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Baryons with two heavy quarks: Masses, production, decays, and detection

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The large number of B_c mesons observed by LHCb suggests a sizable cross section for producing doubly heavy baryons in the same experiment. Motivated by this, we estimate masses of the doubly heavy $J = 1/2$ baryons Ξ_{cc}, Ξ_{bb}, and Ξ_{bc}, and their $J = 3/2$ hyperfine partners, using a method which accurately predicts the masses of ground-state baryons with a single heavy quark. We obtain $M(\Xi_{cc}) = 3627 \pm 12$ MeV, $M(\Xi^*_{cc}) = 3690 \pm 12$ MeV, $M(\Xi_{bb}) = 10162 \pm 12$ MeV, $M(\Xi^*_{bb}) = 10184 \pm 12$ MeV, $M(\Xi_{bc}) = 6914 \pm 13$ MeV, $M(\Xi'_{bc}) = 6933 \pm 12$ MeV, and $M(\Xi^*_{bc}) = 6969 \pm 14$ MeV. As a byproduct, we estimate the hyperfine splitting between B^*_c and B_c mesons to be 68 ± 8 MeV. We discuss P-wave excitations, production mechanisms, decay modes, lifetimes, and prospects for detection of the doubly heavy baryons.

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I. INTRODUCTION

Some simple arguments based on the quark model have been shown to accurately predict the spectrum of baryons containing a single b quark [1,2]. The question then arises: Can such methods be applied to systems with two or more heavy quarks? So far the only experimental evidence for such states comes from the SELEX experiment, which has reported a state at 3520 MeV containing two charm quarks and a down quark [3,4], with a conference report of states at 3460 and 3780 MeV containing two charm quarks and an up quark [5]. Despite several searches [6–10], no other experiment has confirmed this result. On the optimistic side, one should notice that a large number of B_c mesons has been seen both by the Tevatron experiments [11,12] mand by LHCb [13–19]. From this one can infer [20] a substantial cross section for simultaneous production of two pairs of heavy quarks and their subsequent coalescence into a doubly heavy hadron.

In this paper we estimate the mass of the lowest-lying $J = 1/2$ ccu or ccd state, finding a value consistent with many other estimates lying well above the SELEX results. We estimate its branching fractions to various final states and discuss the possibility of observing bcu, bcd, bbu, and bbd ground-state baryons. We also estimate the masses of the hyperfine ($J = 3/2$) partners of these states, comment briefly on P-wave excitations, and discuss production, decays, and detection of these states.

In order to have a self-contained discussion, we review calculations based on similar methods for baryons and mesons containing only u, d, and s quarks (Sec. II) and those containing a single charmed quark (Sec. III) or a single bottom quark (Sec. IV). These last two sections also include for completeness discussions of states with both charm (or beauty) and strangeness. Although we do not discuss ccs, bcs, or bbs states in the present paper, regarding their observation as far in the future, we give enough information that their masses may be readily calculated using the present methods.

In what follows we shall neglect the difference between the masses of u and d, referring to them collectively as q. Masses of states with nonzero isospin are taken to be isospin averages. (Isospin splittings of doubly heavy baryons are expected not to exceed several MeV [21,22].) We calculate the masses of the lowest-lying states of ccq in Sec. V, bbq in Sec. VI, and bcq in Sec. VII, commenting briefly on P-wave excitations in Sec. VIII. Likely decay modes are noted in Sec. IX, some suggestions for observing the states are made in Sec. X, while Sec. XI concludes.

II. STATES CONTAINING ONLY u, d, AND s QUARKS

A. Baryons

The following contributions suffice to describe the ground-state baryons containing u, d, s [23,24]:

(i) The effective masses of the u, d, and s quarks.
(ii) Their mutual hyperfine interactions.

(With the addition of heavy-quark masses, these methods were already used in Refs. [23] and [25] to estimate masses of baryons with two heavy quarks.)

In Table I we summarize that description. For all masses we use values quoted by the Particle Data Group [26]

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TABLE I. Quark model description of ground-state baryons containing u, d, s. Here we take $m_u^b = m_d^b \equiv m_q^b = 363$ MeV, $m_s^b = 538$ MeV, and hyperfine interaction term $a/(m_q^b)^2 = 50$ MeV.

State (mass in MeV)	Spin	Expression for mass [24]	Predicted mass (MeV)
$N(939)$	1/2	$3m_q^b - 3a/(m_q^b)^2$	939
$\Delta(1232)$	3/2	$3m_q^b + 3a/(m_q^b)^2$	1239
$\Lambda(1116)$	1/2	$2m_q^b + m_s^b - 3a/(m_q^b)^2$	1114
$\Sigma(1193)$	1/2	$2m_q^b + m_s^b + a/(m_q^b)^2 - 4a/m_q^b m_s^b$	1179
$\Sigma(1385)$	3/2	$2m_q^b + m_s^b + a/(m_q^b)^2 + 2a/m_q^b m_s^b$	1381
$\Xi(1318)$	1/2	$2m_s^b + m_q^b + a/(m_s^b)^2 - 4a/m_q^b m_s^b$	1327
$\Xi(1530)$	3/2	$2m_s^b + m_q^b + a/(m_s^b)^2 + 2a/m_q^b m_s^b$	1529
$\Omega(1672)$	3/2	$3m_s^b + 3a/(m_s^b)^2$	1682

unless otherwise noted. Effective masses of quarks in baryons and mesons can and do differ from one another [27], so we shall use superscripts b and m to denote the former and latter. The parameters of this table then may be interpreted as summarizing all interactions between qq, qs, and ss. We shall assume these same interactions occur also in a baryon containing one c or b quark. The average magnitude of the errors in this description is about 5 MeV. We shall use a similar method [23,28], with appropriate corrections, to calculate masses of states with one or two heavy quarks.

B. Mesons

A similar approach describes ground-state mesons composed of u, d, and s quarks, as shown in Table II. As effective masses of quarks in mesons and baryons differ from one another, the parameters in Table II will not be directly related to those in Table I. We do not discuss η, η', whose masses are strongly affected by octet-singlet mixing. Here the average magnitude of errors is about 6 MeV.

The overprediction of the ϕ mass may indicate slightly stronger binding between two strange quarks. We should keep this possibility in mind when discussing other states with two strange quarks, but these do not occur for $\Xi_{(cc,bb,bc)}$. Some hint of this effect is also present when comparing the predicted $M(\Xi)$ and $M(\Omega)$ with experiment, though the predicted $M(\Xi^*)$ comes within 1 MeV of the observed value.

TABLE II. Quark model description of ground-state mesons containing u, d, s. Here we take $m_u^m = m_d^m \equiv m_q^m = 310$ MeV, $m_s^m = 483$ MeV, $b/(m_q^m)^2 = 80$ MeV.

State (mass in MeV)	Spin	Expression for mass [24]	Predicted mass (MeV)
$\pi(138)$	0	$2m_q^m - 6b/(m_q^m)^2$	140
$\rho(775)$, $\omega(782)$	1	$2m_q^m + 2b/(m_q^m)^2$	780
$K(496)$	0	$m_q^m + m_s^m - 6b/(m_q^m m_s^m)$	485
$K^*(894)$	1	$m_q^m + m_s^m + 2b/(m_q^m m_s^m)$	896
$\phi(1019)$	1	$2m_s^m + 2b/(m_s^m)^2$	1032

III. STATES WITH ONE CHARMED QUARK

A. Mesons

We discuss mesons first because the $c\bar{s}$ interaction in $D_s^{(*)}$ displays a significant binding effect. This is then related using a simple QCD argument to the cs binding in baryons, which is important to keep in mind when predicting $\Xi_c^{(\prime,*)}$ and $\Omega_c^{(*)}$ masses.

The model of Sec. II predicts

$$M(D(1867.2)) = m_q^m + m_c^m - 6b/(m_q^m m_c^m),$$
$$M(D^*(2008.6)) = m_q^m + m_c^m + 2b/(m_q^m m_c^m). \quad (1)$$

The new parameter in these expressions is m_c^m, which may be estimated using

$$m_c^m = [3M(D^*) + M(D)]/4 - m_q^m = (1973.3 - 310)\ \text{MeV} = 1663.3\ \text{MeV}. \quad (2)$$

Using this value and $b/(m_q^m)^2 = 80$ MeV one estimates the hyperfine splitting between D and D^* to be $M(D^*) - M(D) = 8b/(m_q^m m_c^m) = 119.3$ MeV, to be compared with the observed value of 141.4 MeV. Thus there seems to be a hyperfine enhancement between c and $\bar{q}$ relative to q and $\bar{q}$. This difference does not seem to occur between cq and qq hyperfine interactions, however, as we shall see when discussing charmed baryons.

