Inclusive gauge boson production in the color dipole formalism

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The inclusive production of the gauge bosons W^{\pm} and Z^0 is studied within the color dipole formalism. Gluon saturation effects associated with nonlinear corrections to the QCD dynamics, which are expected to contribute at high energies, are naturally included in such formalism. We estimate the contribution of these effects at LHC energies and compare our results with the next-to-next-to-leading order collinear predictions. A comparison with current experimental data is performed and predictions for higher energies are presented.

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I. INTRODUCTION

The production of the massive gauge bosons W^{\pm} and Z^0 is one of the few processes in pp collisions where the collinear factorization has been rigorously proven and the perturbative quantum chromodynamics (pOCD) predictions are known up to next-to-next-to-leading order (NNLO) [1]. Therefore, its study at Tevatron and LHC energies provides an important test of the Standard Model (SM) as well as on the pQCD and the higher-order corrections. In the collinear factorization approach the gauge boson production is viewed, at leading order (LO), as the fusion of the quark and antiquark that produces a gauge boson, a unique process which offers high sensitivity to the parton distribution in the proton [2]. Recently, the ATLAS [3], CMS [4,5], and LHCb [6] Collaborations have performed W and Z precision measurements of the inclusive cross sections at $\sqrt{s} = 7$ and 8 TeV, with the data well described by the NNLO pQCD predictions. A major theoretical uncertainty on the cross section predictions is due to uncertainties on the parton distribution functions (PDFs) [7]. While its behavior at high values of the Bjorken variable x has been determined from fixed-target and HERA data and confirmed at higher virtualities Q^2 by W and Z production at Tevatron, for smaller x values, the PDFs have been measured by HERA alone and at much lower Q^2 . For the energies probed at LHC, they must be evolved using the Dokshitzer, Gribov, Lipatov, Altarelli, and Parisi (DGLAP) equations [8]. In particular, the measurement of the gauge boson cross section at the LHCb [6], which probes forward rapidities, has a sensitivity to values of x as low as 1.7×10^{-4} and values of $Q^2 \approx M_G^2$, where $G = W^{\pm}$ or Z^0 . Consequently, the current and future experimental data for gauge boson production provide an important test of the QCD dynamics in a new kinematical range, where new dynamical effects can contribute.

One such effect is the replacement of the usual collinear approach [9] by a more general factorization scheme, as, for example, the k_{\perp} -factorization approach [10–12]. While in the collinear factorization approach [9] all partons involved are assumed to be on mass shell, carrying only longitudinal momenta, and their transverse momenta are neglected in the QCD matrix elements, in the k_{\perp} -factorization approach the effects of the finite transverse momenta of the incoming partons are taken into account. In this case the cross sections are now k_{\perp} factorized into an off-shell partonic cross section and a k_{\perp} -unintegrated parton density function $\mathcal{F}_i(x, k_{\perp})$ [10–12]. A sizeable piece of the NLO and some of the NNLO corrections to the LO contributions on the collinear approach, related to the contribution of nonzero transverse momenta of the incident partons, are already included in the LO contribution within the k_{\perp} -factorization approach. Moreover, the coefficient functions and the splitting functions giving the collinear parton distributions are supplemented by all-order $\alpha_s \ln(1/x)$ resummation at high energies [13]. Such approach was applied to the gauge boson production in Refs. [14-18].

Another possible dynamical effect is the gluon saturation associated with nonlinear corrections to the QCD dynamics, which are expected to contribute at high energies (for

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recent reviews see Ref. [19]). In this regime, perturbative QCD predicts that the small-x gluons in a hadron wave function should form a color glass condensate (CGC) [20], which is characterized by a limitation on the maximum phase space allowed in the hadron wave function (parton saturation). The transition is then specified by a typical energy-dependent scale, called saturation scale, Q_{sat} . One of the main implications of the gluon saturation is that it leads to the breakdown of the twist expansion and the factorization schemes [21].

