# Trilinear self-couplings in an $S(3)$ flavored Higgs model

E. Barradas-Guevara*

*Facultad de Ciencias Físico Matemáticas, Benemérita Universidad Autónoma de Puebla, Apartado Postal 1152, Puebla, Puebla 72000, México*

O. Félix-Beltrán†

*Facultad de Ciencias de la Electrónica, Benemérita Universidad Autónoma de Puebla, Apartado Postal 542, Puebla, Puebla 72570, México*

E. Rodríguez-Jáuregui‡

*Departamento de Física, Universidad de Sonora, Apartado Postal 1626, Hermosillo, Sonora 83000, México*

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In this paper, we analyze the Higgs sector of the minimal $S(3)$-invariant extension of the Standard Model. This extension includes three Higgs SU(2) doublets fields and $CP$ invariant. We compute the Higgs boson physical masses in terms of the potential parameters and the scalar Higgs matrix rotation angles $\theta_S$ and $\omega_3$ ($\tan\theta_P = \tan\theta_C = \cot\omega_3$). The angles $\theta_S$, $\theta_P$ and $\theta_C$ are related to the scalar, pseudoscalar and charged Higgs matrix rotation respectively. Furthermore, within this model we can also write down in an explicit form the trilinear self-couplings $\lambda_{ijk}$ in terms of the Higgs masses and two free parameters, $\theta_S$ and $\omega_3$. Moreover, we show that the Higgs masses and trilinear Higgs boson self-couplings are closely linked to the Higgs potential structure given by the discrete symmetry $S(3)$, which can be helpful in distinguishing this model from other extensions. In our analysis the lightest Higgs boson mass is taken to be fixed to 125 GeV. In concordance with the results reported in the literature for other Standard Model extensions, one finds that the numerical values $\lambda_{ijk}$ of the minimal $S(3)$-invariant extension of the Standard Model are significantly different from the trilinear Higgs self-coupling of the Standard Model.

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## I. INTRODUCTION

In the Standard Model (SM) the electroweak gauge bosons and the fundamental matter particles acquire masses through the interaction with a single scalar Higgs boson which is an essential part of the SM upon the electroweak spontaneous symmetry breaking (SSB). The Higgs mechanism [1] of SSB in the context of the SM is being tested for internal consistency and the discovery of the Higgs boson candidate on July 2012 is one of the major goals of the Large Hadron Collider (LHC) [2–4]. Although the mass of the Higgs boson was measured with a good approximation value (125 GeV), several properties as the spin, the Higgs self-couplings and fermionic-bosonic Higgs boson couplings must be measured. Several works on this were done by A. Djouadi *et al.* [5,6], J. Baglio *et al.* [7], P. Osland *et al.* [8–11], as well as [12,13].

These days it remains to be seen if the new boson with a mass of 125 GeV, observed in the CMS experiment at LHC and corresponding to the scalar in the SM, is of ultimate importance to analyze the mechanism of spontaneous symmetry breaking in models which include a larger number of scalar bosons [4,5]. Different aspects of models including three and more Higgs doublets have also been studied, with and without discrete symmetries (see for instance [14]). Several extensions of the SM, such as the $S(3)$ extended Standard Model ($S(3)$SM) [15–18], the Minimal Supersymmetric Standard Model (MSSM) [19], and the general Two Higgs Doublet Model (2HDM) [20,21] have a more complicated Higgs structure.

On the other hand, in the absence of mass the Lagrangian is chiral and invariant with respect to any permutation of the left and right quark and lepton fields. As a consequence, the left and right quark and lepton fields are transformed independently under the flavor group. When the gauge symmetry is broken by the Higgs mechanism, all of the particles acquire mass. In the gauge basis the charged currents are diagonal, but the quark and lepton physical masses are found by diagonalization of their corresponding mass matrix. This way, the charged currents $J_\mu^\pm$ are invariant under the transformations of the flavor symmetry group if and only if the up and down quark fields are transformed with the same flavor group. Thus, the charged current invariance condition under the family symmetry group implies that the up and down quark fields are transformed with the same group.

In the SM when the gauge symmetry is spontaneously broken, the quarks and leptons, as well as the gauge bosons $W^\pm$, $Z$ and the Higgs particle acquire mass. Under exact

*barradas@fcfm.buap.mx
†olga_flix@ece.buap.mx
‡ezequiel.rodriguez@correo.fisica.uson.mx

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$S(3)$ symmetry in the fermion sector and only one $SU(2)$ Higgs doublet field, the mass spectrum for either up or down quark sectors consists of one massive particle (top and bottom quarks) in a singlet irreducible representation (irrep) and a pair of massless particles in a doublet irrep see [22–25]. The $S(3)$ group treats three objects symmetrically, while the hierarchical nature of the mass matrices is a consequence of the representation structure $1_S + 2$ of $S(3)$, which treats the generations differently. The $S(3)$ group is non-Abelian and has a doublet and a symmetric singlet in its irrep. In order to properly generate the mass spectrum for either quark and lepton sectors of one massive particle (top and bottom quarks or neutrino and charged leptons) in a singlet irreducible representation (irrep) and a pair of massive particles in a doublet irrep, three right neutrinos are introduced in the leptonic sector and the Higgs field has to be flavor extended too, introducing three $SU(2)$ Higgs doublet fields $H_i, i = 1, 2, 3$, which also transform as the irrep under $S(3)$ [18]. That is, one Higgs field in a symmetric $S(3)$ singlet and the other two in a doublet irrep of $S(3)$. In this way all of the fundamental quark, lepton and Higgs fields in the model, transform under the same flavor symmetry group $S(3)$.

In the literature we can found many possible extensions of the SM with discrete symmetries it is interesting to notice $S(3)$SM is in good agreement with the latest experimental values. $S(3)$SM is a complete extended flavor Lagrangian model which fits all of the SM predictions. Including three $S(3)$ Higgs fields, the $\chi^2$ fits to the theoretically computed CKM mixing matrix, Jarlskog invariant, and the PMNS mixing matrix, where performed by A. Mondragon *et al.*, in excellent agreement with the latest known experimental values [24–27].

A. Mondragon *et al.* have also found that flavor changing neutral currents (FCNC) are strongly suppressed in the leptonic sector and the contribution of FCNC to the anomaly of the muon's magnetic moment is small, but not negligible [28,29].

An extended Standard Model with three $SU(2)$ Higgs doublets has twelve real scalar Higgs fields, with out imposing any flavor symmetry has a Higgs potential structure with 54 free parameters. The $S(3)$ flavor symmetry imposes some constrains on the Higgs potential and in the end, after spontaneous symmetry breaking of the $S(3)$SM, the $12 \times 12$ Higgs mass matrix can be exactly and analytically diagonalized, the physical Higgs masses become known as functions of ten free parameters and the vacuum expectation values (VEVs). The Higgs spectrum of the $S(3)$SM consists of five neutral and four charged Higgs bosons [30]; three neutral Higgs bosons ($h^0$, $H^0_{1,2}$) are $CP$-even, whereas two neutral Higgs bosons ($A^0_{1,2}$) are $CP$-odd. It is important to notice that from the original twelve degrees of freedom, after SSB, nine give mass to nine Higgs Boson particles and the remaining three give mass to $W^\pm$ and $Z^0$.

Up to now, the particle observed at LHC is a particle in the physical spectrum of the Higgs boson of the SM. It is not known if there are one or many Higgs bosons. An indication of the presence of one Higgs boson or an extended Higgs sector, as the one proposed in the $S(3)$-invariant extension of the Standard Model ($S(3)$SM), could be found at the Large Hadron Collider [31–33].

A precise measurement of the trilinear Higgs self-couplings will also make it possible to test extended Higgs models, which have a different Higgs potential structure, and hence different trilinear Higgs couplings as compared to SM. The $S(3)$ flavor group is the simplest flavor symmetry group; it imposes constrains that makes it even simpler than the 2HDM. In particular, the model can easily accommodate all of the quark and lepton mixing matrices [22,23,25,34–38]. In spite of the complicated nature of the Higgs potential, it is possible to study in detail the trilinear couplings to the $CP$-even, $CP$-odd and charged Higgs bosons in the $S(3)$SM. Thus, in the $CP$-conserving case, there are allowed trilinear Higgs scalar couplings which can be labelled as $\lambda_{ijk}$ with $i, j, k = h^0, H^0_1, H^0_2$, all even. In the number of $A$ involving the $CP$-odd Higgs boson. Thus, in the case of the $S(3)$SM there are a total of ten trilinear Higgs scalar couplings.

