Bounding hadronic uncertainties in $c \to u$ decays

A. J. Bevan[1] and B. Meadows[2]

[1]*Queen Mary University of London, Mile End Road, London E1 4NS, United Kingdom*
[2]*University of Cincinnati, Cincinnati, Ohio 45221, USA*

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Time-dependent CP asymmetry measurements in $D \to h^+h^-$ decays, where $h = \pi$ or ρ can, in principal, be used to constrain the angle β_c of the cu unitarity triangle up to theoretical uncertainties. Here we discuss the theoretical uncertainty from penguin contributions that can be investigated through the use of isospin analyses. We show that uncertainty from penguin pollution on a measurement of β_c (or alternatively the mixing phase) in $D^0 \to \pi^+\pi^-$ ($\rho^+\rho^-$) decays is 2.7° (4.6°). We also comment on the applicability of this method to $D^0 \to \rho\pi$ decays for which measurements of weak phases with a precision below the one degree level may be possible.

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I. INTRODUCTION

The Standard Model (SM) description of CP violation is defined by a single phase in a 3×3 complex, unitary transformation known as the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1,2]. Measurements of time-dependent CP asymmetries (TDCPA) arising from mixing and decay of neutral B_d^0 mesons have been used with great effect to test SM expectations for weak phases observable in the "bd" unitarity triangle illustrated in Fig. 1(a). In this, the smallest angle, "β" is presently known to be 21.5° with a precision of significantly less than 1°. It has been pointed out [3] that a full check of the 3-generation unitary structure of the CKM matrix should include similar tests of predictions for phases in the bs and cu triangles illustrated in Figs. 1(b) and 1(c) governing B_s^0 and D^0 decays, respectively. The smallest angle (β_s) in the former is computed to be approximately 1° [4] and for the latter it is computed in Ref. [5] to be 0.035°. Experimentally, β_s has been measured to a precision of ~2° [6], and is compatible with the SM expectation. The precision of this measurement should be improved to a level that better tests the SM as B_s samples from the LHC grow. A test of the cu triangle is of particular interest since this is the only case involving up-type quarks, but no experimental constraint yet exists. Clearly, even though huge samples of D^0 meson decays to a wide range of decay modes will soon be available from the LHCb, Belle II and BES III experiments, this will be challenging if, indeed, β_c really is so small. However, long range or beyond SM effects might show it to be measurably larger. In this paper, we explore how well β_c might eventually be constrained.

The possibility to measure weak phases in D^0 decays to CP eigenstates from TDCPA was explored in Ref. [5]. The CP asymmetry $A(t) \simeq xt \sin\phi_W$ is approximately linear in decay time t (in lifetime units) where $x = \Delta m/\Gamma$ is the difference in the two D^0 mass eigenstates, Γ is the mean D^0 decay time and ϕ_W is the weak phase measured. Estimates for the precision that could be achieved were made and appear to be close to 1° (modulo systematic and theoretical uncertainties) for the channels $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$.[1] With $\phi_W \sim 1°$, about thirty times the expected SM value, the asymmetry would grow one part per mil in ~5 lifetimes. Such precision would not allow a stringent CKM test, but could provide useful information for elucidation of the effects that penguin amplitudes, other long range effects, or even new physics might have.

TDCPA measurements determine the weak phase $\phi_W = \phi_{MIX} - 2\beta_{\mathrm{c,eff}}$ where ϕ_{MIX} is the D^0 mixing phase, that should become known to about 1° in the next few years, and $\beta_{\mathrm{c,eff}} = \beta_c - \delta\beta_c$ where $\delta\beta_c$ represents theoretical uncertainties. In the absence of such uncertainties, the difference between ϕ_W measured for $D^0 \to K^+K^-$ and for $D^0 \to \pi^+\pi^-$ decays would cancel the effect of ϕ_{MIX}.

One of the sources of theoretical uncertainty that could cause $\beta_{\mathrm{c,eff}}$ to differ from β_c comes from gluonic loop (or penguin) amplitudes with large weak phase (close to $\gamma \simeq 62°$) and unknown magnitude. It was noted in Ref. [5] that "penguin pollution" in $D \to \pi\pi$ decays could be significant, by virtue of the large branching fraction of $D^0 \to \pi^0\pi^0$. One can account for the effects of SM gluonic penguins by performing an isospin (I-spin) analysis [7] of the hh final states ($h = \pi$, ρ) in analogy with the corresponding situations found in B decays [8–11]. Here we present a quantitative analysis of penguin pollution in $D \to \pi\pi$ and $D \to \rho\rho$ decays using existing experimental data, and compare these results with expectations from the CP self-conjugate decays $D \to \pi^+\pi^-\pi^0$, which appear to have small penguin contributions [12–16].