Gluon saturation effects can be naturally described in the color dipole formalism [22]. At high energies color dipoles with a defined transverse separation are eigenstates of the interaction. The main quantity in this formalism is the dipole-target cross section, which is universal and is determined by QCD dynamics at high energies. In particular, it provides a unified description of inclusive and diffractive observables in *ep* processes as well as in the Drell-Yan process, i.e., prompt photon and heavy quark production in hadron-hadron collisions [22-28]. The basic idea present in the description of hadronic collisions using the color dipole formalism is that although cross sections are Lorentz invariant, the partonic interpretation of the microscopic process depends on the reference frame [24]. In particular, in the target rest frame, the gauge boson production can be described in the dipole formalism as a bremsstrahlung process, rather than parton annihilation, with the space-time picture illustrated in Fig. 1. A quark (or an antiquark) of the projectile hadron radiates a gauge boson. The radiation can occur after or before the quark scatters off the target. When the energy is high, the projectile quark is probing potentially dense gluon fields in the target, which implies that multiple scatterings have to be taken into account. In this paper we consider the extension of this formalism for the inclusive massive gauge boson production, as derived in Ref. [29], and calculate the cross sections considering the impact-parameter-dependent color glass condensate (bCGC) dipole model proposed in Ref. [30], which is based on the Balitsky-Kovchegov (BK) nonlinear evolution equation [31,32] and which takes into account the impact-parameter dependence of the saturation scale. As demonstrated in Ref. [33], this model is able to describe the recently released high-precision combined HERA data. We estimate the magnitude of the nonlinear effects at LHC and compare our predictions with those obtained using the collinear factorization and DGLAP equation. In addition, our predictions are compared with the recent experimental data.

This paper is organized as follows. In the next section, we present a brief review of the description of gauge boson production in the color dipole formalism. In Sec. III we present the main aspects of the QCD dynamics at high energies, as well as the models for the scattering amplitude used in our calculations. In Sec. IV we present our predictions for the total cross sections. In particular, we estimate the magnitude of the nonlinear effects at LHC energies. A comparison with the collinear predictions and experimental data is performed. Finally, in Sec. V, our main conclusions are summarized.

II. GAUGE BOSON PRODUCTION IN THE COLOR DIPOLE PICTURE

In the color dipole picture, the production mechanism of a gauge boson looks like a bremsstrahlung [24–26], as represented in Fig. 1. In the high-energy limit, each of the two graphs factorizes into a production vertex for the gauge boson times an amplitude for scattering a quark off the target. The quark scatters at different impact parameters depending on whether the gauge boson is irradiated after or before the scattering. The interference between these scattering amplitudes implies that the squared matrix element for the gauge boson production is expressed in terms of the same dipole-target cross section $\sigma_{a\bar{a}}$ determined by the low-x Deep Inelastic Scattering data. In particular, the cross section for the radiation of a gauge boson G of mass M and transverse (T) or longitudinal (L) polarization from a fast quark of flavor f and mass m_f , which takes a fraction α of the radiating quark energy, is given by

$$\sigma_{T,L}^{f}(qN \to GX) = \int \frac{d\alpha}{\alpha} \int d^{2}\rho |\Psi_{T,L}^{V-A}(\alpha,\rho,M,m_{f})|^{2} \sigma_{q\bar{q}}(\alpha\rho,x) \quad (1)$$



FIG. 1. Diagrams contributing to the process of a gauge boson irradiated by a quark (antiquark) of flavor f either (a) after or (b) before the interaction with the target color field (denoted by a shaded circle). For the W^{\pm} radiation one has $q_k \neq q_f$.

where ρ is the transverse separation between *G* and the final quark and the light cone wave function Ψ describes the electroweak radiation $q \rightarrow Gq$, which can be a vector or axial-vector transition. Moreover, $\sigma_{q\bar{q}}$ is the dipole-target cross section, which is determined by the QCD dynamics at

high energies to be discussed in the next section, with x the Bjorken variable that is directly related to the energy scale. The above equation can be generalized in order to estimate the transverse momentum distribution of the gauge bosons, given by

$$\frac{d\sigma_{T,L}^{f}(qN \to GX)}{d\ln \alpha d^{2}q_{\perp}} = \frac{1}{(2\pi)^{2}} \int d^{2}\rho_{1}d^{2}\rho_{2} \exp[i\vec{q}_{\perp} \cdot (\vec{\rho}_{1} - \vec{\rho}_{2})]\Psi_{T,L}^{V-A}(\alpha, \vec{\rho}_{1}, M, m_{f})\Psi_{T,L}^{V-A,*}(\alpha, \vec{\rho}_{2}, M, m_{f}) \\
\times \frac{1}{2} [\sigma_{q\bar{q}}(\alpha\vec{\rho}_{1}, x) + \sigma_{q\bar{q}}(\alpha\vec{\rho}_{2}, x) - \sigma_{q\bar{q}}(\alpha|\vec{\rho}_{1} - \vec{\rho}_{2}|, x)],$$
(2)

