Semileptonic decays $B_c^+ \to D_{(s)}^{(*)}(l^+\nu_l, l^+l^-, \nu\bar{\nu})$ in the perturbative QCD approach

Wen-Fei Wang,[1,*] Xin Yu,[1] Cai-Dian Lü,[1] and Zhen-Jun Xiao[2]

[1]*Center for Future High Energy Physics, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, People's Republic of China*

[2]*Department of Physics and Institute of Theoretical Physics, Nanjing Normal University, Nanjing, Jiangsu 210023, People's Republic of China*

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In this paper we study the semileptonic decays of $B_c^+ \to D_{(s)}^{(*)}(l^+\nu_l, l^+l^-, \nu\bar{\nu})$ (here l stands for e, μ, or τ). After evaluating the $B_c^+ \to (D_{(s)}, D_{(s)}^*)$ transition form factors $F_{0,+,T}(q^2)$ and $V(q^2)$, $A_{0,1,2}(q^2)$, $T_{1,2,3}(q^2)$ by employing the perturbative QCD factorization approach, we calculate the branching ratios for all these semileptonic decays. Our predictions for the values of the $B_c^+ \to D_{(s)}$ and $B_c^+ \to D_{(s)}^*$ transition form factors are consistent with those obtained by using other methods. The branching ratios of the decay modes with $\bar{\nu}\nu$ are almost an order of magnitude larger than the corresponding decays with l^+l^- after the summation over the three neutrino generations. The branching ratios for the decays with $b \to d$ transitions are much smaller than those decays with the $b \to s$ transitions, due to the Cabibbo-Kobayashi-Maskawa suppression. We define ratios R_D and R_{D^*} for the branching ratios with the τ lepton versus μ, e lepton final states to cancel the uncertainties of the form factors, which could possibly be tested in the near future.

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I. INTRODUCTION

The B_c meson is a pseudoscalar ground state of b and c quarks, and thus the electromagnetic interaction cannot transform the B_c meson into other hadrons containing b and c quarks. The two different quark flavors forbid the meson from being annihilated into gluons, and the fact that it is below the $B - D$ threshold makes the B_c meson stable with regards to the strong interaction. The B_c meson can only decay through weak interactions, so it is an ideal system to study weak decays of heavy quarks. Either of the heavy quarks (b or c) can decay individually, which makes it different from the $B_{u,d}$ or B_s meson. The phase space in the $c \to s$ transition is smaller than that in the $b \to c$ transition, but the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{cs}| \sim 1$ is much larger than the CKM matrix element $|V_{cb}| \sim 0.04$. Thus the c-quark decays provide the dominant contribution (about 70%) to the decay width of the B_c meson [1]. Because the mass of a $B_c\bar{B}_c$ pair exceeds the threshold of $\Upsilon(4S)$, the B_c meson cannot be produced at the B factories. Thus—compared with the $B_{u,d}$ or B_s meson—B_c-meson decays have received much less experimental attention. However, at LHC experiments around 5×10^{10} B_c events per year are expected [1,2] due to the relatively large production cross section, which provides a very good platform to study various B_c-meson decay modes.

Because there is only one hadronic final product, the B_c-meson semileptonic decays among the abundant decay modes are relatively clean in the theoretical treatment. These semileptonic decays provide good opportunities to measure not only the CKM matrix elements, such as $|V_{cb}|$, $|V_{ub}|$, and $|V_{cd}|$, but also the form factors of the B_c-to-bottom and B_c-to-charm meson transitions. The rare semileptonic decays governed by the flavor-changing neutral currents are forbidden at tree level in the standard model (SM). Those decays—which are very sensitive to the contributions of new intermediate particles or interactions—are especially interesting. There are various approaches for working on the semileptonic B_c decays. In Ref. [3], for example, Dhir and Verma presented a detailed analysis of the exclusive semileptonic B_c decays in the Bauer-Stech-Wirbel framework. The authors of Refs. [4–6] studied the semileptonic B_c decays in the relativistic and/or constituent quark model. In Refs. [7,8], $B_c \to D_s^* l^+l^-$ decays were studied in the SM with the fourth-generation and supersymmetric models. The three-point QCD sum rules approach was adopted to investigate $B_c^+ \to D_{(s)}^{*+} l^+l^-$ in Ref. [9] and $B_c^+ \to D_{(s)}^+(l^+l^-, \bar{\nu}\nu)$ in Ref. [10].

In this paper, we will study the semileptonic decays of $B_c^+ \to D_{(s)}^{(*)}(l^+\nu_l, l^+l^-, \nu\bar{\nu})$ (here l stands for the leptons e, μ, or τ) in the perturbative QCD (pQCD) approach [11]. These semileptonic decays are governed by the form factors. At the maximum recoil region, the final-state meson is collinear with a large momentum. The spectator c quark in the B_c meson thus needs a hard gluon to kick it from almost zero momentum to a collinear state. However, when doing integrations of the momentum fractions of valence quarks, an endpoint singularity occurs. A natural

*wangwf@ihep.ac.cn

way to kill this singularity is to pick up the neglected transverse momentum in the collinear factorization. With the additional transverse momentum scale k_T [12], double logarithms appear in the calculation. We have to use the renormalization group equation to perform the resummation (resulting in the so-called Sudakov form factors [13]). and make the perturbative calculation of the hard amplitudes (form factors) infrared safe. The pQCD approach is widely adopted to calculate the transition form factors of the $B_{u,d}$ and B_s mesons [14–16]. Furthermore, various B_c decay modes have also been studied in Refs. [17,18] in the pQCD approach.

The structure of this paper is as follows. After this Introduction, we collect the distribution amplitudes of the B_c, $D^{(*)}$, and $D_s^{(*)}$ mesons in Sec. II. Based on the $k_{\rm T}$ factorization formalism, we calculate and present the expressions for the $B_c \to (D^{(*)}, D_s^{(*)})$ transition form factors in the large recoil regions in Sec. III. The numerical results and relevant discussions are given in Sec. IV, and Sec. V contains a short summary.