In general, this set of decays proceeds via tree (T), W-exchange (E), and three penguin amplitudes (P_d, P_s and P_b). The naive SM expectation is that direct CPV will be very small. In this model, the b-penguin phase relative to

[1]Charge conjugation is implied throughout.

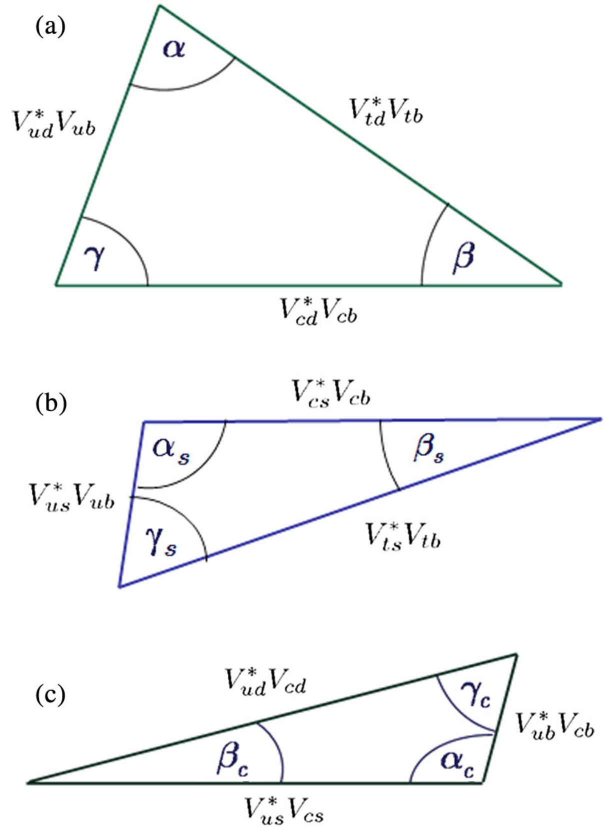

FIG. 1 (color online). Unitarity triangles relating CKM amplitudes for (a) B_d, (b) B_s and (c) D decays (not to scale). We label angles with appropriate suffix as β (smallest), α (closest to 90°) and γ. Note that $\gamma_c = \gamma \simeq 68°$.

that of the tree is large ($\sim\gamma = 68°$) but its magnitude $\mathcal{O}(|V_{cb}V_{ub}/V_{cd}V_{ud}\alpha_s/\pi|) \sim 10^{-4}$ is tiny. E and P_d amplitudes have the same weak phase as T for these modes while the s-penguin phase differs by only $\beta_c \simeq 0.035°$. The CKM couplings for P_d and P_s are large but their sum is equal to the negative of that for P_b (cu triangle unitarity condition). The resulting penguin amplitude is, therefore, $\sim|V_{ub}|$ with magnitude dictated by (U-spin) symmetry breaking $\sim(m_s^2 - m_d^2)/m_c^2$, and is expected to be small.

Prompted by evidence, earlier reported by the LHCb Collaboration [17] for an unexpectedly large direct CP asymmetry difference ($-0.82 \pm 0.24\%$) between $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays, theoretical interest recently centered on the perturbative QCD properties of the light quark amplitudes P_s and P_d and on the possibility for enhancement by such U-spin symmetry breaking [18]. Estimates for these effects from $D^0 \to h^+h^-$ ($h = \pi$ or K) decay rates [19–21] could be tuned to allow CP violating asymmetries up to a few parts per mille in these decays. With larger available data samples of both prompt D^0 's and D^0 's from B decays, however, the LHCb estimate for this asymmetry has shrunk to $0.49 \pm 0.30(\text{stat}) \pm 0.14(\text{syst})\%$ [22], which is now consistent with zero. Experimentally, measurement of the relative influence of P and T amplitudes in D decays can be useful in understanding this confusing picture. We will refer to "P" to encompass P_b, P_d, and P_s amplitudes with their QCD enhancements, weak phase $\sim\gamma$ relative to T and the property of allowing only $\Delta I = 1/2$ transitions (conversion of a c to a u quark). In contrast, we consider "T" to include tree and W-exchange amplitudes, which have the same weak phase and allow both $\Delta I = 1/2$ and $3/2$ transitions.