where q_{\perp} is the transverse momentum of the outgoing gauge boson and $\vec{\rho}_1$ and $\vec{\rho}_2$ are the quark-gauge boson transverse separations in the two radiation amplitudes contributing to the cross section. For an unpolarized initial quark the vector and axial-vector contributions to the wave functions are decorrelated, i.e., their interference is destructive, in a way that [29]

$$\sum_{\text{quark pol}} \Psi_{T,L}^{V-A}(\alpha, \vec{\rho}_1, M, m_f) \Psi_{T,L}^{V-A}(\alpha, \vec{\rho}_2, M, m_f) = \Psi_{T,L}^V(\alpha, \vec{\rho}_1, M, m_f) \Psi_{T,L}^V(\alpha, \vec{\rho}_2, M, m_f) + \Psi_{T,L}^A(\alpha, \vec{\rho}_1, M, m_f) \Psi_{T,L}^A(\alpha, \vec{\rho}_2, M, m_f).$$
(3)

The different Ψ components are written as

$$\begin{split} \Psi_{V}^{T}(\alpha,\vec{\rho}_{1})\Psi_{V}^{T*}(\alpha,\vec{\rho}_{2}) &= \frac{\mathcal{C}_{f}^{2}(g_{v,f}^{G})^{2}}{2\pi^{2}} \left\{ m_{f}^{2}\alpha^{4}\mathrm{K}_{0}(\eta\rho_{1})\mathrm{K}_{0}(\eta\rho_{2}) + [1 + (1 - \alpha)^{2}]\eta^{2}\frac{\vec{\rho}_{1}\cdot\vec{\rho}_{2}}{\rho_{1}\rho_{2}}\mathrm{K}_{1}(\eta\rho_{1})\mathrm{K}_{1}(\eta\rho_{2}) \right\}, \\ \Psi_{V}^{L}(\alpha,\vec{\rho}_{1})\Psi_{V}^{L*}(\alpha,\vec{\rho}_{2}) &= \frac{\mathcal{C}_{f}^{2}(g_{v,f}^{G})^{2}}{\pi^{2}}M^{2}(1 - \alpha)^{2}\mathrm{K}_{0}(\eta\rho_{1})\mathrm{K}_{0}(\eta\rho_{2}), \\ \Psi_{A}^{T}(\alpha,\vec{\rho}_{1})\Psi_{A}^{T*}(\alpha,\vec{\rho}_{2}) &= \frac{\mathcal{C}_{f}^{2}(g_{a,f}^{G})^{2}}{2\pi^{2}} \left\{ m_{f}^{2}\alpha^{2}(2 - \alpha)^{2}\mathrm{K}_{0}(\eta\rho_{1})\mathrm{K}_{0}(\eta\rho_{2}) + [1 + (1 - \alpha)^{2}]\eta^{2}\frac{\vec{\rho}_{1}\cdot\vec{\rho}_{2}}{\rho_{1}\rho_{2}}\mathrm{K}_{1}(\eta\rho_{1})\mathrm{K}_{1}(\eta\rho_{2}) \right\}, \\ \Psi_{A}^{L}(\alpha,\vec{\rho}_{1})\Psi_{A}^{L*}(\alpha,\vec{\rho}_{2}) &= \frac{\mathcal{C}_{f}^{2}(g_{a,f}^{G})^{2}}{\pi^{2}}\frac{\eta^{2}}{M^{2}} \left\{ \eta^{2}\mathrm{K}_{0}(\eta\rho_{1})\mathrm{K}_{0}(\eta\rho_{2}) + \alpha^{2}m_{f}^{2}\frac{\vec{\rho}_{1}\cdot\vec{\rho}_{2}}{\rho_{1}\rho_{2}}\mathrm{K}_{1}(\eta\rho_{1})\mathrm{K}_{1}(\eta\rho_{2}) \right\}, \end{split}$$

where $\eta = (1 - \alpha)M^2 + \alpha^2 m_f^2$ and the coupling factors C_f are

$$\mathcal{C}_{f}^{Z} = \frac{\sqrt{\alpha_{em}}}{\sin 2\theta_{W}}, \qquad \mathcal{C}_{f}^{W^{+}} = \frac{\sqrt{\alpha_{em}}}{2\sqrt{2}\sin\theta_{W}}V_{f_{u}f_{d}}, \qquad \mathcal{C}_{f}^{W^{-}} = \frac{\sqrt{\alpha_{em}}}{2\sqrt{2}\sin\theta_{W}}V_{f_{d}f_{u}},$$

with the vectorial coupling at LO given by

$$g_{v,f_u}^Z = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W, \qquad g_{v,f_d}^Z = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W, \qquad g_{v,f}^W = 1,$$
(4)

and

$$g_{a,f_u}^Z = \frac{1}{2}, \qquad g_{a,f_d}^Z = -\frac{1}{2}, \qquad g_{a,f}^W = 1$$
 (5)

in the axial-vector case.

