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Hidden-charm tetraquarks and charged Z_c states

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Experimentally several charged axial-vector hidden-charm states were reported. Within the framework of the color-magnetic interaction, we have systematically considered the mass spectrum of the hidden-charm and hidden-bottom tetraquark states. It is impossible to accommodate all of the three charged states $Z_c(3900)$, $Z_c(4025)$, and $Z_c(4200)$ within the axial-vector tetraquark spectrum simultaneously. Not all of these three states are tetraquark candidates. Moreover, the eigenvector of the chromomagnetic interaction contains valuable information of the decay pattern of the tetraquark states. The dominant decay mode of the lowest axial-vector tetraquark state is $J/\psi\pi$ while its $D^*\bar{D}$ and $\bar{D}^*D^*$ modes are strongly suppressed, which is in contrast with the fact that the dominant decay mode of $Z_c(3900)$ and $Z_c(4025)$ is $\bar{D}D^*$ and $\bar{D}^*D^*$, respectively. We emphasize that all the available experimental information indicates that $Z_c(4200)$ is a very promising candidate of the lowest axial-vector hidden-charm tetraquark state.

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I. INTRODUCTION

During the past decade, many charmoniumlike states and bottomoniumlike states have been reported by experimental collaborations such as Belle, *BABAR*, CDF, D0, LHCb, BESIII, and CLEOc. $X(3872)$ was first observed by Belle Collaboration in the exclusive decay process $B^\pm \to K^\pm\pi^+\pi^- J/\psi$ [1]. Its mass is very close to the $\bar{D}^0 D^{*0}$ threshold and its width is extremely narrow (< 1.2 MeV). Later LHCb Collaboration determined its $J^{PC} = 1^{++}$ [2]. Many theoretical groups interpreted $X(3872)$ as the molecular candidate of the $\bar{D}D^*$ system [3–6].

Besides $X(3872)$, a family of so-called Y states was also reported. $Y(4260)$ was observed by *BABAR* Collaboration in the invariant mass spectrum of $\pi^+\pi^- J/\psi$ in the initial-state radiation process $e^+e^- \to \gamma_{ISR}\pi^+\pi^- J/\psi$ [7]. Later, Belle Collaboration observed a peak near 4.25 GeV and a new structure around 4.05 GeV which was denoted later as $Y(4008)$ [8,9]. $Y(4360)$ was observed in the reaction $e^+e^- \to \pi^+\pi^-\psi(2S)$ by *BABAR* [10]. Almost at the same time, Belle observed two resonant structures in the $\pi^+\pi^-\psi(2S)$ invariant mass distribution $Y(4360)$ and $Y(4660)$ [11], which was confirmed by *BABAR* via the initial-state radiation process $e^+e^- \to \pi^+\pi^-\psi(2S)$ [12]. $Y(4630)$ was reported as a near-threshold enhancement in the $e^+e^- \to \Lambda_c^+\Lambda_c^-$ process [13].

The group of charged charmoniumlike and bottomoniumlike states is even more exotic. The lightest charged charmoniumlike state $Z_c(3900)$ was observed in the $J/\psi\pi^\pm$ invariant mass in the process $Y(4260) \to J/\psi\pi^+\pi^-$ by BESIII Collaboration [14], by Belle Collaboration with ISR [15], and by using CLEO data [16]. Its decay mode implies that $Z_c(3900)$ is a hidden-charm structure. $Z_c(4025)$ was observed in the $\pi^\mp$ recoil mass spectrum in the process $e^+e^- \to (D^*\bar{D}^*)^\pm\pi^\mp$ [17]. $Z_c(4020)$ was reported in the $\pi^\pm h_c$ mass spectrum in the process $e^+e^- \to \pi^+\pi^- h_c$ [18]. Moreover, two charged bottomoniumlike resonances $Z_b(10610)$ and $Z_b(10650)$ were observed in the $\pi^\pm\Upsilon(nS)$ and $\pi^\pm h_b$ mass spectrum in the $\Upsilon(5S)$ decays [19]. $Z_1(4050)$ and $Z_2(4250)$ were observed in the $\pi^+\chi_{c1}$ invariant mass distribution in the $\bar{B}^0 \to K^-\pi^+\chi_{c1}$ decays [20]. $Z_c(4485)$ was observed by Belle Collaboration in the $\pi^\pm\psi'$ invariant mass distribution in the exclusive $B \to K\pi^\pm\psi'$ decays [21]. Later its spin and parity were determined as $J^P = 1^+$ [22]. The charmoniumlike state $Z_c(4200)$ was observed in the $J/\psi\pi^+$ mode with a significance of 8.2σ when performing the amplitude analysis of $B \to J/\psi K\pi$ [23].

These XYZ states either decay into one charmonium/bottomonium state plus light mesons or into a pair of open-charm/open-bottom heavy mesons. Many of them do not fit into the conventional $q\bar{q}$ meson spectrum in the quark model. Some of them were interpreted as the candidates of the hybrid meson [24], molecular states [3,4,25–29], tetraquark states [30–35], and so on. For example, $Z_c(3900)$ was interpreted as the isovector axial-vector molecular partner of $X(3872)$ [36–38]. Similarly, $Z_c(4025)$ was speculated to be the D^*D^* molecular candidate [39–41]. There are also some other speculations about their nature [42,43]. $Z_b(10610)$ and $Z_b(10650)$ are generally regarded as the candidates of the $\bar{B}B^*$ and $\bar{B}^*B^*$ molecular states [44–47].

However, it is not very natural to explain $Z_c(4200)$ and $Z_c(4485)$ as the S-wave molecular states composed of two

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S-wave heavy mesons. Instead, $Z_c(4485)$ was proposed as the cousin molecular state of $Z_c(3900)$ and $Z_c(4025)$ composed of $D(D^*)$ and its radial excitation [48,49].

Another interesting possibility is that some charged Z_c states might be tetraquark candidates. The light $q\bar{q}q\bar{q}$ tetraquark system was first studied in the MIT bag model [50,51], where the multiquark mass spectrum mostly depends on the chromomagnetic interaction among the quarks. When considering the chromomagnetic interaction, it is convenient to adopt the $SU(6)_{cs}$ representation which is the eigenstate of the color-magnetic (CM) interaction and can be constructed as the direct product of the $SU(3)$ color and the $SU(2)$ spin group. The bag model was later used to discuss the hidden-charm/bottom tetraquark system [52,53]. The hidden-charm tetraquarks were also studied in the constituent quark model (CQM) [33,54].

In this work we will investigate whether some of the charged Z_c states could be the tetraquark candidates. We will discuss the mixing of the hidden-charm tetraquark states in the different color-spin representation and possible mass splitting of the hidden-charm tetraquark states in the framework of the chromomagnetic interaction. We will employ two schemes to fix the strength of the CM interaction and extract the masses and wave functions of the $J^P = 1^+, 0^+, 2^+$ tetraquark systems. Then we compare the hidden-charm tetraquark spectrum with the current experimental data.

The paper is organized as follows. After the introduction, we present the chromomagnetic Hamiltonian and the tetraquark model in Sec. II. In Sec. III, we discuss the masses of the possible tetraquark candidates. We explore the decay pattern of the tetraquark system in Sec. IV. The last section is the discussion and summary.

II. HEAVY TETRAQUARK

A. The chromomagnetic Hamiltonian

For the tetraquark system, we consider the chromomagnetic interaction to derive the mass splitting. The Hamiltonian reads

$$H = \sum_i m_i + H_{\mathrm{CM}}, \tag{1}$$

where m_i is the mass of the ith constituent quark. H_{CM} describes the CM interaction which is derived from one gluon exchange [50,51,55,56],

$$H_{\mathrm{CM}} = -\sum_{i>j} v_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j, \tag{2}$$

where $\vec{\lambda}_i$ is the quark color operator and $\vec{\sigma}_i$ is the spin operator. For the antiquark, $\vec{\lambda}_{\bar{q}} = -\vec{\lambda}^*_q$ and $\vec{\sigma}_{\bar{q}} = -\vec{\sigma}^*_q$. v_{ij} represents the interaction strength between two quarks. Therefore, v_{ij} depends on the wave function of the multiquark system. For example, v_{ij} takes different values in the $q\bar{q}$, qqq, and $q\bar{q}q\bar{q}$ systems. In the bag model, v_{ij} depends on the bag radius and the constituent quark mass. On the other hand, the CQM is very successful in describing the meson and baryon spectrum, where the color-magnetic interaction leads to the mass splitting between the octet and decuplet baryons. We follow the CQM convention and adopt $v_{ij} = v\frac{m_u^2}{m_i m_j}$. The parameter v depends on the multiquark system.

