]f_{\rm in}(p_{\rm in}), \tag{19}$$

$$M'_l \sim i^{2N-1}\bar{f}_{\rm out}(p_{\rm out})\left[\prod_{j=1}^{l-1}\Gamma^j_{\mu_j}S(q_j)\right][-3\Gamma^l_{\mu_l}] \times \left[\prod_{j=l+1}^{N}S(k_{j-1})\Gamma^j_{\mu_j}\right]f_{\rm in}(p_{\rm in}), \tag{20}$$

for $l = 2, \ldots, N-1$

$$M'_1 \sim i^{2N-1}\bar{f}_{\rm out}(p_{\rm out})[-3\Gamma^1_{\mu_1}]\left[\prod_{j=2}^{N}S(k_{j-1})\Gamma^j_{\mu_j}\right]f_{\rm in}(p_{\rm in}) \tag{21}$$

$$M'_N \sim i^{2N-1}\bar{f}_{\rm out}(p_{\rm out})\left[\prod_{j=1}^{N-1}\Gamma^j_{\mu_j}S(q_j)\right][-3\Gamma^N_{\mu_N}]f_{\rm in}(p_{\rm in}). \tag{22}$$

$\Gamma^j_{\mu_j}$ is the SM fermion and gauge boson vertex, $k_l = p_{\rm in} - \sum_{j=l}^{N} p^V_j$, $q_0 = p_{\rm in}$, $q_l = k_l - p_r = p_{\rm out} + \sum_{j=1}^{l} p^V_j$, p^V_j is the vector boson momentum, the upper index V standing for any SM gauge vector boson (Z, W, photon, gluon), p_r is the radion momentum, and m_{f_l}, $m_{f_{\rm in}}$, $m_{f_{\rm out}}$ are the masses of the internal, initial, and final state fermions that may be different in the case of the charged current. Note that the factors i^{2N+1} and i^{2N-1} are different due to the difference by one of the number of the propagators and vertices involved in expressions (17)–(22). This leads to a relative (-1) sign between the two contributions.

From formulas (17)–(22) one can write simple relations taking into account the relation $S^{-1}(q_j)\times S(q_j) = 1$ and the Dirac equations for in and out fermions

$$M_l = M^H_l - \frac{1}{2}M'_l - \frac{1}{2}M'_{l+1} \tag{23}$$

$$M_0 = M^H_0 - \frac{1}{2}M'_1 \tag{24}$$

$$M_N = M^H_N - \frac{1}{2}M'_N, \tag{25}$$

where the Higgs-like contributions M^H_0, M^H_N, M^H_N labeled with the H symbol being proportional to the fermion masses have the form that follows from (17)–(19),

$$M^H_l \sim i^{2N+1}\bar{f}_{\rm out}(p_{\rm out})\left[\prod_{j=1}^{l}\Gamma^j_{\mu_j}S(q_j)\right][m_{f_l}] \times \left[\prod_{j=1+1}^{N}S(k_{j-1})\Gamma^j_{\mu_j}\right]f_{\rm in}(p_{\rm in}), \tag{26}$$

$$M_0^H \sim i^{2N+1}\bar{f}_{\rm out}(p_{\rm out})[m_{f_{\rm out}}]\left[\prod_{j=1}^{N} S(k_{j-1})\Gamma^j_{\mu_j}\right] f_{\rm in}(p_{\rm in}), \tag{27}$$

$$M_N^H \sim i^{2N+1}\bar{f}_{\rm out}(p_{\rm out})\left[\prod_{j=1}^{N}\Gamma^j_{\mu_j}S(q_j)\right][m_{f_{\rm in}}]f_{\rm in}(p_{\rm in}). \tag{28}$$

Summing up the left- and right-hand sides of Eq. (23) from 1 to $N-1$ and adding the left- and right-hand sides of Eqs. (24)–(25) one gets the following equality:

$$\sum_{l=0}^{N} M_l + \sum_{l=1}^{N} M'_l = \sum_{l=0}^{N} M_l^H. \tag{29}$$

The result in formula (29) means that the sum of all the contributions leads to only the Higgs-like type of the contribution and all the other parts are canceled out explicitly.

IV. GENERALIZATION TO THE LOOP CASE

The main result of the previous section can be easily generalized to the loop case. First of all, one should mention that in the way of proving equality (29) nothing special was required for boson lines in the vertices. The boson lines in Fig. 3 could correspond to real particles or virtual propagators in the loops. In this sense all the above considerations are valid for both real and virtual gauge bosons emitted from the fermion current.