In order to estimate the hadronic cross section for the inclusive process $pp \rightarrow GX$ one has to note that the gauge boson carries the momentum fraction x_1 away from the projectile proton, with the light cone fraction of the quark

emitting the gauge boson x_q given by $x_q = x_1/\alpha$. Taking into account that the probability to find a quark (antiquark) with momentum fraction x_q in the proton wave function is described in terms of the quark densities $q_f(\bar{q}_f)$ has the result that the cross section for the inclusive production of a gauge boson of mass M and transverse momentum q_{\perp} is expressed as follows [29]: E. A. F. BASSO, V. P. GONÇALVES, AND M. RANGEL

$$\frac{d^{4}\sigma_{T,L}(pp \to GX)}{d^{2}q_{\perp}dx_{1}} = \frac{1}{2\pi} \sum_{f} \int_{x_{1}}^{1} \frac{d\alpha}{\alpha^{2}} [q_{f}(x_{q},\mu_{F}^{2}) + \bar{q}_{f}(x_{q},\mu_{F}^{2})] \int d^{2}\rho_{1}d^{2}\rho_{2} \exp[i\vec{q}_{\perp} \cdot (\vec{\rho_{1}} - \vec{\rho_{2}})] \\ \times \Psi_{T,L}^{V-A}(\vec{\rho_{1}},\alpha)\Psi_{T,L}^{V-A}(\vec{\rho_{2}},\alpha)\frac{1}{2} [\sigma_{q\bar{q}}(\alpha\rho_{1},x) + \sigma_{q\bar{q}}(\alpha\rho_{2},x) - \sigma_{q\bar{q}}(\alpha|\vec{\rho_{1}} - \vec{\rho_{2}}|,x)],$$
(6)

with the Bjorken variable x given by $x = M^2/\hat{s}$, where $\hat{s} = sx_1/\alpha$ is the quark-proton center-of-mass energy squared, and μ_F is the factorization scale.

It is important to emphasize that although both valence and sea quarks in the projectile are taken into account through the parton distributions, the color dipole accounts only for Pomeron exchange from the target, disregarding its valence content. Therefore, in principle this approach is well suited for high energies and, consequently, small values of x.

III. QCD DYNAMICS

In the color dipole formalism the cross sections are determined by the dipole-proton cross section, $\sigma_{q\bar{q}}$, which encodes all the information about the hadronic scattering, and thus about the nonlinear and quantum effects, in the hadron wave function. It can be expressed by

$$\sigma_{q\bar{q}}(\vec{\rho}, x) = 2 \int d^2 b \mathcal{N}(\vec{b}, \vec{\rho}, Y), \tag{7}$$

where $\mathcal{N}(\vec{b}, \vec{\rho}, Y)$ is the imaginary part of the forward amplitude for the scattering between a small dipole (a colorless quark-antiquark pair) and a dense hadron target, at a given rapidity interval $Y = \ln(1/x)$. The dipole has transverse size given by the vector $\vec{\rho} = \vec{x} - \vec{y}$, where \vec{x} and \vec{x} are the transverse vectors for the quark and antiquark, respectively, and impact parameter $\vec{b} = (\vec{x} + \vec{y})/2$. At high energies the evolution with the rapidity Y of $\mathcal{N}(\vec{b}, \vec{\rho}, x)$ is given by the infinite hierarchy of equations, the so called Balitsky-JIMWLK equations [20,31], which reduces in the mean field approximation to the Balitsky-Kovchegov equation [31,32]. In recent years, the running-coupling corrections to the BK evolution kernel were explicitly calculated [34,35], including the $\alpha_s N_f$ corrections to the kernel to all orders, and its solution studied in detail [36,37]. Basically, one has that the running of the coupling reduces the speed of the evolution to values compatible with experimental e_p HERA data [38,39]. The numerical solutions of the running-coupling BK equation presented in Refs. [38,39] assumed the translational invariance approximation, which implies $\mathcal{N}(\vec{b}, \vec{\rho}, Y) = \mathcal{N}(\vec{\rho}, Y)S(\vec{b})$, with the normalization of the dipole cross section fitted to data. Unfortunately, impact-parameter-dependent numerical solutions to the BK equation are very difficult to obtain [40]. Moreover, the choice of the impact-parameter profile of the