B. Hidden-charm tetraquark wave function

For the $qq\bar{q}\,\bar{q}$ tetraquark system, the CM wave function can be constructed either as $qq \otimes \bar{q}\,\bar{q}$ or $q\bar{q} \otimes q\bar{q}$. We use Q, $\bar{Q}$, and $\tilde{Q}$ to represent the configuration qq, $\bar{q}\,\bar{q}$, and $q\bar{q}$ respectively. We use the notation $|D_6, D_{3c}, S, N\rangle$ to represent the diquark configuration, where D_6, D_{3c}, S, and N are the $SU(6)$ color-spin coupling representations, $SU(3)_c$ color representations, spin, and number of the constituent quarks, respectively. Based on the $SU(6)_{cs} \supset SU(3)_c \otimes SU(2)_s$ group theory, there are four types of representations for the diquark qq: $|21, \bar{3}_c, 0, 2\rangle$, $|21, 6_c, 1, 2\rangle$, $|15, \bar{3}_c, 1, 2\rangle$, and $|15, \bar{6}_c, 0, 2\rangle$. For the antidiquark $\bar{q}\,\bar{q}$, there are also four types of representations: $|\bar{21}, 3_c, 0, 2\rangle$, $|\bar{21}, \bar{6}_c, 1, 2\rangle$, $|\bar{15}, 3_c, 1, 2\rangle$, and $|\bar{15}, 6_c, 0, 2\rangle$. For the $q\bar{q}$ system, there are also four types of representations: $|1, 1_c, 0, 2\rangle$, $|35, 1_c, 1, 2\rangle$, $|35, 8_c, 1, 2\rangle$, and $|35, 8_c, 0, 2\rangle$.

In the previous work [51,57], the coefficient describing the magnetic-interaction strength is fixed based on the bag model, and it depends on the quark masses and on the properties of the spatial wave function. They are extracted from the other multiquark systems when their masses are fitted by certain models including all types of interaction, such as the spin-orbital, spin-spin interaction and the confinement potential. Our choice of the coefficient in Eq. (2) is based on the CQM model; the coefficient $v_{ij} = v\frac{m_u^2}{m_i m_j}$ depends on the constituent quark mass and v can be extracted from the experimental data as we did in Eq. (19).

For the tetraquark system $q_1 q_2 \bar{q}_3 \bar{q}_4$, if the color-spin and flavor symmetry is exact, the parameters $v_{ij} = v$ are the same for each diquark sector. Then the interaction H_{CM} matrix element can be expressed in terms of the Casimir operators as

$$H_{\mathrm{CM}} = \frac{v}{2}[\bar{C}(\mathrm{tot}) - 2\bar{C}(Q) - 2\bar{C}(\tilde{Q}) + 16N]. \tag{3}$$

We want to emphasize that this formula is derived assuming the exact SU(3) flavor and SU(6) color-spin symmetry. In our case, we considered that the SU(3) flavor symmetry is broken since the strange quark is heavier than the up/down quark. The above formula cannot be used any more for the tetraquark system. In order to perform the numerical calculation, we explicitly expand the matrix element of the interaction H_{CM} between two $SU(6)_{cs}$ eigenstates $|k\rangle$ and $|l\rangle$ as

$$V_{\rm CM}(q_1q_2\bar{q}_3\bar{q}_4) = \langle k|H_{\rm CM}|l\rangle = V_{12}(q_1q_2) + V_{13}(q_1\bar{q}_3) + V_{14}(q_1\bar{q}_4) + V_{23}(q_2\bar{q}_3) + V_{24}(q_2\bar{q}_4) + V_{34}(\bar{q}_3\,\bar{q}_4) \quad (4)$$

where

$$V_{ij}(Q) = -\frac{v_{ij}}{2}[\bar{C}(Q) - 16N], \quad (5)$$

which is the eigenvalue of CM operator $v_{ij}\vec{\lambda}_i\cdot\vec{\lambda}_j\vec{\sigma}_i\cdot\vec{\sigma}_i$ and

$$V_{ij}(\tilde{Q}) = \frac{v_{ij}}{2}[\bar{C}(\tilde{Q}) - 16N], \quad (6)$$

which is the eigenvalue of CM operator $v_{ij}\vec{\lambda}_i\cdot\vec{\lambda}^*_j\vec{\sigma}_i\cdot\vec{\sigma}^*_j$. For the diquark system we have

$$\bar{C}(Q) = \bar{C}(\tilde{Q}) = C_6 - C_3 - \frac{8}{3}S(S+1), \quad (7)$$

where C_6 and C_3 are the eigenvalues of the quadratic Casimir operators of $SU(6)_{cs}$ and $SU(3)_c$ groups. S is the spin operator. The above equation is equivalent to Eq. (3) when the flavor and color-spin symmetry are exact.

Based on the $SU(6)_{cs}$ group decomposition, the color-spin wave function of the $J^p = 1^+$ tetraquark $SU(6)_{cs}$ eigenstates can be constructed in the $Q\otimes\bar{Q}$ form

$$|35,1_c,1,4\rangle = |21,6_c,1,2\rangle\otimes|\bar{2}1,\bar{6}_c,1,2\rangle \quad (8)$$

$$|35,1_c,1,4\rangle = |15,\bar{3}_c,1,2\rangle\otimes|\bar{1}5,3_c,1,2\rangle \quad (9)$$

$$|35,1_c,1,4\rangle = \sqrt{\frac{1}{3}}|21,\bar{3}_c,0,2\rangle\otimes|\bar{1}5,3_c,1,2\rangle - \sqrt{\frac{2}{3}}|21,6_c,1,2\rangle\otimes|\bar{1}5,\bar{6}_c,1,2\rangle \quad (10)$$

$$|280,1_c,1,4\rangle = \sqrt{\frac{2}{3}}|21,\bar{3}_c,0,2\rangle\otimes|\bar{1}5,3_c,1,2\rangle + \sqrt{\frac{1}{3}}|21,6_c,1,2\rangle\otimes|\bar{1}5,\bar{6}_c,1,2\rangle \quad (11)$$

$$|35,1_c,1,4\rangle = \sqrt{\frac{1}{3}}|\bar{2}1,3_c,0,2\rangle\otimes|15,\bar{3}_c,1,2\rangle - \sqrt{\frac{2}{3}}|\bar{2}1,\bar{6}_c,1,2\rangle\otimes|15,6_c,1,2\rangle \quad (12)$$

$$|280,1_c,1,4\rangle = \sqrt{\frac{2}{3}}|\bar{2}1,3_c,0,2\rangle\otimes|15,\bar{3}_c,1,2\rangle + \sqrt{\frac{1}{3}}|\bar{2}1,\bar{6}_c,1,2\rangle\otimes|15,6_c,1,2\rangle. \quad (13)$$

The CM wave functions of the $J^P = 0^+$ and $J^P = 2^+$ tetraquark states are listed in the Appendix. These wave functions are the eigenstates of the CM interaction $V_{ij}(Q)$ and $V_{ij}(\bar{Q})$. The CM interaction $V_{\rm CM}$ also has the form $V_{ij}(\tilde{Q})$. In order to get their eigenstates, we need to do the recoupling from $Q\otimes\bar{Q}$ to $\tilde{Q}\otimes\tilde{Q}$. Based on Wigner and Racah coefficients of $SU(6)_{cs}\supset SU(3)_c\otimes SU(2)_s$ [58,59], the 1^+ $SU(6)_{cs}$ eigenstates in terms of $q_1\bar{q}_3\otimes q_2\bar{q}_4$ are

$$|35,1_c,1,4\rangle = \frac{\sqrt{3}}{3}|q_1\bar{q}_3 1,1_c,0,2\rangle\otimes|q_2\bar{q}_4 35,1_c,1,2\rangle + \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35,8_c,0,2\rangle\otimes|q_2\bar{q}_4 35,8_c,1,2\rangle + \frac{\sqrt{3}}{3}|q_1\bar{q}_3 35,1_c,1,2\rangle\otimes|q_2\bar{q}_4 1,1_c,0,2\rangle + \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35,8_c,1,2\rangle\otimes|q_2\bar{q}_4 35,8_c,0,2\rangle \quad (14)$$

$$|35,1_c,1,4\rangle = \frac{\sqrt{6}}{6}|q_1\bar{q}_3 1,1_c,0,2\rangle\otimes|q_2\bar{q}_4 35,1_c,1,2\rangle - \frac{\sqrt{3}}{3}|q_1\bar{q}_3 35,8_c,0,2\rangle\otimes|q_2\bar{q}_4 35,8_c,1,2\rangle + \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35,1_c,1,2\rangle\otimes|q_2\bar{q}_4 1,1_c,0,2\rangle - \frac{\sqrt{3}}{3}|q_1\bar{q}_3 35,8_c,1,2\rangle\otimes|q_2\bar{q}_4 35,8_c,0,2\rangle \quad (15)$$