II. KINEMATICS AND THE WAVE FUNCTIONS

The lowest-order diagrams for $B_c^+ \to (D^{(*)}, D_s^{(*)})$ transitions are displayed in Fig. 1, where M stands for a $D^{(*)}$ or $D_s^{(*)}$ meson and $\otimes$ is the weak vertex for the leptonic pairs to come out. In the rest frame of the B_c meson—where m_{B_c} is the mass of the B_c meson and m is the mass of the $D_{(s)}$ or $D^*_{(s)}$ meson—the momenta of the B_c and $D_{(s)}^{(*)}$ mesons are defined in the light-cone coordinates as [18,19]

$$p_1 = \frac{m_{B_c}}{\sqrt{2}}(1, 1, 0_\perp), \qquad p_2 = \frac{m_{B_c}}{\sqrt{2}}(r\eta^+, r\eta^-, 0_\perp), \tag{1}$$

with $r = m/m_{B_c}$ and $\eta^\pm = \eta \pm \sqrt{\eta^2 - 1}$. As for the η in $\eta^\pm$, the expression

$$\eta = \frac{1}{2r}\left[1 + r^2 - \frac{q^2}{m_{B_c}^2}\right] \tag{2}$$

can be evaluated with $q^2 = (p_1 - p_2)^2$ which is the invariant mass of the lepton pairs. The momenta of the spectator quarks in the B_c and $D_{(s)}^{(*)}$ mesons are parametrized as

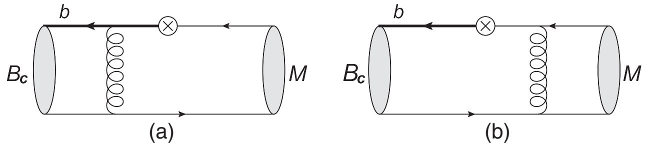

FIG. 1. The leading-order Feynman diagrams for the transition of $B_c^+ \to (D^{(*)}, D_s^{(*)})$, where M stands for a $D^{(*)}$ or $D_s^{(*)}$ meson, and $\otimes$ is the weak vertex.

$$k_1 = \left(x_1 \frac{m_{B_c}}{\sqrt{2}}, x_1 \frac{m_{B_c}}{\sqrt{2}}, k_{1\perp}\right),$$
$$k_2 = \left(x_2 \frac{m_{B_c}}{\sqrt{2}} r\eta^+, x_2 \frac{m_{B_c}}{\sqrt{2}} r\eta^-, k_{2\perp}\right). \tag{3}$$

We define the polarization vector ϵ of the $D^*_{(s)}$ mesons as

$$\epsilon_L = \frac{1}{\sqrt{2}}(\eta^+, -\eta^-, 0_\perp), \qquad \epsilon_T = (0, 0, 1), \tag{4}$$

where ϵ_L and ϵ_T denote the longitudinal and transverse polarization of the $D^*_{(s)}$ mesons, respectively.

In this work, we use the same distribution amplitude for the B_c meson as that used in Refs. [18,20–22],

$$\Phi_{B_c}(x) = \frac{i}{\sqrt{2N_c}}[(\not p + m_{B_c})\gamma_5 \phi_{B_c}(x)]_{\alpha\beta}, \tag{5}$$

with

$$\phi_{B_c}(x) = \frac{f_{B_c}}{2\sqrt{2N_c}}\delta(x - m_c/m_{B_c})\exp[-\omega_{B_c}^2 b^2/2], \tag{6}$$

where m_c is the mass of the c quark. Because the B_c meson consists of two heavy quarks b and c (just like a heavy quarkonium), the nonrelativistic QCD framework can be applied, which means the leading-order wave function should be just the zero-point wave function shown in Eq. (6).

For the $D_{(s)}^{(*)}$ mesons, up to twist-3 accuracy, the two-parton light-cone distribution amplitudes are defined as [19,23]

$$\begin{aligned}
&\langle D_{(s)}(p)|q_\alpha(z)\bar c_\beta(0)|0\rangle \\
&\quad = \frac{i}{\sqrt{2N_C}}\int_0^1 dx e^{ixp\cdot z}[\gamma_5(\not p + m)\phi_{D_{(s)}}(x, b)]_{\alpha\beta}, \\
&\langle D^*_{(s)}(p)|q_\alpha(z)\bar c_\beta(0)|0\rangle \\
&\quad = -\frac{1}{\sqrt{2N_C}}\int_0^1 dx e^{ixp\cdot z}[\not\epsilon_L(\not p + m)\phi_{D^*_{(s)}}(x, b) \\
&\qquad + \not\epsilon_T(\not p + m)\phi_{D^*_{(s)}}(x, b)]_{\alpha\beta},
\end{aligned} \tag{7}$$

where

$$\int_0^1 dx \phi_{D_{(s)}}(x, 0) = \frac{f_{D_{(s)}}}{2\sqrt{2N_c}},$$
$$\int_0^1 dx \phi_{D^*_{(s)}}(x, 0) = \frac{f_{D^*_{(s)}}}{2\sqrt{2N_c}} \tag{8}$$

are the normalization conditions. We adopt $f_D = 206.7 \pm 8.9$ MeV and $f_{D_s} = 260.0 \pm 5.6$ MeV (from the Particle Data Group [24]) as the experimental averages for D and D_s mesons, respectively. For the D^* or D_s^* meson, we adopt the same decay constant and distribution amplitude for the longitudinal and transverse components. Since there is no experimental data, we use $f_{D^*} = 270$ MeV and $f_{D_s^*} = 310$ MeV for the D^* and D_s^* mesons (given the results in Refs. [25]) and assume a 10% uncertainty. The distribution amplitude for the $D_{(s)}$ meson is

$$\phi_{D_{(s)}} = \frac{1}{2\sqrt{2N_C}} f_{D_{(s)}} 6x(1-x)[1 + C_{D_{(s)}}(1-2x)] \times \exp\left[-\frac{\omega_{D_{(s)}}^2 b^2}{2}\right], \tag{9}$$

which is a k_T-dependent form, with $C_D = 0.5$, $\omega_D = 0.1$ and $C_{D_s} = 0.4$, $\omega_{D_s} = 0.2$ for the D and D_s mesons, respectively [23]. In this work, we also adopt the same distribution amplitude for both the vector meson $D_{(s)}^*$ and pseudoscalar meson $D_{(s)}$ because of their small mass difference [23].