In the case of a fermion loop one can follow a similar logic as was given above. In the loop case there is an additional fermion propagator instead of the external spinors at the tree level. Correspondingly the expressions for various fermion loop contributions with emission of N gauge bosons have the following form by analogy with (17), (19), and (20)–(22).

$$\begin{aligned} M_l \sim i^{2N+1}{\rm Tr}\Bigg\{&\left[\prod_{j=1}^{l}\Gamma^j_{\mu_j}S(q_j)\right] \\ &\times\left[-\frac{3}{2}(S^{-1}(q_l)+S^{-1}(k_l))+m_{f_l}\right] \\ &\times\left[\prod_{j=1+1}^{N} S(k_{j-1})\Gamma^j_{\mu_j}\right]S(p)\Bigg\}, \end{aligned} \tag{30}$$

for $l = 1, \ldots, N-1$,

$$\begin{aligned} M_N \sim i^{2N+1}{\rm Tr}\Bigg\{&\left[\prod_{j=1}^{N}\Gamma^j_{\mu_j}S(q_j)\right] \\ &\times\left[-\frac{3}{2}(S^{-1}(q_N)+S^{-1}(p_{\rm in}))+m\right]S(p)\Bigg\}, \end{aligned} \tag{31}$$

$$\begin{aligned} M'_l \sim i^{2N-1}{\rm Tr}\Bigg\{&\left[\prod_{j=1}^{l-1}\Gamma^j_{\mu_j}S(q_j)\right][-3\Gamma^l_{\mu_l}] \\ &\times\left[\prod_{j=l+1}^{N} S(k_{j-1})\Gamma^j_{\mu_j}\right]S(p)\Bigg\}, \end{aligned} \tag{32}$$

for $l = 2, \ldots, N-1$

$$M'_1 \sim i^{2N-1}{\rm Tr}\left\{[-3\Gamma^1_{\mu_1}]\left[\prod_{j=2}^{N} S(k_{j-1})\Gamma^j_{\mu_j}\right]S(p)\right\}, \tag{33}$$

$$M'_N \sim i^{2N-1}{\rm Tr}\left\{\left[\prod_{j=1}^{N-1}\Gamma^j_{\mu_j}S(q_j)\right][-3\Gamma^N_{\mu_N}]S(p)\right\}, \tag{34}$$

where the momenta are expressed as $k_l = p - \sum_{j=l}^{N} p_j^V$, $q_l = k_l - p_r = p + \sum_{j=1}^{l-1} p_j^V$. Note that the contribution M_0 is not present now since it coincides with M_N.

In the same manner as in the previous section we get

$$M_l = M_l^H - \frac{1}{2}M'_l - \frac{1}{2}M'_{l+1} \tag{35}$$

$$M_N = M_N^H - \frac{1}{2}M'_N - \frac{1}{2}M'_1. \tag{36}$$

From relations (35)–(36) it is easy to show that

$$\sum_{l=1}^{N} M_l = \sum_{l=1}^{N-1} M_l + M_N = \sum_{l=1}^{N} M_l^H - \sum_{l=1}^{N} M'_l \tag{37}$$

and therefore one gets once again the equality

$$\sum_{l=1}^{N} M_l + \sum_{l=1}^{N} M'_l = \sum_{l=1}^{N} M_l^H, \tag{38}$$

which demonstrates that all the contributions except for the Higgs-like type are canceled out in the case of a fermion loop.

V. CONCLUSIONS

In all the main radion production processes, there are contributing Feynman diagrams involving off-shell fermions. The radion is emitted from various fermion currents containing off-shell fermion propagators in the radion strahlung or the vector boson fusion production in e^+e^- collisions, as well as in the vector boson fusion, associated radion and vector boson production, associated radion, and the top-quark pair production processes in hadron collisions at the LHC. The off-shell fermions participate in fermion loops for gluon-gluon fusion production process at the LHC and in gg, $\gamma\gamma$, γZ decay modes of the radion. The additional to the Higgs boson case nontrivial vertices (even for massless fermions) of fermion-radion interactions follows from the structure of the trace of the gauge invariant SM energy-momentum tensor as given in Appendixes A–B.