dipole amplitude entails intrinsically nonperturbative physics, which is beyond the OCD weak coupling approach of the BK equation. In fact, the BK equation generates a power-law Coulomb-like tail, which is not confining at large distances and, therefore, can violate the unitarity bound. It is important to emphasize that although a complete analytical solution of the BK equation is still lacking, its main properties at fixed \vec{b} are known: (a) for the interaction of a small dipole ($\rho \ll 1/Q_{\text{sat}}$), $\mathcal{N}(\vec{b}, \vec{\rho}, Y) \approx \rho^2$, implying that this system is weakly interacting; (b) for a large dipole ($\rho \gg 1/Q_{sat}$), the system is strongly absorbed and therefore $\mathcal{N}(\vec{b}, \vec{\rho}, Y) \approx 1$. The typical momentum scale, $Q_{\rm sat}^2 \propto x^{-\lambda} (\lambda \approx 0.3)$, is the so-called saturation scale. This property is associated with the large density of saturated gluons in the hadron wave function. In the last years, several groups have constructed phenomenological models that satisfy the asymptotic behaviors of the BK equation in order to fit the HERA and RHIC data (see, e.g., Refs. [30,41–47]). In particular, in Ref. [30] the authors have proposed a generalization of the CGC model [42] with the inclusion of the impact-parameter dependence in the dipole-proton scattering amplitude. This bCGC dipole model is based on the Balitsky-Kovchegov nonlinear evolution equation and improves the CGC model by incorporating the impact-parameter dependence of the saturation scale. The corresponding dipole-proton scattering is given by [30]

$$\mathcal{N}(\vec{b},\vec{\rho},Y) = \begin{cases} \mathcal{N}_0 \left(\frac{\rho Q_{\text{sat}}}{2}\right)^{2(\gamma_s + \frac{\ln(2/\rho Q_{\text{sat}})}{\kappa AY})} & \rho Q_{\text{sat}} \le 2\\ 1 - \exp^{-A\ln^2(B\rho Q_{s,p})} & \rho Q_{\text{sat}} > 2, \end{cases}$$
(8)

with $Y = \ln(1/x)$ and $\kappa = \chi''(\gamma_s)/\chi'(\gamma_s)$, where χ is the LO Balitsky, Fadin, Kuraev, and Lipatov (BFKL) characteristic function. The coefficients *A* and *B* are determined uniquely from the condition that $\mathcal{N}(\vec{b}, \vec{\rho}, Y)$, and its derivative with respect to ρQ_{sat} , are continuous at $\rho Q_{\text{sat}} = 2$. In this model, the proton saturation scale Q_{sat} depends on the impact parameter,

$$Q_{\text{sat}} \equiv Q_{\text{sat}}(x, \vec{b}) = \left(\frac{x_0}{x}\right)^{\frac{1}{2}} \left[\exp\left(-\frac{b^2}{2B_{\text{CGC}}}\right)\right]^{\frac{1}{2\gamma_s}}.$$
 (9)

The parameter $B_{\rm CGC}$ was adjusted to give a good description of the *t* dependence of exclusive J/ψ photoproduction. In addition, the factors \mathcal{N}_0 and γ_s were taken to be free. In

this way a very good description of F_2 data was obtained. Recently this model has been improved by fitting its free parameters in consideration of the high-precision combined HERA data [33]. The set of parameters that will be used here are the following: $\gamma_s = 0.6599$, $B_{CGC} = 5.5 \text{ GeV}^{-2}$, $\mathcal{N}_0 = 0.3358$, $x_0 = 0.00105 \times 10^{-5}$, and $\lambda = 0.2063$. The value of γ_s deserves some comment. In the CGC model, in which the impact-parameter dependence of the saturation scale is disregarded, the authors assumed a fixed value of the anomalous dimension, $\gamma_s = 0.63$, which is the value predicted by the BFKL dynamics at leading order. Moreover, the old HERA data were fitted considering only three light flavors. Later, in Ref. [48], the charm contribution was included in the analysis assuming γ_s as a free parameter. In this case, a value of $\gamma_s = 0.7376$ was obtained. As demonstrated in [33], a slightly higher value for $\gamma_s = 0.76$ is favored by the recent HERA data. Such higher values of γ_s are rather close to what we expect from NLO BFKL, which implies $\gamma_s \ge 0.7$ [48]. In contrast, as demonstrated in Refs. [30,33], the inclusion of the impactparameter dependence in the saturation scale reduces the extracted value of the anomalous dimension. The main distinction between these two studies is that the analysis performed in Ref. [33] using the recent HERA data seem to favor a larger value than that obtained in Ref. [30] using the old HERA data; the value obtained in [33] is close to the LO BFKL value. Another important difference between these previous models and the bCGC model used in our studies is related to the distinct prediction for the energy behavior of the saturation scale. As demonstrated in Ref. [33], the recent HERA data implies a steeper energy dependence in comparison to that obtained considering the old HERA data, which is directly associated with the larger value for the parameter $\lambda = 0.206$ in comparison to the previous one ($\lambda = 0.119$). At central collisions b = 0, the predictions of the CGC and rcBK dipole models are steeper than the bCGC one (see Figs. 2 and 3 in Ref. [33]), with Q_s^2 being predicted by the bCGC model to be $\approx 2 \text{ GeV}^2$ at $x \leq 10^{-6}$.

