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Photon-neutrino scattering and the *B*-mode spectrum of CMB photons

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On the basis of the quantum Boltzmann equation governing the time evolution of the density matrix of polarized cosmic microwave background (CMB) photons in the primordial scalar perturbations of metric, we calculate the *B*-mode spectrum of polarized CMB photons contributed from the scattering of CMB photons and cosmic neutrino background neutrinos. We show that such a contribution to the *B*-mode spectrum is negligible for small ℓ; however, it is made significantly larger for $50 < \ell < 200$ by plotting our results together with the BICEP2 data. Our study and results imply that in order to theoretically better understand the origin of the observed *B*-mode spectrum of polarized CMB photons (r parameter), it should be necessary to study the relevant and dominated processes in both tensor and scalar perturbations.

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I. INTRODUCTION

It is known that in the inflation cosmology, the power-law spectrum of either metric scalar perturbation $P_S(k) = A_S(k/k_0)^{n_S-1}$ or tensor perturbation $P_T(k) = A_T(k/k_0)^{n_T}$ has been produced in the inflationary era of the early universe [1], where A_S and A_T are the amplitudes of scalar and tensor perturbations, and $n_{S,T}$ are their spectral indices. A_S and n_S have been determined through the measurements of microwave background temperature anisotropy [2–4]. The amplitude of metric tensor perturbation is characterized by the tensor-scalar ratio $r = P_T/P_S$, relating to the *B*-mode spectrum of polarized cosmic microwave background (CMB) photons imprinted by the metric tensor perturbations of primordial gravitational waves. The BICEP2 Collaboration recently reported $r = 0.20^{+0.07}_{-0.05}$ [5]. If this report is verified, it is regarded as an important result that may reveal the existence of metric tensor perturbations in the inflationary era of the early universe.

However, there are alternative explanations of the BICEP2 data—whether the BICEP2 data could be explained by the vector and tensor modes from primordial magnetic fields [6]. Some authors speculate that the BICEP2 observed *B*-mode polarization is the result of a primordial Faraday rotation of the *E*-mode polarization [7,8]. In this article, using the result of the photon polarization generated by the photon-neutrino scattering [9], we investigate the possible contribution to the observed *B*-mode spectrum by considering the interaction between CMB photons and cosmic neutrino background (CNB) in the background of scalar perturbations, without tensor perturbations. In order to quantitatively calculate such a contribution in the scalar perturbation, we solve the quantum Boltzmann equation for the time evolution of the matrix density (Stokes parameters) of polarized CMB photons which are involved in the Compton and photon-neutrino scattering as the collision terms of the quantum Boltzmann equation. Our result is shown together with the BICEP2 data and its implication on the interpretation of the BICEP2 data is discussed.

II. THE PHOTON POLARIZATION FROM COMPTON AND PHOTON-NEUTRINO SCATTERING

The linear and circular polarizations of an ensemble of photons can be described by the density operator

$$\hat{\rho}_{ij} = \frac{1}{\mathrm{tr}(\hat{\rho})}\int \frac{d^3k}{(2\pi)^3}\rho_{ij}(k)D_{ij}(k),$$
$$\hat{\rho}_{ij}(k) = \frac{1}{2}\begin{pmatrix} I+Q & U-iV \\ U+iV & I-Q \end{pmatrix}, \tag{1}$$

where $\rho_{ij}(k)$ represents the density matrix in terms of the Stokes parameters I, Q, U, and V in the 2×2 polarization space (i,j) of one photon of energy-momentum "k." The number operator $D_{ij}(k) = a_i^\dagger(k)a_j(k)$ and its expectation value

$$\langle D_{ij}(k)\rangle \equiv \mathrm{tr}[\hat{\rho}_{ij}D_{ij}(k)] = (2\pi)^3\delta^3(0)(2k^0)\rho_{ij}(k). \tag{2}$$