We examine the use of methods originally developed for the corresponding B physics problem to constrain the penguin pollution corrections to unitarity triangle phase measurements in $c \to d$ transitions. Using currently available D decay measurements, we then attempt to estimate the precision with which such measurements of P and T could be made.

II. ISOSPIN ANALYSIS OF $D \to \pi\pi$ DECAYS

The prescription given here parallels the one presented in Ref. [7] which outlines how to measure the bd unitarity triangle angle α from $B \to \pi\pi$ decays by constraining the penguin pollution. In this strategy, the T and P amplitudes are distinguished on the basis of their I-spin structures. Bose symmetry dictates that, for either B or D decays, the two-pion (or two-ρ) final states can be in an $I = 0$ or $I = 2$ final state, calling for specific fractions of both $\Delta I = 1/2$ and $\Delta I = 3/2$ components. I-spin symmetry, ignoring any small ($\sim 1\%$) electromagnetic contributions, calls for a triangular relationship between amplitudes $A^{ij}(\bar{A}^{ij})$ for $D(\bar{D}) \to h^i h^j$ decays ($h = \pi$ or $h = \rho$):

$$\frac{1}{\sqrt{2}}A^{+-} = A^{+0} - A^{00},$$
$$\frac{1}{\sqrt{2}}\bar{A}^{-+} = \bar{A}^{-0} - \bar{A}^{00}, \qquad (1)$$

where the charges are $i, j = +1, -1, 0$. These two triangles (shown in Fig. 2) can be aligned, by a rotation $2\beta_c$ of one of the triangles, with a common base given by $A^{+0} = \bar{A}^{-0}$ where $|A^{+0}| = |\bar{A}^{-0}|$. By convention the rotated amplitudes are usually referred to as $\tilde{A}^{ij}$. Penguin operators only allow $\Delta I = 1/2$ transitions so, in the limit of I-spin symmetry, the amplitudes A^{+0} and $\bar{A}^{-0}$ are pure tree ($I = 2$), whereas the other amplitudes are a combination of tree ($I = 0, 2$)

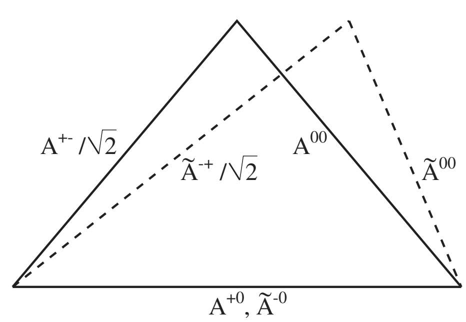

FIG. 2. The I-spin triangle relations given in Eq. (1).

and penguin ($I = 0$) contributions. In this case the angle between A^{+-} and $\tilde{A}^{-+}$, the shift in the measured phase resulting from penguin contributions, is $2\delta\beta_c$. Neglecting other sources of theoretical uncertainty one has $2\delta\beta_c = 2(\beta_c - \beta_{c,\rm eff})$. A similar relation exists for time-dependent asymmetry measurements in the h^0h^0 mode between A^{00} and $\tilde{A}^{00}$.

One must measure the rates and CP asymmetries for $D^0 \to h^+h^-$, $D^\pm \to h^\pm h^0$, and $D^0 \to h^0h^0$ in order to extract the weak phase of interest, β_c. The amplitude of sinusoidal oscillation in the time dependence of $D \to \pi^+\pi^-$ decays is related to $\sin(\phi_{MIX} - 2\beta_{c,\rm eff})$ as discussed in Ref. [5], where ϕ_{MIX} is the D^0 mixing phase. The proposed I-spin analysis would enable one to translate a measurement of $\beta_{c,\rm eff}$ to a constraint on β_c, given a precise determination of the mixing phase and the aforementioned $D \to hh$ amplitudes. Also, with a measurement of the time-dependent CP asymmetry in $D^0 \to \pi^+\pi^-$ decays, using the method outlined in [23] (used by Belle for their $B \to \pi^+\pi^-$ constraint on the unitarity triangle angle α [24]) one could also constrain the $|P/T|$ ratio. The penguin constraint from either of these methods could also be used, under the assumption of SU(3) to constrain the penguin content of $D \to KK$ decays. Alternatively one can study the I-spin decomposition for $D \to KK$ decays as presented in [25].