Charmed-strange mesons display an effect of enhanced $c\bar{s}$ binding. Anticipating this, we may write

$$M(D_s(1968.5)) = B(c\bar{s}) + m_s^m + m_c^m - 6b/(m_s^m m_c^m),$$
$$M(D_s^*(2112.3)) = B(c\bar{s}) + m_s^m + m_c^m + 2b/(m_s^m m_c^m), \quad (3)$$

allowing one to solve for the binding term

$$B(c\bar{s}) = [3M(D_s^*) + M(D_s)]/4 - m_s^m - m_c^m = -69.9\ \text{MeV}. \quad (4)$$

This quantity will be related to the binding between c and s quarks when we discuss charmed-strange baryons. This term represents the additional binding to c of the heavier $\bar{s}$ quark in comparison with that of the $\bar{u}$ or $\bar{d}$, due to the shorter Compton wavelength of the $\bar{s}$ which allows it to sit more deeply in the interquark potential.

Comparing Eqs. (1) and (3), one would conclude that

$$M(D_s^*) - M(D_s) = (m_q^m/m_s^m)[M(D^*) - M(D)] = 90.6\ \text{MeV}, \quad (5)$$

TABLE III. Relative attraction or repulsion $\langle T_1 \cdot T_2 \rangle$ of quarks $Q\bar{Q}$ or QQ in various states.

State	Color	$\langle T_1 \cdot T_2 \rangle$
$Q\bar{Q}$	1	$-4/3$
$Q\bar{Q}$	8	$1/6$
QQ	3^*	$-2/3$
QQ	6	$1/3$

a factor of 0.63 times the observed value of 143.8 MeV which is almost the same as $M(D^*) - M(D)$. The scaling of the wave function describing the $c\bar{s}$ or $c\bar{q}$ bound state in a confining potential accounts for this behavior [29]. We shall estimate the cs hyperfine interaction in baryons directly from the $\Omega_c^* - \Omega_c$ splitting, finding a similar enhancement with respect to the nominal value implied by Table I.

B. Baryons

An approach to charmed baryon masses similar to that leading to the predictions for u, d, s baryons in Table I must take account of enhanced cs binding and an enhanced cs hyperfine interaction. The effect of cs binding may be related to $c\bar{s}$ binding by means of a color-SU(3) argument. The interactions between two quarks in various color states are summarized in Table III. The quarks in a $c\bar{s}$ meson are in a color singlet, while a cs pair in a baryon is in a color antitriplet. The cs interaction strength in a color triplet is half that of $c\bar{s}$ in a color singlet, so we shall assume, for every cs pair in a charmed-strange baryon, that

$$B(cs) = B(c\bar{s})/2 = -35.0 \text{ MeV}. \quad (6)$$

As we shall see, this provides a contribution of reasonable magnitude.

The scaling of energy levels linearly with coupling strength is not an automatic feature. In a power-law central potential of the form $V(r) = \lambda r^\nu$, spacings ΔE of energy levels depend on λ via the relation [30] $\Delta E \propto \lambda^{2/(2+\nu)}$. Thus, in the Coulomb potential ($\nu = -1$) the Rydberg scales as α^2; harmonic oscillator level spacings ($\nu = 2$) scale as the square root of the force constant; and $\Delta E \propto \lambda$ for a logarithmic potential, which has been shown to interpolate not only between charmonium and bottomonium interactions [30], but also to apply approximately to $s\bar{s}$ excitations [31].

The hyperfine splitting between Ω_c^* and Ω_c would be given by $6a/(m_s^b m_c^b)$, but we shall parametrize it independently by replacing a with a_{cs}. Accounting for enhanced cs binding and hyperfine interaction, the predictions for baryon masses then may be summarized in Table IV. Here we have used the experimental value of $M(\Lambda_c)$ in Table IV to estimate $m_c^b = M(\Lambda_c) - 2m_q^b + 3a/(m_q^b)^2 = 1710.5$ MeV.

The hyperfine splitting between Σ_c^* and Σ_c is predicted to be $6a/(m_q^b m_c^b) = 63.7$ MeV, to be compared with the observed value of 64.5 MeV. Thus there does not seem to be an enhancement of the hyperfine interaction between c and q over the value inferred from Table I.

The states Ξ_c and Ξ'_c will mix with one another as a result of SU(3) breaking. This effect, leading to mass shifts of the order of several MeV [32], has been ignored.

The naive hyperfine term $6a/(m_s^b m_c^b) = 43.0$ MeV is 0.61 times a term $6a_{cs}/(m_s^b m_c^b) = 70.7$ MeV evaluated using the splitting between Ω_c^* and Ω_c. Thus the cs hyperfine interaction in baryons undergoes the same enhancement with regard to the naive value as does the $c\bar{s}$ hyperfine interaction in mesons.

The average magnitude of the errors in the predictions of Table IV is about 9 MeV, not much higher than that for the light-quark baryons in Table I.

IV. STATES WITH ONE b QUARK

A. Mesons

We discuss B_s and B_s^* mesons in order to estimate binding effects of a b quark with an $\bar{s}$ antiquark, so as to assess bs binding in a baryon, and in order to obtain an

TABLE IV. Quark model description of ground-state baryons containing one charmed quark. Here we take $m_u^b = m_d^b \equiv m_q^b = 363$ MeV, $m_s^b = 538$ MeV, $m_c^b = 1710.5$ MeV, and $a/(m_q^b)^2 = 50$ MeV. The spin of the qs pair is taken to be zero in Ξ_c and one in Ξ'_c.

State (M in MeV)	Spin	Expression for mass	Predicted M (MeV)
$\Lambda_c(2286.5)$	1/2	$2m_q^b + m_c^b - 3a/(m_q^b)^2$	Input
$\Sigma_c(2453.4)$	1/2	$2m_q^b + m_c^b + a/(m_q^b)^2 - 4a/(m_q^b m_c^b)$	2444.0
$\Sigma_c^*(2518.1)$	3/2	$2m_q^b + m_c^b + a/(m_q^b)^2 + 2a/(m_q^b m_c^b)$	2507.7
$\Xi_c(2469.3)$	1/2	$B(cs) + m_q^b + m_s^b + m_c^b - 3a/(m_q^b m_s^b)$	2475.3
$\Xi'_c(2575.8)$	1/2	$B(cs) + m_q^b + m_s^b + m_c^b + a/(m_q^b m_s^b) - 2a/(m_q^b m_c^b) - 2a_{cs}/(m_s^b m_c^b)$	2565.4
$\Xi_c^*(2645.9)$	3/2	$B(cs) + m_q^b + m_s^b + m_c^b + a/(m_q^b m_s^b) + a/(m_q^b m_c^b) + a_{cs}/(m_s^b m_c^b)$	2632.6
$\Omega_c(2695.2)$	1/2	$2B(cs) + 2m_s^b + m_c^b + a/(m_s^b)^2 - 4a_{cs}/(m_s^b m_c^b)$	2692.1[a]
$\Omega_c^*(2765.9)$	3/2	$2B(cs) + 2m_s^b + m_c^b + a/(m_s^b)^2 + 2a_{cs}/(m_s^b m_c^b)$	2762.8[a]

[a]Difference between experimental values used to determine $6a_{cs}/(m_s^b m_c^b) = 70.7$ MeV.

effective mass of a b quark in a meson. The model of Sec. II predicts

$$M(B(5279.4)) = m_q^m + m_b^m - 6b/(m_q^m m_b^m),$$
$$M(B^*(5325.2)) = m_q^m + m_b^m + 2b/(m_q^m m_b^m). \quad (7)$$

By a calculation similar to that in Sec. III, one finds

$$m_b^m = [3M(B^*) + M(B)]/4 - m_q^m$$
$$= (5313.8 - 363)\ \text{MeV} = 5003.8\ \text{MeV}. \quad (8)$$

The predicted hyperfine splitting is $M(B^*) - M(B) =$ 39.7 MeV, a factor of 0.87 times the observed value of 45.8 MeV. For comparison, the predicted hyperfine splitting $M(D^*) - M(D)$ was found in the previous section to be 119.3 MeV, a factor of 0.84 times the observed value of 141.4 MeV. This near-equality is a consequence of the often-quoted relation

$$(45.78 \pm 0.35)\ \text{MeV} = M(B^*) - M(B)$$
$$= (m_c^m/m_b^m)[M(D^*) - M(D)]$$
$$= (47.0 \pm 0.1)\ \text{MeV}, \quad (9)$$

in which light-quark masses do not appear.

Allowing for a binding term $B(b\bar{s})$, the pseudoscalar and vector $b\bar{s}$ states have masses

$$M(B_s(5366.77 \pm 0.24))$$
$$= B(b\bar{s}) + m_s^m + m_b^m - 6b/(m_s^m m_b^m),$$
$$M(B_s^*(5415.4^{+2.4}_{-2.1}))$$
$$= B(b\bar{s}) + m_s^m + m_b^m + 2b/(m_s^m m_b^m), \quad (10)$$

where we have indicated errors on masses in MeV because those of B_s^* are non-negligible. Repeating the calculation of the previous section, we find

$$B(b\bar{s}) = [3M(B_s^*) + M(B_s)]/4 - m_b^m - m_s^m$$
$$= (-83.6 \pm 1.8)\ \text{MeV}. \quad (11)$$

This binding term is slightly larger than the value of $B(c\bar{s})$ found above, because the reduced mass of the $b\bar{s}$ system is greater than that of $c\bar{s}$, leading to a shorter Compton wavelength and a more deeply bound system.