For comparison, in what follows we also will use the Golec-Biernat and Wusthof (GBW) model [41], which assumes $\mathcal{N}(\vec{b}, \vec{\rho}, Y) = \mathcal{N}(\vec{\rho}, Y)S(\vec{b})$ with the scattering amplitude parametrized as follows:

$$\mathcal{N}_{\text{GBW}}(\rho, Y) = 1 - e^{-\rho^2 Q_{\text{sat}}^2(Y)/4},$$
 (10)

where the saturation scale is given by $Q_{\text{sat}}^2 = Q_0^2 (x_0/x)^{\lambda}$, x_0 is the value of the Bjorken x in the beginning of the evolution, and λ is the saturation exponent. In comparison to the bCGC model, the GBW model implies a steeper energy dependence for the saturation scale, with $Q_s^2 \approx 4 \text{ GeV}^2$ at $x \leq 10^{-6}$. Our motivation to use this phenomenological model is associated with the fact that its linear limit is known, given by

$$\mathcal{N}_{\text{GBW-lin}}(\rho, Y) = \frac{\rho^2 Q_{\text{sat}}^2(Y)}{4}.$$
 (11)

It allows us to compare the full and linear predictions for the gauge boson production cross sections and thus estimate the magnitude of the nonlinear corrections for these observables.

IV. RESULTS

In what follows we present our predictions for the *W*- and *Z*-boson production obtained using the color dipole formalism and the three phenomenological models for the dipole-proton scattering amplitude discussed above. Following Ref. [41] we will assume that the quark masses are given by $m_u = m_d = m_s = 0.14$ GeV, $m_c = 1.4$ GeV, and $m_b = 4.5$ GeV. Moreover, we will assume that the factorization scale is equal to the gauge boson mass and we



FIG. 2 (color online). Energy dependence of the total cross sections predicted by the linear (GBW linear) and nonlinear (GBW and CGC) models. The NNLO collinear predictions obtained in Refs. [52,53] are presented for comparison.



FIG. 3 (color online). Energy dependence of the total cross section predicted by the bCGC and GBW linear models. Data from [3,4,55–59].

TABLE I. Comparison between the linear and nonlinear predictions for the total cross sections for different values of the center-ofmass energy. The NLLO predictions obtained in Refs. [52,53] are also presented, as well as experimental results obtained in Refs. [3–5,57–59]. Cross sections in nb.

\sqrt{s} (TeV)	GBW	GBW linear	bCGC	MSTW	dynJR14	DATA (nb)				
$Z_0 o \ell^+ \ell^-$										
1.8	0.21	0.25	0.21		0.2456	0.231 ± 0.012 (CDF) 0.221 ± 0.012 (D0)				
1.96	0.23	0.28	0.23	0.2507		0.2549 ± 0.016 (CDF)				
7	0.89	1.08	0.89		0.9689	0.937 ± 0.037 (ATLAS) 0.974 ± 0.044 (CMS)				
8	1.02	1.23	1.00		1.1271	1.15 ± 0.37 (CMS)				
10	1.26	1.47	1.23	1.429						
14	1.73	1.91	1.65	2.051	2.0658					
				$W^{+} + W^{-}$	$\rightarrow \ell \mu_{\ell}$					
1.8	2.40	2.80	2.34		2.5659	2.49 ± 0.12 (CDF) 2.310 ± 0.11 (D0)				
1.96	2.64	3.08	2.59	2.747		2.749 ± 0.17 (CDF)				
7	9.62	11.56	9.46		10.2976	10.207 ± 0.403 (ATLAS) 10.3 ± 0.43 (CMS)				
8	10.91	13.14	10.70		11.8966	12.21 ± 0.40 (CMS)				
10	13.39	16.24	13.07	15.35						
14	18.16	22.21	17.51	21.72	21.32					