In D meson decays the magnitude of CP asymmetries are expected to be smaller by a factor $\sim 10^{-2}$ than for B's, so electroweak penguin (EWP) processes, that violate I-spin conservation, can play a more important role. EWPs are, however, suppressed by a factor α/α_s and they do preserve CP symmetry. Since the I-spin relations used work mostly with differences between D and $\bar{D}$ amplitudes, such effects are largely removed, at least to first order [26]. An estimate of the uncertainties in $\delta\beta_c$ can, therefore, be made ignoring EWPs.

Current experimental constraints (i.e. branching fractions) on the inputs necessary to perform an I-spin analysis of $D \to \pi\pi$ decays can be found in Ref. [27]. At this time there are no experimentally measured CP asymmetries, however it is safe to assume in the SM that direct CP asymmetries in these modes are small so that, in effect, the triangle relations of Eq. (1) are equivalent up to a possible reflection in A^{+0}. Hence there is an ambiguity in the value of $2\delta\beta_c$. With the large samples of data in these modes that will soon be available at BES III, Belle II and possibly also at LHCb, one would be able to refine our knowledge of the inputs to this I-spin analysis, and eventually relax the assumptions on CP asymmetries.

We create ensembles of Monte Carlo simulated experiments, based on the results given in Ref. [27], in order to compute $2\delta\beta_c$, noting the ambiguity in the orientation of the two relations given in Eq. (1). As a cross-check we also use the same method as in Ref. [28] to verify the numerical estimates obtained. On performing the I-spin analysis using existing data one extracts the constraint shown in Fig. 3, where the uncertainty on each of the solutions is 5.4°. Hence the uncertainty from penguin contributions on a measurement of β_c from $D \to \pi\pi$ decays is found to be 2.7° using current data. The $\pi^0\pi^0$ final state is required in the I-spin analysis, so it may not be possible to improve this estimate using data collected solely from hadron collider experiments. These results rely on measurements of decay rates limited by systematic uncertainty in π^0 efficiency. Measurement of the CP asymmetries of these rates are less limited, however. We estimate, assuming a similar detection performance is achievable to that found at *BABAR* and Belle, that results from high statistics e^+e^- based experiments could be able to measure $\delta\beta_c$ at the level of $\mathcal{O}(1.3°)$ in $D \to \pi\pi$ decays. Possible improvements in tracking and calorimetry should also be investigated to understand if a sub-1° level could be achievable.

III. ISOSPIN ANALYSIS OF $D \to \rho\rho$ DECAYS

The I-spin analysis for $D \to \rho\rho$ decays parallels the procedure described for $D \to \pi\pi$, with the exception that there may be an $I = 1$ contribution to the (4-pion) final states as discussed in Ref. [29] for the corresponding $B \to \rho\rho$ case. It is possible to test for an $I = 1$ component by measuring $\beta_{c,\rm eff}$ as a function of the $\pi\pi$ invariant mass in the ρ region. Any observed variation as a function of the difference in these masses would be an indication of such a contribution.

Experimentally the situation is not quite as straightforward as the $D \to \pi\pi$ case. The presence of two broad ρ resonances in the final state will result in larger backgrounds than in the $\pi\pi$ case. Furthermore the four-pion $\rho\rho$ final state has to be distinguished from other possible resonant and nonresonant contributions, taking into account any interference that may occur. In particular there could be visible interference effects between ρ^0 and ω apparent in the ρ^0 line shape for the $\rho^+\rho^0$ and $\rho^0\rho^0$ modes. As noted in Ref. [5], we expect the fraction of longitudinally polarized events f_L to be nontrivial ($f_L \sim 0.83$), so one has to perform a transversity analysis in order to extract CP-even and CP-odd components of the final state. We note that experimental constraints on f_L for $\rho^0\rho^0$ are consistent with naive expectations discussed in Ref. [5]. Having done this, one can then perform an I-spin analysis for each of the three signal components (i.e. the three transversity amplitudes), where in principle one could have a different level of penguin pollution for each component. Current experimental constraints on the inputs necessary to perform an I-spin analysis of $D \to \rho\rho$ decays can be found in Ref. [27], where we interpret the four-pion final state as being dominated by $\rho\rho$ decays. As before it is safe to assume that direct CP asymmetries in the $\rho\rho$ modes are small. In the following we assume that only the longitudinally polarized events are used in order to constrain $\beta_{c,\rm eff}$. On performing the I-spin analysis one extracts the constraint on $2\delta\beta_c$ shown as the dashed line in Fig. 3. The