On the other hand, the introduction of additional symmetries to SM open up to understand the replication of families in that. Interesting extended models have been proposed in the literature (see for instance [39] and references therein, for a review on the subject). One flavor discrete symmetry such as $S(3)$ is the minimal discrete group that reproduces the fermionic masses and mixing matrices [25,37,40].

We study quantitatively the trilinear Higgs couplings, and compare these couplings to the corresponding Standard Model trilinear Higgs coupling in some regions of the parameter space. In particular, we are interested on the trilinear ($\lambda^{SM}_{HHH}$) and quartic ($\lambda^{SM}_{HHHH}$) self-couplings. As we know, SM has just one trilinear self-coupling $\lambda^{SM}_{HHH}$, which is given by $\lambda^{SM}_{HHH} = 3m^2_H/v$, where $v$ is the vacuum expectation value ($v = 246$ GeV). Writing the free parameters as a function of the physical Higgs masses, the trilinear Higgs couplings of the model may be determined in terms of seven Higgs mass eigenvalues, one free parameter $\theta_S$ and the VEV ratio $\tan\omega_3 \equiv 2v_2/v_3$.

In this paper we present the trilinear self-couplings of the $S(3)$ extended Standard Model, including an extended $S(3)$ Higgs boson sector. In the literature there are other extensions of the SM, such as the two Higgs doublet model (THDM) [20] which also has 10 free parameters plus two VEVs, in this model there are several trilinear Higgs couplings, with a more complicated dependence on the underlying masses. We assume that spontaneous $CP$ violation does not occur.

This paper is organized as follows: in Sec. II we present some remarks about the $S(3)$ flavor symmetry and its matter content. The Yukawa and Higgs sectors of the $S(3)$SM, and the form of the Higgs mass matrix are given in Sec. III; in Sec. IV we focus on the trilinear self-couplings of neutral Higgs bosons whereas the details of the numerical results are presented in Sec. V, and finally we present our conclusions in Sec. VI.

## II. $S(3)$SM MATTER CONTENT

The ingredients of the extension of the Standard Model are the following: To associate each family to an irreducible representation of the flavor group, and to extend the flavor and family concepts to the Higgs sector. To construct a Lagrangian invariant under the action of the $SU(3)_c \times SU(2) \times U(1) \times S(3)^f$ group. The group $S(3)$ has two one-dimensional irreducible representations and a two dimensional irreducible representation, one dimensional representations: $\mathbf{1}_A$ antisymmetric singlet, $\mathbf{1}_s$ symmetric singlet and bidimensional doublet $\mathbf{2}$. Direct products of an $S(3)$ irreducible representations are given as [18]

$$\mathbf{1}_s \otimes \mathbf{1}_s = \mathbf{1}_s, \quad \mathbf{1}_s \otimes \mathbf{1}_A = \mathbf{1}_A, \quad \mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_s,$$
$$\mathbf{1}_s \otimes \mathbf{2} = \mathbf{2}, \quad \mathbf{1}_A \otimes \mathbf{2} = \mathbf{2}, \quad \mathbf{2} \otimes \mathbf{2} = \mathbf{1}_s \oplus \mathbf{1}_A \oplus \mathbf{2}.$$

Then, a direct doublet product $\mathbf{p_D} \otimes \mathbf{q}_D = r_s \oplus r_A \oplus \mathbf{r}_D$ with $\mathbf{p_D}^T = (p_{D1}, p_{D2})$ and $\mathbf{q}_D{}^T = (q_{D1}, q_{D2})$. It has two singlets, $r_s$ (invariant), $r_A$ (not invariant), and just one doublet $r_D^T$ as follows:

$$r_s = p_{D1}q_{D1} + p_{D2}q_{D2}, \qquad r_D^T = \begin{pmatrix} p_{D1}q_{D2} + p_{D2}q_{D1} \\ p_{D1}q_{D1} - p_{D2}q_{D2} \end{pmatrix}.$$
$$r_A = p_{D1}q_{D2} - p_{D2}q_{D1},$$

Under $SU(2)_L \otimes U(1)_Y$ gauge symmetry, the quark and lepton fields for one family are given by $Q_{1L}^T = (u_L, d_L), u_R, d_R, L_{1L}^T = (\nu_L, e_L), e_R, \nu_R$. Under the flavor symmetry $S(3)$ group, including tree fermion families, it may be written as

$$F_s = \frac{1}{\sqrt{3}}(f_1 + f_2 + f_3), \quad \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(f_1 - f_2) \\ \frac{1}{\sqrt{6}}(f_1 + f_2 - 2f_3) \end{pmatrix},$$
$$f_i = u_i, d_i, e_i, \nu_i \quad i = 1, 2, 3.$$

Then, the Higgs sector is modified $\Phi \to H = (\Phi_a, \Phi_b, \Phi_c)^T$, and $H$ is a reducible representation to $\mathbf{1_s} \oplus \mathbf{2}$ of $S(3)$; that is,

$$H_s = \frac{1}{\sqrt{3}}(\Phi_a + \Phi_b + \Phi_c),$$
$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\Phi_a - \Phi_b) \\ \frac{1}{\sqrt{6}}(\Phi_a + \Phi_b - 2\Phi_c) \end{pmatrix}.$$

In this extension of the SM, all the quark, lepton and Higgs fields have three species (flavors) and belong to a representation reducible to $\mathbf{1_S} \oplus \mathbf{2}$ of $S(3)$.

## III. $S(3)$SM LAGRANGIAN

After the matter content is defined above, we can give the Yukawa Lagrangian of this model, which can be written following Ref. [18] as

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_u} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu},$$

where $\mathcal{L}_{Y_{D,U}}$ and $\mathcal{L}_{Y_{E,\nu}}$ correspond to quark and leptonic sectors, respectively. The explicit form of each terms is (see Ref. [18])

$$\begin{aligned}
\mathcal{L}_{Y_D} &= -Y_1^d \bar{Q}_I H_S d_{IR} - Y_3^d \bar{Q}_3 H_S d_{3R} \\
&\quad - Y_2^d [\bar{Q}_I \kappa_{IJ} H_1 d_{JR} + \bar{Q}_I \eta_{IJ} H_2 d_{JR}] \\
&\quad - Y_4^d \bar{Q}_3 H_I d_{IR} - Y_5^d \bar{Q}_I H_I d_{3R} + \text{H.c} \\
\mathcal{L}_{Y_U} &= -Y_1^u \bar{Q}_I (i\sigma_2) H_S^* u_{IR} - Y_3^u \bar{Q}_3 (i\sigma_2) H_S^* u_{3R} \\
&\quad - Y_2^u [\bar{Q}_I \kappa_{IJ} (i\sigma_2) H_1^* u_{JR} + \eta \bar{Q}_I \eta_{IJ} (i\sigma_2) H_2^* u_{JR}] \\
&\quad - Y_4^u \bar{Q}_3 (i\sigma_2) H_I^* u_{IR} - Y_5^u \bar{Q}_I (i\sigma_2) H_I^* u_{3R} + \text{H.c.}, \\
\mathcal{L}_{Y_E} &= -Y_1^e \bar{L}_I H_S e_{IR} - Y_3^e \bar{L}_3 H_S e_{3R} \\
&\quad - Y_2^e [\bar{L}_I \kappa_{IJ} H_1 e_{JR} + \bar{L}_I \eta_{IJ} H_2 e_{JR}] \\
&\quad - Y_4^e \bar{L}_3 H_I e_{IR} - Y_5^e \bar{L}_I H_I e_{3R} + \text{H.c.}, \\
\mathcal{L}_{Y_\nu} &= -Y_1^\nu \bar{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \bar{L}_3 (i\sigma_2) H_S^* \nu_{3R} \\
&\quad - Y_2^\nu [\bar{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \bar{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR}] \\
&\quad - Y_4^\nu \bar{L}_3 (i\sigma_2) H_I^* \nu_{IR} - Y_5^\nu \bar{L}_I (i\sigma_2) H_I^* \nu_{3R} + \text{H.c.}, \qquad (1)
\end{aligned}$$

The index $s$ (or 3) denotes a singlet, and the indices $I, J = 1, 2$ denote doublets; the matrices $\kappa$ and $\eta$ are given as

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Furthermore, the model allows the mass terms for the Majorana neutrinos through the see-saw mechanism and the corresponding Majorana Lagrangian is

$$\mathcal{L}_M = -M_1 \nu_{IR}^T C \nu_{IR} - M_3 \nu_{3R}^T C \nu_{3R}.$$

here $C$ is the charge matrix.