The time evolution of the number operator $D_{ij}(k)$ obeys the Heisenberg equation

$$\frac{d}{dt}D_{ij}(k) = i[H_I, D_{ij}(k)], \tag{3}$$

where H_I is an interacting Hamiltonian. Using Eqs. (1), (2), and (3), one obtains the time evolution of $\rho_{ij}(k)$, quantum Boltzmann equation [10],

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$$(2\pi)^3\delta^3(0)(2k^0)\frac{d\rho_{ij}(k)}{dt} = i\langle[H_I(t), D_{ij}(k)]\rangle - \frac{1}{2}\int dt\langle[H_I(t),[H_I(0),D_{ij}(k)]]\rangle, \tag{4}$$

where $H_I(t)$ is the interacting Hamiltonian. On the right-hand side of Eq. (4), the first and second terms respectively represent the forward scattering and higher order collision terms.

There are a lot of papers which investigate the effects of the Compton scattering on the anisotropy and polarization of CMB (see for example [10–12]). In this article, we attempt to study the CMB photon polarization by considering the contribution of the photon-neutrino scattering to the polarization density matrix of photons obtained recently [9]

$$2k^0\frac{d\rho_{ij}}{dt} = -\frac{\sqrt{2}}{6\pi}\alpha G^F\int d\mathbf{q}[\rho_{s'j}(\mathbf{k})\delta_{is} - \rho_{is}(\mathbf{k})\delta_{js'}]f_\nu(x,q) \times (q^2\epsilon_{s'}\cdot\epsilon_s + 2\mathbf{q}\cdot\epsilon_{s'}\mathbf{q}\cdot\epsilon_s - \varepsilon_{\mu\nu\rho\sigma}\epsilon_s^\mu\epsilon_{s'}^\nu k^\rho q^\sigma), \tag{5}$$

where $d\mathbf{q} = (2E_\nu)^{-1}d^3q/(2\pi)^3$ is the integration over the neutrino four-momentum ($q^0 = E_\nu \approx |\mathbf{q}|$) with the distribution function $f_\nu(x,q)$, and the polarization four-vectors $\epsilon_{i\mu}(\mathbf{k})$ and their indices $i, j, s, s' = 1, 2$, represent two transverse polarizations of the photon $k^0 = |\mathbf{k}|$. G^F and α are Fermi the coupling constant and electromagnetic fine-structure constant.

Note that we consider massive neutrino (m_ν) in our calculations. Here we only calculate the leading order contribution to the polarization density matrix, i.e., the forward scattering term in the rhs of Eq. (4), which does not depend on neutrino masses. The next-leading contribution from higher order collision terms in the rhs of Eq. (4) depends on neutrino masses; see for example Ref. [13]. In order for readers' convenience, we give a brief clarification in the following of how our approximate calculations are done by using Eq. (4).

The scattering rate of photon-neutrino energy-momentum states is indeed very small due to their cross section of about $(\alpha G^F)^2$; see Ref. [13]. However, in our case the time evaluation (4) of the Stokes parameters ρ_{ij} can be simply written as

$$\frac{d\rho_{ij}}{dt} = i[H^0,\rho_{ij}] + \frac{1}{2}\int dt[H^0,[H^0,\rho_{ij}]]. \tag{6}$$

It has been discussed in Ref. [9] that the amplitude of forward scattering $[H^0,\rho_{ij}]$, which affects only photon polarizations without changing photon momenta, is of the order of H^0 or (αG^F). In other words, the rate of generation of polarization due to the forward scattering amplitude of photon-neutrino interaction cannot simply be neglected. This is the main result of this article and will be shown in detail. Whereas the high order terms $[H^0, [H^o,\rho_{ij}]]$ for the scattering cross section of photon-neutrino energy-momentum states is of the order of $(\alpha G^F)^2$, which is thus negligible in comparison with the forward scattering amplitude.

As discussed in previous points, the leading forward scattering (αG^F) of CMB photons and CNB neutrinos which we considered only affects the polarizations of photons. This leading contribution neither depends on neutrino masses nor changes the direction of photon propagation (momentum); thus, it does not change the anisotropy of CMB photons. Therefore, our calculations do meet other cosmological constraints. However the next-leading order contribution $(\alpha G^F)^2$ that could matter is very small and negligible.