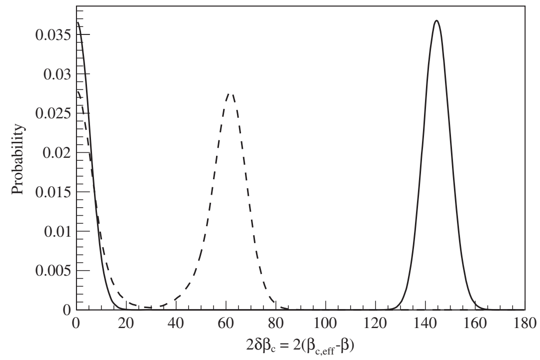

FIG. 3. The constraint on $2\delta\beta_c$ obtained from (solid) $D \to \pi\pi$ and (dashed) $D \to \rho\rho$ decays using existing data. Mirror solutions arising from ambiguities in the orientation of the I-spin triangles also exist for $-180° \le 2\delta\beta_c < 0$.

penguin uncertainty is broader than obtained for the $\pi\pi$ case, and for $\rho\rho$ the two different solutions have an uncertainty of 9.2° and overlap slightly. Hence there is an uncertainty of 4.6° on a future measurement of β_c from this source. The fraction of resonant $\rho\rho$ events required to minimize the level of unphysical results is one, highlighting the need for improved experimental input in this area. As with the $\pi\pi$ case one needs to reconstruct final states with neutral particles in order to perform this I-spin analysis. Thus it is important that experiments such as BES III and Belle II study the $D \to \rho\rho$ final states in detail in order to provide the appropriate inputs for the I-spin analysis. However, we expect that LHCb should be able to contribute significantly via measurement of $D^0 \to \rho^0\rho^0$ decays.

IV. ROLE OF PENGUINS IN $D^0 \to \pi^+\pi^-\pi^0$ DECAYS

Time-dependent CP asymmetry measurements of D^0 decays to the CP-even state $\rho^0\pi^0$ also provide information on $\beta_{c,\mathrm{eff}}$ [5]. These decays have contributions from Cabibbo suppressed exchange (C) and color suppressed tree diagrams (T), each with the weak phase of $V_{cd}^* V_{ud}$ (following the usual convention) that could contribute to a CP asymmetry. Again, the penguin diagram can play a role in the weak phase, so some estimate of it needs to be made.

The $\rho^0\pi^0$ mode represents almost 30% of the 3-body decays $D^0 \to \pi^+\pi^-\pi^0$ whose branching fraction is $(1.43 \pm 0.06)\%$, over ten times larger than that for $D^0 \to \pi^+\pi^-$. The advantage arising from this additional source of data, despite an increase in experimental complexity of the I-spin makeup of the $\rho\pi$ states, is even further enhanced by additional information from the other $\rho\pi$ charge modes. It can be used to place interesting constraints on the value for β_c directly, using the method outlined in [30] for the extraction of α from the corresponding B^0 decay.

Amplitude analyses have been made by the CLEO-c [12] and *BABAR* [13] Collaborations. The *BABAR* analysis, which used a sample almost 20 times larger than CLEO's, revealed, in addition to the 3 $\rho\pi$ modes, the presence of radial excitations of the ρ as well as other CP-odd eigenstates ($f_0\pi^0$ and $f_2\pi^0$ with $I = 1$). An I-spin study [14,15] of the *BABAR* amplitudes has also shown that $I = 0$ is dominant in the three $D^0 \to \rho\pi$ modes. This somewhat surprising observation has been interpreted as being consistent with $\mathrm{SU(3)_{flav}}$ relationships [16] with only C and T amplitudes from other $D^0 \to P + V$ (pseudoscalar + vector) decays. The effect of penguin contributions was not required in this interpretation. Larger event samples are yet to come from all these channels that may be able to clarify the role of penguins further.