$$|35,1_c,1,4\rangle = \frac{1}{2}|q_1\bar{q}_3 1,1_c,0,2\rangle\otimes|q_2\bar{q}_4 35,1_c,1,2\rangle - \frac{1}{2}|q_1\bar{q}_3 35,1_c,1,2\rangle\otimes|q_2\bar{q}_4 1,1_c,0,2\rangle - \frac{2}{3}|q_1\bar{q}_3 35,8_c,1,2\rangle\otimes|q_2\bar{q}_4 35,8_c,1,2\rangle - \frac{\sqrt{2}}{6}|q_1\bar{q}_3 35,1_c,1,2\rangle\otimes|q_2\bar{q}_4 35,1_c,1,2\rangle \quad (16)$$

$$|280,1_c,1,4\rangle = -\frac{1}{2}|q_1\bar{q}_3 35,8_c,0,2\rangle\otimes|q_2\bar{q}_4 35,8_c,1,2\rangle + \frac{1}{2}|q_1\bar{q}_3 35,8_c,1,2\rangle\otimes|q_2\bar{q}_4 35,8_c,0,2\rangle - \frac{\sqrt{2}}{6}|q_1\bar{q}_3 35,8_c,1,2\rangle\otimes|q_2\bar{q}_4 35,8_c,1,2\rangle + \frac{2}{3}|q_1\bar{q}_3 35,1_c,1,2\rangle\otimes|q_2\bar{q}_4 35,1_c,1,2\rangle \quad (17)$$

$$|35,1_c,1,4\rangle = -\frac{1}{2}|q_1\bar{q}_3 1,1_c,0,2\rangle\otimes|q_2\bar{q}_4 35,1_c,1,2\rangle + \frac{1}{2}|q_1\bar{q}_3 35,1_c,1,2\rangle\otimes|q_2\bar{q}_4 1,1_c,0,2\rangle - \frac{2}{3}|q_1\bar{q}_3 35,8_c,1,2\rangle\otimes|q_2\bar{q}_4 35,8_c,1,2\rangle - \frac{\sqrt{2}}{6}|q_1\bar{q}_3 35,1_c,1,2\rangle\otimes|q_2\bar{q}_4 35,1_c,1,2\rangle \quad (18)$$

$$
\begin{aligned}
|280, 1_c, 1, 4\rangle = &\frac{1}{2}|q_1\bar{q}_3 35, 8_c, 0, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \\
&-\frac{1}{2}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 0, 2\rangle \\
&-\frac{\sqrt{2}}{6}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \\
&+\frac{2}{3}|q_1\bar{q}_3 35, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 1_c, 1, 2\rangle.
\end{aligned} \tag{19}
$$

According to the $SU(3)_c$ and $SU(2)_s$ symmetry, the $SU(6)_{cs}$ eigenstates in terms of $q_2\bar{q}_3 \otimes q_1\bar{q}_4$ have the same form with those of $q_1\bar{q}_3 \otimes q_2\bar{q}_4$ in Eqs. (14), (16), and (17). There appears an extra minus sign in the tetraquark states in Eqs. (15), (18), and (19) when we change the basis from $q_2\bar{q}_3 \otimes q_1\bar{q}_4$ to $q_1\bar{q}_3 \otimes q_2\bar{q}_4$. For the $J^p = 0^+$ and $J^P = 2^+$ tetraquark states, the $SU(6)_c s$ eigenstates in the form of $\tilde{Q} \otimes \tilde{Q}$ are listed in the Appendix.

Using the above $SU(6)_{cs}$ eigenstates in Eqs. (8)–(19), we can calculate each individual term in Eq. (4), obtain the eigenvalues of the CM interaction matrix $V_{\rm CM}$, and derive the wave function and mass of the tetraquark system.

III. POSSIBLE TETRAQUARK CANDIDATES AMONG VARIOUS Z_c STATES

In order to extract the tetraquark mass, we need the values of the constituent quark mass and the parameter v. Recall that the charmonium J/ψ and η_c can be treated as the $SU(6)_{cs}$ diquark $c\bar{c}$ state $|35, 1_c, 1, 2\rangle$ and $|1, 1_c, 0, 2\rangle$. Similarly, the charmed mesons D^* and D can be treated as $SU(6)_{cs}$ diquark $c\bar{u}$ state $|35, 1_c, 1, 2\rangle$ and $|1, 1_c, 0, 2\rangle$. With Eq. (5) and the meson masses from PDG [60], we can extract the masses of the u, c, s, and b constituent quarks.

$$
\begin{aligned}
M(J/\psi) &= 2m_c + \frac{16}{3}v_{c\bar{c}}\left(\frac{m_u}{m_c}\right)^2; \\
M(\eta_c) &= 2m_c - 16v_{c\bar{c}}\left(\frac{m_u}{m_c}\right)^2; \\
M(D^*) &= m_u + m_c + \frac{16}{3}v_{c\bar{u}}\frac{m_u}{m_c}; \\
M(D) &= m_u + m_c - 16v_{c\bar{u}}\frac{m_u}{m_c}; \\
M(D_s^*) &= m_s + m_c + \frac{16}{3}v_{c\bar{s}}\frac{m_u}{m_c}\frac{m_u}{m_s}; \\
M(D_s) &= m_s + m_c - 16v_{c\bar{s}}\frac{m_u}{m_c}\frac{m_u}{m_s}; \\
M(\Upsilon) &= 2m_b + 16v_{b\bar{b}}\left(\frac{m_u}{m_b}\right)^2 \approx 2m_b.
\end{aligned} \tag{20}
$$

From the above equation, we get

TABLE I. The eigenvalues of $V_{\rm CM}$ for the tetraquark configuration $qc\bar{q}\,\bar{c}$, $qc\bar{s}\,\bar{c}$, and $sc\bar{s}\,\bar{c}$ with $J^P = 0^+, 1^+, 2^+$.

Configuration	J^P	$V_{\rm CM}$					
$qc\bar{q}\,\bar{c}$	0^+	$-18.6v$	$-7.4v$	$0.8v$	$8.3v$		
	1^+	$-15.9v$	$-4.1v$	$-1.5v$	$1.7v$	$5.6v$	$5.8v$
	2^+	$2.7v$	$5.8v$				
$qc\bar{s}\,\bar{c}$	0^+	$-15.6v$	$-6.5v$	$0.5v$	$7.1v$		
	1^+	$-12.8v$	$-3.8v$	$-1.3v$	$1.3v$	$4.6v$	$4.8v$
	2^+	$2.5v$	$4.7v$				
$sc\bar{s}\,\bar{c}$	0^+	$-13.1v$	$-5.7v$	$0.3v$	$6.1v$		
	1^+	$-10.3v$	$-3.2v$	$-1.3v$	$1.0v$	$3.8v$	$3.9v$
	2^+	$2.3v$	$3.9v$				

$$
\begin{aligned}
m_c &= 1534 \text{ MeV}; \\
m_u &= 437 \text{ MeV}; \\
m_s &= 542 \text{ MeV}; \\
m_b &= 4730 \text{ MeV}.
\end{aligned} \tag{21}
$$

According to Eq. (4), $V_{\rm CM}(qc\bar{q}\,\bar{c})$ reads

$$
\begin{aligned}
V_{\rm CM}(qc\bar{q}\,\bar{c}) = &\frac{m_u}{m_c}V_{12} + V_{13} + \frac{m_u}{m_c}V_{14} \\
&+ \frac{m_u}{m_c}V_{23} + \left(\frac{m_u}{m_c}\right)^2 V_{24} + \frac{m_u}{m_c}V_{34}.
\end{aligned} \tag{22}
$$

After diagonalizing the mass matrix $V_{\rm CM}$ for the $J^P = 1^+$ $qc\bar{q}\,\bar{c}$ tetraquark states, we get six eigenvalues: $-15.9v$, $-4.1v$, $-1.5v$, $1.7v$, $5.6v$, $5.8v$, which are listed in Table I. Sometimes we use the eigenvalues to denote the state. In the following we discuss two schemes to fix the parameter v and extract the tetraquark spectrum.

A. Scheme I: using the mass of one of the Z_c states as input

Assuming that $Z_c(3900)$ is one of the six tetraquark states, the parameter v can be fixed. Similarly, $Z_c(4025)$ and $Z_c(4200)$ can also be used as input to extract the value of v. Throughout our discussion, we require v to be positive. Then we use the obtained v to calculate the masses of the other eigenstates, which are listed in Table II.