We have shown that all the contributions to perturbative amplitudes of physical processes with a single radion are canceled out due to the corresponding additional terms for both massless and massive off-shell fermions. We demonstrated, first, the cancellation in a simple example of the radion-strahlung process in e^+e^- collisions. Then we presented a general proof of the cancellation for an arbitrary fermion current radiating a single radion and any number of the SM gauge bosons. This proof was generalized to the case of the amplitudes containing closed fermion loops and an arbitrary number of the gauge bosons.

The proof also means that, in calculating the amplitudes of processes with a single radion, the terms with the covariant derivatives of the fermion fields in Lagrangian (4) can be replaced by the mass terms in accordance with the equation of motion for the fermion fields, which is not *a priori* justified in the gauge theory, where the quantization is performed with the help of the path integration. Thus the additional fermion-radion terms in the interaction Lagrangian do not alter any production and decay properties of a single radion compared to those of the Higgs boson.

It is worth noting that the observed cancellation property, being a result of the gauge invariant structure of the SM energy-momentum trace, is also valid for any scalar particle (not only the radion) which interacts with the SM particles via the trace.

In our study we concentrated on physical processes with emitted single radion and an arbitrary number of gauge bosons. In the case of the radion and the Higgs boson emission there are differences in comparison with the Higgs boson pair production. In the radion and Higgs case the contributions due to off-shell fermions do not cancel each other, leading to remaining terms that are absent in the Higgs pair production case. Another difference comes from the diagrams involving the triple hhr vertex. As shown in Appendix B, the vertex hhr contains an additional, compared to the hhh vertex, momentum depending part. As for the radion pair production, the amplitude of the process is of the order Λ_r^{-2}. In this case some contributions come from corrections to the interactions of the radion and the SM fields of the same Λ_r^{-2} order, such as the two fermions–two radions vertex.

However, a realistic analysis of this pair production processes in experiments at colliders, particularly at the LHC, is problematic. The Higgs boson pair production process has a rather small rate even at the LHC, and a production process leading to various final states is difficult to be extracted from the backgrounds (see, e.g., [28]). Taking into account that the rate of the pair production processes involving the radion is even smaller than the rate of the Higgs boson pair production, a delicate analysis including backgrounds is needed to understand the influence and the impact of the mentioned differences between the processes on their observability.

An interesting question arises wether the observed cancellations take place in other variants of brane world models, in particular, when the SM fields are allowed to propagate in the bulk leading to the appearance of the corresponding KK towers of the fermion and boson states. In this case the interaction Lagrangian of the radion and the SM fields is more complicated, because it includes a contribution from the energy-momentum tensor in the extra dimension [18]. The structure of this interaction is different from the one when the radion couples to the SM energy-momentum trace in four dimensions. Thus, the cancellation of the off-shell fermion contributions including the fermion KK towers is not obvious, and the corresponding analysis deserves a separate study.

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APPENDIX A: TRACE OF THE SM ENERGY-MOMENTUM TENSOR

The trace of the SM energy-momentum tensor calculated as the variation of the SM Lagrangian with respect to the metric [29,30] can be written in the unitary gauge as follows:

$$\begin{aligned}T^\mu_\mu = &-(\partial_\mu h)(\partial^\mu h) + 2m_h^2 h^2\left(1+\frac{h}{2v}\right)^2 - 2m_W^2 W^+_\mu W^{\mu-}\left(1+\frac{h}{v}\right)^2 - m_Z^2 Z_\mu Z^\mu\left(1+\frac{h}{v}\right)^2\\ &+\sum_f\left\{-\frac{i3}{2}[\bar f\gamma^\mu(\partial_\mu f)-(\partial_\mu\bar f)\gamma^\mu f]+4m_f\bar f f\right\}+\frac{4h}{v}\sum_f m_f\bar f f - 3eA_\mu\sum_f q_f\bar f\gamma^\mu f\\ &-\frac{3}{2}\frac{m_Z}{v}Z_\mu\sum_f\bar f\gamma^\mu[a_f+b_f\gamma_5]f-\frac{3}{\sqrt2}\frac{m_W}{v}(W^-_\mu\bar\nu_j U^{\rm PMNS}_{jk}\gamma^\mu[1-\gamma_5]e_k+\text{h.c.})\\ &-\frac{3}{\sqrt2}\frac{m_W}{v}(W^-_\mu\bar u_j\gamma^\mu[1-\gamma_5]V^{\rm CKM}_{jk}d_k+\text{h.c.})-3g_c(\bar u_j\gamma^\mu\hat G_\mu u_j+\bar d_j\gamma^\mu\hat G_\mu d_j)+\frac{\beta(e)}{2e}F_{\mu\nu}F^{\mu\nu}+\frac{\beta(g_s)}{2g_s}G^{ab}_{\mu\nu}G^{\mu\nu}_{ab},\end{aligned}$$

where U^{PMNS} is the Pontecorvo-Maki-Nakagawa-Sakata matrix, and V^{CKM} is the Cabibbo-Kobayashi-Maskawa matrix. The last two terms take the anomalies into account; they describe the interaction of the radion with the photon and the gluon fields.