The I-spin structure of the $\rho\pi$ final states differs from that of either $\pi\pi$ or $\rho\rho$ states [31] that each consist primarily of pairs of identical bosons with even I. The $\rho\pi$ states allow for $I = 0$, 1 or 2, and all but $I = 2$ are accessible to penguin transitions. The decay amplitudes can be written [26] in terms of I-spin amplitudes $\mathcal{A}_1$ and $\mathcal{B}_1$, $\Delta I = 1/2$ transitions, respectively, to $I = 1$ and $I = 0$ three pion states, and $\mathcal{A}_3$ and $\mathcal{B}_3$, ($\Delta I = 3/2$) transitions to $I = 2$ and $I = 1$ states. SM penguins cannot contribute to $\Delta I = 3/2$ processes and we ignore $\Delta I = 5/2$ transitions that are not expected in the SM. Then

$$\begin{aligned}
A^{+-} &= \mathcal{A}_3 + \mathcal{B}_3 + \frac{1}{\sqrt{2}}\mathcal{A}_1 + \mathcal{B}_1 \\
&= T^{+-} + P_1 + P_0 \\
A^{-+} &= \mathcal{A}_3 - \mathcal{B}_3 - \frac{1}{\sqrt{2}}\mathcal{A}_1 + \mathcal{B}_1 \\
&= T^{-+} - P_1 + P_0 \\
A^{00} &= 2\mathcal{A}_3 - \mathcal{B}_1 \\
&= [T^{+-} + T^{-+} - T^{+0} - T^{0+}]/2 - P_0 \\
A^{+0} &= \frac{3}{\sqrt{2}}\mathcal{A}_3 - \frac{1}{\sqrt{2}}\mathcal{B}_1 + \mathcal{A}_1 \\
&= [T^{+0} + 2P_1]/\sqrt{2} \\
A^{0+} &= \frac{3}{\sqrt{2}}\mathcal{A}_3 + \frac{1}{\sqrt{2}}\mathcal{B}_1 - \mathcal{A}_1 \\
&= [T^{0+} - 2P_1]/\sqrt{2}
\end{aligned}$$

where the first superscript in the decay A and tree T amplitudes is the charge state for the ρ, and the second is that for the π. P_0 and P_1 are, respectively, penguin amplitudes leading to $I = 0$ and $I = 1$ states. A sum rule involving all five decay amplitudes

$$A^{+-} + A^{-+} + 2A^{00} = \sqrt{2}(A^{+0} + A^{0+}) \tag{2}$$

can be inferred, and is represented graphically in Fig. 4. The corresponding antiparticle amplitudes $\tilde{A}$ are similarly related though they each can differ from the D^0 ones. Each side of Eq. (2) corresponds to a sum of amplitudes with

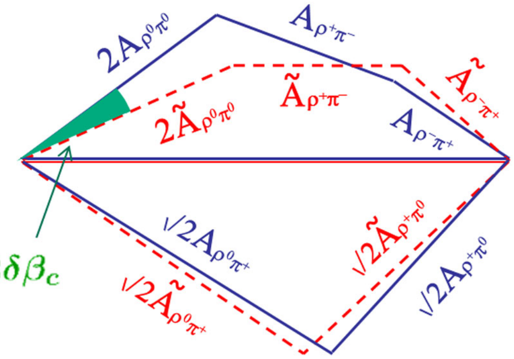

FIG. 4 (color online). The I-spin pentagon relation for the $D \to \rho\pi$ modes given in Eq. (2). Amplitudes for D^0 (solid) and $\bar{D}^0$ (dotted) are shown. The $\bar{D}^0$ pentagon is rotated so that the sums on each side of Eq. (2) coincide with those for D^0 decays. These rotated amplitudes are labeled with a tilde. The effect of penguins that can contribute a change, $\delta\beta_c$, in the phase of the $\rho^0\pi^0$ amplitude is illustrated.

$\Delta I = 3/2$ (tree only). The asymmetry between these sums for D^0 and $\bar{D}^0$ should therefore be negligible in the SM. Even if I-spin symmetry is broken, the difference between the D^0 and $\bar{D}^0$ sums should be very small [26]. These sums, therefore, play a similar role to the $\pi^+\pi^0$ and $\rho^+\rho^0$ modes discussed earlier, and a similar rotation of the charged $\bar{D}$ amplitudes through $2\beta_{c,\mathrm{eff}}$, illustrated in Fig. 4, aligns them. Penguin contributions lead to a difference in phase $2\delta\beta_c$ to $\tilde{A}^{00}$ (the rotated $\bar{D}^0 \to \rho^0\pi^0$ amplitude) relative to A^{00}.