In the two BESIII experiments [14,17], the width of $Z_c(3900)$ is about 46 MeV and $Z_c(4025)$ is about 25 MeV, while their mass errors are even smaller, such as around 3 MeV. In the reference [23], the width of $Z_c(4200)$ is 370 MeV and its mass error is about 30 MeV. We consider the error or even the width to evaluate our assignments.

If $Z_c(3900)$ is appointed as the state with the eigenvalue $-15.9v$, $-4.1v$, and $-1.5v$, it is quite difficult to accommodate either $Z_c(4025)$ or $Z_c(4200)$ among the six states. If $Z_c(4025)$ is appointed as the state with the eigenvalue $1.7v$, the mass of the state with the eigenvalue $5.6v$ is

TABLE II. The masses of the six axial-vector $qc\bar{q}\,\bar{c}$ tetraquark states when the parameter v is fixed by the mass of $Z_c(3900)$, $Z_c(4025)$, and $Z_c(4200)$. The eigenvalue is used to denote the state as the subscript.

v	$-15.9v$	$-4.1v$	$-1.5v$	$1.7v$	$5.6v$	$5.8v$
$v_{-15.9}^{Z_c(3900)} = 2.6$	3900	3931.2	3938.0	3946.5	3956.8	3957.3
$v_{-4.1}^{Z_c(3900)} = 10.2$	3779.1	3900	3926.6	3959.4	3999.4	4001.4
$v_{-1.5}^{Z_c(3900)} = 28.0$	3496.8	3827.2	3900	3989.6	4098.8	4104.4
$v_{1.7}^{Z_c(4025)} = 48.8$	3165.7	3741.8	3868.8	4025	4215.4	4225.2
$v_{5.6}^{Z_c(4025)} = 14.8$	3706.3	3881.2	3919.8	3967.2	4025	4028.0
$v_{5.8}^{Z_c(4025)} = 14.3$	3714.5	3883.3	3920.5	3966.3	4022.1	4025
$v_{5.6}^{Z_c(4200)} = 46.1$	3209.4	3753.1	3872.9	4020.3	4200	4209.2
$v_{5.8}^{Z_c(4200)} = 44.5$	3234.7	3759.6	3875.3	4017.6	4191.1	4200

4215.4 MeV which is close to $Z_c(4200)$, while the mass of the state with the eigenvalue $-1.5v$ is 3868.8 MeV which is 30 MeV lower than $Z_c(3900)$. Unfortunately, the lowest axial-vector tetraquark state is around 3166 MeV. Such a scheme is not realistic.

If $Z_c(4200)$ is appointed as the state with the eigenvalue $5.6v$, the mass of the state with the eigenvalue $1.7v$ is 4020.3 MeV which is close to $Z_c(4020)$. The mass of the state with the eigenvalue $-1.5v$ is 3872.9 MeV, which is 28 MeV lower than $Z_c(3900)$. In this case the lowest state is around 3210 MeV, which is also quite unrealistic. It is almost impossible to accommodate all of the three charged states $Z_c(3900)$, $Z_c(4025)$, and $Z_c(4200)$ within the axial-vector tetraquark spectrum simultaneously. In other words, at least one or two of these states is not a tetraquark candidate.

TABLE III. The masses of the 1^+ $qc\bar{q}\,\bar{c}$ tetraquark states when the parameter v are fixed by the mass difference of two Z_c states.

$-15.9v$	$-4.1v$	$-1.5v$	$1.7v$	$5.6v$	$5.8v$
$Z_c(3900) \to -15.9v,\ Z_c(4025) \to -4.1v,\ v = 10.6$					
3900	4025	4052.5	4086.4	4127.8	4129.9
$Z_c(3900) \to -15.9v,\ Z_c(4025) \to -1.5v,\ v = 8.7$					
3900	4002.4	4025	4052.8	4086.6	4088.4
$Z_c(3900) \to -15.9v,\ Z_c(4025) \to 1.7v,\ v = 7.1$					
3900	3983.8	4002.3	4025	4052.7	4054.1
$Z_c(3900) \to -15.9v,\ Z_c(4025) \to 5.6v,\ v = 5.81$					
3900	3968.6	3983.7	4002.3	4025	4026.2
$Z_c(3900) \to -15.9v,\ Z_c(4025) \to 5.8v,\ v = 5.76$					
3900	3968	3983	4001.4	4023.9	4025
$Z_c(3900) \to -1.5v,\ Z_c(4025) \to 1.7v,\ v = 39.1$					
3337.5	3798.4	3900	4025	4177.3	4185.2
$Z_c(3900) \to -1.5v,\ Z_c(4025) \to 5.6v,\ v = 17.6$					
3646.5	3854.2	3900	3956.3	4025	4028.5
$Z_c(4025) \to 1.7v,\ Z_c(4200) \to 5.6v,\ v = 44.9$					
3235.3	3764.7	3881.4	4025	4200	4209
$Z_c(4025) \to 1.7v,\ Z_c(4200) \to 5.8v,\ v = 42.7$					
3273.8	3777.4	3888.4	4025	4191.5	4200
$Z_c(3900) \to -1.5v,\ Z_c(4200) \to 5.6v,\ v = 42.3$					
3291.6	3790.1	3900	4035.2	4200	4208.5
$Z_c(3900) \to -15.9v,\ Z_c(4200) \to 5.6v,\ v = 14$					
3900	4064.7	4100.9	4145.8	4200	4202.8

B. Scheme II: using the mass splitting of two Z_c states as input

The parameter v can be extracted from the mass splitting if we assume two of the three states $Z_c(3900)$, $Z_c(4025)$, and $Z_c(4200)$ are the 1^+ $qc\bar{q}\,\bar{c}$ tetraquark states. As pointed out in Sec. IV, the state with the eigenvalue $-4.1v$ does not decay to $J/\psi\pi$. Thus, it is not appropriate to assign it as $Z_c(3900)$. Therefore, we only assume $Z_c(3900)$ as the state either with the eigenvalue -15.9 or $-1.5v$. Once the value

TABLE IV. The masses of the $qc\bar{q}\,\bar{c}$, $qc\bar{s}\,\bar{c}$, and $sc\bar{s}\,\bar{c}$ tetraquark states with $J^P = 0^+, 1^+, 2^+$. The parameter v is fixed assuming $Z(4025)$ as the tetraquark state with the eigenvalue $1.7v$.

		0^+	1^+	2^+
$qc\bar{q}\,\bar{c}$	V_{CM}	$-18.6v$	$-15.9v$	$2.7v$
	M (MeV)	3033.9	3165.7	4073.8
	V_{CM}	$-7.4v$	$-4.1v$	$5.8v$
	M (MeV)	3580.7	3741.8	4225.2
$qc\bar{s}\,\bar{c}$	V_{CM}	$-15.6v$	$-12.8v$	$2.5v$
	M (MeV)	3285.4	3422.1	4169.1
	V_{CM}	$-6.5v$	$-3.8v$	$4.7v$
	M (MeV)	3729.7	3861.5	4276.5
$sc\bar{s}\,\bar{c}$	V_{CM}	$-13.1v$	$-10.3v$	$2.3v$
	M (MeV)	3512.4	3649.1	4264.3
	V_{CM}	$-5.7v$	$-3.2v$	$3.9v$
	M (MeV)	3873.7	3995.8	4342.4

TABLE V. The eigenvalues of the $qb\bar{q}\,\bar{b}$, $qb\bar{s}\,\bar{b}$, and $sb\bar{s}\,\bar{b}$ tetraquark states with $J^P = 0^+, 1^+, 2^+$.

Configuration	J^P	V_{CM}					
$qb\bar{q}\,\bar{b}$	0^+	$-16.2v$	$-3.0v$	$1.8v$	$5.8v$		
	1^+	$-16.0v$	$-1.8v$	$-0.8v$	$2.0v$	$5.3v$	$5.4v$
	2^+	$0.4v$	$5.4v$				
$qb\bar{s}\,\bar{b}$	0^+	$-13.1v$	$-2.7v$	$1.5v$	$4.7v$		
	1^+	$-12.9v$	$-1.6v$	$-0.6v$	$1.6v$	$4.31v$	$4.34v$
	2^+	$0.5v$	$4.3v$				
$sb\bar{s}\,\bar{b}$	0^+	$-10.7v$	$-2.3v$	$1.1v$	$3.9v$		
	1^+	$-10.4v$	$-1.3v$	$-0.5v$	$1.3v$	$3.48v$	$3.51v$
	2^+	$0.5v$	$3.5v$				

of v is extracted, we obtain the whole spectrum. The results are listed in Table III.