The predicted hyperfine splitting between B_s and B_s^* is $8a/(m_b m_s) = 25.5$ MeV, to be compared with the observed value of $48.7^{+2.3}_{-2.1}$ MeV. Alternatively, one may evaluate this quantity to be m_q^m/m_s^m times the observed value of $M(B^*) - M(B) = 45.8$ MeV, giving 29.4 MeV or a factor of 0.60 ± 0.03 times the observed value. For comparison, the same scaling argument applied in Sec. III gave $M(D_s^*) - M(D_s)$ a factor of 0.63 times its observed value. Thus the relation

$$48.7^{+2.3}_{-2.1}\ \text{MeV} = M(B_s^*) - M(B_s)$$
$$\simeq (m_c^m/m_b^m)[M(D_s^*) - M(D_s)]$$
$$= 47.8\ \text{MeV}, \quad (12)$$

in which light-quark masses do not appear, holds quite well.

B. Baryons

Recent progress in b-flavored baryon studies has been so great that we have found it necessary to construct our own averages of masses. These are summarized in Table V. We have omitted measurements superseded by those of higher statistics by the same collaboration, and measurements older than 2011.

We start with a value of the b quark mass in baryons obtained from the observed value of $M(\Lambda_b) =$ 5619.5 ± 0.3 MeV:

$$m_b^b = M(\Lambda_b) - 2m_q + 3a/(m_q^b)^2 = 5043.5\ \text{MeV}. \quad (13)$$

The observed and calculated masses of the ground-state b-flavored baryons are summarized in Table VI. We note several points.

TABLE V. Averages of b-baryon masses based on recent experiments.

Baryon	Reference	Mass (MeV)
Λ_b	[33]	5619.30 ± 0.34
	[34]	$5620.15 \pm 0.31 \pm 0.47$
	[35]	$5619.7 \pm 0.7 \pm 1.1$
	Average	5619.5 ± 0.3
Σ_b^+	[36]	$5811.3^{+0.9}_{-0.8} \pm 1.7$
Σ_b^-	[36]	$5815.5^{+0.6}_{-0.5} \pm 1.7$
Average[a]	(Over charges)	5814.26 ± 1.76
Σ^{*+}	[36]	$5832.1 \pm 0.7^{+1.7}_{-1.8}$
Σ^{*-}	[36]	$5835.1 \pm 0.6^{+1.7}_{-1.8}$
Average[a]	(Over charges)	5833.83 ± 1.81
Ξ_b^0	[37]	5793.5 ± 2.3
	[34]	$5788.7 \pm 4.3 \pm 1.4$
	[38]	$5791.80 \pm 0.39 \pm 0.17 \pm 0.26$
	Average	5791.84 ± 0.50
Ξ_b^-	[39]	$5795.8 \pm 0.9 \pm 0.4$
	[34]	$5793.4 \pm 1.8 \pm 0.7$
	Average	5795.30 ± 0.88
Average	(Over charges)	5792.68 ± 0.43
Ξ_b^{*0}	[40]	5949.71 ± 1.25[b]
Ω_b^-	[39]	$6046.0 \pm 2.2 \pm 0.5$
	[34]	$6047.5 \pm 3.8 \pm 0.6$
	Average	6046.38 ± 1.95

[a]Common systematic error added in quadrature.
[b]Reference [40] quotes $M(\Xi_b^{*0}) - M(\Lambda_b) - M(\pi^+) =$ $(14.84 \pm 0.74 \pm 0.28)$ MeV.

TABLE VI. Quark model description of ground-state baryons containing one bottom quark. Here we take $m_u^b = m_d^b \equiv m_q^b = 363$ MeV, $m_s^b = 538$ MeV, $m_b^b = 5043.5$ MeV, and $a/(m_q^b)^2 = 50$ MeV. The spin of the qs pair is taken to be zero in Ξ_b and one in Ξ'_b. The parameter a_{bs} is rescaled from a in the same manner as for charmed baryons: $a_{bs} = a_{cs} = (70.7/43.0)a$.

State (M in MeV)	Spin	Expression for mass	Predicted M (MeV)
$\Lambda_b(5619.5)$	1/2	$2m_q^b + m_b^b - 3a/(m_q^b)^2$	Input
$\Sigma_b(5814.3)$	1/2	$2m_q^b + m_b^b + a/(m_q^b)^2 - 4a/(m_q^b m_b^b)$	5805.1
$\Sigma_b^*(5833.8)$	3/2	$2m_q^b + m_b^b + a/(m_q^b)^2 + 2a/(m_q^b m_b^b)$	5826.7
$\Xi_b(5792.7)$	1/2	$B(bs) + m_q^b + m_s^b + m_b^b - 3a/(m_q^b m_s^b)$	5801.5
$\Xi'_b(-)$	1/2	$B(bs) + m_q^b + m_s^b + m_b^b + a/(m_q^b m_s^b) - 2a/(m_q^b m_b^b) - 2a_{bs}/(m_s^b m_b^b)$	5921.3
$\Xi_b^*(5949.7)$	3/2	$B(bs) + m_q^b + m_s^b + m_b^b + a/(m_q^b m_s^b) + a/(m_q^b m_b^b) + a_{bs}/(m_s^b m_b^b)$	5944.1
$\Omega_b(6046.4)$	1/2	$2B(bs) + 2m_s^b + m_b^b + a/(m_s^b)^2 - 4a_{bs}/(m_s^b m_b^b)$	6042.8
$\Omega_b^*(-)$	3/2	$2B(bs) + 2m_s^b + m_b^b + a/(m_s^b)^2 + 2a_{bs}/(m_s^b m_b^b)$	6066.7

(i) Although the predicted Σ_b and Σ_b^* masses are a bit below the observed ones, their predicted hyperfine splitting is 21.6 MeV, while the observed value is 19.6 ± 0.7 MeV (neglecting a common systematic error of 1.7 MeV). Thus there is no evidence for enhancement of the term $a/(m_q^b m_b^b)$ beyond the value based on Table I.

(ii) The rescaling of $a/(m_s^b m_b^b)$ to $a_{bs}/(m_s^b m_b^b)$ is taken to be identical to that for the cs hyperfine interaction in baryons, which we saw was very close to that for the $c\bar{s}$ and $b\bar{s}$ mesons. It could be tested in principle using the hyperfine difference prediction

$$M(\Omega_b^*) - M(\Omega_b) = 6a_{bs}/(m_s^b m_b^b) = 24.3 \text{ MeV}, \tag{14}$$

but this involves detection of the very soft photon in the decay $\Omega_b^* \to \gamma\Omega_b$, which is probably impossible. The enhancement of a_{cs} and a_{bs} with respect to a is due to the deeper binding of the cs and bs system in comparison with cq or bq, but a quantitative relation between $B(cs)$ and a_{cs} or between $B(bs)$ and a_{bs} does not seem obvious to us. A possible reason for lack of such a relation is that $B(cs)$ and $B(bs)$ parametrize spin-independent binding, while a_{cs} and a_{bs} measure the strength of a spin-dependent interaction between the relevant quarks.

(iii) The predictions for $M(\Xi_b)$ and $M(\Omega_b)$ are not far from those of Ref. [1]: 5795 ± 5 MeV and 6052.1 ± 5.6 MeV, respectively. In that work some use was made of potential models, whereas in the present estimates such effects are parametrized by binding terms or modification of hyperfine interactions.

(iv) The average magnitude of errors in predictions of Table VI is about 8 MeV, a bit below that for charmed baryons in Table IV. We shall use these two errors and those in Table I to extrapolate to the case of two heavy quarks, estimating prediction errors of 12 MeV for $M(\Xi_{cc})$ and $M(\Xi_{bb})$. For $M(\Xi_{bc})$ an additional systematic error is associated with ignorance of the B_c–B_c^* splitting.

V. CALCULATION OF *ccq* MASS

The mass of the ccq state may be regarded as the sum of the following contributions:

(i) The masses of the two charmed quarks.
(ii) Their binding energy in a color 3^* state.
(iii) Their mutual hyperfine interaction.
(iv) Their hyperfine interaction with the light quark q.
(v) The mass of the light quark q.

When more than one heavy quark is present, one must take into account the binding energy between them. We do this by comparing the sum of the charm quark masses in the $1S$ charmonium levels η_c and J/ψ with their spin-weighted mass

$$\bar{M}(c\bar{c}:1S) \equiv [3M(J/\psi) + M(\eta_c)]/4 = 3068.6 \text{ MeV}. \tag{15}$$

We estimated the effective charm quark mass in a meson to be $m_c^m = 1663.3$ MeV. The binding energy in $1S$ charmonium is thus $[3068.6 - 2(1663.3)]$ MeV $= -258.0$ MeV. Using the color-SU(3) relations in Table III we then estimate the cc binding energy in a baryon to be -129.0 MeV.

The cc hyperfine interaction $a_{cc}/{m_c}^2$ is estimated as follows. The $\bar{c}c$ hyperfine splitting in the meson sector is given by $M(J/\psi) - M(\eta_c) = 113.2 \text{ MeV} = 4a_{\bar{c}c}/(m_c^m)^2$. Assuming that the quark-quark interaction a_{cc} is half of the quark-antiquark interaction $a_{\bar{c}c}$, and neglecting the small difference between m_c^m and m_c^b, we have $a_{cc}/(m_c^b)^2 = 1/2 \cdot [M(J/\psi) - M(\eta_c)]/4 = 14.2$ MeV [41].