will use the CTEQ6L parameterization [49] for the parton distribution functions. We have verified that our results do not depend on these choices. It is important to emphasize that the expression for the gauge boson production in the color dipole formalism was derived at leading order in log(1/x), disregarding the contributions of valence quarks and larger-x corrections. Such contributions can modify the normalization and the behavior at low energies of our predictions. One possibility to estimate the magnitude of these corrections is to leave the normalization of cross sections as a free parameter to be fixed by the experimental data, usually denoted as the K factor. However, as in previous calculations for the Drell-Yan [50] and heavy quark [51] production using the color dipole formalism, we will not include a K factor. Consequently, our predictions will be parameter free. For comparison, we will compare our results with the NNLO collinear predictions available in



FIG. 4 (color online). Energy dependence of the ratio between the W and Z cross sections. Data from [3-5,58]. The Monte Carlo for FeMtobarn processes (MCFM) and Martin, Stirling, Thorne, and Watt (MSTW) NNLO predictions are presented for comparison.

the literature [52–54] and with current experimental data [3–6,55–59]. In particular, we compare our predictions with recent data from the ATLAS and CMS Collaborations, which have performed W- and Z-boson measurements in the inclusive Drell-Yan processes to high precision for pp collisions at 7 and 8 TeV. These measurements have been made for the decay channels $W \rightarrow l\nu$ and $Z \rightarrow l\bar{l}$, with l denoting a lepton, for the available phase volume and have been extrapolated to the entire kinematical region.

In Fig. 2 we present our predictions for the energy dependence of the total cross sections. We observe that the GBW and bCGC predictions are similar. The GBW linear predictions, which disregard gluon saturation effects, are $\approx 20\%$ larger than the nonlinear predictions. In comparison with the NNLO collinear predictions presented in Refs. [52,53], we obtain that the GBW linear model predicts similar values for the total cross sections. Our results indicate that the NNLO predictions can be quite well



FIG. 5 (color online). Energy dependence of the total cross sections obtained assuming that the gauge boson *G* is produced in the LHCb kinematical range $(2 \le \eta(G) \le 4.5)$.



FIG. 6 (color online). Rapidity distribution for the *Z*-boson production in the LHCb kinematical range. Data from [6].

reproduced using the color dipole formalism with K = 1 if the nonlinear effects are disregarded, i.e., if we assume that the behavior $\rho^2 x^{-\lambda}$ for the dipole-proton scattering amplitude is valid for all values of ρ . A more detailed comparison can be made analyzing Table I, where we present the results for some typical values of the center-of-mass energy. In addition, in Fig. 3 we present the energy dependence of our predictions and provide a comparison with current experimental data [3–5,55–59] (see also Table I). We observe that the GBW and bCGC predictions are below the experimental data. Considering that the bCGC describes the HERA data quite well, we can interpret this result as an indication that the DGLAP evolution, disregarded in this model, is important for the description of the gauge boson production or that higher-order corrections and/or valence quarks contributions for the color dipole formalism should be included in our calculations.

In Fig. 4 we compare our predictions for the ratio between the *W* and *Z* cross sections, denoted $R_{W/Z}$, with the experimental data from the CDF, ATLAS, and CMS Collaborations [3,5,58] and theoretical NNLO predictions [52,54]. We observe that our predictions strongly increase at small values of energy, in disagreement with the data and the theoretical expectations for $R_{W/Z}$ in this kinematical regime. This behavior is directly associated with the limitation of the color dipole formalism, which disregarded the valence quark contribution in the gauge boson cross section. In contrast, at larger values of energy, the GBW

linear prediction is similar to the NNLO one, with the bCGC one being smaller by $\approx 18\%$, in agreement with our previous results for the total cross sections.

In Fig. 5 we present our predictions for the energy dependence of the W- and Z-boson cross sections obtained assuming that the gauge boson G is produced in the LHCb kinematical range $(2 \le \eta(G) \le 4.5)$. For comparison we also present the MCFM predictions [54] which are obtained considering the NNLO corrections for the total cross sections. As already observed for the total cross sections, we obtain that the CGC and GBW predictions for the LHCb kinematical range are similar, being $\approx 20\%$ smaller than the GBW linear and MCFM ones. The LHCb Collaboration has determined the Z-production cross section in three leptonic decay channels $(\mu^+\mu^-, e^+e^-, \tau^+\tau^-)$, with all events selected by requiring a single muon or a single electron trigger. Let us now compare our results with the experimental data for the rapidity distribution of the Z-boson and the corresponding MCFM predictions (see Fig. 6). We obtain that the GBW linear prediction is very similar to the MCFM one and describes the data quite well. In contrast, the nonlinear predictions underestimate the data at $y_7 \approx 3$. In order to compare our predictions with the recent LHCb data for the cross sections [6], it is necessary to apply the experimental cuts in the leptons in the final state. Following the procedure discussed in Refs. [6], we obtain the results presented in Table II, where we compare our predictions with the LHCb data. As expected from our previous analysis, the color dipole formalism describes the data quite well if we use the GBW linear dipole-proton scattering amplitude.