The Lagrangian $\mathcal{L}_H$ of the $S(3)$ extended Higgs sector $S(3)$SM includes three complex scalar SU(2) doublets fields. Such a theory is based on the successful fits on the neutrino and quark mixing matrices and masses to the experimental allowed values. In view of the family replication of the elementary fermion spectrum one can speculate that this flavor symmetry is the symmetry of the fundamental particles, and an analogous flavor symmetry principle might work for the Higgs sector as well:

$$\mathcal{L}_H = [D_\mu H_S]^2 + [D_\mu H_1]^2 + [D_\mu H_2]^2 - V(H_1, H_2, H_S), \tag{2}$$

where $D_\mu$ is the usual covariant derivative, $D_\mu = (\partial_\mu - \frac{i}{2} g_2 \tau_a W_\mu^a - \frac{i}{2} g_1 B_\mu)$, from here we obtain the $W^\pm$ and $Z^0$ mass, one obtains $m_W^2 = (g_2^2/4) \sum_{i=1}^3 v_i^2$, with $g_1$ and $g_2$ standing for the $U(1)$ and $SU(2)$ coupling constants, and $m_Z^2 = [(g_2^2 + g_1^2)/4] \sum_{i=1}^3 v_i^2$. The couplings of the Higgs fields to the gauge bosons $W^\pm$ and $Z^0$, in the normal minimum (see Ref. [30]), are SM like $\sum_{i=1,2,s} [\frac{g_2^2}{8} (v_i + h_i)^2 \times (W_\mu^1 + W_\mu^2)^2]$ and $\sum_{i=1,2,s} [\frac{1}{8} (v_i + h_i)^2 (g_2 W_\mu^3 - g_1 Y_H B_\mu)^2]$, respectively. The scalar potential in Eq. (2), $V(H_1, H_2, H_S)$ is the most general Higgs potential invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y \times S(3)$ [41,42], it can be written as:

$$\begin{aligned} V = {} & \mu_1^2 (H_1^\dagger H_1 + H_2^\dagger H_2) + \mu_0^2 (H_S^\dagger H_S) + a (H_S^\dagger H_S)^2 + b (H_S^\dagger H_S)(H_1^\dagger H_1 + H_2^\dagger H_2) + c (H_1^\dagger H_1 + H_2^\dagger H_2)^2 \\ & + d (H_1^\dagger H_2 - H_2^\dagger H_1)^2 + e f_{ijk} ((H_S^\dagger H_i)(H_j^\dagger H_k)) + f \{ (H_S^\dagger H_1)(H_1^\dagger H_S) + (H_S^\dagger H_2)(H_2^\dagger H_S) \} \\ & + g \{ (H_1^\dagger H_1 - H_2^\dagger H_2)^2 + (H_1^\dagger H_2 + H_2^\dagger H_1) \} + h \{ (H_S^\dagger H_1)(H_S^\dagger H_1) + (H_S^\dagger H_2)(H_S^\dagger H_2) + (H_1^\dagger H_S)(H_1^\dagger H_S) \\ & + (H_2^\dagger H_S)(H_2^\dagger H_S) \}, \end{aligned} \tag{3}$$

where $f_{112} = f_{121} = f_{211} = -f_{222} = 1$, and in our case, the discrete flavor symmetry $S(3)$ Higgs doublets (1) for this model are:

$$H_1 = \begin{pmatrix} \phi_1 + i\phi_4 \\ \phi_7 + i\phi_{10} \end{pmatrix}, \qquad H_2 = \begin{pmatrix} \phi_2 + i\phi_5 \\ \phi_8 + i\phi_{11} \end{pmatrix}, \qquad H_S = \begin{pmatrix} \phi_3 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix}. \tag{4}$$

The numbering of the real scalar $\phi_i$ fields is chosen for convenience when writing the mass matrices for the scalar particles, and the subscript $S$ is the flavor index for the Higgs field singlet under $S(3)$. $H_i$ with $i = 1, 2$ are the components of the $S(3)$ doublet field. In the analysis, it is better to introduce nine real quadratic forms $x_i$ invariant under $SU(2) \times U(1)$

$$\begin{aligned} x_1 &= H_1^\dagger H_1, & x_4 &= \mathcal{R}(H_1^\dagger H_2), & x_7 &= \mathcal{I}(H_1^\dagger H_2), \\ x_2 &= H_2^\dagger H_2, & x_5 &= \mathcal{R}(H_1^\dagger H_S), & x_8 &= \mathcal{I}(H_1^\dagger H_S), \\ x_3 &= H_S^\dagger H_S, & x_6 &= \mathcal{R}(H_2^\dagger H_S), & x_9 &= \mathcal{I}(H_2^\dagger H_S). \end{aligned} \tag{5}$$

Now, it is a simple matter to write down the $S(3)$SM potential

$$\begin{aligned} V = {} & \mu_1^2 (x_1 + x_2) + \mu_0^2 x_3 + a x_3^2 + b (x_1 + x_2) x_3 \\ & + c (x_1 + x_2)^2 - 4 d x_7^2 + 2e [(x_1 - x_2) x_6 + 2 x_4 x_5] \\ & + f (x_5^2 + x_6^2 + x_8^2 + x_9^2) + g [(x_1 - x_2)^2 + 4 x_4^2] \\ & + 2h (x_5^2 + x_6^2 - x_8^2 - x_9^2). \end{aligned} \tag{6}$$

where the $\mu_{0,1}^2$ parameters have dimensions of mass squared and the eight real couplings $a, \ldots, h$ are dimensionless free parameters.

### A. S(3)-Higgs mass matrices

The $S(3)$ invariant Higgs potential in Eq. (6) has a minimum at

$$\begin{aligned} &\phi_7 = v_1, \qquad \phi_8 = v_2, \qquad \phi_9 = v_3, \\ &\phi_i = 0, \qquad i \neq 7, 8, 9, \end{aligned} \tag{7}$$

where we have adopted for convenience VEVs $v_i$ ($i = 1, 2, 3$) which do not have any complex relative phase, $v_i \in \Re$. Such a minimum determines vector boson masses through the Higgs mechanism. In a standard notation we can write $v_i = v \cos \omega_i$ where $v = 246$ GeV is the electroweak scale and their $\omega_i{}'s$ are three free parameters. Now we can rewrite the potential $V$ and express it in a simple matrix form as

$$V(\mathbf{X}) = \mathbf{A}^T \mathbf{X} + \frac{1}{2} \mathbf{X}^T \mathbf{B} \mathbf{X}, \tag{8}$$

with the vector $\mathbf{X}$ given by

$$\mathbf{X}^T = (x_1, x_2, x_3, \ldots, x_9), \tag{9}$$

$\mathbf{A}$ is a mass parameter vector,

$$\mathbf{A}^T = (\mu_1^2, \mu_1^2, \mu_0^2, 0, 0, 0, 0, 0, 0), \tag{10}$$

and $\mathbf{B}$ is a $9 \times 9$ real parameter symmetric matrix,

$$\mathbf{B}=\begin{pmatrix} 2(c+g) & 2(c-g) & b & 0 & 0 & 2e & 0 & 0 & 0 \\ 2(c-g) & 2(c+g) & b & 0 & 0 & -2e & 0 & 0 & 0 \\ b & b & 2a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8g & 4e & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4e & 2(f+2h) & 0 & 0 & 0 & 0 \\ 2e & -2e & 0 & 0 & 0 & 2(f+2h) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8d & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2(f-2h) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2(f-2h) \end{pmatrix}. \quad (11)$$

The minimization conditions give us three equations determined by demanding the vanishing of $\partial V/\partial\phi_i$. Then we get the mass parameters $\mu_1$ and $\mu_0$ given as

$$\mu_1^2=-(b+f+2h)v_3^2-2(c+g)(v_1^2+v_2^2)+\frac{3e(v_1^2-2v_1v_2-v_2^2)v_3}{v_1-v_2}, \quad (12)$$

and

$$\mu_0^2=-\left[2av_3^2+(b+f+2h)(v_1^2+v_2^2)-e\left(\frac{3v_1^2-v_2^2}{v_3}\right)v_2\right]. \quad (13)$$