III. FORM FACTORS OF SEMILEPTONIC DECAYS

The form factors $F_+(q^2)$, $F_0(q^2)$ for the B_c to pseudoscalar meson $D_{(s)}$ transition induced by the vector current can be defined as [26,27]

$$\langle D_{(s)}(p_2)|\bar{q}(0)\gamma_\mu b(0)|B_c(p_1)\rangle = \left[(p_1+p_2)_\mu - \frac{m_{B_c}^2 - m^2}{q^2} q_\mu\right] F_+(q^2) + \frac{m_{B_c}^2 - m^2}{q^2} q_\mu F_0(q^2), \tag{10}$$

where $q = p_1 - p_2$ is the momentum of the lepton pairs. In order to cancel the poles at $q^2 = 0$, $F_+(0)$ should be equal to $F_0(0)$. For the sake of convenience, we define the auxiliary form factors $f_1(q^2)$ and $f_2(q^2)$,

$$\langle D_{(s)}(p_2)|\bar{q}(0)\gamma_\mu b(0)|B_c(p_1)\rangle = f_1(q^2)p_{1\mu} + f_2(q^2)p_{2\mu}. \tag{11}$$

In terms of $f_1(q^2)$ and $f_2(q^2)$, the form factors $F_+(q^2)$ and $F_0(q^2)$ are

$$F_+(q^2) = \frac{1}{2}[f_1(q^2) + f_2(q^2)],$$
$$F_0(q^2) = \frac{1}{2} f_1(q^2)\left[1 + \frac{q^2}{m_{B_c}^2 - m^2}\right] + \frac{1}{2} f_2(q^2)\left[1 - \frac{q^2}{m_{B_c}^2 - m^2}\right]. \tag{12}$$

The form factor $F_T(q^2)$ for the $B_c \to D_{(s)}$ transition induced by the tensor current can be defined as [27]

$$\langle D_{(s)}(p_2)|\bar{q}(0)\sigma_{\mu\nu} b(0)|B_c(p_1)\rangle = i[p_{2\mu}q_\nu - q_\mu p_{2\nu}]\frac{2F_T(q^2)}{m_{B_c}+m}. \tag{13}$$

There are seven form factors—$V(q^2)$, $A_{0,1,2}(q^2)$, and $T_{1,2,3}(q^2)$—that are needed for the $B_c \to D_{(s)}^*$ transition in this work. The form factors $V(q^2)$ and $A_{0,1,2}(q^2)$ are defined by [27–29]

$$\langle D_{(s)}^*(p_2)|\bar{q}(0)\gamma_\mu b(0)|B_c(p_1)\rangle = \epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu*} p_1^\alpha p_2^\beta \frac{2V(q^2)}{m_{B_c}+m}, \tag{14}$$

$$\langle D_{(s)}^*(p_2)|\bar{q}(0)\gamma_\mu\gamma_5 b(0)|B_c(p_1)\rangle = i\left[\epsilon_\mu^* - \frac{\epsilon^*\cdot q}{q^2}q_\mu\right](m_{B_c}+m)A_1(q^2) - i\left[(p_1+p_2)_\mu - \frac{m_{B_c}^2 - m^2}{q^2}q_\mu\right](\epsilon^*\cdot q)\frac{A_2(q^2)}{m_{B_c}+m} + i\frac{2m(\epsilon^*\cdot q)}{q^2}q_\mu A_0(q^2), \tag{15}$$

where ϵ^* is the polarization vector of the $D_{(s)}^*$ meson. The form factors $T_{1,2,3}$ are defined by [27,30]

$$\langle D_{(s)}^*(p_2)|\bar{q}(0)\sigma_{\mu\nu}q^\nu(1+\gamma_5)b(0)|B_c(p_1)\rangle = i\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p_1^\alpha p_2^\beta 2T_1(q^2) + [\epsilon_\mu^*(m_{B_c}^2 - m^2) - (\epsilon^*\cdot q)(p_1+p_2)_\mu]T_2(q^2) + (\epsilon^*\cdot q)\left[q_\mu - \frac{q^2}{m_{B_c}^2 - m^2}(p_1+p_2)_\mu\right]T_3(q^2), \tag{16}$$

where $T_1(0) = T_2(0)$ is implied by the identity

$$\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}. \tag{17}$$

In the transverse configuration b space, and by including the Sudakov form factors and the threshold resummation effects, we obtain the $B_c \to D_{(s)}$ form factors $f_1(q^2)$, $f_2(q^2)$, and $F_T(q^2)$ as follows:

$$f_1(q^2) = 16\pi m_{B_c}^2 r C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\phi_{D_{(s)}}(x_2, b_2)\{[1 - rx_2]\cdot h_1(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_1)\exp[-S_{ab}(t_1)] - [r + 2x_1(1-\eta)]\cdot h_2(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_2)\exp[-S_{ab}(t_2)]\}, \quad (18)$$

$$f_2(q^2) = 16\pi m_{B_c}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\phi_{D_{(s)}}(x_2, b_2)\{[1 - 2rx_2(1-\eta)]\cdot h_1(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_1)\exp[-S_{ab}(t_1)] + [2r - x_1]\cdot h_2(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_2)\exp[-S_{ab}(t_2)]\}, \quad (19)$$

$$F_T(q^2) = 8\pi m_{B_c}^2 C_F (1+r) \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\phi_{D_{(s)}}(x_2, b_2)\{[1 - rx_2]\cdot h_1(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_1)\exp[-S_{ab}(t_1)] + [2r - x_1]\cdot h_2(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_2)\exp[-S_{ab}(t_2)]\}, \quad (20)$$

where $C_F = 4/3$ is the color factor. The functions h_1 and h_2, the scales t_1 and t_2, and the Sudakov factors S_{ab} are the same as those given in Refs. [18,19].