If $Z_c(3900)$ and $Z_c(4025)$ are assigned as the state with the eigenvalue $-1.5v$ and $1.7v$, respectively, the resulting mass of the state with the eigenvalue $5.6v$ is 4177.3 MeV, which is close to $Z_c(4200)$. Unfortunately the lowest state is around 3338 MeV, which is unrealistic. Similarly, if $Z_c(3900)$ and $Z_c(4200)$ are assigned as the state with the eigenvalue $-1.5v$ and $5.6v$, respectively, the mass of the state with the eigenvalue $1.7v$ is 4035.2 MeV, which is close to $Z_c(4025)$. If $Z_c(4025)$ and $Z_c(4200)$ are treated as the state with the eigenvalue $1.7v$ and $5.6v$, respectively, the mass of the state with the eigenvalue $-1.5v$ is 3881. MeV, which is close to $Z_c(3900)$. Now the lowest state is around 3235 MeV. Although we could accommodate all three charged states $Z_c(3900)$, $Z_c(4025)$, and $Z_c(4200)$ as the axial-vector tetraquark candidates, the resulting mass of the lowest state is always too low and unrealistic. In other words, not all these three states are tetraquark candidates, which is consistent with the conclusion in the previous subsection.

C. The $qc\bar{s}\,\bar{c}$, $sc\bar{s}\,\bar{c}$ and hidden-bottom tetraquark states

We assume $Z_c(4025)$ as the $qc\bar{q}\,\bar{c}$ tetraquark state with the eigenvalue $1.7v$ to fix the parameter v and collect the numerical results for the $qc\bar{s}\,\bar{c}$ and $sc\bar{s}\,\bar{c}$ tetraquark states in Table IV.

We extend the same formalism to investigate the hidden-bottom tetraquark states. The results are collected in Tables V, VI, and VII. The resulting lowest states listed in the above tables also seem too light.

TABLE VII. The eigenvalues and masses of the $qb\bar{q}\,\bar{b}$, $qb\bar{s}\,\bar{b}$, and $sb\bar{s}\,\bar{b}$ tetraquark states with $J^P = 0^+, 1^+, 2^+$. The parameter v is fixed assuming $Z(4025)$ as the tetraquark state with the eigenvalue $1.7v$.

		0^+	1^+	2^+
$qb\bar{q}\,\bar{b}$	V_{CM}	$-16.2v$	$-16.0v$	$0.4v$
	M (MeV)	9543.1	9552.8	10353.5
	V_{CM}	$-3.0v$	$-1.8v$	$5.4v$
	M (MeV)	10187.5	10246.1	10597.6
$qb\bar{s}\,\bar{b}$	V_{CM}	$-13.1v$	$-12.9v$	$0.5v$
	M (MeV)	9799.4	9809.2	10463.4
	V_{CM}	$-2.7v$	$-1.6v$	$4.3v$
	M (MeV)	10307.2	10360.9	10648.9
$sb\bar{s}\,\bar{b}$	V_{CM}	$-10.7v$	$-10.4v$	$0.5v$
	M (MeV)	10021.6	10036.2	10568.4
	V_{CM}	$-2.3v$	$-1.3v$	$3.5v$
	M (MeV)	10431.7	10480.5	10714.9

IV. DECAY PATTERNS OF HIDDEN-CHARM TETRAQUARKS

The eigenvalues of the CM interaction matrix V_{CM} can be used to derive the mass of the tetraquark system, while the eigenvectors of V_{CM} contain important information on their decay pattern. Therefore, we carefully investigate the eigenvectors of the tetraquark systems with the configuration $qc\bar{q}\,\bar{c}$, $qc\bar{s}\,\bar{c}$, and $sc\bar{s}\,\bar{c}$ and $J^P = 0^+, 1^+, 2^+$. We first list the eigenvalues of V_{CM} for the $qc\bar{q}\,\bar{c}$, $qc\bar{s}\,\bar{c}$, and $sc\bar{s}\,\bar{c}$ tetraquark configuration in Table I.

For the $J^P = 0^+, 1^+$ case, we only list the eigenvectors with the negative eigenvalues. When we present the eigenvectors using the diquark representation $q\bar{q} \otimes q\bar{q}$, we omit the N in the diquark representation $|D_6, D_{3c}, S, N\rangle$

TABLE VI. The masses of the 1^+ $qb\bar{q}\,\bar{b}$ tetraquark states. The parameter v is fixed using the Z_c mass as input.

v	$-16.0v$	$-1.8v$	$-0.8v$	$2.0v$	$5.3v$	$5.4v$
$v_{-15.9v}^{Z_c(3900)} = 2.6$	10292.4	10329.3	10331.9	10339.2	10347.8	10348
$v_{-4.1}^{Z_c(3900)} = 10.2$	10170.8	10315.6	10325.8	10354.4	10388.1	10389.1
$v_{-1.5}^{Z_c(3900)} = 28.0$	9886	10283.6	10311.6	10390	10482.4	10485.2
$v_{1.7}^{Z_c(4025)} = 48.8$	9553.2	10246.2	10295	10431.6	10592.6	10597.5
$v_{5.6}^{Z_c(4025)} = 14.8$	10097.2	10307.4	10322.2	10363.6	10412.4	10413.9
$v_{5.8}^{Z_c(4025)} = 14.3$	10105.2	10308.3	10322.6	10362.6	10409.8	10411.2
$v_{5.6}^{Z_c(4200)} = 46.1$	9596.4	10251	10297.1	10426.2	10578.3	10582.9
$v_{5.8}^{Z_c(4200)} = 44.5$	9622	10253.9	10298.4	10423	10569.9	10574.3

for brevity since $N = 2$. We present the expressions of the eigenvectors for the $qc\bar{q}\,\bar{c}$, $qc\bar{s}\,\bar{c}$, and $sc\bar{s}\,\bar{c}$ tetraquark systems in Tables VIII–XIII, and XIV–XVI respectively.

We notice that J/ψ and η_c can also be expressed as the $SU(6)_{cs}$ $c\bar{c}$ state $|35, 1_c, 1\rangle$ and $|1, 1_c, 0\rangle$. Similarly, D^* and D can be treated as the $SU(6)_{cs}$ $c\bar{u}$ state $|35, 1_c, 1\rangle$ and $|1, 1_c, 0\rangle$. Therefore, we can identify the decay patterns of the tetraquark states from the expression of their CM interaction eigenvectors. The branching fraction of each decay mode is proportional to the square of the coefficient of the corresponding component in the eigenvectors if we ignore the phase space difference. From the very beginning, we want to emphasize the following point: so long as the phase space allows, the $\psi'\pi$ decay mode is also allowed if $J/\psi\pi$ is one of the allowed decay modes.

For the $J^P = 0^+$ state, the lowest state corresponds to the eigenvalue $-18.6v$. From Table IX, its dominant decay mode is $\eta_c\pi$. The $\bar{D}D$ mode is also important. The $\bar{D}^*D^*$ mode is suppressed by a factor of eight if we compare the coefficients of the $\bar{D}D$ and $\bar{D}^*D^*$ components only. In fact, the $\bar{D}^*D^*$ mode is further suppressed by phase space.

TABLE VIII. The eigenvectors of $V_{\rm CM}$ for the $qc\bar{q}\,\bar{c}$ tetraquark states with $J^P = 1^+$.

$V_{\rm CM}$	$q\bar{q} \otimes c\bar{c}$	$c\bar{q} \otimes q\bar{c}$
$-15.9v$	$+0.99\|1, 1_c, 0\rangle \otimes \|35, 1_c, 1\rangle$	$+0.24\|1, 1_c, 0\rangle \otimes \|35, 1_c, 1\rangle$
	$+0.14\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$	$+0.44\|35, 8_c, 0\rangle \otimes \|35, 8_c, 1\rangle$
		$+0.24\|35, 1_c, 1\rangle \otimes \|1, 1_c, 0\rangle$
		$+0.44\|35, 8_c, 1\rangle \otimes \|35, 8_c, 0\rangle$
		$-0.69\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$
		$-0.15\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$
$-4.1v$	$+\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$	$+0.67\|1, 1_c, 0\rangle \otimes \|35, 1_c, 1\rangle$
		$-0.24\|35, 8_c, 0\rangle \otimes \|35, 8_c, 1\rangle$
		$-0.67\|35, 1_c, 1\rangle \otimes \|1, 1_c, 0\rangle$
		$+0.24\|35, 8_c, 1\rangle \otimes \|35, 8_c, 0\rangle$
$-1.5v$	$+0.12\|1, 1_c, 0\rangle \otimes \|35, 1_c, 1\rangle$	$-0.65\|1, 1_c, 0\rangle \otimes \|35, 1_c, 1\rangle$
	$-0.56\|35, 8_c, 0\rangle \otimes \|35, 8_c, 1\rangle$	$+0.18\|35, 8_c, 0\rangle \otimes \|35, 8_c, 1\rangle$
	$-0.22\|35, 1_c, 1\rangle \otimes \|1, 1_c, 0\rangle$	$-0.65\|35, 1_c, 1\rangle \otimes \|1, 1_c, 0\rangle$
	$-0.79\|35, 8_c, 1\rangle \otimes \|35, 8_c, 0\rangle$	$+0.18\|35, 8_c, 1\rangle \otimes \|35, 8_c, 0\rangle$
		$-0.17\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$
		$-0.23\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$

TABLE IX. The eigenvectors of $V_{\rm CM}$ for the $qc\bar{q}\,\bar{c}$ tetraquark states with $J^P = 0^+$.