From these, the following relationship among the Higgs VEVs is obtained Ref. [30]:

$$v_1=\sqrt{3}v_2. \quad (14)$$

This relationship between $v_1$ and $v_2$ is similar to the condition imposed in Ref. [25] to generate the nearest neighbor interaction (NNI) mass matrices in the context of the SM $\otimes$ $S(3)$ with two texture zeroes. Equations (12)–(14) reduce the number of free parameters from thirteen to ten. In this model, the Higgs boson masses are obtained by diagonalizing the $12\times 12$ mass matrix,

$$(\mathcal{M}_H^2)_{ij}=\frac{1}{2}\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\bigg|_{\min}, \quad (15)$$

with $i,j=\overline{1,12}$. We have

$$\mathcal{M}_H^2=\mathrm{diag}(\mathbf{M}_C^2,\mathbf{M}_C^2,\mathbf{M}_S^2,\mathbf{M}_P^2). \quad (16)$$

The $3\times 3$ symmetric and Hermitian submatrices $\mathbf{M}_C^2$, $\mathbf{M}_S^2$, $\mathbf{M}_P^2$, are, respectively, the charged, scalar, and pseudoscalar Higgs mass matrix. After diagonalizing the mass matrices, the masses of the physical charged, scalar, and pseudoscalar Higgs bosons are obtained. From the minimization condition (14), expressing the VEVs of the Higgs fields as $v_i=v\cos\omega_i$ and the relationship $v^2=\sum_{i=1}^3 v_i^2$, the mass matrix $\mathcal{M}_H^2$ can be parametrized with eight free parameters and $\omega_3$ given as

$$\tan\omega_3=\frac{2v_2}{v_3}, \quad (17)$$

where $\sin\omega_3=2v_2/v$ and $\cos\omega_3=v_3/v$.

The $S(3)$SM involves 12 independent scalar fields; three of them can be identified with the would-be Goldstone bosons $W^\pm$, $Z^0$ and the remaining nine correspond to physical Higgs particles. The proper Goldstone and Higgs fields are found through a diagonalization of the Higgs mass matrix. The Higgs boson masses in this model are obtained by diagonalizing the exact and explicit $12\times 12$ mass matrix in Eq. (16),

$$M_C^2=\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ \square & c_{22} & c_{23} \\ \square & \square & c_{33} \end{pmatrix}, \quad M_S^2=\begin{pmatrix} s_{11} & s_{12} & s_{13} \\ \square & s_{22} & s_{23} \\ \square & \square & s_{33} \end{pmatrix}, \quad M_P^2=\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ \square & p_{22} & p_{23} \\ \square & \square & p_{33} \end{pmatrix}, \quad (18)$$

with the following matrix entries,

$$
\begin{aligned}
c_{11} &= -4gv_2^2 - (4ev_2 + f'v_3)v_3 & s_{11} &= 12(c+g)v_2^2 & p_{11} &= -4((d+g)v_2^2 + ev_2v_3 + hv_3^2) \\
c_{12} &= 2\sqrt{3}(2gv_2 + ev_3)v_2 & s_{12} &= 2\sqrt{3}(2(c+g)v_2 + 3ev_3)v_2 & p_{12} &= 2\sqrt{3}(2(d+g)v_2 + ev_3)v_2 \\
c_{13} &= \sqrt{3}(2ev_2 + f'v_3)v_3)v_2 & s_{13} &= 2\sqrt{3}(3ev_2 + (b+f')v_3)v_2 & p_{13} &= 2\sqrt{3}(ev_2 + 2hv_3)v_2 \\
c_{22} &= -12gv_2^2 - (8ev_2 + f'v_3)v_3 & s_{22} &= 4((c+g)v_2 - 3ev_3)v_2 & p_{22} &= -12(d+g)v_2^2 - 8ev_2v_3 - 4hv_3^2 \\
c_{23} &= (2ev_2 + f'v_3)v_2 & s_{23} &= 2(3ev_2 + (b+f')v_3)v_2 & p_{23} &= 2(ev_2 + 2hv_3)v_2 \\
c_{33} &= -\frac{4v_2^2}{v_3}(2ev_2 + f'v_3). & s_{33} &= -\frac{8ev_2^2}{v_3} + 4av_3^2. & p_{33} &= -\frac{8v_2^2}{v_3}(ev_2 + 2hv_3),
\end{aligned} \tag{19}
$$

where $f' \equiv f + 2h$. The physical Higgs masses are found from the diagonalization process,

$$
[\mathcal{M}^2_{\rm diag}]_i = R_i^T \mathcal{M}_i^2 R_i \qquad i = C, S, P, \tag{20}
$$

where the indices $C, S, P$ stand for charged, scalar, and pseudoscalar, respectively. Then, the rotation matrices $R_i$ are

$$
R_i = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot Q_i, \quad i = C, S, P, \tag{21}
$$

the mixing matrix $Q_i$ is

$$
Q_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_i & \sin\theta_i \\ 0 & -\sin\theta_i & \cos\theta_i \end{pmatrix} \quad i = C, S, P \tag{22}
$$

with

$$
\tan\theta_S = \frac{2s_{13}}{\sqrt{3}(m^2_{H_1^0} - s_{33})}, \qquad \tan\theta_C = \tan\theta_P = \cot\omega_3, \tag{23}
$$

where $s_{13}$ and $s_{33}$ are obtained from Eq. (19).

We start by considering the Higgs mass matrix $M_S^2$ for the $CP$-even Higgs. Defining the physical mass eigenstates $m^2_{h^0}$, $m^2_{H_1^0}$, and $m^2_{H_2^0}$, the physical Higgs masses are found from the diagonalization process. The physical masses for the $CP$-even Higgs boson scalars are:

$$
\begin{aligned}
m^2_{h^0} &= -18ev_2v_3, \\
m^2_{H_1^0,H_2^0} &= (\mathcal{M}_a^2 + \mathcal{M}_c^2) \pm \sqrt{(\mathcal{M}_a^2 - \mathcal{M}_c^2)^2 + (\mathcal{M}_b^2)^2},
\end{aligned} \tag{24}
$$

where

$$
\begin{aligned}
\mathcal{M}_a^2 &= v_2(8(c+g)v_2 + 3ev_3), \\
\mathcal{M}_b^2 &= 4v_2(3ev_2 + (b+f+2h)v_3), \\
\mathcal{M}_c^2 &= -\frac{4ev_2^3}{v_3} + 2av_3^2.
\end{aligned} \tag{25}
$$

The scalar Higgs mass matrix $M_S^2$ is diagonalized by a rotation matrix $R_S$ given by Eqs. (21)–(22) which is parametrized with

$$
\tan\theta_S = \frac{\mathcal{M}_b^2}{2\mathcal{M}_a^2 - m^2_{H_2}}. \tag{26}
$$

The charged Higgs mass matrix $M_C^2$ is diagonalized by a rotation matrix $R_C$ in Eq. (21), and the charged Higgs boson masses are

$$
\begin{aligned}
m^2_{H_1^\pm} &= -(10ev_2 + (f+2h)v_3)v_3 - 16gv_2^2, \\
m^2_{H_2^\pm} &= -\frac{v^2}{v_3}(2ev_2 + (f+2h)v_3).
\end{aligned}
$$

The resulting squared masses for the $CP$-odd pseudoscalar bosons Higgs are

$$
\begin{aligned}
m^2_{A_1^0} &= -16(d+g)v_2^2 - 10ev_2v_3 - 4hv_3^2, \\
m^2_{A_2^0} &= -\frac{2(ev_2 + 2hv_3)(4v_2^2 + v_3^2)}{v_3}.
\end{aligned} \tag{27}
$$

Of the original twelve scalar degrees of freedom, three Goldstone bosons ($G^\pm$ and $G^0$) are absorbed by $W^\pm$ and $Z^0$. The remaining nine physical Higgs particles are three $CP$—even scalars ($h^0$ and $H_1^0$, $H_2^0$, with $m_{h^0}, m_{H_1^0}, m_{H_2^0}$), two $CP$-odd scalars ($A_1^0$, and $A_2^0$, with $m_{A_1^0}, m_{A_2^0}$), and two charged Higgs pairs ($H^\pm_{1,2}$, mass degenerate).