Using the Stokes parameters in Eq. (1), the total intensity I, linear polarizations intensities Q and U, as well as the V indicating the difference between left- and right-circular polarizations intensities, we consider both the Compton and photon-neutrino scattering and write Eq. (4) as follows:

$$\frac{dI}{dt} = C^I_{e\gamma}$$
$$\frac{d}{dt}(Q\pm iU) = C^\pm_{e\gamma}\mp i\dot{\kappa}_\pm(Q\pm iU) + \mathcal{O}(V)$$
$$\frac{dV}{dt} = C^V_{e\gamma} + \dot{\kappa}_Q Q + \dot{\kappa}_U U, \tag{7}$$

here $C^I_{e\gamma}$, $C^\pm_{e\gamma}$, and $C^V_{e\gamma}$ respectively indicate the contributions from the Compton scattering to the time evaluation of I, $Q\pm iU$, and V parameters; their expressions can be found from the literature for example [10–12]. Whereas the contributions from the photon-neutrino scattering (5) are given by

$$\dot{\kappa}_\pm = -\frac{\sqrt{2}}{6\pi k^0}\alpha G^F\int d\mathbf{q}f_\nu(x,q)\times(\varepsilon_{\mu\nu\rho\sigma}\epsilon_2^\mu\epsilon_1^\nu k^\rho q^\sigma)$$
$$\dot{\kappa}_Q = -\frac{\sqrt{2}}{3\pi k^0}\alpha G^F n_\nu\langle v_\alpha q_\beta\rangle\epsilon_2^\alpha\epsilon_1^\beta$$
$$\dot{\kappa}_U = -\frac{\sqrt{2}}{6\pi k^0}\alpha G^F n_\nu(\langle v_\alpha q_\beta\rangle\epsilon_1^\alpha\epsilon_1^\beta - \langle v_\alpha q_\beta\rangle\epsilon_2^\alpha\epsilon_2^\beta), \tag{8}$$

where we define the neutrino average velocity

$$\langle v_\alpha\rangle = \frac{1}{n_\nu}\int\frac{d^3q}{(2\pi)^3}\frac{q_\alpha}{q_0}f_\nu(x,q),$$
$$\langle v_\alpha q_\beta\rangle = \frac{1}{n_\nu}\int\frac{d^3q}{(2\pi)^3}\frac{q_\alpha}{q_0}q_\beta f_\nu(x,q), \tag{9}$$

where the neutrino number density $n_\nu(x) = \int d^3q/(2\pi)^3 f_\nu(x,q)$ and energy density $\epsilon_\nu(x) = \int d^3q/(2\pi)^3 q^0 f_\nu(x,q)$. In the second equation of Eq. (7), we will neglect the small contribution $\mathcal{O}(V)$ from the circular polarization. In Eq. (8), the first equation $\dot{\kappa}_\pm$ yields

$$\dot{\kappa}_\pm = \frac{\sqrt{2}}{6\pi k^0}\alpha G^F \int d\mathbf{q} f_\nu(x,q) \times [q^0 \mathbf{k}\cdot(\epsilon_1\times\epsilon_2) + k^0 \mathbf{q}\cdot(\epsilon_1\times\epsilon_2)] = \frac{\sqrt{2}}{6\pi}\alpha G^F \frac{n_\nu}{2}[1+\langle \mathbf{v}\rangle\cdot(\epsilon_1\times\epsilon_2)] \approx \frac{\sqrt{2}}{6\pi}\alpha G^F \frac{n_\nu}{2}, \quad (10)$$

where $\mathbf{k}\cdot(\epsilon_1\times\epsilon_2) = |\mathbf{k}|$. In this article, we apply Eqs. (7)–(10) to the case of photons scattering with CNB, whose average velocities (9) are small, and will be discussed in the next section. As a result of the leading order approximation, the dominated contribution of photon-neutrino scattering to photon polarization comes from the first term of Eq. (10).