The expressions for the form factors $V(q^2)$, $A_{0,1,2}(q^2)$, and $T_{1,2,3}(q^2)$ for the $B_c \to D^*_{(s)}$ transition in the pQCD approach are

$$V(q^2) = 8\pi m_{B_c}^2 C_F (1+r) \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\phi^T_{D^*_{(s)}}(x_2, b_2)\{[1 - rx_2]\cdot h_1(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_1)\exp[-S_{ab}(t_1)] + r\cdot h_2(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_2)\exp[-S_{ab}(t_2)]\}, \quad (21)$$

$$A_0(q^2) = 8\pi m_{B_c}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\phi^L_{D^*_{(s)}}(x_2, b_2)\{[1 - rx_2(r-2\eta) + r(1-2x_2)] \times h_1(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_1)\exp[-S_{ab}(t_1)] + [r^2 + x_1(1-2r\eta)]\cdot h_2(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_2)\exp[-S_{ab}(t_2)]\}, \quad (22)$$

$$A_1(q^2) = 16\pi m_{B_c}^2 C_F \frac{r}{1+r} \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\phi^T_{D^*_{(s)}}(x_2, b_2) \times \{[1 + rx_2\eta - 2rx_2 + \eta]\cdot h_1(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_1)\exp[-S_{ab}(t_1)] + [r\eta - x_1]\cdot h_2(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_2)\exp[-S_{ab}(t_2)]\}, \quad (23)$$

$$A_2(q^2) = \frac{(1+r)^2(\eta - r)}{2r(\eta^2 - 1)}\cdot A_1(q^2) - 8\pi m_{B_c}^2 C_F \frac{1+r}{\eta^2 - 1} \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1) \times \phi^L_{D^*_{(s)}}(x_2, b_2)\cdot\{[\eta(1 - r^2 x_2) - rx_2(1 - 2\eta^2 - 2r) + (1-r) - r\eta(1+2x_2)] \times h_1(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_1)\exp[-S_{ab}(t_1)] + [r(1 - x_1 + 2x_1\eta^2) - \eta(r^2 + x_1)] h_2(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_2)\exp[-S_{ab}(t_2)]\}, \quad (24)$$

$$T_1(q^2) = 8\pi m_{B_c}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\phi^T_{D^*_{(s)}}(x_2, b_2) \times \{[1 + r(1 - x_2(2 + r - 2\eta))]\cdot h_1(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_1)\exp[-S_{ab}(t_1)] + r[1 - x_1]\cdot h_2(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_2)\exp[-S_{ab}(t_2)]\}, \quad (25)$$

$$T_2(q^2) = 16\pi m_{B_c}^2 C_F \frac{r}{1-r^2} \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\phi^T_{D^*_{(s)}}(x_2, b_2)\{[(1-r)(1+\eta) + 2rx_2(r-\eta) + rx_2(2\eta^2 - r\eta - 1)] \times h_1(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_1)\exp[-S_{ab}(t_1)] + [r(1+x_1)\eta - r^2 - x_1]\cdot h_2(x_1, x_2, b_1, b_2)\cdot \alpha_s(t_2)\exp[-S_{ab}(t_2)]\}, \quad (26)$$

$$T_3(q^2) = \frac{r+\eta}{r} \cdot \frac{1-r^2}{2(\eta^2-1)} \cdot T_2(q^2) - \frac{1-r^2}{(\eta^2-1)} 8\pi m_{B_c}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1) \phi^L_{D^*_{(s)}}(x_2, b_2)$$
$$\times \{[1 + r x_2(\eta-2) + \eta] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \exp[-S_{ab}(t_1)] + [x_1\eta - r] \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp[-S_{ab}(t_2)]\}. \quad (27)$$

One should note that the expressions for the form factors $f_{1,2}(q^2)$, $F_T(q^2)$, $V(q^2)$, $A_{0,1,2}(q^2)$, and $T_{1,2,3}(q^2)$ given in Eqs. (18)–(27) are the results at leading order of the pQCD approach. The next-to-leading-order contributions to the form factors of $B \to (\pi, K, \eta^{(\prime)})$ transitions given in Refs. [14,15,31] are not available here because of the large mass of the c quark and the $(D_{(s)}, D^*_{(s)})$ mesons.

One should note that the pQCD predictions for the considered form factors are reliable only for small values of q^2. For the form factors in the large-q^2 region, one has to make an extrapolation from the low-q^2 region to large-q^2 region. In this work we make the extrapolation by using the formula from Refs. [18,32],

$$F(q^2) = F(0) \cdot \exp[a \cdot q^2 + b \cdot (q^2)^2], \quad (28)$$

where F stands for the form factors $F_{0,+,T}$, V, $A_{0,1,2}$, and $T_{1,2,3}$, and a, b are constants that are to be determined by the fitting procedure.

The $B_c^- \to \bar{D}^0 l^- \bar{\nu}_l$ and $B_c^- \to \bar{D}^{*0} l^- \bar{\nu}_l$ decays are from the quark-level $b \to u l^- \bar{\nu}$ charged-current transition. The effective Hamiltonian for such a transition is [33]

$$\mathcal{H}_{\rm eff}(b \to u l^- \bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{ub} \bar{u} \gamma_\mu (1-\gamma_5) b \cdot \bar{l} \gamma^\mu (1-\gamma_5) \nu_l, \quad (29)$$

where $G_F = 1.16637 \times 10^{-5}$ GeV^{-2} is the Fermi coupling constant and V_{ub} is one of the CKM matrix elements. With the form factors calculated in Eqs. (18), (19), and (21)–(24), one can easily get the differential decay width expression for $B_c^- \to \bar{D}^0 l^- \bar{\nu}_l$ and $B_c^- \to \bar{D}^{*0} l^- \bar{\nu}_l$.

The flavor-changing neutral-current one-loop decay modes, such as $B_c \to D^{(*)} l^+ l^-$ and $B_c \to D_s^{(*)} l^+ l^-$, are transitions of $b \to d l^+ l^-$ and $b \to s l^+ l^-$ at the quark level, respectively. The effective Hamiltonians and the corresponding differential decay widths are more complicated; we refer the reader to Refs. [14,30,34–36].