We may then summarize the contributions to $M(\Xi_{cc})$ in Table VII. The third line gives the contribution of the hyperfine interaction between the two charmed quarks, while the fourth gives their total hyperfine interaction with the light-quark q. The predicted value $M(\Xi_{cc}) = 3627 \pm 12$ MeV lies among a number of other estimates

TABLE VII. Contributions to the mass of the lightest doubly charmed baryon Ξ_{cc}.

Contribution	Value (MeV)
$2m_c^b + m_q^b$	3783.9
cc binding	−129.0
$a_{cc}/(m_c^b)^2$	14.2
$-4a/m_q^b m_c^b$	−42.4
Total	3627 ± 12

summarized in Table VIII, but well above the values claimed for Ξ_{cc}^{+} and Ξ_{cc}^{++} by the SELEX Collaboration.

The hyperfine splitting is given by $M(\Xi_{cc}^*) - M(\Xi_{cc}) = 6a/m_q m_c = 63.7$ MeV, yielding $M(\Xi_{cc}^*) = 3690 \pm 12$ MeV. This state lies too close in mass to Ξ_{cc} to decay to it by pion emission, so it must decay radiatively.

VI. CALCULATION OF *bbq* MASS

One may apply very similar methods to calculate the mass of the lowest-lying Ξ_{bb} state. The spin-weighted average of the $b\bar{b}$:$1S$ levels is

$$\bar{M}(b\bar{b}:\ 1S) \equiv [3M(\Upsilon) + M(\eta_b)]/4 = 9444.7\ \text{MeV}. \tag{16}$$

The spin-weighted average of the ground-state bottom mesons is

$$\bar{M}(b\bar{q}:\ 1S) \equiv [3M(B^*) + M(B)]/4 = 5313.8\ \text{MeV}. \tag{17}$$

Subtracting $m_q^m = 310$ MeV, we arrive at $m_b^m = 5003.8$ MeV. The binding energy in $1S$ bottomonium is thus $[9444.7 - 2(5003.8)]$ MeV $= -562.8$ MeV. By arguments similar to those in the previous section, we then calculate the binding energy between the two b quarks in Ξ_{bb} to be half this, or -281.4 MeV.

The mass of a bottom quark in a baryon, $m_b^b = 5043.5$ MeV, was obtained in Sec. IV. By the same approach as for Ξ_{cc}, the bb hyperfine interaction term $a_{bb}/(m_b^b)^2$ may be taken as $(1/8)\cdot[M(\Upsilon) - M(\eta_b)] = 7.8$ MeV [41].

We summarize the contributions to $M(\Xi_{bb})$ in Table IX. The resulting value $M(\Xi_{bb}) = 10162 \pm 12$ MeV tends to

TABLE VIII. Comparison of predictions for $M(\Xi_{cc})$. Here KS stands for Kogut-Susskind; LGT stands for lattice gauge theory.

Reference	Value (MeV)	Method
Present work	3627 ± 12	
[23]	3550–3760	QCD-motivated quark model
[25]	3668 ± 62	QCD-motivated quark model
[28]	3651	QCD-motivated quark model
[42]	3613	Potential and bag models
[43]	3630	Potential model
[44]	3610	Heavy-quark effective theory
[45]	3660 ± 70	Feynman-Hellmann + semiempirical formulas
[46]	3676	Mass sum rules
[47]	3660	Relativistic quasipotential quark model
[48]	3607	Three-body Faddeev equations.
[49]	3527	Bootstrap quark model + Faddeev Eqs
[50]	ucc: 3649 ± 12, dcc: 3644 ± 12	Quark model
[51]	3480 ± 50	Potential approach + QCD sum rules
[52]	3690	Nonperturbative string
[53]	3620	Relativistic quark-diquark
[54]	3520	Bag model
[55]	3643	Potential model
[56]	3642	Relativistic quark model + Bethe-Salpeter
[57]	3612^{+17}	Variational
[58]	3678	Quark model
[59]	3540 ± 20	Instantaneous approx + Bethe-Salpeter
[60]	4260 ± 190	QCD sum rules
[61]	$3608(15)\binom{13}{35}$, $3595(12)\binom{21}{22}$	Quenched lattice
[62]	3549(13)(19)(92)	Quenched lattice
[63]	$3665 \pm 17 \pm 14^{+0}_{-78}$	Lattice, domain-wall + KS fermions
[64]	3603(15)(16)	Lattice, $N_f = 2 + 1$
[65]	3513(23)(14)	LGT, twisted mass ferm., $m_\pi = 260$ MeV
[66]	3595(39)(20)(6)	LGT, $N_f = 2 + 1$, $m_\pi = 200$ MeV
[67]	3568(14)(19)(1)	LGT, $N_f = 2 + 1$, $m_\pi = 210$ MeV

TABLE IX. Contributions to the mass of the lightest baryon Ξ_{bb} with two bottom quarks.

Contribution	Value (MeV)
$2m_b^b + m_q^b$	10450.0
bb binding	−281.4
$a_{bb}/(m_b^b)^2$	7.8
$-4a/m_q^b m_b^b$	−14.4
Total	10162 ± 12

TABLE X. Comparison of predictions for $M(\Xi_{bb})$.

Reference	Value (MeV)	Method
Present work	10162 ± 12	
[25]	10294 ± 131	QCD-motivated quark model
[28]	10235	QCD-motivated quark model
[43]	10210	Potential models
[45]	10340 ± 100	Feynman-Hellmann+ semiempirical formulas
[47]	10230	Relativistic quasipotential quark model
[51]	10090 ± 50	Potential approach and QCD sum rules
[52]	10160	Nonperturbative string
[54]	10272	Bag model
[58]	10322	Quark model
[59]	10185 ± 5	Instantaneous approx + Bethe-Salpeter
[60]	9780 ± 70	QCD sum rules
[68]	10045	Coupled channel formalism

lie a bit below some (but not all) estimates, as seen in Table X.

The hyperfine splitting is given by $M(\Xi_{bb}^*) - M(\Xi_{bb}) = 6a/m_q^b m_b^b = 21.6$ MeV, yielding $M(\Xi_{bb}^*) = 10184 \pm 12$ MeV. This state decays radiatively to Ξ_{bb}.

VII. CALCULATION OF *bcq* MASS

The methods of the previous two sections may be applied to calculate the ground-state mass of Ξ_{bc}, with one qualification. The $^3S_1$ state of $b\bar{c}$, the B_c^*, has not yet been observed, so we shall have to estimate its mass. One method is to note that hyperfine interactions between quarks with masses m_1 and m_2 are proportional to $|\Psi(0)|^2/(m_1 m_2)$, so we need to evaluate the magnitude of $|\Psi(0)|^2$ for the $b\bar{c}$ system by interpolating between $c\bar{c}$ and $b\bar{b}$.

A convenient parametrization is to assume that $|\Psi(0)|^2$ behaves as some power p of the reduced mass $\mu_R \equiv (m_1 m_2)/(m_1 + m_2)$. With the quark masses $m_c^m = 1663.3$ MeV and $m_b^m = 5003.8$ MeV and the hyperfine splittings

$$M(J/\psi) - M(\eta_c) = 113.2 \text{ MeV},$$
$$M(\Upsilon) - M(\eta_b) = 62.3 \text{ MeV}, \quad (18)$$

one finds this power to be 1.46, very close to the value of 1.5 that one would expect from a logarithmic potential. Such a potential has been shown to successfully interpolate between the charmonium and bottomonium spectra [30], and now seems to give approximately the correct spacing between the 1S and 2S of the B_c system as well [69]. With this power, the hyperfine splitting between b and $\bar{c}$ in the ground state is then estimated to be 68.0 MeV. (This quantity also may be estimated by taking the geometric mean of the charmonium and bottomonium hyperfine splittings, with the result of 84.0 MeV. The 16 MeV difference between these two estimates can be viewed as an indication of the error associated with determining $b\bar{c}$ hyperfine splitting.) The spin-weighted average ground state $b\bar{c}$ mass is then

$$\bar{M}(b\bar{c}: 1S) = M(B_c) + (3/4)(68.0 \text{ MeV}) = (6274.5 + 52.0) \text{ MeV} = 6325.5 \text{ MeV}. \quad (19)$$

The rest of the calculations proceed as in the previous two sections. The binding energy in the spin-weighted average $b\bar{c}$ ground state is $6325.5 - 5003.8 - 1663.2 = -341.5$ MeV, so in a bc baryon it is half this, or -170.8 MeV. The bcq mass (before accounting for binding and hyperfine interactions) is

$$m_b^b + m_c^b + m_q^b = (5043.5 + 1710.5 + 363) \text{ MeV} = 7117.0 \text{ MeV}. \quad (20)$$

The error associated with $c\bar{s}$ binding may be taken to be 3/4 times that of the hyperfine splitting between b and $\bar{c}$, or $(3/4)(16 \text{ MeV}) = 12$ MeV. We then take the error on the cs binding to be 6 MeV. The strength of the bc hyperfine interaction is determined by the same approach as for Ξ_{cc} and Ξ_{bb}, i.e., $a_{bc}/(m_b^b m_c^b) = (1/8) \cdot b\bar{c}$ hyperfine splitting. As a result, a small error also is introduced to the bc hyperfine interaction.