The couplings of the Higgs fields to the fermion-antifermion pairs in terms of the model parameters are obtained after spontaneous gauge symmetry breaking. First, we perform a rotation to the physical lepton basis

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}_{L,R} = \mathbf{R}^{\dagger}_{L,R} \begin{pmatrix} e'_1 \\ e'_2 \\ e'_3 \end{pmatrix}_{L,R}$$

where $\mathbf{R}_{L,R}$ is the fermion rotation matrix [25,26]. On the other hand, from Eq. (2) one can compute the fermion mass matrix as follows:

$$\mathbf{M}_l = \mathbf{R}_L Y_{H_s} v_3 \mathbf{R}^{\dagger}_R + \mathbf{R}_L Y_{H_1} v_1 \mathbf{R}^{\dagger}_R + \mathbf{R}_L Y_{H_2} v_2 \mathbf{R}^{\dagger}_R + \text{H.c.} \tag{28}$$

Defining the Yukawa rotated coupling matrix as

$$\tilde{Y}^l_{H_i} = \mathbf{R}_L Y^l_{H_i} \mathbf{R}^{\dagger}_R \qquad i = 1, 2, s$$

and including the Higgs mass eigenstates

$$\begin{pmatrix} \phi_7 \\ \phi_8 \\ \phi_9 \end{pmatrix} = R^{\dagger}_S \begin{pmatrix} h^0 \\ H^0_1 \\ H^0_2 \end{pmatrix}, \qquad \begin{pmatrix} \phi_{10} \\ \phi_{11} \\ \phi_{12} \end{pmatrix} = R^{\dagger}_P \begin{pmatrix} G_0 \\ A^0_1 \\ A^0_2 \end{pmatrix}$$

where $G^0$ is a Goldstone boson and $R_S$ and $R_P$ diagonalizes the neutral Higgs mass matrix, see Eq. (21). We obtain the neutral Higgs Lagrangian write down with scalars $h^0, H^0_1, H^0_2$ and pseudoscalars $A^0_1, A^0_2$ physical Higgs fields. Although, this model contain either charged and neutral Higgs couplings to charged fermion-antifermion, we show just the corresponding neutral Higgs couplings to the fermions pairs.

$$\mathcal{L}_Y{}^{\text{neutral}} = -i(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L)(\tilde{Y}_{H_1} R^{\dagger}_S, \tilde{Y}_{H_2} R^{\dagger}_S, \tilde{Y}_s R^{\dagger}_S) \begin{pmatrix} h^0 \\ H^0_1 \\ H^0_2 \end{pmatrix} \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} - i(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L)(\tilde{Y}_{H_1} R^{\dagger}_P, \tilde{Y}_{H_2} R^{\dagger}_P, \tilde{Y}_s R^{\dagger}_P) \begin{pmatrix} G^0 \\ A^0_1 \\ A^0_2 \end{pmatrix} \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} + i(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L) \mathbf{M}^{\text{Diag}}_l \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} + \text{H.c.}$$

We display here only the leptonic part of the Yukawa Lagrangian in terms of ($\tilde{\xi}^l = \tilde{Y}_{H_l} R^{\dagger}_S$) and ($\tilde{\Xi}^l = \tilde{Y}_{H_l} R^{\dagger}_P$) which contains the physical information of the model and will be studied else where. The remaining fermion sector can be studied in an analogous way giving similar expressions.

### B. The $S(3)$SM free parameter space

The Higgs mass matrix $\mathcal{M}^2_H$ in Eq. (16), contains ten real free independent parameters, namely, $\mu_0$, $\mu_1$, eight dimensionless $a, \ldots, h$ parameters, and the three vacuum expectation values $v_1 = v\cos\omega_1,\ v_2 = v\cos\omega_2,\ v_3 = v\cos\omega_3$. These thirteen free parameters may be readily expressed in terms of two free parameters and seven Higgs mass eigenvalues as follows: first, we use the minimization conditions to eliminate $\mu_0, \mu_1$ and $v_1$, it reduces the number of free parameters from thirteen to ten, we then use the VEV relationship $v^2_1 + v^2_2 + v^2_3 = v^2 = (246\ \text{GeV})^2$ and the seven squared Higgs mass eigenvalue equations. We end up with two free parameters namely the VEV $\omega_3$ and the scalar Higgs mixing angle $\theta_S$.

Higgs boson masses are not determined *a priori* within the theory and their decay patterns depend strongly on the masses. In order to determine the possible decay modes and branching ratios, it is necessary to investigate the change of mass spectrum with respect to the quadrilinear $a, \ldots, h$ parameters. For simplicity, we assume in our numerical analysis the following values for these dimensionless parameters $a = 1, b = 1, c = 1, d = -1, e = -1, f = 3/2, g = 1, h = 1/2$, which guarantees that matrix $B$, Eq. (11), will be positive definite, and $v_2 = 246(\sin\omega_3)/2$ GeV, $v_3 = 246\cos\omega_3$ GeV, $-\pi \leq \omega_3 \leq \pi$. Figure 1 shows the masses of three $CP$-even Higgs scalars with respect to $\omega_3$. The quadrilinear $a, \ldots, h$ couplings are functions of $\theta_S$ and $\omega_3$ and are related to the masses of the Higgs bosons by

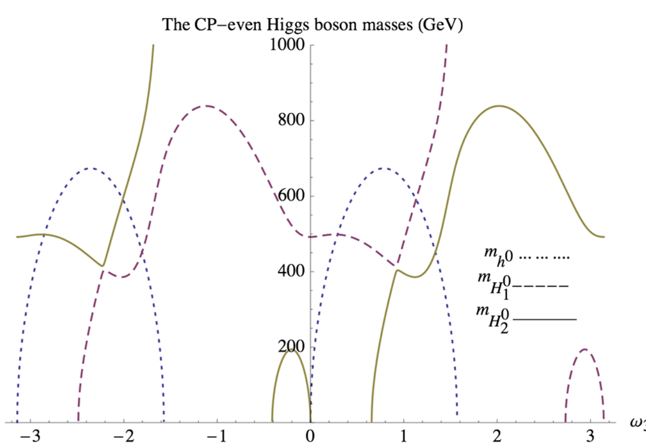


FIG. 1 (color online). The masses of three $CP$-even Higgs scalars with respect to $\omega_3$, $-\pi \leq \omega_3 \leq \pi$. The dashed line is for $H^0_1$, the solid line is for $H^0_2$ and the dotted line is for $h^0$.

$$a = \frac{9\mathcal{M}_c^2 v_3^2 - 2m_{h^0}^2 v_2^2}{18 v_3^4}$$

$$b = \frac{2m_{h^0}^2 v^2 + 9\mathcal{M}_b^2 v_3}{36 v_2 v_3^2} + \frac{m_{H_2^\pm}^2}{v^2}$$

$$c = \frac{m_{H_2^\pm}^2}{4v^2} - \frac{m_{h^0}^2 - 9(m_{H_1^\pm}^2 - m_{H_2^\pm}^2 + 2\mathcal{M}_a^2)}{144 v_2^2}$$

$$d = \frac{1}{16}\left(\frac{m_{A_2^0}^2 - m_{A_1^0}^2 + m_{H_1^\pm}^2 - m_{H_2^\pm}^2}{v_2^2} - \frac{4(m_{A_2^0}^2 - m_{H_2^\pm}^2)}{v^2}\right)$$

$$e = -\frac{m_{h^0}^2}{(18 v_2 v_3)}$$

$$f = \frac{1}{18}\left(\frac{9(m_{A_2^0}^2 - 2m_{H_2^\pm}^2)}{v^2} + \frac{m_{h^0}^2}{v_3^2}\right)$$

$$g = \frac{4m_{h^0}^2 - 9(m_{H_1^\pm}^2 - m_{H_2^\pm}^2)}{144 v_2^2} - \frac{m_{H_2^\pm}^2}{4v^2}$$

$$h = \frac{1}{36}\left(\frac{m_{h^0}^2}{v_3^2} - \frac{9m_{A_2^0}^2}{v^2}\right), \tag{29}$$

where

$$\mathcal{M}_a^2 = \frac{1}{4}(m_{H_1^0}^2 + m_{H_2^0}^2 + (m_{H_1^0}^2 - m_{H_2^0}^2)\cos 2\theta_S)$$
$$\mathcal{M}_b^2 = (\mathcal{M}_a^2 - \mathcal{M}_c^2)\tan 2\theta_S$$
$$\mathcal{M}_c^2 = \frac{1}{4}(m_{H_1^0}^2 + m_{H_2^0}^2 - (m_{H_1^0}^2 - m_{H_2^0}^2)\cos 2\theta_S). \tag{30}$$

Figure 2 shows the behavior of the quadrilinear parameters in Eq. (29) where we have taken $\theta_S = \pi/3$ and $\omega_3$ as a free parameter [Eq. (17)]. It is possible to specify a value for $\omega_3$ and from this determine a mass spectrum of Higgs bosons. For example, for $\omega_3 = 1$, quadrilinear parameters take the values $a = 5.33$, $b = 0.16$, $c = 0.78$, $d = -0.22$, $e = -1.94$, $f = 3.28$, $g = 0.93$, $h = 0.46$, fixing the Higgs mass spectrum to $m_{h^0} = 693$, $m_{H_1^0} = 790$, $m_{H_2^0} = 125$, $m_{H_1^\pm} = 181$, $m_{H_2^\pm} = 268$, $m_{A_1^0} = 334$, $m_{A_2^0} = 265$ GeV.