$V_{\rm CM}$	$q\bar{q} \otimes c\bar{c}$	$c\bar{q} \otimes q\bar{c}$
$-18.6v$	$+0.94\|1, 1_c, 0\rangle \otimes \|1, 1_c, 0\rangle$	$+0.45\|1, 1_c, 0\rangle \otimes \|1, 1_c, 0\rangle$
	$+0.33\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$	$-0.16\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$
		$+0.35\|35, 8_c, 0\rangle \otimes \|35, 8_c, 0\rangle$
		$-0.80\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$
$-7.4v$	$+0.33\|1, 1_c, 0\rangle \otimes \|1, 1_c, 0\rangle$	$-0.85\|1, 1_c, 0\rangle \otimes \|1, 1_c, 0\rangle$
	$-0.34\|35, 8_c, 0\rangle \otimes \|35, 8_c, 0\rangle$	$-0.25\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$
	$+0.88\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$	$-0.39\|35, 8_c, 0\rangle \otimes \|35, 8_c, 0\rangle$
		$-0.27\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$

TABLE X. The eigenvectors of $V_{\rm CM}$ for the $qc\bar{q}\,\bar{c}$ tetraquark states with $J^P = 2^+$.

$V_{\rm CM}$	$q\bar{q} \otimes c\bar{c}$	$c\bar{q} \otimes q\bar{c}$
$5.8v$	$+\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$	$+0.94\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$
		$+0.33\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$
$2.7v$	$-\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$	$+0.33\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$
		$-0.94\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$

TABLE XI. The eigenvectors of $V_{\rm CM}$ for the $qc\bar{s}\,\bar{c}$ tetraquark states with $J^P = 1^+$.

$V_{\rm CM}$	$q\bar{s} \otimes c\bar{c}$	$c\bar{s} \otimes q\bar{c}$
$-12.8v$	$+0.99\|1, 1_c, 0\rangle \otimes \|35, 1_c, 1\rangle$	$+0.24\|1, 1_c, 0\rangle \otimes \|35, 1_c, 1\rangle$
	$+0.16\|35, 8_c, 1\rangle \otimes \|35, 8_c, 0\rangle$	$+0.43\|35, 8_c, 0\rangle \otimes \|35, 8_c, 1\rangle$
		$+0.25\|35, 1_c, 1\rangle \otimes \|1, 1_c, 0\rangle$
		$+0.43\|35, 8_c, 1\rangle \otimes \|35, 8_c, 0\rangle$
		$-0.69\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$
		$-0.14\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$
$-3.8v$	$-0.16\|35, 8_c, 0\rangle \otimes \|35, 8_c, 1\rangle$	$+0.5\|1, 1_c, 0\rangle \otimes \|35, 1_c, 1\rangle$
	$-0.14\|35, 8_c, 1\rangle \otimes \|35, 8_c, 0\rangle$	$-0.2\|35, 8_c, 0\rangle \otimes \|35, 8_c, 1\rangle$
	$-0.97\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$	$-0.8\|35, 1_c, 1\rangle \otimes \|1, 1_c, 0\rangle$
		$+0.25\|35, 8_c, 1\rangle \otimes \|35, 8_c, 0\rangle$
$-1.3v$	$+0.13\|1, 1_c, 0\rangle \otimes \|35, 1_c, 1\rangle$	$-0.78\|1, 1_c, 0\rangle \otimes \|35, 1_c, 1\rangle$
	$-0.57\|35, 8_c, 0\rangle \otimes \|35, 8_c, 1\rangle$	$+0.24\|35, 8_c, 0\rangle \otimes \|35, 8_c, 1\rangle$
	$-0.24\|35, 1_c, 1\rangle \otimes \|1, 1_c, 0\rangle$	$-0.49\|35, 1_c, 1\rangle \otimes \|1, 1_c, 0\rangle$
	$-0.74\|35, 8_c, 1\rangle \otimes \|35, 8_c, 0\rangle$	$-0.09\|35, 8_c, 1\rangle \otimes \|35, 8_c, 0\rangle$
	$+0.22\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$	$-0.21\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$
		$-0.2\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$

TABLE XII. The eigenvectors of $V_{\rm CM}$ for the $qc\bar{s}\,\bar{c}$ tetraquark states with $J^P = 0^+$.

$V_{\rm CM}$	$q\bar{s} \otimes c\bar{c}$	$c\bar{s} \otimes q\bar{c}$
$-15.6v$	$+0.92\|1, 1_c, 0\rangle \otimes \|1, 1_c, 0\rangle$	$+0.5\|1, 1_c, 0\rangle \otimes \|1, 1_c, 0\rangle$
	$-0.38\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$	$-0.14\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$
		$+0.33\|35, 8_c, 0\rangle \otimes \|35, 8_c, 0\rangle$
		$-0.79\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$
$-6.5v$	$+0.37\|1, 1_c, 0\rangle \otimes \|1, 1_c, 0\rangle$	$-0.83\|1, 1_c, 0\rangle \otimes \|1, 1_c, 0\rangle$
	$-0.35\|35, 8_c, 0\rangle \otimes \|35, 8_c, 0\rangle$	$-0.24\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$
	$+0.85\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$	$+0.4\|35, 8_c, 0\rangle \otimes \|35, 8_c, 0\rangle$
		$-0.31\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$

TABLE XIII. The eigenvectors of $V_{\rm CM}$ for the $qc\bar{s}\,\bar{c}$ tetraquark states with $J^P = 2^+$.

$V_{\rm CM}$	$q\bar{s} \otimes c\bar{c}$	$c\bar{s} \otimes q\bar{c}$
$4.7v$	$+\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$	$+0.94\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$
		$+0.33\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$
$2.5v$	$+\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$	$+0.33\|35, 8_c, 1\rangle \otimes \|35, 8_c, 1\rangle$
		$-0.94\|35, 1_c, 1\rangle \otimes \|35, 1_c, 1\rangle$

TABLE XIV. The eigenvectors of V_{CM} for the $sc\bar{s}\,\bar{c}$ tetraquark states with $J^P = 1^+$.

V_{CM}	$s\bar{s} \otimes c\bar{c}$	$c\bar{s} \otimes s\bar{c}$
$-10.3v$	$+0.98\|1,1_c,0\rangle \otimes \|35,1_c,1\rangle$	$+0.25\|1,1_c,0\rangle \otimes \|35,1_c,1\rangle$
	$+0.17\|35,8_c,1\rangle \otimes \|35,8_c,0\rangle$	$+0.43\|35,8_c,0\rangle \otimes \|35,8_c,1\rangle$
		$+0.26\|35,1_c,1\rangle \otimes \|1,1_c,0\rangle$
		$+0.43\|35,8_c,1\rangle \otimes \|35,8_c,0\rangle$
		$-0.69\|35,8_c,1\rangle \otimes \|35,8_c,1\rangle$
		$-0.13\|35,1_c,1\rangle \otimes \|35,1_c,1\rangle$
$-3.2v$	$+\|35,8_c,1\rangle \otimes \|35,8_c,1\rangle$	$+0.67\|1,1_c,0\rangle \otimes \|35,1_c,1\rangle$
		$-0.24\|35,8_c,0\rangle \otimes \|35,8_c,1\rangle$
		$-0.67\|35,1_c,1\rangle \otimes \|1,1_c,0\rangle$
		$+0.24\|35,8_c,1\rangle \otimes \|35,8_c,0\rangle$
$-1.3v$	$+0.14\|1,1_c,0\rangle \otimes \|35,1_c,1\rangle$	$-0.65\|1,1_c,0\rangle \otimes \|35,1_c,1\rangle$
	$-0.63\|35,8_c,0\rangle \otimes \|35,8_c,1\rangle$	$+0.14\|35,8_c,0\rangle \otimes \|35,8_c,1\rangle$
	$-0.31\|35,1_c,1\rangle \otimes \|1,1_c,0\rangle$	$-0.65\|35,1_c,1\rangle \otimes \|1,1_c,0\rangle$
	$-0.7\|35,8_c,1\rangle \otimes \|35,8_c,0\rangle$	$-0.14\|35,8_c,1\rangle \otimes \|35,8_c,0\rangle$
		$-0.29\|35,8_c,1\rangle \otimes \|35,8_c,1\rangle$
		$-0.15\|35,1_c,1\rangle \otimes \|35,1_c,1\rangle$

TABLE XV. The eigenvectors of V_{CM} for the $sc\bar{s}\,\bar{c}$ tetraquark states with $J^P = 0^+$.