The presence of three distinct quarks in $\Xi_{bc} = bcq$ means that there are two ways of coupling them up to spin 1/2 in an S-wave ground state. Taking the basis defined by the combined spin of the two lightest quarks, as was done for the $\Xi_c = csq$ and $\Xi_b = bsq$, we call the state with $S(cq) = 0$ the Ξ_{bc} and that with $S(cq) = 1$ the Ξ'_{bc}. Tables XI and XII show the respective contributions to their masses, and Tables XIII and XIV compare our predictions with others. The Ξ'_{bc} will decay radiatively to Ξ_{bc}. The uncertainties on the masses of these two states are calculated by adding in quadrature the spread between the two masses in each table and the global error assumed to be 12 MeV.

The mass of the $J = 3/2$ state is given by $M(\Xi_{bc}^*) = M(\Xi'_{bc}) + 3a/(m_q^b m_b^b) + 3a_{bc}/(m_b^b m_c^b) = M(\Xi'_{bc}) +$ 36.3 MeV. Using the $M(\Xi'_{bc})$ value in the first column of Table XII we then obtain $M(\Xi_{bc}^*) = 6969 \pm 14$ MeV. As in previous cases, this state decays radiatively to the $J = 1/2$ ground state.

TABLE XI. Contributions to the mass of the lightest baryon Ξ_{bc} with one bottom and one charmed quark and the cq pair in a spin-singlet state.

Contribution	Value (MeV) from $\lvert\Psi(0)\rvert^2 \sim \mu_R^{1.46}$	Value (MeV) from $\sqrt{\mathrm{HF}(\bar{b}b)\cdot\mathrm{HF}(\bar{c}c)}$
$m_b^b + m_c^b + m_q^b$	7117.0	7117.0
bc binding	−170.8	−164.8
$-3a/(m_c^b m_q^b)$	−31.8	−31.8
Total	6914 ± 13	6920 ± 13

TABLE XII. Contributions to the mass of the lightest baryon Ξ'_{bc} with one bottom and one charmed quark and the cq pair in a spin-triplet state.

Contribution	Value (MeV) from $\lvert\Psi(0)\rvert^2 \sim \mu_R^{1.46}$	Value (MeV) from $\sqrt{\mathrm{HF}(\bar{b}b)\cdot\mathrm{HF}(\bar{c}c)}$
$m_b^b + m_c^b + m_q^b$	7117.0	7117.0
bc binding	−170.8	−164.8
$a/(m_c^b m_q^b)$	10.6	10.6
$-2a/(m_b^b m_q^b)$ $-2a_{bc}/(m_b^b m_c^b)$	−24.2	−28.2
Total	6933 ± 12	6935 ± 12

VIII. P-WAVE EXCITATIONS

In the event that a $\Xi_{(cc,bb,bc)}$ state is accompanied by a pion nearby in phase space, the two can have come from a P-wave excitation. Let us take the example of Ξ_{cc}.

TABLE XIII. Comparison of predictions for $M(\Xi_{bc})$.

Reference	Value (MeV)	Method
Present work	6914 ± 13	
[25]	6916 ± 139	QCD-motivated quark model
[28]	6938	QCD-motivated quark model
[43]	6930	Potential models
[45]	6990 ± 90	Feynman-Hellmann +semiempirical formulas
[46]	7029	Mass sum rules
[47]	6950	Relativistic quasipotential quark model
[48]	6915	Three-body Faddeev equations.
[51]	6820 ± 50	Potential approach and QCD sum rules
[52]	6960	Nonperturbative string
[53]	6933	Relativistic quark-diquark
[54]	6800	Bag model
[57]	6919	Variational
[58]	7011	Quark model
[59]	6840 ± 10	Instantaneous approx +Bethe-Salpeter
[60]	6750 ± 50	QCD sum rules
[68]	6789	Coupled channel formalism

TABLE XIV. Comparison of predictions for $M(\Xi'_{bc})$.

Reference	Value (MeV)	Method
Present work	6933 ± 12	
[25]	6976 ± 99	QCD-motivated quark model
[28]	6971	QCD-motivated quark model
[45]	7040 ± 90	Feynman-Hellmann+ semi-empirical formulas
[46]	7053	Mass sum rules
[47]	7000	Relativistic quasipotential quark model
[51]	6850 ± 50	Potential approach and QCD sum rules
[53]	6963	Relativistic quark-diquark
[54]	6870	Bag model
[57]	6948	Variational
[58]	7047	Quark model
[60]	6950 ± 80	QCD sum rules
[68]	6818	Coupled channel formalism

Heavy-quark symmetry implies that in transitions involving a single pion the cc state maintains its spin of 1, while in such P-wave states the light-quark q couples with a unit of orbital angular momentum to form a state of total light-quark angular momentum $j = 1/2$ or $j = 3/2$. We can then expect a rich family of P-wave states with

$$(j = 1/2) \otimes (J(cc) = 1) \to J_{\rm tot} = 1/2, \quad 3/2;$$
$$(j = 3/2) \otimes (J(cc) = 1) \to J_{\rm tot} = 1/2, \quad 3/2, \quad 5/2. \tag{21}$$

The parity of the Ξ_{cc} is positive, whereas that of the states in Eq. (21) is negative. Heavy-quark symmetry predicts that the states with $j = 1/2$ will decay via S-wave pion emission, whereas states with $j = 3/2$ will decay via D-wave pion emission, and hence will be narrower. This is particularly true of the $J_{\rm tot} = 5/2$ state, which is pure $j = 3/2$ and hence immune from mixing.

Let us neglect the fine-structure interaction between the $j = 3/2$ light-quark system and the heavy cc diquark. Even in P-wave mesons with a single heavy quark, this interaction gives rise to a splitting of only 41 MeV between $D_1(2421)$ and $D_2^*(2462)$, and 20 MeV between $B_1(5723)$ and $B_2^*(5743)$. The spin-weighted average of $D_1(2421)$ and $D_2^*(2462)$ masses is 2446 MeV, lying 473 MeV above the spin-weighted average of D and D^* masses. The spin-weighted average of $B_1(5723)$ and $B_2^*(5743)$ masses is 5736 MeV, lying 422 MeV above the spin-weighted average of B and B^* masses. The cc diquark is intermediate in mass between the c and b quarks, so one might expect the narrow P-wave excitations of Ξ_{cc} to occupy an interval of no more than a few tens of MeV, lying between 420 and 470 MeV above the spin-weighted average of Ξ_{cc} and Ξ_{cc}^* masses.

IX. LIKELY DECAY MODES AND LIFETIMES

Many of the references quoted in Tables VIII, X, XIII, and XIV also discuss likely branching ratios and production mechanisms. In addition, we note early suggestions by Bjorken [70,71] and Moinester [72]. Here we give some general guidelines, avoiding specific calculations depending on details of form factors and fragmentation. We pay special attention to those modes which can show up in the online selection criteria ("triggers") of experiments at e^+e^- colliders, the Tevatron, and the LHC. We concentrate on those decays involving the most-favored Cabibbo-Kobayashi-Maskawa matrix elements, such as $c \to sW^{*+}$ and $b \to cW^{*-}$. In lifetime estimates we shall neglect the effects of Pauli interference, concentrating on effects of factorized decays and $2 \to 2$ internal transitions. Although we do not present detailed branching fractions, Tables 9–18 through 9–20 of Ref. [28] are a useful guide.

A. $\Xi_{cc}^{++} = ccu$

The decay of Ξ_{cc}^{++} begins with the decay of either charm quark to a strange quark and a virtual W^+ ("W^{*+}"). In this and other processes, a virtual W^+ gives rise to a positively charged hadronic state limited only by available phase space. In this case the minimum mass of the csu remnant is that of the $\Xi_c(2469)$. Given our prediction of $M(\Xi_{cc}) = (3627 \pm 12)$ MeV, one has 1158 MeV of available energy for the W^{*+} products, which can then be π^+, ρ^+, or the low-energy tail of the a_1^+.

The csu remnant has the quantum numbers of the Ξ_c^+. It may decay via virtual W^+ emission to an ssu remnant which is either a Ξ^0 (hard to detect) or an excited state of it (decaying to $\Xi^-\pi^+$). Alternatively, the csu remnant may fragment into states such as $\Lambda_c^+ K^- \pi^+$, with the Λ_c^+ decaying to such final states as $pK^-\pi^+$.

The decay chain $\Xi_{cc}^{++} \to \pi^+ \Xi_c^+ \to 3\pi^+ \Xi^-$ leads to pions all of the same sign. The Collider Detector at Fermilab (CDF) trigger based on two displaced tracks accepts only a pair of opposite-sign tracks, and would miss such a signature [73]. One might be able to pick up opposite-sign tracks from higher-multiplicity decays giving rise to a π^+ and π^- or K^-, but one pays a price in higher multiplicity because such tracks are often soft and below the accepted transverse momentum threshold.