Charged $D \to \rho\pi$ decays are observed in two different Dalitz plots, $D^+ \to \pi^+\pi^0\pi^0$ ($\rho^+\pi^0$) and $D^+ \to \pi^+\pi^-\pi^0$ ($\rho^0\pi^+$). It is, therefore, impossible to determine their phases relative to each other, or to any of the three neutral $\rho\pi$ modes. For a similar reason, it is difficult to relate their magnitudes in light of uncertainties in the normalization of amplitudes from these Dalitz plots. The charged modes, therefore, can only add a somewhat weak I-spin constraint. Fortunately, as pointed out by Quinn and Snyder [30], a time-dependent amplitude analysis of the $\pi^+\pi^-\pi^0$ Dalitz plot alone (just the upper part of Fig. 4) is sufficient to extract the weak decay phase $\beta_{c,\mathrm{eff}}$, as well as the P/T ratio from the time dependence of all three $D^0 \to \rho\pi$ modes. The relative phase of the amplitudes A^{00} and $\tilde{A}^{00}$ is $2\delta\beta_c$, $\beta_{c,\mathrm{eff}}$ can also be determined, and is obtained from the time dependence of the $D^0(\bar{D}^0) \to \rho^0\pi^0$ amplitude and $\delta\beta_c$ from the $\rho^\pm$ modes.

The phase β_c can also be obtained directly. A further advantage of analyzing just the three $B^0 \to (\rho\pi)^0$ channels is that only their amplitude ratios and relative phases are required, and information on these reside entirely within the same Dalitz plot. The precision of their measurements, therefore, should all be reduced with larger sample sizes, with no limiting systematic uncertainty from π^0 detection efficiency.

To estimate the precision in β_c, we take as a model the results from the *BABAR* time-integrated amplitude analysis of $D^0 \to \pi^+\pi^-\pi^0$ decays [13]. At this statistical level, interferences in the Dalitz plot allow measurement precision of relative phases of A^{-+}, A^{+-} and A^{00} of $\sim 1°$ and of relative magnitudes at the level of $\sim 1\%$. The *BABAR* analysis combined both D^0 and $\bar{D}^0$ samples in a single fit, so in our study, we take random fluctuations about the same model (i.e. no direct *CPV*). Our expectation, then is that the central value we find for β_c is zero and that any penguin will be much smaller than the corresponding tree amplitude. In a time-dependent analysis, mixing brings the D^0 and $\bar{D}^0$ into interference, allowing measurement of their relative decay phases. The time-integrated *BABAR* analysis did not, therefore, provide any information from mixing-induced *CPV* on the central value for $\beta_{c,\mathrm{eff}}$, so we set this to be zero.

To estimate the precision in both β_c and in the ratio $|P/T|$ for the $\rho\pi$ modes, we follow the Quinn and Snyder [30] procedure, as used for B^0 decays by the B factories (e.g., Ref. [32]). Using the *BABAR* model as input, we construct a sample of $D^0 \to \pi^+\pi^-\pi^0$ decays and scan a set of values for β_c around 0°. At each value, we obtain the p-value from the minimum χ^2 fit and the ratio of $|P/T|$ (for the neutral ρ). The p-values are plotted in Figs. 5(a) and 5(b). As expected, a peak centered at $\beta_c = 0°$ is seen in the p-value and that this has a width of close to 1°. At this value for β_c the $|P/T|$ ratio is zero, with an uncertainty of ± 0.01. It is seen that β_c can be measured with a sample comparable to that of *BABAR* with a precision of $\sim 1°$ and $|P/T|$ with precision $\sim \pm 0.01$.

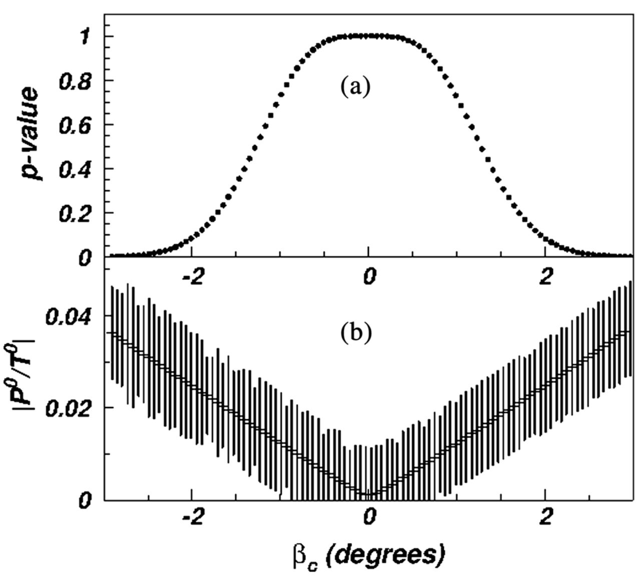

FIG. 5. (a) p-values for minimum χ^2 fits to *BABAR* data for $-3° < \beta_c < 3°$ about zero, the value expected as outlined in the text. A second, identical peak centered at 180° is also observed in the full β_c scan, but is not shown here. (b) The corresponding values for $|P|/|T|$, the ratio of magnitudes for the penguin and tree amplitudes for the $\rho^0\pi^0$ decays. As expected, this ratio is zero at $\beta_c = 0$ in the absence of *CPV*, with an uncertainty of ± 0.01.