## IV. TRILINEAR SELF-COUPLINGS OF NEUTRAL HIGGS BOSONS

The measurement of the Higgs self-coupling is crucial to determine the Higgs potential. Self-couplings are uniquely determined in the SM by the mass of the Higgs boson, which is related to the quadrilinear coupling $\lambda$ through

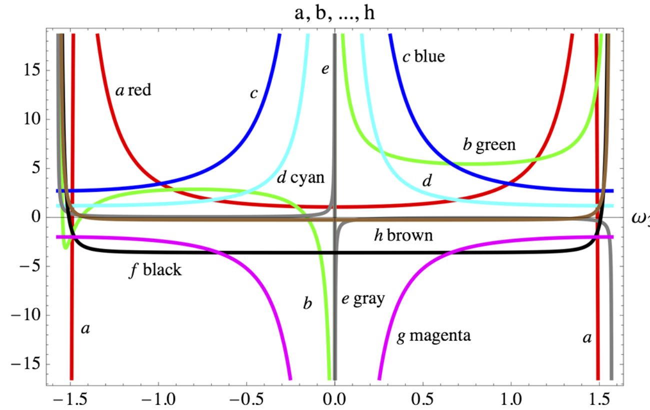


FIG. 2 (color online). Quadrilinear parameters $a, \ldots, h$ with respect to $\omega_3$, in the range $-\pi/2 \leq \omega_3 \leq \pi/2$. If $\omega_3$ take values close to zero, the parameters are on the order of tenths, then $\tan\omega_3 \sim 0.09$. When $\omega_3$ approaches to $\pi/2$, $\tan\omega_3$ diverges. Higher values are acceptable for $\tan\omega_3 = 5$. Also, considering $\tan\omega_3 = 1$, the numerical values of the parameters are $a \sim 1$, $b \sim 0.5$, $c \sim 1.3$, $d \sim -5.4$, $e \sim -1.7$, $f \sim 3.4$, $g \sim 1$, $h \sim 0.2$, and the mass spectrum of Higgs bosons is $m_{h^0} = 677$, $\mathrm{m}_{H_1^0} = 516$, $\mathrm{m}_{H_2^0} = 125$, $\mathrm{m}_{A_1^0}$, $\mathrm{m}_{A_2^0} = 248$, $\mathrm{m}_{H_2^\pm} = 175$, $m_{H_2^\pm} = 350$ GeV.

$M_H = \sqrt{2\lambda v}$. The trilinear and quadrilinear vertices of the Higgs field $H$ are given by the coefficients

$$\lambda_{HHH} = \lambda v = \frac{M_H^2}{2v}, \qquad \lambda_{HHHH} = \frac{\lambda}{4} = \frac{M_H^2}{8v^2}. \tag{31}$$

The following definitions are often used,

$$\lambda_{ijk} = \frac{-i\partial^3 V}{\partial H_i \partial H_j \partial H_k}, \tag{32}$$

which are most easily obtained from the corresponding derivatives of $V$ in Eq. (6) with respect to the fields $\{\phi_i\}$ with $i = 1, \ldots, 12$. We can then write the trilinear couplings in terms of the derivatives of the potential (6) with respect to $\phi_i$ and the elements of the rotation matrix $R_{il}$, Eqs. (21)–(22), as

$$\lambda_{ijk} = N\sum_{lmn} R_{il} R_{jm} R_{kn} \frac{\partial^3 V}{\partial\phi_i \partial\phi_j \partial\phi_k}, \tag{33}$$

the indices $l$, $m$, $n$ refer to the weak field basis, and $l \leq m \leq n = 1, 2, 3$, $N$ is a factor of $n!$ for $n$ identical fields. We now proceed to obtain these couplings in an explicit form. The trilinear self-couplings $a_{lmn} = \frac{\partial^3 V}{\partial\phi_i \partial\phi_j \partial\phi_k}$, among the neutral Higgs bosons can be written as

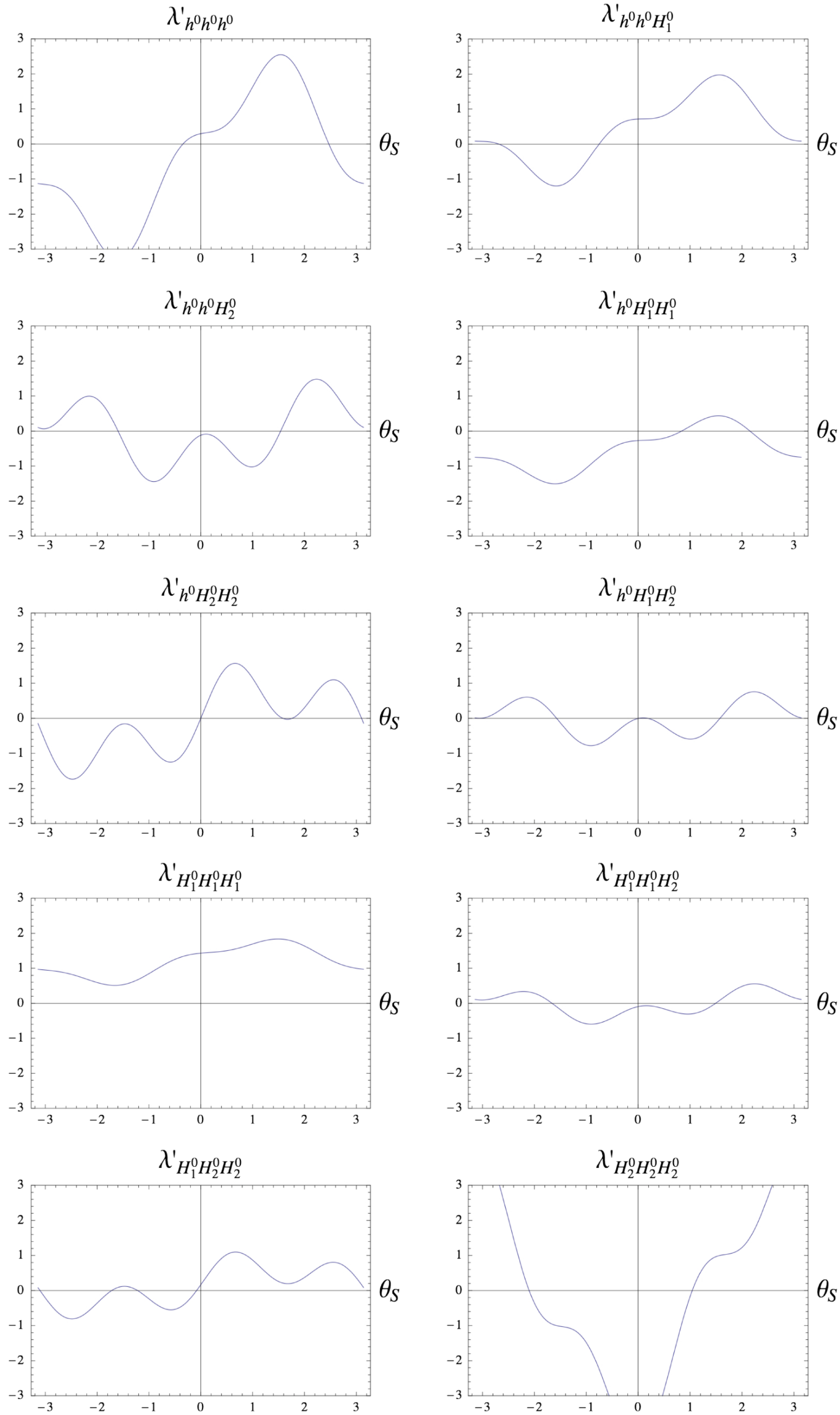


FIG. 3 (color online). The trilinear couplings by $CP$-even Higgs bosons, $\lambda_{ijk}{}' = \lambda_{ijk}/(125\ \mathrm{GeV})^2$ with $i, j, k = h^0, H^0_{1,2}$, as a function of the mixing angle $-\pi \leq \theta_S \leq \pi$, Eq. (26), where $\tan\omega_3 = 1$.