A crude estimate of the lifetime of the Ξ_{cc}^{++} may be obtained by considering the two c quarks to decay independently. Bjorken [70,71] and Fleck and Richard [42] estimate $\tau(\Xi_{cc}^{++}) \simeq 200$ fs by this method. We reproduce this value by assuming an initial state with $M(\Xi_{cc}) = 3627$ MeV, a final state with $M(\Xi_c) = 2469$ MeV, a weak current giving rise to $e\nu, \mu\nu$, and three colors of $u\bar{d}$, a kinematic suppression factor

$$F(x) = 1 - 8x + 8x^3 - x^4 + 12x^2 \ln(1/x),$$
$$x_{cc} \equiv [M(\Xi_c)/M(\Xi_{cc})]^2 = 0.4634, \tag{22}$$

and a factor of 2 to count each decaying c quark. The resulting decay rate is

$$\Gamma(\Xi_{cc}^{++}) = \frac{10 G_F^2 M(\Xi_{cc})^5}{192\pi^3} F(x_{cc}) = 3.56 \times 10^{-12}\ \text{GeV}, \tag{23}$$

leading to a predicted lifetime of $\tau(\Xi_{cc}^{++}) = 185$ fs. In this calculation two compensating effects have been neglected: (i) a form factor for the weak transition $\Xi_{cc} \to \Xi_c$, and (ii) the excitation of csu states above Ξ_c^+. Here and elsewhere we have assumed $V_{ud} = V_{cs} = 1$ for favored elements of the Cabibbo-Kobayashi-Maskawa matrix. A similar approach to semileptonic decays of hadrons containing a single heavy quark has been shown to reproduce observed rates with an accuracy of about 10% [74].

B. $\Xi_{cc}^{+} = ccd$

We treat this final state separately because, in addition to decaying via the subprocess $c \to su\bar{d}$ discussed in the previous subsection, it may decay via the subprocess $cd \to su$. The decays of $\Lambda_c = cud$ ($\tau = 200 \pm 6$ fs) and $\Xi_c^0 = csd$ ($\tau = 112_{-10}^{+13}$ fs) are probably enhanced by this subprocess with respect to those of $\Xi_c^+ = csu$ ($\tau = 442 \pm 26$ fs), where it cannot occur. By comparing the Ξ_c^+ and Ξ_c^0 decay rates, and including a factor of 2 for the two charmed quarks participating in $cd \to su$, the enhancement to the decay rate becomes 8.78×10^{-12} GeV and the lifetime becomes $\tau(\Xi_{cc}^+) = 53$ fs. Bjorken [70,71] and Fleck and Richard [42] predict about 100 fs.

The subprocess $cd \to su$ in $\Xi_{cc}^+ = ccd$ leads to an excited csu state without the π^+ emitted in Ξ_{cc}^{++} decay. The rest of the discussion proceeds as for Ξ_{cc}^{++}, but with slightly more available phase space. In particular, the fragmentation of csu into $\Lambda_c^+ K^- \pi^+$ gives rise to a slightly more energetic K^-, advantageous for the CDF two-opposite-sign-track trigger.

C. $\Xi_{bc}^{+} = bcu$

A factorization approach similar to that described for the Ξ_{cc} states may be used to estimate one set of contributions to $\Xi_{bc} = bcq$ decays. There are two contributing subprocesses: $b \to cd\bar{u}$ and $c \to su\bar{d}$. In the case of the first, the weak current can produce not only $e\nu$, $\mu\nu$, and $\bar{u}d$, but also $\tau\nu$ and $\bar{c}s$. An interesting consequence of the last is the decay $\Xi_{bc} \to J/\psi \Xi_c$, allowed for both charge states of Ξ_{bc}. The rate for this decay should not exceed the total in which the weak current produces a $\bar{c}s$ pair. For the sake of a very crude estimate, we shall neglect the masses of *all* allowed states produced by the weak current.

The $b \to cW^{*-}$ subprocess, under assumptions similar to those in the previous subsections, gives rise to a partial decay rate

$$\Gamma(\Xi_{bc} \to W^{*-}\Xi_{cc}) = \frac{9G_F^2 M(\Xi_{bc})^5}{192\pi^3} F\{[M(\Xi_{cc})/M(\Xi_{bc})]^2\}|V_{cb}|^2 = 6.87 \times 10^{-13}\ \text{GeV}, \quad (24)$$

where we have used $|V_{cb}| = 0.04$ and have assumed massless final states of $e\nu$, $\mu\nu$, $\tau\nu$, three colors of $\bar{u}d$, and three colors of $\bar{c}s$. The $c \to sW^{*+}$ subprocess gives rise to a larger partial rate:

$$\Gamma(\Xi_{bc} \to W^{*+}\Xi_b) = \frac{5G_F^2 M(\Xi_{bc})^5}{192\pi^3} F\{[M(\Xi_b)/M(\Xi_{bc})]^2\} = 2.01 \times 10^{-12}\ \text{GeV}. \quad (25)$$

In principle for $\Xi_{bc}^+ = bcu$ there should be a third contribution from the subprocess $bu \to cd$. However, the near-equality of the lifetimes of $\Xi_b^0 = bsu$ and $\Xi_b^- = bsd$ [34,75,76], as summarized in Table XV, suggests that this process carries little weight, so we shall neglect it. The sum of the two contributions to the Ξ_{bc}^+ decay rate is then 2.70×10^{-12} GeV, yielding a lifetime of $\tau(\Xi_{bc}^+) = 244$ fs.

For the $b \to cW^{*-}$ subprocess, contributing to the decay of both Ξ_{bc} states, the virtual W can easily produce a negative pion. Subsequent decays of the ccq intermediate state easily lead to a positive pion, so the CDF trigger should be able to respond to a pair of opposite-sign displaced tracks coming from Ξ_{bc} decays.

One effect which we have not considered is the internal $2 \to 2$ transition $bc \to cs$. For both $\Xi_{bc} = bcq$ states, this leads to a final csq state, an excited version of $\Xi_c^{(+,0)}$ which can decay to the same products as $\Xi_c^{(+,0)}$ or hadronically to states like $\Lambda D^{0,+}$. In principle one could relate the $bc \to cs$ process in Ξ_{bc} to the $b\bar{c} \to W^{*-}$ annihilation process in B_c^- decay.

D. $\Xi_{bc}^0 = bcd$

In addition to the contributions just calculated to the decay rate of Ξ_{bc}^+, we have seen the subprocess $cd \to su$ to be important in the difference between Ξ_c^0 and Ξ_c^+ lifetimes. If we take the additional contribution to the Ξ_c^0 decay rate to be the same here, that provides an additional term of 4.39×10^{-12} GeV, leading to

$$\Gamma(\Xi_{bc}^0) = 7.09 \times 10^{-12}\ \text{GeV}, \qquad \tau(\Xi_{bc}^0) = 93\ \text{fs}. \quad (26)$$

TABLE XV. Lifetimes of Ξ_b baryons (ps).

State	Ref. [75]	Ref. [34]	Ref. [76]
Ξ_b^-	$1.55^{+0.16}_{-0.09} \pm 0.03$	$1.32 \pm 0.14 \pm 0.02$	$1.36 \pm 0.15 \pm 0.02$
Ξ_b^0	$1.477 \pm 0.026 \pm 0.014 \pm 0.013$		

The intermediate state produced by $cd \to su$ is that of an excited bsu ("Ξ_b^{*0}") with the mass of Ξ_{bc}. The dominant subsequent decay is governed by the subprocess $b \to cW^{*-}$, with enough phase space that the virtual W^- can produce all three lepton pairs, $\bar{u}d$, and $\bar{c}s$. The last process can lead to J/ψ production, for example in the decay $\Xi_{bc}^0 \to J/\psi \Xi^0$ or $\Xi_{bc}^0 \to J/\psi \Xi^- \pi^+$.

E. $\Xi_{bb} = bbq$

Although the $2 \to 2$ process $bu \to cd$ is possible in principle for $\Xi_{bb}^0 = bbu$, we have seen that it seems to play little role in generating a lifetime difference between Ξ_b^0 and Ξ_b^-. Hence we may treat Ξ_{bb}^0 and Ξ_{bb}^- generically as $\Xi_{bb} = bbq$ in what follows.

The initial process in a Ξ_{bb} decay is the process $bbq \to bcq + W^{*-}$, where the minimum mass of the bcq remnant is that of the Ξ_{bc}, or 6914 MeV. As the predicted mass of Ξ_{bb} is 10162 MeV, there is enough phase space for the weak current to produce all three lepton pairs, $\bar{u}d$, and $\bar{c}s$. Neglecting all of their masses, the total decay rate is calculated to be

$$\Gamma(\Xi_{bb}) = \frac{18G_F^2 M(\Xi_{bb})^5}{192\pi^3} F\{[M(\Xi_{bc})/M(\Xi_{bb})]^2\}|V_{cb}|^2 = 1.78 \times 10^{-12}\ \text{GeV}, \quad (27)$$

leading to a predicted lifetime $\tau(\Xi_{bb}) = 370$ fs.