The effective Hamiltonian for the decay modes $B_c \to D_s^{(*)} \nu\bar{\nu}$ is [33]

$$\mathcal{H}_{\rm eff}(b \to s\nu\bar{\nu}) = \frac{G_F}{\sqrt{2}} \frac{\alpha_{\rm em}}{2\pi \sin^2(\theta_W)} V_{tb} V_{ts}^* \eta_X X(x_t) \times [\bar{s}\gamma^\mu(1-\gamma_5) b][\bar{\nu}\gamma_\mu(1-\gamma_5)\nu], \quad (30)$$

where θ_W is the Weinberg angle with $\sin^2(\theta_W) = 0.231$ [24], V_{tb} and V_{ts} are CKM matrix elements, and $\alpha_{\rm em} \approx 1/137$ is the fine-structure constant. The function $X(x_t)$ can be found in Ref. [33], while $\eta_X \approx 1$ is the QCD radiative correction factor [33]. As for the decay modes $B_c \to D^{(*)} \nu\bar{\nu}$, their effective Hamiltonian can be obtained by the simple replacement $s \to d$ in Eq. (30). The corresponding differential decay widths for $B_c \to D_{(s)} \nu\bar{\nu}$ are the same as those for $B \to \pi(K)\nu\bar{\nu}$ in Ref. [14], except with the replacements $m_B \to m_{B_c}$ and $m_P \to m$. The differential decay width for the decay modes $B_c \to D_s^* \nu\bar{\nu}$ is [37]

$$\frac{d\Gamma(B_c \to D_s^* \nu\bar{\nu})}{dq^2} = \frac{G_F^2 \alpha_{\rm em}^2}{2^{10}\pi^5 m_{B_c}^3} \cdot \left| \frac{X(x_t)}{\sin^2(\theta_w)} \right|^2 \cdot \eta_X^2 \cdot |V_{tb} V_{ts}^*|^2 \lambda^{\frac{1}{2}} \left\{ 8\lambda q^2 \frac{V^2}{(m_{B_c}+m)^2} + \frac{1}{m^2} \left[\lambda^2 \frac{A_2^2}{(m_{B_c}+m)^2} + (m_{B_c}+m)^2 (\lambda + 12 m^2 q^2) \cdot A_1^2 - 2\lambda (m_{B_c}^2 - m^2 - q^2) \cdot {\rm Re}[A_1^* A_2] \right] \right\}, \quad (31)$$

where V, A_1, and A_2 are the form factors of the $B_c \to D_s^*$ transition, and the phase-space factor

$$\lambda = (m_{B_c}^2 + m^2 - q^2)^2 - 4 m_{B_c}^2 m^2. \quad (32)$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

In the numerical calculations we adopt the following input parameters [24]:

$$m_{B_c^-} = 6.277 \text{ GeV}, \quad m_{\bar{D}^0} = 1.865 \text{ GeV}, \quad m_{D^-} = 1.870 \text{ GeV},$$
$$m_{\bar{D}^{*0}} = 2.007 \text{ GeV}, \quad m_{D^{*-}} = 2.010 \text{ GeV}, \quad m_{D_s^-} = 1.969 \text{ GeV},$$
$$m_{\bar{D}_s^{*-}} = 2.112 \text{ GeV}, \quad m_\tau = 1.777 \text{ GeV}, \quad m_c = 1.275 \pm 0.025 \text{ GeV},$$
$$\tau_{B_c} = (0.45 \pm 0.04) \text{ ps}. \quad (33)$$

For the CKM matrix element V_{ub} we adopt the value used in Refs. [14,38], and we use $|V_{tb}| = 0.999$, $|V_{ts}| = 0.0404$, and $|V_{td}/V_{ts}| = 0.211$ [24] in this work. As for the decay constant of the B_c meson, we adopt 0.489 GeV [39] as its central value, and give it an uncertainty of 0.050 GeV.

The numerical values of the $B_c \to D$ and $B_c \to D_s$ transition form factors $F_{0,+,T}$ at $q^2 = 0$ and their fitted parameters a, b are listed in Table I. The numerical values of the form factors V, $A_{0,1,2}$, and $T_{1,2,3}$ at $q^2 = 0$ for the $B_c \to D^*$ and $B_c \to D_s^*$ transitions are listed in Table II. The first error of the pQCD predictions for the form factors in Table I and Table II is induced by the B_c-meson wave-function parameter $\omega_{B_c} = 1.0 \pm 0.1$; the second error comes from the uncertainty of the decay constant f_{B_c}; the third error comes from the uncertainty of the decay constants of the $D_{(s)}^{(*)}$ mesons; the fourth error in Tables I and II comes from the uncertainty of the $D_{(s)}^{(*)}$ wave function $C_{D^{(*)}} = 0.5 \pm 0.1$ or $C_{D_s^{(*)}} = 0.4 \pm 0.1$; and the fifth error comes from $m_c = 1.275 \pm 0.025$ GeV. The errors from the uncertainty of $\omega_{D^{(*)}} = 0.10 \pm 0.02$ or $\omega_{D_s^{(*)}} = 0.20 \pm 0.04$ are very small and have been neglected.

Unlike the form factors at maximum recoil, the extrapolation parameters a, b of the form factors are less sensitive to the decay constant and wave function of the $D_{(s)}^{(*)}$ meson. In Tables I and II, we only show uncertainties for the parameters a and b from the B_c-meson wave-function parameter ω_{B_c}, and from the quark-mass uncertainty $m_c = 1.275 \pm 0.025$ GeV. As a comparison, we also present some results obtained by other authors based on different methods in Table III. It is easy to see that our results are consistent with the results in the literature.

With the form factors given, it is straightforward to calculate the branching ratios for all the considered semileptonic decays by performing the numerical integration over the whole range of q^2. For the $b \to u$ charged-current process, with $l = (e, \mu)$, the decay rates are the following:

TABLE I. The pQCD predictions for the form factors F_0, F_+, and F_T at $q^2 = 0$, and the parametrization constants a and b for $B_c \to D$ and $B_c \to D_s$ transitions.

	$F(0)$	a	b
$F_0^{B_c \to D}$	$0.19 \pm 0.02 \pm 0.02 \pm 0.01 \pm 0.01 \pm 0.01$	$0.038 \pm 0.001 \pm 0.000$	0.0013 ± 0.0001
$F_+^{B_c \to D}$	$0.19 \pm 0.02 \pm 0.02 \pm 0.01 \pm 0.01 \pm 0.01$	$0.059 \pm 0.001 \pm 0.001$	0.0020 ± 0.0001
$F_T^{B_c \to D}$	$0.20 \pm 0.02 \pm 0.02 \pm 0.01 \pm 0.01 \pm 0.01$	$0.070 \pm 0.001 \pm 0.001$	$0.0021^{+0.0000}_{-0.0001}$
$F_0^{B_c \to D_s}$	$0.27 \pm 0.03 \pm 0.03 \pm 0.02 \pm 0.01 \pm 0.01$	$0.039 \pm 0.002 \pm 0.001$	$0.0015^{+0.0001}_{-0.0000}$
$F_+^{B_c \to D_s}$	$0.27 \pm 0.03 \pm 0.03 \pm 0.02 \pm 0.01 \pm 0.01$	$0.061 \pm 0.002 \pm 0.001$	$0.0023^{+0.0001}_{-0.0000}$
$F_T^{B_c \to D_s}$	$0.28 \pm 0.03 \pm 0.03 \pm 0.02 \pm 0.01 \pm 0.01$	$0.073 \pm 0.002 \pm 0.001$	$0.0025^{+0.0000}_{-0.0001}$

TABLE II. The pQCD predictions for form factors $A_{0,1,2}$, V, and $T_{1,2,3}$ at $q^2 = 0$, and the parametrization constants a and b for $B_c \to D^*$ and $B_c \to D_s^*$ transitions.