$$
\begin{aligned}
&a_{1,1,1} = 6\sqrt{6}(c+g)v_2, && a_{1,1,2}, = \sqrt{2}(2(c+g)v_2 + 3ev_3),\\
&a_{1,1,3} = \sqrt{2}(3ev_2 + (b+f+2h)v_3), && a_{1,2,2} = 2\sqrt{6}(c+g)v_2,\\
&a_{1,2,3} = 3\sqrt{6}ev_2, && a_{1,3,3} = \sqrt{6}(b+f+2h)v_2,\\
&a_{2,2,2} = 3\sqrt{2}(2(c+g)v_2 - ev_3), && a_{2,2,3} = \sqrt{2}((b+f+2h)v_3 - 3ev_2),\\
&a_{2,3,3} = \sqrt{2}(b+f+2h)v_2, && a_{3,3,3} = 6\sqrt{2}av_3. \qquad (34)
\end{aligned}
$$

Then, here we have ten trilinear Higgs scalar self-couplings and substituting for the elements of the rotation matrix, Eq. (21), one obtains

$$
\begin{aligned}
&\lambda_{1,1,1} = 6v(\lambda_1 s\omega_3 + \lambda_2 c\omega_3), && \lambda_{2,2,2} = 6v(\lambda_3 s\omega_3 + \lambda_4 c\omega_3),\\
&\lambda_{3,3,3} = 6v(\lambda_5 s\omega_3 + \lambda_6 c\omega_3), && \lambda_{1,1,2} = 2v(\lambda_7 s\omega_3 + \lambda_8 c\omega_3),\\
&\lambda_{1,1,3} = 2v(\lambda_9 s\omega_3 + \lambda_{10} c\omega_3), && \lambda_{1,2,2} = 2v(\lambda_{11} s\omega_3 + \lambda_{12} c\omega_3),\\
&\lambda_{1,2,3} = v(\lambda_{13} s\omega_3 + \lambda_{14} c\omega_3), && \lambda_{1,3,3} = 2v(\lambda_{15} s\omega_3 + \lambda_{16} c\omega_3),\\
&\lambda_{2,2,3} = 2v(\lambda_{17} s\omega_3 + \lambda_{18} c\omega_3), && \lambda_{2,3,3} = 2v(\lambda_{19} s\omega_3 + \lambda_{20} c\omega_3). \qquad (35)
\end{aligned}
$$

$\lambda_1, \ldots, \lambda_{20}$ depend on the quadrilinear coupling parameters of the Higgs potential Eq. (6) and the mixing angle $\theta_S$ Eq. (26). For example, one can use the results of the appendix to compute the first $\lambda_{h^0h^0h^0} = \lambda_{1,1,1}$, these results are given in Appendix A,

$$
\begin{aligned}
\lambda_{h^0h^0h^0} = 3\frac{\sqrt{3}}{2}v[2\{18as_{\theta_S}^3 + (b+f+2h)(3c_{\theta_S}^2+1)s_{\theta_S} - 9ec_{\theta_S}^3 + 3ec_{\theta_S}\}c\omega_3\\
+\{3(b+f+2h)(c_{\theta_S}-1)s_{\theta_S}^2 + 2(c+g)(9c_{\theta_S}^3 - 3c_{\theta_S}^2 + c_{\theta_S} - 3) - 3e(3c_{\theta_S}(c_{\theta_S}+1)-1)s_{\theta_S}\}s\omega_3], \qquad (36)
\end{aligned}
$$

where $s_{\theta_S} = \sin\theta_S$, $c_{\theta_S} = \cos\theta_S$, $s\omega_3 = \sin\omega_3$, and $c\omega_3 = \cos\omega_3$.

## V. NUMERICAL RESULTS

Here, we present numerical results considering that one of the $CP$-even Higgs scalar boson behaves like the Higgs boson of the Standard Model, where the mass of the lightest Higgs scalar is significantly smaller than the masses of the other six Higgs bosons of the model. We assume that it is light and its mass is around 125 GeV. In our analysis we propose $H_2^0$ as our candidate, since this is the physical scalar Higgs boson whose self-coupling $\lambda_{H_2^0H_2^0H_2^0}$ is like the SM self-coupling $\lambda_{HHH}^{\mathrm{SM}}$, and in turn has a mass of 125 GeV. The other Higgs bosons have consequently masses of the order of ~500 GeV and without loss of generality and for the sake of simplicity, we consider that their masses are not equal. Figure 3 shows the trilinear couplings by $CP$-even Higgs bosons as a function of the mixing angle $\theta_S$, Eq. (26), where we have fixed $\tan\omega_3 = 1$. The higher strength coupling corresponds to $\lambda_{h^0h^0h^0}$ and $\lambda_{H_2^0H_2^0H_2^0}$, whereas that $\lambda_{H_1^0H_1^0H_1^0}$ became very small,

$$\lambda_{h^0h^0h^0} \sim \lambda_{H_2^0H_2^0H_2^0} > \lambda_{H_1^0H_1^0H_1^0},$$

followed by $\lambda_{h^0h^0H_2^0}$, $\lambda_{h^0h^0H_1^0}$ $(\lambda_{h^0H_2^0H_2^0})$ and others. We proposed that the lightest is $H_2^0$ and its trilinear coupling has the lowest intensity. Although there are couplings equal in shape, they are different in intensity. Comparing these

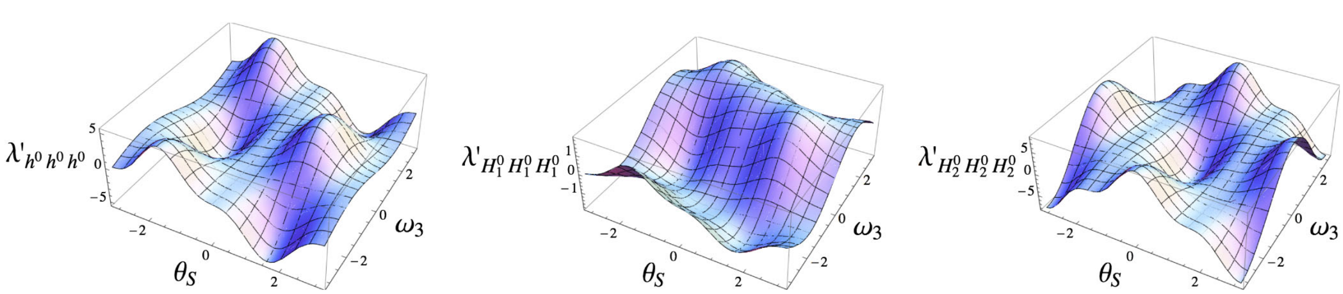


FIG. 4 (color online). The trilinear couplings by $CP$-even Higgs bosons, $\lambda_{ijk}{}' = \lambda_{ijk}/(125\ \mathrm{GeV})^2$ with $i, j, k = h^0, H_{1,2}^0$, as a function of the mixing angle $-\pi \le \theta_S \le \pi$, and $-\pi \le \omega_3 \le \pi$.

results only with the SM trilinear coupling, we see that our values are within acceptance ranges in the literature, but they are richer for a Higgs potential analysis, enabling better understanding of the spontaneous weak symmetry breaking. Furthermore, when comparing the self-couplings of the Higgs boson as a function of the two mixing angles $\theta_S$ and $\omega_3$, there is a symmetry; see Fig. 4. This allows us to characterize different Higgs bosons. The mass of the Higgs boson is fixed through $\omega_3$ and couplings are also determined by mixing Higgs bosons depending on $\theta_S$. Our results are normalized ($M = 125$ GeV) in Fig. 3, but with either parametrization three scalar bosons dependence is observed. See Fig. 4.

## VI. CONCLUSIONS

We studied only the scalar sector assuming the pseudoscalars to be too heavy to be relevant. In this work we have analyzed the complete scalar sector of an $S(3)$ flavor model. We deal with three $CP$-even, two $CP$-odd, and two sets of charged scalar particles. We have improved our potential minimization technique which enabled us to explore a larger region of the allowed parameter space. We have studied in detail the trilinear couplings of the lightest Higgs boson of this model. Within the allowed domain of the parameter space of the model, the trilinear Higgs couplings have a strong dependence on $\tan\omega_3 = 2v_2/v_3$ and $\tan\theta_S$. The extended Higgs spectrum in $S(3)$ models gives rise to numbers of trilinear couplings. The $h^0h^0h^0$ coupling can be measured in $h^0h^0$ continuum production linear colliders as at $e^+e^-$.