III. CNB NEUTRINO DISTRIBUTION FUNCTION AND AVERAGE VELOCITY

We discuss the CNB neutrino distribution function $f_\nu(x,q)$ and average velocity $\langle \mathbf{v}\rangle$ by using the Boltzmann equation for massive neutrinos [14,15]. It is convenient to write the phase space distribution of neutrino $f_\nu(x,q)$ as a zeroth-order distribution $f_{\nu 0}(x,q)$ plus a perturbation $\Psi(x,q)$ as the following:

$$f_\nu(x,q) = f_{\nu 0}(x,q)[1+\Psi(x,q)], \quad (11)$$

where $\vec{q} = q\hat{n}$, and $\hat{n}$ indicates the direction of neutrino velocity. This phase space distribution evolves according to the collisionless Boltzmann equation which is given as the following in terms of our variables $(\vec{x}, q, \hat{n}, \tau)$:

$$\frac{\partial\Psi}{\partial\tau} + i\frac{q}{\epsilon_\nu}(\vec{K}.\hat{n})\Psi + \frac{d\ln f_{\nu 0}}{d\ln(q)}\left[\dot{\phi} - i\frac{\epsilon_\nu}{q}(\vec{K}.\hat{n})\psi\right] = 0, \quad (12)$$

where $\vec{K}$ (wave number) is the Fourier conjugate of $\vec{X}$; the collision terms on the rhs of the above equation are neglected due to the weak interactions of neutrinos and two scalar potentials, ϕ and ψ, that characterize the metric perturbations,

$$ds^2 = a^2(\tau)\{-(1+2\psi)d\tau^2 + (1+2\varphi)dx_i dx^i\}. \quad (13)$$

Notice, Eq. (12) shows that the Boltzmann equation depends on the direction $\hat{n}$ of the neutrino momentum only through its angle with $\vec{K}$, i.e., $(\mu' = \hat{K}\cdot\hat{n})$. The conformal Newtonian gauge (also known as the longitudinal gauge) advocated in Ref. [16] is a particularly simple gauge to use for the scalar mode of metric perturbations, and we neglect the tensor perturbations here. By considering the collisionless Boltzmann equation (12) and expanding the angular dependence of the perturbation Ψ in a series of Legendre polynomials $P_l(\mu')$ we have the following:

$$\Psi(\vec{K}, q, \mu', \tau) = \sum_{l=0}(-i)^l(2l+1)\Psi_l(\vec{K},\tau)P_l(\mu'). \quad (14)$$

In addition, in the following calculations, we select that $\hat{\mathbf{K}}$ and $\hat{z}$ are in the same direction; the components of the photon momentum $\mathbf{k}$ and the neutrino momentum $\mathbf{q}$ are the following:

$$\hat{\mathbf{k}} = \{\cos\theta, \sin\theta\cos\phi_k, \sin\theta\sin\phi_k\},$$
$$\hat{\mathbf{q}} = \{\cos\theta', \sin\theta'\cos\phi_q, \sin\theta'\sin\phi_q\}. \quad (15)$$

In this case we have for the second term of Eq. (10):

$$\int q^2 dq \frac{q}{\epsilon_\nu} f_{\nu 0}(q)\int d\Omega(\hat{\mathbf{q}}\cdot\hat{\mathbf{k}})\Psi(\mu') = \int q^2 dq \frac{q}{\epsilon_\nu} f_{\nu 0}(q)\Psi_1 \cong n_\nu \mu\langle v\rangle. \quad (16)$$

The time evaluation of Ψ_l is given

$$\dot{\Psi}_0 = -\frac{qK}{\epsilon_\nu}\Psi_1 - \dot{\varphi}\frac{d\ln f_{\nu 0}}{d\ln q},$$
$$\dot{\Psi}_1 = \frac{qK}{3\epsilon_\nu}(\Psi_0 - 2\Psi_2) + \frac{\epsilon_\nu K}{3q}\frac{d\ln f_{\nu 0}}{d\ln q},$$
$$\dot{\Psi}_l = \frac{qK}{(2l+1)\epsilon_\nu}(l\Psi_{l-1} - (l+1)\Psi_{l+1}). \quad (17)$$

Notice that $\langle v\rangle \propto \Psi_1$ and $\Psi_l \leq \Psi \sim \frac{\Delta T}{T}|_\nu$ for CNB neutrinos. As a result, the second term on the rhs of Eq. (10) depends on the neutrino average velocity $\langle v\rangle \propto \Psi_1$, being proportional to the first perturbation of neutrino distribution function. Therefore, we can neglect this term.