	$F(0)$	a	b
$A_0^{B_c \to D^*}$	$0.17 \pm 0.02 \pm 0.02 \pm 0.02 \pm 0.01 \pm 0.00$	$0.063 \pm 0.001 \pm 0.001$	$0.0024 \pm 0.0000 \pm 0.0000$
$A_1^{B_c \to D^*}$	$0.18 \pm 0.02 \pm 0.02 \pm 0.02 \pm 0.01 \pm 0.01$	$0.043 \pm 0.001 \pm 0.001$	$0.0018 \pm 0.0001 \pm 0.0001$
$A_2^{B_c \to D^*}$	$0.20 \pm 0.02 \pm 0.02 \pm 0.02 \pm 0.01 \pm 0.01$	$0.067 \pm 0.001 \pm 0.001$	$0.0026 \pm 0.0001 \pm 0.0001$
$V^{B_c \to D^*}$	$0.25 \pm 0.03 \pm 0.03 \pm 0.03 \pm 0.01 \pm 0.01$	$0.073 \pm 0.002 \pm 0.001$	$0.0029 \pm 0.0001 \pm 0.0001$
$T_1^{B_c \to D^*}$	$0.22 \pm 0.02 \pm 0.02 \pm 0.02 \pm 0.01 \pm 0.01$	$0.063 \pm 0.001 \pm 0.001$	$0.0027 \pm 0.0001 \pm 0.0001$
$T_2^{B_c \to D^*}$	$0.22 \pm 0.02 \pm 0.02 \pm 0.02 \pm 0.01 \pm 0.01$	$0.038 \pm 0.001 \pm 0.001$	$0.0017 \pm 0.0001 \pm 0.0001$
$T_3^{B_c \to D^*}$	$0.20 \pm 0.02 \pm 0.02 \pm 0.02 \pm 0.01 \pm 0.01$	$0.077 \pm 0.002 \pm 0.001$	$0.0049 \pm 0.0001 \pm 0.0001$
$A_0^{B_c \to D_s^*}$	$0.21 \pm 0.02 \pm 0.02 \pm 0.02 \pm 0.01 \pm 0.01$	$0.064 \pm 0.001 \pm 0.001$	$0.0031 \pm 0.0002 \pm 0.0001$
$A_1^{B_c \to D_s^*}$	$0.23 \pm 0.02 \pm 0.02 \pm 0.02 \pm 0.01 \pm 0.01$	$0.044 \pm 0.002 \pm 0.001$	$0.0022 \pm 0.0002 \pm 0.0001$
$A_2^{B_c \to D_s^*}$	$0.25 \pm 0.03 \pm 0.03 \pm 0.03 \pm 0.01 \pm 0.01$	$0.069 \pm 0.002 \pm 0.001$	$0.0035 \pm 0.0002^{+0.0002}_{-0.0001}$
$V^{B_c \to D_s^*}$	$0.33 \pm 0.03 \pm 0.03 \pm 0.03 \pm 0.02 \pm 0.01$	$0.075 \pm 0.002 \pm 0.001$	$0.0039 \pm 0.0002 \pm 0.0001$
$T_1^{B_c \to D_s^*}$	$0.28 \pm 0.03 \pm 0.03 \pm 0.03 \pm 0.02 \pm 0.01$	$0.064 \pm 0.001 \pm 0.001$	$0.0035 \pm 0.0002 \pm 0.0001$
$T_2^{B_c \to D_s^*}$	$0.28 \pm 0.03 \pm 0.03 \pm 0.03 \pm 0.02 \pm 0.01$	$0.039 \pm 0.001 \pm 0.001$	$0.0023 \pm 0.0002 \pm 0.0001$
$T_3^{B_c \to D_s^*}$	$0.27 \pm 0.03 \pm 0.03 \pm 0.03 \pm 0.02 \pm 0.01$	$0.082 \pm 0.002 \pm 0.001$	$0.0068 \pm 0.0002 \pm 0.0002$

TABLE III. Comparison of $B_c \to D_{(s)}^{(*)}$ transition form factors at $q^2 = 0$ evaluated in this paper with other methods.

$B_c \to D^{(*)}$	$F_+(0) = F_0(0)$	$F_T(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$	$V(0)$	$T_1(0) = T_2(0)$	$T_3(0)$
pQCD	0.19	0.20	0.17	0.18	0.20	0.25	0.22	0.20
Ref. [32]	0.16	–	0.09	0.08	0.07	0.13	–	–
Ref. [40]	0.14	–	0.14	0.17	0.19	0.18	–	–
Ref. [41]	0.189	–	0.284	0.146	0.158	0.296	–	–
Ref. [42]	0.35	–	0.05	0.32	0.57	0.57	–	–
Ref. [43]	0.32	–	0.35	0.43	0.51	1.66	–	–
Ref. [44]	0.075	–	0.081	0.095	0.11	0.16	–	–
$B_c \to D_s^{(*)}$	$F_+(0) = F_0(0)$	$F_T(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$	$V(0)$	$T_1(0) = T_2(0)$	$T_3(0)$
pQCD	0.27	0.28	0.21	0.23	0.25	0.33	0.28	0.27
Ref. [32]	0.28	–	0.17	0.14	0.12	0.23	–	–
Ref. [43]	0.45	–	0.47	0.56	0.65	2.02	–	–
Ref. [44]	0.15	–	0.16	0.18	0.20	0.29	–	–

TABLE IV. The pQCD predictions for the branching ratios of the considered decays ($l = e, \mu$).