V_{CM}	$s\bar{s} \otimes c\bar{c}$	$c\bar{s} \otimes s\bar{c}$
$-13.1v$	$+0.91\|1,1_c,0\rangle \otimes \|1,1_c,0\rangle$	$+0.53\|1,1_c,0\rangle \otimes \|1,1_c,0\rangle$
	$-0.41\|35,8_c,1\rangle \otimes \|35,8_c,1\rangle$	$-0.13\|35,1_c,1\rangle \otimes \|35,1_c,1\rangle$
		$+0.31\|35,8_c,0\rangle \otimes \|35,8_c,0\rangle$
		$-0.78\|35,8_c,1\rangle \otimes \|35,8_c,1\rangle$
$-5.6v$	$+0.41\|1,1_c,0\rangle \otimes \|1,1_c,0\rangle$	$-0.81\|1,1_c,0\rangle \otimes \|1,1_c,0\rangle$
	$-0.36\|35,8_c,0\rangle \otimes \|35,8_c,0\rangle$	$-0.24\|35,1_c,1\rangle \otimes \|35,1_c,1\rangle$
	$+0.83\|35,8_c,1\rangle \otimes \|35,8_c,1\rangle$	$+0.4\|35,8_c,0\rangle \otimes \|35,8_c,0\rangle$
	$+0.11\|35,1_c,1\rangle \otimes \|35,1_c,1\rangle$	$-0.35\|35,8_c,1\rangle \otimes \|35,8_c,1\rangle$

TABLE XVI. The eigenvectors of V_{CM} for the $sc\bar{s}\,\bar{c}$ tetraquark states with $J^P = 2^+$.

V_{CM}	$s\bar{s} \otimes c\bar{c}$	$c\bar{s} \otimes s\bar{c}$
$3.9v$	$+\|35,1_c,1\rangle \otimes \|35,1_c,1\rangle$	$+0.94\|35,8_c,1\rangle \otimes \|35,8_c,1\rangle$
		$+0.33\|35,1_c,1\rangle \otimes \|35,1_c,1\rangle$
$2.3v$	$+\|35,8_c,1\rangle \otimes \|35,8_c,1\rangle$	$+0.33\|35,8_c,1\rangle \otimes \|35,8_c,1\rangle$
		$-0.94\|35,1_c,1\rangle \otimes \|35,1_c,1\rangle$

The $J^P = 0^+$ state with the eigenvalue $-7.4v$ also decays into $\eta_c\pi$, $\bar{D}D$, and $\bar{D}^*D^*$. However, $\bar{D}D$ becomes its dominant decay mode. The $\bar{D}^*D^*$ mode is also severely suppressed.

From Table X, the $J^P = 2^+$ state with the eigenvalue $5.8v$ mainly decays into $J/\psi\rho$ while its $\bar{D}^*D^*$ mode is suppressed. In contrast, the $J^P = 2^+$ state with the eigenvalue $2.7v$ decays into $\bar{D}^*D^*$ only. Its $J/\psi\rho$ mode is forbidden.

Experimentally several charged axial-vector hidden-charm states were reported in different decay channels. All the four charged axial-vector states $Z_c(3900)$, $Z_c(4025)$, $Z_c(4200)$, and $Z_c(4485)$ were observed in the $J/\psi\pi$ channel. $Z_c(4485)$ was also observed in the $\psi'\pi$ mode. The dominant decay mode of $Z_c(3900)$ and $Z_c(4025)$ is $\bar{D}D^*$ and $\bar{D}^*D^*$, respectively. Up to now, the dominant decay mode of $Z_c(4200)$ and $Z_c(4485)$ has not been established yet. Moreover, $Z_c(4025)$ does not decay into $\bar{D}D^*$.

It is very interesting to investigate the decay patterns of the low lying tetraquark states and compare their typical decay modes with the available experimental data. From Table VIII, the lowest axial-vector $qc\bar{q}\,\bar{c}$ tetraquark state corresponds to the eigenvalue $-15.9v$. Its dominant decay mode is $J/\psi\pi$. The $\bar{D}^*D$ mode is suppressed by a factor of sixteen if we compare the coefficients of the $J/\psi\pi$ and $D^*\bar{D}$ components and ignore the phase space difference. The $\bar{D}^*D^*$ mode is further suppressed roughly by a factor of two compared with the $D^*\bar{D}$ mode. Considering the decay phase space, the $D^*\bar{D}$ and $\bar{D}^*D^*$ modes are further suppressed. This state mainly decays into $J/\psi\pi$, which is in strong contrast with the fact that the dominant decay mode of $Z_c(3900)$ and $Z_c(4025)$ is $\bar{D}D^*$ and $\bar{D}^*D^*$, respectively. In other words, neither $Z_c(3900)$ nor $Z_c(4025)$ is a good candidate of this lowest lying axial-vector tetraquark state. On the other hand, either $Z_c(4200)$ or $Z_c(4485)$ could be a candidate of this tetraquark state. In fact, $Z_c(4200)$ is a very promising tetraquark candidate.

The second axial-vector tetraquark state with the eigenvalue $-4.1v$ decays into $D^*\bar{D}$ only. It is quite particular that this state neither decays into $J/\psi\pi$ nor into $\bar{D}^*D^*$ even if phase space allows. Since all of the four charged Z_c states decay into the $J/\psi\pi$ mode, none of them is the candidate of this tetraquark state.

The third $J^P = 1^+$ state corresponds to the eigenvalue $-1.5v$, which decays into $J/\psi\pi$, $\eta_c\rho$, $D^*\bar{D}$, and $\bar{D}^*D^*$. Its dominant decay mode is $D^*\bar{D}$. For comparison, both the $\bar{D}^*D^*$ and $\eta_c\rho$ modes are suppressed roughly by a factor of eight if we ignore the phase space difference. In contrast, the $J/\psi\pi$ mode is strongly suppressed. If we ignore the phase space difference, the suppression factor is roughly 25 compared with the dominant $D^*\bar{D}$ mode. Based on the current experimental information, all the three $Z_c(3900)$, $Z_c(4200)$, and $Z_c(4485)$ can be assigned as this third tetraquark state with the eigenvalue $-1.5v$. Especially, the characteristic decay pattern of this third axial-vector tetraquark state matches well with that of $Z_c(3900)$. With such an assignment, we would expect two more axial-vector tetraquark states with the eigenvalues $-15.9v$ and $-4.1v$, which are very close to (or even below) the open charm threshold and lie below $Z_c(3900)$. Their decay patterns are listed in the previous paragraphs.

V. SUMMARY

Within the framework of the color-magnetic interaction, we have systematically considered the mass spectrum of the hidden-charm and hidden-bottom tetraquark states with the configurations $qc\bar{q}\,\bar{c}$, $qc\bar{s}\,\bar{c}$, $sc\bar{s}\,\bar{c}$, $qb\bar{q}\,\bar{b}$, $qb\bar{s}\,\bar{b}$, $sb\bar{s}\,\bar{b}$, and $J^P = 1^+, 0^+, 2^+$.

Experimentally several charged axial-vector hidden-charm states were reported. We have adopted two schemes to fix the parameter v and extracted the tetraquark spectrum. We first tried to assume one of Z_c states is a tetraquark state and use its mass as input to determine v and the masses of the other tetraquark states. We notice that it is impossible to accommodate all the three charged states $Z_c(3900)$, $Z_c(4025)$, and $Z_c(4200)$ within the axial-vector tetraquark spectrum simultaneously. Then we tried to use the mass splitting between two Z_c states as input. With the second scheme we could accommodate all three charged states $Z_c(3900)$, $Z_c(4025)$, and $Z_c(4200)$ as the axial-vector tetraquark candidates simultaneously. However, the resulting mass of the lowest axial-vector tetraquark state is always too low and unrealistic. We have to conclude that not all these three states are tetraquark candidates. Instead of being a tetraquark candidate, at least one or two of these states is probably a molecular state or some other structure.