A comment is in order here. As discussed previously, the color dipole formalism used in our calculations disregards valence quark contributions as well as next-to-leading corrections. Both contributions can modify the normalization of the cross sections and rapidity distributions. A simplistic way to include these corrections is to multiply the cross section by a *K* factor fixed by the data in order to obtain the correct normalization of the cross section. If it is made, we have checked that for $K \approx 1.2$, independent of the total cross section as well as the LHCb data for the rapidity distributions. However, it is important to emphasize that the inclusion of a common value for the *K* factor, independent of the gauge boson produced, does not improve the

TABLE II. Comparison between the linear and nonlinear predictions for the cross sections considering the LHCb experimental cuts. Data from Ref. [6]. Cross sections in pb.

	LHCb data	bCGC	GBW	GBW linear	
$\sigma_Z \text{Br}(Z \to e^+ e^-)$	$76.0 \pm 0.8 \pm 2.0 \pm 2.6$	59.35	58.91	73.61	
$\sigma_Z \operatorname{Br}(Z \to \mu^+ \mu^-)$	$76.7 \pm 1.7 \pm 3.3 \pm 2.7$	59.50	59.20	73.90	
$\sigma_Z \operatorname{Br}(Z \to \tau^+ \tau^-)$	$71.4 \pm 3.5 \pm 2.8 \pm 2.5$	59.50	59.20	73.90	
$\sigma_{W^+} \operatorname{Br}(W^+ \to \mu^+ \nu_{\mu})$	$831\pm9\pm27\pm29$	659.07	667.59	836.78	
$\sigma_{W^-} \mathrm{Br}(W^- \to \mu^- \nu_{\mu})$	$656\pm8\pm19\pm23$	599.19	593.90	710.23	

description of the ratio between the W and Z cross sections presented in Fig. 5. Therefore, this subject deserves more detailed study.

V. CONCLUSIONS

The study of the production of the massive gauge bosons W^{\pm} and Z^{0} in proton-proton collisions provides an important test of the perturbative OCD. The description of this process is usually made using the collinear factorization and by taking into account the perturbative contributions up to next-to-next-to-leading order, with the corresponding predictions describing the data quite well. However, the current and future experimental data for the gauge boson production also provide an important test of the QCD dynamics at high energies (small x), where gluon saturation effects can be present. Such effects are naturally described in the color dipole formalism. This formalism also resums leading logarithms in the energy and takes into account higher twist contributions for the cross sections, which are disregarded by the collinear factorization approach. Thus, no matter if the gluon saturation effects are or are not important for the gauge boson production, it is possible to obtain a description of the W^{\pm} and Z^0 cross sections in this framework. The first step was performed in Ref. [29], generalizing previous analyses of the Drell-Yan process in the color dipole formalism [24–27]. Our goal in this paper was to improve that analysis, performing a systematic study of the inclusive production and considering different models for the dipole-proton cross section, as well as a detailed comparison of its predictions with the current experimental data. Through the comparison between the full and linear predictions of the GBW model, we have obtained that the gluon saturation effects contribute to the gauge boson production at LHC energies by approximately 20%. However, our results indicate that current experimental data for the gauge boson production can only be described in the color dipole formalism if the nonlinear effects are disregarded. In particular, the bGCC model, which successfully describes the recent high-precision combined HERA data, fails to describe the recent ATLAS, CMS, and LHCb data. In principle, it can be associated with the fact that the bCGC model disregards the DGLAP evolution in the description of the dipole-target cross section at small values of the dipole pair separation, which dominates the gauge boson production. Another possible interpretation is that valence quark contributions and/or higher-order corrections to the formalism should be taken into account. Both possibilities deserve more detailed study, which we plan to perform in the future. Finally, as the saturation scale increases with the atomic number, it will be interesting to generalize our results for protonnucleus collisions, estimating the gauge boson production in this process at LHC energies and making a comparison between our predictions with those that come from the collinear factorization with nuclear effects included in the nuclear parton distributions [60].

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