Decay modes	pQCD predictions
$\mathrm{Br}(B_c^- \to D^- l^+ l^-)$	$(3.79^{+1.16}_{-0.86}(\omega_{B_c})^{+0.81}_{-0.74}(f_{B_c})^{+0.35}_{-0.32}(m_c)^{+0.37}_{-0.35}(C_D)^{+0.33}_{-0.32}(f_D) \pm 0.34(\tau_{B_c})) \times 10^{-9}$
$\mathrm{Br}(B_c^- \to D^- \tau^+ \tau^-)$	$(1.03^{+0.38}_{-0.27}(\omega_{B_c})^{+0.22}_{-0.20}(f_{B_c})^{+0.12}_{-0.10}(m_c)^{+0.09}_{-0.08}(C_D) \pm 0.09(f_D) \pm 0.09(\tau_{B_c})) \times 10^{-9}$
$\mathrm{Br}(B_c^- \to D^- \bar{\nu}\nu)$	$(3.13^{+0.96}_{-0.71}(\omega_{B_c})^{+0.67}_{-0.61}(f_{B_c})^{+0.30}_{-0.26}(m_c)^{+0.31}_{-0.29}(C_D)^{+0.28}_{-0.26}(f_D) \pm 0.28(\tau_{B_c})) \times 10^{-8}$
$\mathrm{Br}(B_c^- \to D_s^- l^+ l^-)$	$(1.56^{+0.46}_{-0.36}(\omega_{B_c})^{+0.33}_{-0.30}(f_{B_c})^{+0.17}_{-0.15}(m_c)^{+0.13}_{-0.12}(C_{D_s}) \pm 0.07(f_{D_s}) \pm 0.14(\tau_{B_c})) \times 10^{-7}$
$\mathrm{Br}(B_c^- \to D_s^- \tau^+ \tau^-)$	$(0.38^{+0.13}_{-0.10}(\omega_{B_c})^{+0.08}_{-0.07}(f_{B_c})^{+0.05}_{-0.04}(m_c) \pm 0.03(C_{D_s}) \pm 0.02(f_{D_s}) \pm 0.03(\tau_{B_c})) \times 10^{-7}$
$\mathrm{Br}(B_c^- \to D_s^- \bar{\nu}\nu)$	$(1.29^{+0.39}_{-0.30}(\omega_{B_c})^{+0.28}_{-0.25}(f_{B_c})^{+0.14}_{-0.12}(m_c)^{+0.11}_{-0.10}(C_{D_s}) \pm 0.06(f_{D_s}) \pm 0.11(\tau_{B_c})) \times 10^{-6}$
$\mathrm{Br}(B_c^- \to D^{*-} l^+ l^-)$	$(1.21^{+0.36}_{-0.28}(\omega_{B_c})^{+0.26}_{-0.23}(f_{B_c})^{+0.14}_{-0.12}(m_c) \pm 0.11(C_{D^*})^{+0.25}_{-0.23}(f_{D^*}) \pm 0.11(\tau_{B_c})) \times 10^{-8}$
$\mathrm{Br}(B_c^- \to D^{*-} \tau^+ \tau^-)$	$(0.16^{+0.05}_{-0.04}(\omega_{B_c}) \pm 0.03(f_{B_c}) \pm 0.02(m_c) \pm 0.01(C_{D^*})^{\pm}0.03(f_{D^*}) \pm 0.01(\tau_{B_c})) \times 10^{-8}$
$\mathrm{Br}(B_c^- \to D^{*-} \bar{\nu}\nu)$	$(1.10^{+0.34}_{-0.26}(\omega_{B_c})^{+0.24}_{-0.21}(f_{B_c})^{+0.13}_{-0.11}(m_c) \pm 0.10(C_{D^*})^{+0.23}_{-0.21}(f_{D^*}) \pm 0.10(\tau_{B_c})) \times 10^{-7}$
$\mathrm{Br}(B_c^- \to D_s^{*-} l^+ l^-)$	$(4.40^{+1.40}_{-1.05}(\omega_{B_c})^{+0.95}_{-0.85}(f_{B_c})^{+0.72}_{-0.57}(m_c)^{+0.32}_{-0.31}(C_{D_s^*})^{+0.92}_{-0.84}(f_{D_s^*}) \pm 0.39(\tau_{B_c})) \times 10^{-7}$
$\mathrm{Br}(B_c^- \to D_s^{*-} \tau^+ \tau^-)$	$(0.52^{+0.18}_{-0.13}(\omega_{B_c})^{+0.11}_{-0.10}(f_{B_c})^{+0.10}_{-0.08}(m_c) \pm 0.03(C_{D_s^*})^{+0.11}_{-0.10}(f_{D_s^*}) \pm 0.05(\tau_{B_c})) \times 10^{-7}$
$\mathrm{Br}(B_c^- \to D_s^{*-} \bar{\nu}\nu)$	$(4.04^{+1.30}_{-0.97}(\omega_{B_c})^{+0.87}_{-0.78}(f_{B_c})^{+0.68}_{-0.53}(m_c)^{+0.29}_{-0.28}(C_{D_s^*})^{+0.85}_{-0.77}(f_{D_s^*}) \pm 0.36(\tau_{B_c})) \times 10^{-6}$

$$\begin{aligned}
\mathrm{Br}(B_c^- \to \bar{D}^0 l^- \bar{\nu}_l) &= (3.15^{+0.97}_{-0.72}(\omega_{B_c})^{+0.68}_{-0.61}(f_{B_c})^{+0.29}_{-0.27}(m_c)^{+0.31}_{-0.29}(C_D)^{+0.28}_{-0.27}(f_D) \pm 0.28(\tau_{B_c})) \times 10^{-5}, \\
\mathrm{Br}(B_c^- \to \bar{D}^0 \tau^- \bar{\nu}_\tau) &= (2.16^{+0.72}_{-0.52}(\omega_{B_c})^{+0.46}_{-0.42}(f_{B_c})^{+0.22}_{-0.19}(m_c)^{+0.20}_{-0.19}(C_D)^{+0.19}_{-0.18}(f_D) \pm 0.19(\tau_{B_c})) \times 10^{-5}, \\
\mathrm{Br}(B_c^- \to \bar{D}^{*0} l^- \bar{\nu}_l) &= (1.09^{+0.34}_{-0.26}(\omega_{B_c})^{+0.23}_{-0.21}(f_{B_c})^{+0.13}_{-0.11}(m_c) \pm 0.10(C_{D^*})^{+0.23}_{-0.21}(f_{D^*}) \pm 0.10(\tau_{B_c})) \times 10^{-4}, \\
\mathrm{Br}(B_c^- \to \bar{D}^{*0} \tau^- \bar{\nu}_\tau) &= (0.64^{+0.20}_{-0.16}(\omega_{B_c})^{+0.14}_{-0.12}(f_{B_c})^{+0.08}_{-0.07}(m_c)^{+0.06}_{-0.05}(C_{D^*})^{+0.13}_{-0.12}(f_{D^*}) \pm 0.06(\tau_{B_c})) \times 10^{-4},
\end{aligned} \tag{34}$$