We implicitly assume that the D^0 mixing parameters are well known, as we anticipate they will be when the analyses outlined here are made. Perhaps the mixing phase ($\phi_{MIX} = arg[q/p]$) is one of these parameters that most directly affects any conclusions on possible constraints on the cu unitarity triangle. This phase is not well understood theoretically, and the time-dependent CP asymmetry measurements discussed here determine the combination $\phi_{MIX} - 2\beta_c$. It is estimated [33] that ϕ_{MIX} can be measured in a super flavor factory or LHCb with precision of order 1°. Since then, more recent measurements from LHCb indicate that this remains so [34]. So constraints on the cu unitarity triangle will probably lie in the 1° level, permitting observation of deviations from the SM expectations at this level.

V. SUMMARY

The analysis of penguin pollution discussed here is the first step toward resolving the theoretical uncertainties associated with a measurement of the mixing phase in the near term, and constraining the cu unitarity triangle phase β_c. Additional theoretical uncertainties should be investigated, including those related to long distance amplitudes contributing to the direct decay of a D^0 or $\bar{D}^0$ meson into the final state, as well as possible I-spin breaking terms arising from electroweak penguin contributions. As noted in Ref. [5], the penguin contribution in $D \to \pi\pi$ decays is potentially large, however current experimental inputs are precise enough to enable one to constrain the uncertainty from penguins on β_c to a few degrees. Super flavor factories should be able to further reduce this uncertainty with improved measurements of the $\pi\pi$ final states. Assuming that the four-pion final states measured in charged and neutral decays of D mesons are dominated by resonant $\rho\rho$ contributions, we find that the maximum value of the penguin pollution contributions in $D \to \rho\rho$ decays is smaller than that for $\pi\pi$, however current experimental data are of limited value and we obtain a worse precision for the penguin pollution uncertainty. A detailed experimental analysis of all of the $\rho\rho$ final states is needed in order to improve on the estimate we obtain here. While BES III will not be able to contribute to the time-dependent asymmetry inputs required to extract an estimate of β_c, this experiment should be able to make significant contributions to the knowledge of time-integrated rates and asymmetries used as inputs to the I-spin analysis. Time-dependent analyses are likely to provide measurements of $\beta_{c,\mathrm{eff}}$ from $D^0 \to \rho^0\pi^0$ decays with precision in the 1° range using the large samples expected from the super favor factories. In the meantime, BES III should be able to provide large samples of such decays allowing for more precise studies to be made of time-integrated Dalitz plots. It is also possible that CP asymmetries and I-spin analyses of the various charge states of the $\rho\pi$ systems can be used to estimate penguin contributions.

The $D^0 \to \rho\pi$ modes are especially encouraging. In particular, full time-dependent amplitude analyses of the neutral $D^0 \to 3\pi$ Dalitz plots should allow internally compatible measurements for magnitudes and phases for each of the three charge modes for both D^0 and $\bar{D}^0$ decays. These measurements should not be limited by systematic uncertainties in π^0 detection efficiency, so resolution in $\delta\beta_c$ could shrink considerably below the 1° level as larger samples are used.

The different constraints on penguin pollution evident in Figs. 3 and 5 are a clear indication that some of the ambiguities involved in a measurement of β_c from $D \to hh$ decays can be resolved when results from different final states are combined. A recent result from Lattice QCD by the RBC-UKQCD Collaboration [35] states that penguins should be small in this system. This prediction can be tested using time-dependent asymmetry measurements of $D \to hh$ decays combined with the isospin analysis discussed here. We conclude that with sufficient data it is feasible to perform a measurement of the cu unitarity triangle phase β_c to a precision less than 1°, comparable to that achievable by the B factories and LHCb for the bd (bs) unitarity triangles, thus limiting the contribution of any new physics to this phase.

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