An interesting decay involving the subprocess $b \to J/\psi s$ *twice* is the chain

$$\Xi_{bb} \to J/\psi \Xi_b^{(*)} \to J/\psi J/\psi \Xi^{(*)}, \quad (28)$$

where $\Xi_b^{(*)}$ denotes a (possibly excited) state with the minimum mass of $\Xi_b(5792)$, while $\Xi^{(*)}$ denotes a (possibly excited) state with the minimum mass of Ξ. Although this state is expected to be quite rare and one has to pay the penalty of two J/ψ leptonic branching fractions, it has a distinctive signature and is worth looking for.

F. Lifetime summary and discussion

We summarize our lifetime predictions and compare them with others in Table XVI. There is quite a spread in predicted values, but in all cases lifetimes are shortened when the $2 \to 2$ process $cd \to su$ is permitted, as in the case of the Λ_c^+, while the $2 \to 2$ process $bu \to cd$ seems to have little effect. Our very short lifetime for Ξ_{cc}^+ stems from two main effects: (i) the difference between the Ξ_c^0 and Ξ_c^+ lifetimes (112 vs 442 fs), used to estimate the effect of the $cd \to su$ subprocess, and (ii) the factor of 2 in the $cd \to su$ rate because the Ξ_{cc}^+ has two charmed quarks.

TABLE XVI. Summary of lifetime predictions for baryons containing two heavy quarks. Values given are in fs.

Baryon	This work	[28]	[51]	[71]	[72]
$\Xi_{cc}^{++} = ccu$	185	430 ± 100	460 ± 50	500	~200
$\Xi_{cc}^{+} = ccd$	53	120 ± 100	160 ± 50	150	~100
$\Xi_{bc}^{+} = bcu$	244	330 ± 80	300 ± 30	200	⋯
$\Xi_{bc}^{0} = bcd$	93	280 ± 70	270 ± 30	150	⋯
$\Xi_{bb}^{0} = bbu$	370	⋯	790 ± 20	⋯	⋯
$\Xi_{bb}^{-} = bbd$	370	⋯	800 ± 20	⋯	⋯

X. PROSPECTS FOR DETECTION

Production of baryons containing two heavy quarks requires simultaneous production of two heavy quark-antiquark pairs. Subsequently, a heavy quark from one pair needs to coalesce with a heavy quark from the other pair, forming together a color antitriplet heavy diquark. The heavy diquark then needs to pick up a light quark to finally hadronize as a doubly heavy baryon. The coalescence of the two heavy quarks requires that they be in each other's vicinity in both ordinary space and in rapidity space. Computation of the corresponding cross section from first principles is difficult [28,77–86], and is subject to considerable uncertainties due to nonperturbative effects. Instead, we use existing data [11–13] and theoretical estimates [87–89] of the closely related process of B_c production.

The two processes are closely related because production of B_c also requires simultaneous production of two heavy quark-antiquark pairs. *A priori*, B_c production has a somewhat higher probability, since in B_c production a heavy quark from one pair needs to coalesce with a heavy antiquark (rather than a quark) from the other pair and there is no need to pick up an additional light quark. There is no suppression associated with the latter, as once the color antitriplet heavy diquark is formed it can only hadronize by picking up a light quark. On the other hand, the attraction between a quark and an antiquark is two times stronger than the attraction between two quarks and we need to estimate the corresponding suppression factor. In order to see if Ξ_{bc} and B_c production rates are comparable, it would be useful to compare the analogous production rates of Ξ_c and D_s (or Ξ_b and B_s) in experiments with large enough E_{CM}, whether in e^+e^-, $\bar{p}p$, or pp collisions.

Although it is not directly related, one may consider the relative probability of a b quark produced at high energy fragmenting into a meson (picking up a light antiquark) and a baryon (picking up a light diquark). The Heavy Flavor Averaging Group (HFAG) [90] has tabulated these quantities as measured in Z decays and the Tevatron, as shown in Table XVII.

According to the HFAG analysis, depending on the production mechanism, the b quark turns into a baryon between about 10 and 25% of the time. Fragmentation into a baryon is somewhat favored at low transverse momentum [90] in hadron collisions.

TABLE XVII. Fractions of different b-hadron species arising from b quarks from Ref. [90].

Quantity	Z decays	Tevatron
B^+ or B^0 fraction $f_u = f_d$	0.403 ± 0.009	0.330 ± 0.030
B_s^0 fraction	0.103 ± 0.009	0.102 ± 0.012
b-baryon fraction	0.090 ± 0.015	0.236 ± 0.067

More recently, LHCb has carried out a thorough analysis of the b quark fragmentation into mesons and baryons [91–94]. In particular, the rather striking Fig. 4 in Ref. [94] shows that the ratio of Λ_b production to B^0 meson production for p_T below 10 GeV is above 0.3 and goes above 0.5 for lower p_T.

A crude conclusion which we might draw from this comparison is that a baryon composed of two heavy quarks could be produced with at least 10% of the B_c production rate. An even more optimistic estimate, supported by the above LHCb fragmentation data, is provided by an explicit calculation [28] which predicts the production rates for Ξ_{cc} and Ξ_{bc} to be as large as 50% of that for $(B_c + B_c^*)$ at the Tevatron, of the order of several nb. The cross section for Ξ_{bb} is estimated in that work to be about a factor of 10 less.

The inclusive production cross section of the B_c^+ at the LHC, including the contribution from excited states, was estimated to be ~1 μb for $\sqrt{s} = 14$ TeV, and ~0.4 μb for $\sqrt{s} = 7$ TeV [89], based on a dominant contribution from gg fusion: $gg \to B_c + b + \bar{c}$, computed by the complete order-α_s^4 approach and by the fragmentation approach.

As a figure of merit, for 1 fb^{-1} integrated luminosity 1 μb translates to $\sim 10^9$ B_c^+ mesons being produced at the LHC, one order of magnitude more than at the Tevatron. This number is considerably reduced by triggering on specific decay modes and folding in the detector efficiency, but nevertheless it leaves a sufficiently large number of B_c mesons to carry out a detailed study of the B_c^+ properties.

Based on 0.37 fb^{-1} of data collected in pp collisions at $\sqrt{s} = 7$ TeV LHCb has reported [9] the ratio of the production cross section times branching fraction between the $B_c^+ \to J/\psi\pi^+$ and the $B^+ \to J/\psi K^+$ decays,

$$\frac{\sigma(pp \to B_c + X) \cdot \mathcal{B}(B_c^+ \to J/\psi\pi^+)}{\sigma(pp \to B^+ + X) \cdot \mathcal{B}(B^+ \to J/\psi K^+)} = (0.68 \pm 0.10(\mathrm{stat}) \pm 0.03(\mathrm{syst}) \pm 0.05(\mathrm{lifetime})) \times 10^{-2}, \tag{29}$$

for B_c^+ and B^+ mesons with transverse momenta $p_{\mathrm{T}} > 4$ GeV$/c$ and pseudorapidities $2.5 < \eta < 4.5$, corresponding to 162 ± 18 $B_c^+ \to J/\psi\pi^+$ signal events. We may use this last figure to estimate the total number of B_c^+ produced within the LHCb acceptance.

A number of calculations of B_c branching fractions are compared with one another in Ref. [95]. This reference is the one which best reproduces the observed ratio [15]

$$\frac{\mathcal{B}(B_c^+ \to J/\psi\pi^+)}{\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu)} = 0.0469 \pm 0.0028 \pm 0.0046, \quad (30)$$

so we shall quote its result $\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu) = 1.36\%$, which we have corrected using a recent measurement [16], $\tau(B_c^+) = (509 \pm 8 \pm 12)$ fs. With the measured ratio (30) this implies $\mathcal{B}(B_c^+ \to J/\psi\pi^+) = 6.4 \times 10^{-4}$.

With the above one can now compute the total B_c production cross section directly from data[1]: the total B^+ production cross section at LHCb is $38.9 \pm 0.3(\text{stat}) \pm 2.5(\text{syst}) \pm 1.3(\text{norm})\ \mu$b [96] and $\mathcal{B}(B^+ \to J/\psi K^+) = (1.028 \pm 0.031) \times 10^{-3}$ [26]. Putting this all together, we obtain

$$\begin{aligned} &\sigma(pp \to B_c + X) \\ &\quad \approx \sigma(pp \to B^+ + X) \cdot \frac{\mathcal{B}(B^+ \to J/\psi K^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} \cdot 0.68 \times 10^{-2} \\ &\quad = \frac{38.9 \cdot 1.028 \times 10^{-3} \cdot 0.68 \times 10^{-2}}{6.4 \times 10^{-4}}\ \mu\text{b} = 0.4\ \mu\text{b} \end{aligned} \quad (31)$$

for $4 < p_T < 40$ GeV and $2.5 < \eta < 4.5$, whereas Ref. [89] predicts this value for the whole of phase space. With 162 ± 18 $B_c^+ \to J/\psi\pi^+$ events $\mathcal{B}(B_c^+ \to J/\psi\pi^+) = 6.4 \times 10^{-4}$ indicates a total of

$$\frac{162 \pm 18}{(6.4 \times 10^{-4})(0.0593 \pm 0.0006)} \sim 4.3 \times 10^6\ B_c \quad (32)$$

produced within the LHCb acceptance, where the second number in the denominator is $\mathcal{B}(J/\psi \to \mu^+\mu^-)$. With an observed B_c production cross section 0.4 μb in 0.37 fb^{-1} there is a total of about 1.5×10^8 B_c produced overall, indicating an acceptance a bit below 3%. One might expect the Ξ_{cc} production cross section to be at most a tenth of this, or 40 nb, at 7 TeV.