APPENDIX B: INTERACTION VERTICES OF RADION AND SM FIELDS

Radion and two fermions:

$$r(k_r)\bar{f}(p_{\bar{f}})f(p_f) \qquad \frac{i}{\Lambda_r}\left\{\frac{3}{2}[\not{p}_{\bar{f}} - \not{p}_f] + 4m_f\right\} = \frac{i}{\Lambda_r}\left\{\frac{3}{2}[(\not{p}_{\bar{f}} + m_f) - (\not{p}_f - m_f)] + m_f\right\}.$$

Radion, two fermions, and photon:

$$r(k_r)\gamma(k_\gamma)\bar{f}(p_{\bar{f}})f(p_f) \qquad -i\frac{3eq_f}{\Lambda_r}\gamma^\mu.$$

Radion, two fermions, and Z boson:

$$r(k_r)Z(k_Z)\bar{f}(p_{\bar{f}})f(p_f) \qquad -i\frac{1}{\Lambda_r}\frac{3m_Z}{2v}\gamma^\mu[a_f + b_f\gamma_5].$$

Radion, two fermions, and W boson:

$$r(k_r)W(k_W)\bar{u}(p_u)d(p_d) \qquad -i\frac{1}{\Lambda_r}\frac{3m_W}{\sqrt{2}v}V^{\mathrm{CKM}}_{jk}\gamma^\mu[1-\gamma_5]$$

$$r(k_r)W(k_W)\bar{\nu}(p_\nu)e(p_e) \qquad -i\frac{1}{\Lambda_r}\frac{3m_W}{\sqrt{2}v}U^{\mathrm{PMNS}}_{jk}\gamma^\mu[1-\gamma_5].$$

Radion, two fermions, and Higgs boson:

$$r(k_r)h(k_h)\bar{f}(p_{\bar{f}})f(p_f) \qquad i\frac{1}{\Lambda_r}\frac{4m_f}{v}.$$

Radion and two Higgs bosons:

$$r(k_r)h(p_1)h(p_2) \qquad i\frac{1}{\Lambda_r}\{p_{1\mu}p_2^\mu - 2m_h^2\}.$$

Radion and three Higgs bosons:

$$r(k_r)h(p_1)h(p_2)h(p_3) \qquad -i\frac{1}{\Lambda_r}\frac{12m_h^2}{v}.$$

Radion and four Higgs bosons:

$$r(k_r)h(p_1)h(p_2)h(p_3)h(p_4) \qquad -i\frac{1}{\Lambda_r}\frac{12m_h^2}{v^2}.$$

Radion and two Z bosons:

$$r(k_r)Z(p_1)Z(p_2) \qquad -i\frac{2m_Z^2}{\Lambda_r}g^{\mu\nu}.$$

Radion, two Z bosons, and Higgs boson:

$$r(k_r)Z(p_1)Z(p_2)h(p_3) \qquad -i\frac{1}{\Lambda_r}\frac{4m_Z^2}{v}g^{\mu\nu}.$$

Radion, two Z bosons, and two Higgs bosons:

$$r(k_r)Z(p_1)Z(p_2)h(p_3)h(p_4) \qquad -i\frac{1}{\Lambda_r}\frac{4m_Z^2}{v^2}g^{\mu\nu}.$$

Radion and two W bosons:

$$r(k_r)W(p_1)W(p_2) \qquad -i\frac{2m_W^2}{\Lambda_r}g^{\mu\nu}.$$

Radion, two W bosons, and Higgs boson:

$$r(k_r)W(p_1)W(p_2)h(p_3) \qquad -i\frac{1}{\Lambda_r}\frac{4m_W^2}{v}g^{\mu\nu}.$$

Radion, two W bosons, and two Higgs bosons:

$$r(k_r)W(p_1)W(p_2)h(p_3)h(p_4) \qquad -i\frac{1}{\Lambda_r}\frac{4m_W^2}{v^2}g^{\mu\nu}.$$

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