Moreover, the eigenvectors of the chromomagnetic interaction contain valuable information of the decay pattern of the tetraquark states. For example, the dominant decay mode of the lowest axial-vector $qc\bar{q}\,\bar{c}$ tetraquark state is $J/\psi\pi$. Its $D^*\bar{D}$ and $\bar{D}^*D^*$ modes are strongly suppressed. Recall that the dominant decay mode of $Z_c(3900)$ and $Z_c(4025)$ is $\bar{D}D^*$ and $\bar{D}^*D^*$, respectively. We tend to conclude that neither $Z_c(3900)$ nor $Z_c(4025)$ is a good candidate of the lowest lying axial-vector tetraquark state. In fact, $Z_c(3900)$ and $Z_c(4025)$ are close to the $\bar{D}D^*$ and $\bar{D}^*D^*$ mass threshold. They are good molecular candidates. Their mass and decay pattern agree with the naive expectation within the molecular picture.

On the other hand, the charmoniumlike charged state $Z_c(4200)$ is observed in the $J/\psi\pi$ channel with significance 8.2σ. Its mass is far away from the mass threshold of two S-wave heavy mesons. In fact, the axial-vector hidden-charm tetraquark state was predicted to lie around 4.2 GeV several years ago [61]. As expected as a tetraquark candidate, $Z_c(4200)$ is very broad with a width around 370 MeV. All the available experimental information indicates that $Z_c(4200)$ is a very promising candidate of the lowest axial-vector hidden-charm tetraquark state. Future experimental investigations of this state will be very desirable.

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APPENDIX: THE RECOUPLING COEFFICIENTS

The $SU(6)_{cs}$ eigenstates of the 0^+ tetraquark in the $Q\otimes\bar{Q}$ form are

$$|1,1_c,0,4\rangle = \sqrt{\frac{6}{7}}|21,6_c,1,2\rangle\otimes|\bar{21},\bar{6}_c,1,2\rangle + \sqrt{\frac{1}{7}}|21,\bar{3}_c,0,2\rangle\otimes|\bar{21},3_c,0,2\rangle \tag{A1}$$

$$|405,1_c,0,4\rangle = \sqrt{\frac{1}{7}}|21,6_c,1,2\rangle\otimes|\bar{21},\bar{6}_c,1,2\rangle - \sqrt{\frac{6}{7}}|21,\bar{3}_c,0,2\rangle\otimes|\bar{21},3_c,0,2\rangle \tag{A2}$$

$$|1,1_c,0,4\rangle = \sqrt{\frac{3}{5}}|15,\bar{3}_c,1,2\rangle\otimes|\bar{15},3_c,1,2\rangle + \sqrt{\frac{2}{5}}|15,6_c,0,2\rangle\otimes|\bar{15},\bar{6}_c,0,2\rangle \tag{A3}$$

$$|189,1_c,0,4\rangle = \sqrt{\frac{2}{5}}|15,\bar{3}_c,1,2\rangle\otimes|\bar{15},3_c,1,2\rangle - \sqrt{\frac{3}{5}}|15,6_c,0,2\rangle\otimes|\bar{15},\bar{6}_c,0,2\rangle. \tag{A4}$$

The $SU(6)_{cs}$ eigenstates of the 0^+ tetraquark in the $q_1\bar{q}_3\otimes q_2\bar{q}_4$ form are

$$\begin{aligned}|1,1_c,0,4\rangle = {}&\frac{\sqrt{21}}{6}|q_1\bar{q}_3 1,1_c,0,2\rangle\otimes|q_2\bar{q}_4 1,1_c,0,2\rangle\\&+\frac{\sqrt{7}}{14}|q_1\bar{q}_3 35,1_c,1,2\rangle\otimes|q_2\bar{q}_4 35,1_c,1,2\rangle\\&+\frac{\sqrt{42}}{21}|q_1\bar{q}_3 35,8_c,0,2\rangle\otimes|q_2\bar{q}_4 35,8_c,0,2\rangle\\&-\frac{\sqrt{14}}{7}|q_1\bar{q}_3 35,8_c,1,2\rangle\otimes|q_2\bar{q}_4 35,8_c,1,2\rangle\end{aligned} \tag{A5}$$

$$\begin{aligned}|405,1_c,0,4\rangle = {}&\frac{2\sqrt{42}}{21}|q_1\bar{q}_3 35,1_c,1,2\rangle\otimes|q_2\bar{q}_4 35,1_c,1,2\rangle\\&+\frac{3\sqrt{7}}{14}|q_1\bar{q}_3 35,8_c,0,2\rangle\otimes|q_2\bar{q}_4 35,8_c,0,2\rangle\\&+\frac{5\sqrt{21}}{42}|q_1\bar{q}_3 35,8_c,1,2\rangle\otimes|q_2\bar{q}_4 35,8_c,1,2\rangle\end{aligned} \tag{A6}$$

$$\begin{aligned}|1,1_c,0,4\rangle = {}&\frac{15}{6}|q_1\bar{q}_3 1,1_c,0,2\rangle\otimes|q_2\bar{q}_4 1,1_c,0,2\rangle\\&+\frac{5}{10}|q_1\bar{q}_3 35,1_c,1,2\rangle\otimes|q_2\bar{q}_4 35,1_c,1,2\rangle\\&-\frac{\sqrt{30}}{15}|q_1\bar{q}_3 35,8_c,0,2\rangle\otimes|q_2\bar{q}_4 35,8_c,0,2\rangle\\&+\frac{\sqrt{10}}{5}|q_1\bar{q}_3 35,8_c,1,2\rangle\otimes|q_2\bar{q}_4 35,8_c,1,2\rangle\end{aligned} \tag{A7}$$

$$|189,1_c,0,4\rangle = -\frac{2\sqrt{30}}{15}|q_1\bar{q}_3 35,1_c,1,2\rangle \otimes |q_2\bar{q}_4 35,1_c,1,2\rangle - \frac{3\sqrt{5}}{10}|q_1\bar{q}_3 35,8_c,0,2\rangle \otimes |q_2\bar{q}_4 35,8_c,0,2\rangle - \frac{\sqrt{15}}{30}|q_1\bar{q}_3 35,8_c,1,2\rangle \otimes |q_2\bar{q}_4 35,8_c,1,2\rangle. \tag{A8}$$

According to the $SU(3)_c$ and $SU(2)_s$ symmetry, the $SU(6)_{cs}$ eigenstates of the $q_2\bar{q}_3 \otimes q_1\bar{q}_4$ form are the same as those of the $q_1\bar{q}_3 \otimes q_2\bar{q}_4$ form in the first two tetraquark states, while there appears an extra minus sign in the last two tetraquark states.

The $SU(6)_{cs}$ eigenstates of the 2^+ tetraquark in the $Q \otimes \bar{Q}$ form are

$$|405,1_c,0,4\rangle = |21,6_c,1,2\rangle \otimes |\bar{21},\bar{6}_c,1,2\rangle \tag{A9}$$

$$|189,1_c,2,4\rangle = |15,\bar{3}_c,1,2\rangle \otimes |\bar{15},3_c,1,2\rangle. \tag{A10}$$

The $SU(6)_{cs}$ eigenstates of the 2^+ tetraquark in the $q_1\bar{q}_3 \otimes q_2\bar{q}_4$ form are

$$|405,1_c,2,4\rangle = \frac{1}{3}|q_1\bar{q}_3 35,8_c,1,2\rangle \otimes |q_2\bar{q}_4 35,8_c,1,2\rangle + \frac{2}{3}|q_1\bar{q}_3 35,1_c,1,2\rangle \otimes |q_2\bar{q}_4 35,1_c,1,2\rangle \tag{A11}$$

$$|189,1_c,2,4\rangle = -\frac{\sqrt{2}}{3}|q_1\bar{q}_3 35,8_c,1,2\rangle \otimes |q_2\bar{q}_4 35,8_c,1,2\rangle - \frac{3\sqrt{1}}{3}|q_1\bar{q}_3 35,1_c,1,2\rangle \otimes |q_2\bar{q}_4 35,1_c,1,2\rangle. \tag{A12}$$

According to the $SU(3)_c$ and $SU(2)_s$ symmetry, the $SU(6)_{cs}$ eigenstate of the $q_2\bar{q}_3 \otimes q_1\bar{q}_4$ form is the same as that of the $q_1\bar{q}_3 \otimes q_2\bar{q}_4$ form in the first tetraquark state, while there appears an extra minus sign in the second tetraquark state.

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