There is an interesting question whether Ξ_{cc} is LHCb's best bet for discovering doubly heavy baryons. The point is that because of Cabibbo suppression the b quark lifetime is about 7 times longer than the c quark, even though the b quark is more than three times heavier and the phase space for weak quark decay of a heavy quark scales like $(m_b/m_c)^5$ times a kinematic function of the final and initial masses. Thus $\tau(\Lambda_b) \approx 1.5 \times 10^{-12}$ s vs $\tau(\Lambda_c) \approx 2 \times 10^{-13}$ s, etc. The difference between actual Ξ_{cc} and Ξ_{bc} lifetimes, as shown in Table XVI, is not so pronounced. Longer lifetime makes it much easier to identify the secondary vertex. On the other hand, the cross section for producing bottom quarks is of course much smaller than for charmed quarks. So there is a tradeoff.

For sake of completeness, we also provide here a brief update on the status of search for doubly charmed baryons in e^+e^- experiments. The most recent and most stringent limits in this case come from Belle [10]. They used a 980 fb^{-1} data sample to search for Ξ_{cc}^+ and Ξ_{cc}^{++} decaying into $\Lambda_c^+K^-\pi^+(\pi^+)$ and $\Xi_c^0\pi^+(\pi^+)$ final states.

Theoretical predictions for the inclusive cross section $\sigma(e^+e^- \to \Xi_{cc} + X)$ at Belle center-of-mass (CM) energy, $\sqrt{s} = 10.58$ GeV, vary over a rather wide range, from 70 [85] to 230 fb [86].

Belle did not find any significant Ξ_{cc} signal and set a 95% C.L. upper limit on $\sigma(e^+e^- \to \Xi_{cc}^{+(+)} + X) \times \mathcal{B}(\Xi_{cc}^{+(+)} \to \Lambda_c^+K^-\pi^+(\pi^+))$ with the scaled momentum $0.5 < x_p < 1.0$: 4.1–25.0 fb for Ξ_{cc}^+ and 2.5–26.5 fb for Ξ_{cc}^{++}. They also set a 95% C.L. upper limit on $\sigma(e^+e^- \to \Xi_{cc}^{+(+)} + X) \times \mathcal{B}(\Xi_{cc}^{+(+)} \to \Xi_c^0\pi^+(\pi^+)) \times \mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+)$ with the scaled momentum $0.45 < x_p < 1.0$: 0.076–0.35 fb for the Ξ_{cc}^+ and 0.082–0.40 fb for the Ξ_{cc}^{++}.

The CM energy of the B factories is sufficient only for production of Ξ_{cc}, as Ξ_{bc} and Ξ_{bb} are too heavy. So within the foreseeable future the latter can only be produced at LHC and perhaps at RHIC.

As in the case of doubly heavy baryon production in LHCb, there is a significant uncertainty in theoretical predictions for the inclusive cross section $\sigma(e^+e^- \to \Xi_{cc} + X)$. Therefore, we suggest another approach, similar in spirit to what we proposed for LHCb. This approach is again directly based on observables which are in principle accessible in e^+e^- machines.

One can make a rough estimate of the doubly charmed baryon production rate by assuming that the suppression of ccq baryons Ξ_{cc} vs csq baryons Ξ_c is of the same order of magnitude as the suppression of Ξ_c vs ssq baryons Ξ. The physical content of this assumption is that the suppression due to replacing an s quark in a baryon by a much heavier c quark is approximately independent of the spectator quarks in the baryon:

$$\begin{aligned} &\sigma(e^+e^- \to \Xi_{cc} + X) \\ &\quad \sim \sigma(e^+e^- \to \Xi_c + X) \cdot \frac{\sigma(e^+e^- \to \Xi_c + X)}{\sigma(e^+e^- \to \Xi + X)}. \end{aligned} \quad (33)$$

Information on inclusive Ξ production in e^+e^- annihilation at CM energy very close to Belle energy is readily available. The ARGUS experiment has measured [97] the following Ξ^- rates per multihadronic event at $\sqrt{s} = 10$ GeV:

$$\begin{aligned} &(2.06 \pm 0.17 \pm 0.23) \times 10^{-2} \quad \text{in direct } \Upsilon \text{ decays} \quad \text{and} \\ &(0.67 \pm 0.06 \pm 0.07) \times 10^{-2} \quad \text{in the continuum.} \end{aligned} \quad (34)$$

[1]We thank Vanya Belyaev for pointing out that the total B^+ production cross section at LHCb is available and can be used for this purpose.

The situation with inclusive Ξ_c production is less simple. Belle has seen Ξ_c only in some specific channels, so what they measure is (production rate) × (branching fractions into specific channels). The latter are not known well, so it is not easy to determine the production rate itself.

Nevertheless, for our purpose it is sufficient to estimate the Ξ_{cc} production rate to within a factor $2 \div 4$, which should be possible even within the existing uncertainties about Ξ_c branching fractions.

The approximate formula in Eq. (33) and its generalizations to Ξ_{bc} and Ξ_{bb} production should also apply to pp collisions:

$$\sigma(pp \to \Xi_{bc} + X) \sim \sigma(pp \to \Xi_b + X) \cdot \frac{\sigma(pp \to \Xi_c + X)}{\sigma(pp \to \Xi + X)} \sim \sigma(pp \to \Xi_c + X) \cdot \frac{\sigma(pp \to \Xi_b + X)}{\sigma(pp \to \Xi + X)} \quad (35)$$

as well as

$$\sigma(pp \to \Xi_{bb} + X) \sim \sigma(pp \to \Xi_b + X) \cdot \frac{\sigma(pp \to \Xi_b + X)}{\sigma(pp \to \Xi + X)}. \quad (36)$$

XI. CONCLUSIONS

The conclusive observation of baryons with two heavy quarks is long overdue. The weight of theoretical and experimental evidence suggests that whatever the SELEX experiment has reported [3,4], it is not the Ξ_{cc}: Its mass lies below almost all expectations, the isospin splitting between $\Xi_{cc}^{++}(3460)$ and $\Xi_{cc}^{+}(3520)$ candidates is implausibly large, and no other experiment has seen the effect. We have predicted $M(\Xi_{cc}) = 3627 \pm 12$ MeV and made several suggestions for its observation, including the decay to $\pi^+\Xi_c$, where both states of $\Xi_c^{+,0}$ have been identified in previous studies. We also predict the masses of other states summarized in Table XVIII, and have estimated lifetimes for these states as summarized in Table XVI.

We also estimate the hyperfine splitting between B_c^* and B_c mesons to be 68 MeV, with an alternate method giving 84 MeV. P-wave excitations of the Ξ_{cc} with light-quark total angular momentum $j = 3/2$, the analog of those observed for D and B mesons, are estimated to lie around 420–470 MeV above the spin-weighted average of the Ξ_{cc} and Ξ_{cc}^* masses. Production rates could be as large as 50% of those for B_c, which also requires the production of two heavy-quark pairs. We are optimistic that with the increased data samples soon to be available in hadronic and e^+e^- collisions, the first baryons with two heavy quarks will finally be seen.

TABLE XVIII. Summary of our mass predictions (in MeV) for lowest-lying baryons with two heavy quarks. States without a star have $J = 1/2$; states with a star are their $J = 3/2$ hyperfine partners. The quark q can be either u or d. The square or curved brackets around cq denote coupling to spin 0 or 1.

State	Quark content	$M(J = 1/2)$	$M(J = 3/2)$
$\Xi_{cc}^{(*)}$	ccq	3627 ± 12	3690 ± 12
$\Xi_{bc}^{(*)}$	$b[cq]$	6914 ± 13	6969 ± 14
Ξ'_{bc}	$b(cq)$	6933 ± 12	$\cdots$
$\Xi_{bb}^{(*)}$	bbq	10162 ± 12	10184 ± 12

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Note added.—After this work had been completed, a new set of lattice results appeared in Ref. [98]. As noted by the authors, in several cases their results are quite close to ours: $M(\Xi_{cc}) = 3610(23)(22)$ MeV, $M(\Xi_{cc}^*) = 3692(28)(21)$ MeV, $M(\Xi_{bb}) = 10143(30)(23)$ MeV, $M(\Xi_{bb}^*) = 10178(30)(24)$ MeV, $M(\Xi_{bc}) = 6943(33)(28)$ MeV, $M(\Xi'_{bc}) = 6959(36)(28)$ MeV, and $M(\Xi_{bc}^*) = 6985(36)(28)$ MeV.

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