where the errors come from the uncertainties of $\omega_{B_c} = 1.0 \pm 0.1$, $f_{B_c} = 0.489 \pm 0.050$ GeV, $m_c = (1.275 \pm 0.025)$ GeV, $C_{D^{(*)}} = 0.5 \pm 0.1$, $f_D = (206.7 \pm 8.9)$ MeV or $f_{D^*} = (270 \pm 27)$ MeV, and $\tau_{B_c} = (0.45 \pm 0.04)$ ps, respectively.

For the flavor-changing neutral-current processes, after performing the numerical integration over the whole range of $4m_l^2 \le q^2 \le (m_{B_c} - m)^2$, we get the pQCD predictions for the branching ratios of the considered decay modes, which are listed in Table IV. The errors of the pQCD predictions in Table IV come from the uncertainties of ω_{B_c}, m_c, $C_{D^{(*)}}$ or $C_{D_s^{(*)}}$, $f_{D^{(*)}}$ or $f_{D_s^{(*)}}$, and τ_{B_c}, respectively.

From the pQCD predictions for the form factors $F_{0,+,T}$ in Table I, the form factors V, $A_{0,1,2}$, and $T_{1,2,3}$ in Table II, and the pQCD predictions for the branching ratios as listed in Eq. (34) and in Table IV, we have the following points:

(i) All the form factors for the transitions $B_c \to D_s^{(*)}$ are larger than the corresponding values for the transitions $B_c \to D^{(*)}$ at $q^2 = 0$, which characterizes the SU(3)-breaking effect.

(ii) By definition, $F_0(0)$ is equal to $F_+(0)$ for the $B_c \to D$ or $B_c \to D_s$ transition, but they have different q^2 dependencies due to the different parameters (a, b). $T_1(0)$ is equal to $T_2(0)$ for the $B_c \to D^*$ or $B_c \to D_s^*$

transition [by Eq. (17)]—as shown in Table II—although their expressions are different in Eqs. (25) and (26).

(iii) Because of the phase-space suppression, the branching ratios of the decay modes with a τ in the final product are smaller than decay modes with an electron or muon in the final product for the charged-current process. For the flavor-changing neutral-current processes with two τ's in the final product, the branching ratios are much smaller than the corresponding decays with electron or muon pairs in the final product.

(iv) The branching ratios of the decay modes with $\bar{\nu}\nu$ are almost an order of magnitude larger than the corresponding decays with l^+l^- after the summation over the three neutrino generations. Because of the strong suppression of the CKM factor $|V_{td}/V_{ts}|^2 = |0.211|^2$ [24], the branching ratios for the decay modes with $b \to d$ transitions are much smaller than those with $b \to s$ transitions.

In order to reduce the theoretical uncertainty of the form-factor calculations, we define two ratios R_D and R_{D^*} among the branching ratios for the charged-current processes,

$$R_D = \frac{\mathrm{Br}(B_c^- \to \bar{D}^0 \tau^- \bar{\nu}_\tau)}{\mathrm{Br}(B_c^- \to \bar{D}^0 l^- \nu_l)} = 0.69 \pm 0.01(\omega_{B_c})^{+0.01}_{-0.00}(m_c), \quad (35)$$

$$R_{D^*} = \frac{\mathrm{Br}(B_c^- \to \bar{D}^{*0} \tau^- \bar{\nu}_\tau)}{\mathrm{Br}(B_c^- \to \bar{D}^{*0} l^- \nu_l)} = 0.59^{+0.00}_{-0.01}(\omega_{B_c})^{+0.00}_{-0.01}(m_c), \quad (36)$$

with $l = (e, \mu)$. These two relations will be tested by experiments.

V. SUMMARY

In this paper we studied the $B_c \to (D_{(s)}, D^*_{(s)})$ transition form factors $F_{0,+,T}(q^2)$ and $V(q^2)$, $A_{0,1,2}(q^2)$, $T_{1,2,3}(q^2)$ in the pQCD factorization approach based on k_T factorization. The pQCD predictions for the values of the $B_c \to D_{(s)}$ and $B_c \to D^*_{(s)}$ transition form factors agree with those obtained using other methods. Utilizing these form factors, we calculated the branching ratios for all of the semileptonic decays of $B_c^+ \to D^{(*)}_{(s)}(l^+\nu_l, l^+l^-, \nu\bar{\nu})$. Because of the phase-space suppression, the production ratios of the decay modes with a τ lepton in the final product are smaller than the corresponding decays with an electron or muon in the final product. The branching ratios of the decay modes with $\bar{\nu}\nu$ are almost an order of magnitude larger than the corresponding decays with l^+l^- after the summation over the three neutrino generations. The branching ratios for the decays with $b \to d$ transitions are much smaller than those with $b \to s$ transitions.

In order to reduce the theoretical uncertainty of the pQCD predictions, we defined two ratios R_D and R_{D^*} among the branching ratios for the charged-current processes. The pQCD predictions are

$$R_D = \frac{\mathrm{Br}(B_c^- \to \bar{D}^0 \tau^- \bar{\nu}_\tau)}{\mathrm{Br}(B_c^- \to \bar{D}^0 l^- \nu_l)} \approx 0.7, \quad (37)$$

$$R_{D^*} = \frac{\mathrm{Br}(B_c^- \to \bar{D}^{*0} \tau^- \bar{\nu}_\tau)}{\mathrm{Br}(B_c^- \to \bar{D}^{*0} l^- \nu_l)} \approx 0.6, \quad (38)$$

with $l = (e, \mu)$. It would be possible to test these predictions at the LHCb and the forthcoming Super B experiments.

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