IV. TIME EVOLUTION OF POLARIZED CMB PHOTONS

In this section, we focus on the linear polarization of CMB (*E*- and *B*-modes) due to the Compton and photon-neutrino (CNB) scattering in company with primordial scalar perturbations only. As usual, the CMB radiation transfer in the conformal time η is described by the multipole moments of temperature (I) and polarization (P)

$$\Delta_{I,P}(\eta, K, \mu) = \sum_{\ell=0}^{\infty}(2\ell+1)(-i)^l\Delta^\ell_{I,P}(\eta,K)P_l(\mu),$$

where $\mu = \hat{\mathbf{n}}\cdot\hat{\mathbf{K}} = \cos\theta$, θ is the angle between the CMB photon direction $\hat{\mathbf{n}} = \mathbf{k}/|\mathbf{k}|$ and the wave vectors $\mathbf{K}$ of Fourier modes of scalar perturbations, and $P_\ell(\mu)$ is the Legendre polynomial of rank ℓ. We adopt the following Boltzmann equation obeyed by $\Delta_{I,P}(\eta,K,\mu)$, and expand the primordial scalar perturbations (S) of the metric field in Fourier modes characterized by the wave vector $\mathbf{K}$. For a given Fourier mode, one can select a coordinate system where $\mathbf{K}\|\hat{\mathbf{z}}$ and $(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2) = (\hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_\phi)$. For each plane wave, the scattering can be described as the transport through a plane parallel medium [17,18], and Boltzmann equations are

$$\frac{d}{d\eta}\Delta_I^{(S)} + iK\mu\Delta_I^{(S)} + 4[\dot{\psi} - iK\mu\varphi] = \dot{\tau}\left[-\Delta_I^{(S)} + \Delta_I^{0(S)} + i\mu v_b + \frac{1}{2}P_2(\mu)\Pi\right] \quad (18)$$

$$\frac{d}{d\eta}\Delta_P^{\pm(S)} + iK\mu\Delta_P^{\pm(S)} = \dot{\tau}\left[-\Delta_P^{\pm(S)} - \frac{1}{2}[1-P_2(\mu)]\Pi\right] \mp ia(\eta)\dot{\kappa}_\pm\Delta_P^{\pm(S)}, \quad (19)$$

where the opacity $\dot{\tau} \equiv d\tau/d\eta$, the normalized scaling factor $a(\eta)|_{\eta_0} = 1$ at the present time η_0, $\Pi \equiv \Delta_I^{2(S)} + \Delta_P^{2(S)} + \Delta_P^{0(S)}$, and the polarization anisotropy is defined by

$$\Delta_P^{\pm(S)} = Q^{(S)} \pm iU^{(S)}. \quad (20)$$

In the rhs of Eqs. (18) and (19), the scattering parts are determined by the Compton and photon-neutrino scattering terms in Eq. (7), in particular, the contribution of photon-neutrino (CNB) scattering to CMB polarization comes from the $\dot{\kappa}_\pm$ terms in Eq. (7). The temperature anisotropy Δ_I^S depends on the metric perturbations φ and ψ and baryon velocity term v_b in Eq. (18). Equation (19) for the polarization anisotropy can be written as follows:

$$\frac{d}{d\eta}[\Delta_P^{\pm(S)}e^{iK\mu\eta\pm i\tilde{\kappa}(\eta,\mu)+\tilde{\tau}(\eta)}] = -e^{iK\mu\eta\pm i\tilde{\kappa}(\eta)+\tilde{\tau}(\eta)}\left(\frac{1}{2}\dot{\tau}[1-P_2(\mu)]\Pi\right), \quad (21)$$

where

$$\tilde{\kappa}(\eta,\mu) \equiv \int_0^\eta d\eta a(\eta)\dot{\kappa}_\pm, \qquad \tilde{\tau}(\eta) \equiv \int_0^\eta d\eta\dot{\tau}. \quad (22)$$