## ACKNOWLEDGMENTS

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## APPENDIX: TRILINEAR COUPLINGS $\lambda_{i,j,k}$

For completeness, we show the explicit neutral scalar Higgs bosons self-couplings:

$$
\begin{aligned}
\lambda_{h^0h^0h^0} &= 6v(\lambda_1 s\omega_3 + \lambda_2 c\omega_3),\\
\lambda_1 &= \frac{\sqrt{3}}{4}[3(b+f+2h)(c_{\theta_S}-1)s_{\theta_S}^2 + 2(c+g)(9c_{\theta_S}^3 - 3c_{\theta_S}^2 + c_{\theta_S} - 3) - 3e(3c_{\theta_S}(c_{\theta_S}+1)-1)s_{\theta_S}]\\
\lambda_2 &= \frac{\sqrt{3}}{2}(18as_{\theta_S}^3 + [(b+f+2h)s_{\theta_S} - 3ec_{\theta_S}](3c_{\theta_S}^2+1))c_{\theta_S}\\
\lambda_{H_1^0H_1^0H_1^0} &= 6v(\lambda_3 s\omega_3 + \lambda_4 c\omega_3),\\
\lambda_3 &= \frac{1}{4}[(b+f+2h)(c_{\theta_S}+3)s_{\theta_S}^2 + 6(c+g)(c_{\theta_S}^3 + c_{\theta_S}^2 + c_{\theta_S} + 9) - 3e((c_{\theta_S}-3)c_{\theta_S}-3)s_{\theta_S}]\\
\lambda_4 &= \frac{1}{2}[6as_{\theta_S}^3 + (b+f+2h)(c_{\theta_S}^2+3)s_{\theta_S} - 3ec_{\theta_S}(c_{\theta_S}^2-3)]\\
\lambda_{H_2^0H_2^0H_2^0} &= 6v(\lambda_5 s\omega_3 + \lambda_6 c\omega_3),\\
\lambda_5 &= 2[(b+f+2h)c_{\theta_S}^2 + 6(c+g)s_{\theta_S}^2 + 3ec_{\theta_S}s_{\theta_S}]\\
\lambda_6 &= -2[6ac_{\theta_S}^3 + s_{\theta_S}^2((b+f+2h)c_{\theta_S} + 3es_{\theta_S})]\\
\lambda_{h^0h^0H_1^0} &= 2v(\lambda_7 s\omega_3 + \lambda_8 c\omega_3),\\
\lambda_7 &= \frac{1}{4}[3(b+f+2h)(3c_{\theta_S}+1)s_{\theta_S}^2 + 2(c+g)(c_{\theta_S}(3c_{\theta_S}(9c_{\theta_S}+1)-5)+27) - 3e(9c_{\theta_S}^2 - 3c_{\theta_S} + 5)s_{\theta_S}]\\
\lambda_8 &= \frac{1}{2}[54as_{\theta_S}^3 + (b+f+2h)(9c_{\theta_S}^2-5)s_{\theta_S} - 3ec_{\theta_S}(9c_{\theta_S}^2+5)]
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
\lambda_{h^0h^0H_2^0} &= 2v(\lambda_9 s\omega_3 + \lambda_{10} c\omega_3),\\
\lambda_9 &= \frac{1}{2}(2s_{\theta_S}(3(b(1-c_{\theta_S}) + (9c-f+9g-2h)c_{\theta_S} - 2c + f - 2g + 2h)c_{\theta_S} + g)\\
&\quad + 3(b+f+2h)s_{\theta_S}^3 + 2cs_{\theta_S} - 9e(2c_{\theta_S}+1)s_{\theta_S}^2 + 3ec_{\theta_S}(3c_{\theta_S}(c_{\theta_S}+1)-1))\\
\lambda_{10} &= 6(-9a+b+f+2h)c_{\theta_S}s_{\theta_S}^2 - (b+f+2h)(3c_{\theta_S}^3 + c_{\theta_S}) - 3e(9c_{\theta_S}^2-1)s_{\theta_S}\\
\lambda_{h^0H_1^0H_1^0} &= 2v(\lambda_{11} s\omega_3 + \lambda_{12} c\omega_3),
\end{aligned}
$$

$$\lambda_{11} = \frac{\sqrt{3}}{4}((b+f+2h)(3c_{\theta_S}+5)s_{\theta_S}^2 + 2(c+g)(9c_{\theta_S}^3 + 5c_{\theta_S}^2 + c_{\theta_S} - 27) + 3e((5-3c_{\theta_S})c_{\theta_S}+1)s_{\theta_S})$$

$$\lambda_{12} = \frac{\sqrt{3}}{2}(18as_{\theta_S}^3 + (b+f+2h)(3c_{\theta_S}^2+1)s_{\theta_S} - 9ec_{\theta_S}^3 + 3ec_{\theta_S})$$

$$\lambda_{h^0 H_1^0 H_2^0} = v(\lambda_{13} s\omega_3 + \lambda_{14} c\omega_3),$$

$$\lambda_{13} = \sqrt{3}(-2((b(1+c_{\theta_S}) - 9cc_{\theta_S} - 2c + c_{\theta_S} f - 9c_{\theta_S} g + 2(c_{\theta_S}+1)h + f - 2g)c_{\theta_S} + g)s_{\theta_S}$$
$$+ (b+f+2h)s_{\theta_S}^3 - 2cs_{\theta_S} + 3e(1-2c_{\theta_S})s_{\theta_S}^2 + 3ec_{\theta_S}((c_{\theta_S}-1)c_{\theta_S}+1))$$

$$\lambda_{14} = 2\sqrt{3}((2(-9a+b+f+2h)s_{\theta_S}^2 - (b+f+2h)(c_{\theta_S}^2-1) - 9ec_{\theta_S}s_{\theta_S})c_{\theta_S} - 3es_{\theta_S}) \quad \text{(A2)}$$

$$\lambda_{h^0 H_2^0 H_2^0} = 2v(\lambda_{15} s\omega_3 + \lambda_{16} c\omega_3),$$

$$\lambda_{15} = \sqrt{3}(-2((b+f-9g+2h)c_{\theta_S} - 9cc_{\theta_S} + c + g)s_{\theta_S}^2 + (b+f+2h)(c_{\theta_S}-1)c_{\theta_S}^2 + 3ec_{\theta_S}(2c_{\theta_S}+1)s_{\theta_S} - 3es_{\theta_S}^3)$$

$$\lambda_{16} = 2\sqrt{3}s_{\theta_S}(18ac_{\theta_S}^2 - (b+f+2h)(2c_{\theta_S}^2 - s_{\theta_S}^2) - 9ec_{\theta_S}s_{\theta_S})$$

$$\lambda_{H_1^0 H_1^0 H_2^0} = 2v(\lambda_{17} s\omega_3 + \lambda_{18} c\omega_3),$$

$$\lambda_{17} = \frac{1}{2}(-2((b(c_{\theta_S}+3) + (-9c+f-9g+2h)c_{\theta_S} - 6c + 3f - 6g + 6h)c_{\theta_S} - 3g)s_{\theta_S}$$
$$+ (b+f+2h)s_{\theta_S}^3 + 6cs_{\theta_S} + 3e(3-2c_{\theta_S})s_{\theta_S}^2 + 3ec_{\theta_S}((c_{\theta_S}-3)c_{\theta_S}-3))$$

$$\lambda_{18} = 2(-9a+b+f+2h)c_{\theta_S}s_{\theta_S}^2 - (b+f+2h)(c_{\theta_S}^2+3)c_{\theta_S} - 9e(c_{\theta_S}^2-1)s_{\theta_S}$$

$$\lambda_{H_1^0 H_2^0 H_2^0} = 2v(\lambda_{19} s\omega_3 + \lambda_{20} c\omega_3),$$

$$\lambda_{19} = 2(-(b+f-9g+2h)c_{\theta_S} + c(9c_{\theta_S}+3) + 3g)s_{\theta_S}^2 + (b+f+2h)(c_{\theta_S}+3)c_{\theta_S}^2 + 3ec_{\theta_S}(2c_{\theta_S}-3)s_{\theta_S} - 3es_{\theta_S}^3$$

$$\lambda_{20} = 2(18ac_{\theta_S}^2 - (b+f+2h)(2c_{\theta_S}^2 - s_{\theta_S}^2) - 9ec_{\theta_S}s_{\theta_S}). \quad \text{(A3)}$$


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