With the initial condition $\Delta_P^{\pm(S)}(0,K,\mu) = 0$, the integration of Eq. (21) along the line of sight up to the present time η_0 yields [19]

$$\Delta_P^{\pm(S)}(\eta_0,K,\mu) = \frac{3}{4}(1-\mu^2)\int_0^{\eta_0} d\eta e^{ix\mu\pm i\kappa(\eta)-\tau}\dot{\tau}\Pi(\eta,K), \quad (23)$$

where $x = K(\eta_0 - \eta)$ and

$$\kappa(\eta) = \int_\eta^{\eta_0} d\eta a(\eta)\dot{\kappa}_\pm(\eta). \quad (24)$$

These are analogous to the optical depth $\tau(\eta)$ with respect to the Compton scattering

$$\dot{\tau} = an_e x_e\sigma_T, \qquad \tau(\eta) = \int_\eta^{\eta_0}\dot{\tau}(\eta)d\eta, \quad (25)$$

where n_e is the electron density, x_e is the ionization fraction, and σ_T is the Thomson cross section.

V. THE B-MODE POWER SPECTRUM OF POLARIZED CMB PHOTONS

One can separate the CMB polarization $\Delta_P^{\pm(S)}(\eta_0,K,\mu)$ into the divergence-free part [B mode $\Delta_B^{(S)}$] and curl-free part [E mode $\Delta_E^{(S)}$] as the following [12]:

$$\Delta_E^{(S)}(\eta_0,K,\mu) \equiv -\frac{1}{2}[\bar{\eth}^2\Delta_P^{+(S)}(\eta_0,K,\mu) + \eth^2\Delta_P^{-(S)}(\eta_0,K,\mu)] \quad (26)$$

$$\Delta_B^{(S)}(\eta_0,K,\mu) \equiv \frac{i}{2}[\bar{\eth}^2\Delta_P^{+(S)}(\eta_0,K,\mu) - \eth^2\Delta_P^{-(S)}(\eta_0,K,\mu)], \quad (27)$$

where $\eth$ and $\bar{\eth}$ are spin raising and lowering operators, respectively, and one assumes scalar perturbations to be axially symmetric around $\mathbf{K}$ so that

$$\bar{\eth}^2\Delta_P^{\pm(S)}(\eta_0,K,\mu) = \partial_\mu^2[(1-\mu^2)\Delta_P^{\pm(S)}(\eta_0,K,\mu)], \quad (28)$$

where $\partial_\mu = \partial/\partial\mu$. From Eqs. (23) and (28), we obtain the E and B modes

$$\Delta_E^{(S)}(\eta_0,K,\mu) = -\frac{3}{4}\int_0^{\eta_0} d\eta g(\eta)\Pi(\eta,K)\partial_\mu^2 \times [(1-\mu^2)^2 e^{ix\mu}\cos\kappa(\eta)], \quad (29)$$

$$\Delta_B^{(S)}(\eta_0,K,\mu) = \frac{3}{4}\int_0^{\eta_0} d\eta g(\eta)\Pi(\eta,K)\partial_\mu^2 \times [(1-\mu^2)^2 e^{ix\mu}\sin\kappa(\eta)], \quad (30)$$

where $g(\eta) = \dot{\tau}e^{-\tau}$. Equation (30) shows that the photon-neutrino scattering ($\kappa \neq 0$) results in the nontrivial B mode $\Delta_B^{(S)}$ and the modifications of the E mode $\Delta_E^{(S)}$. This agrees that the Compton scattering only cannot generate B modes without taking into account the tensor type of metric perturbations [12,20–22].

Using Eq. (23), we can obtain the value of $\Delta_{E,B}^{(S)}(\hat{\mathbf{n}})$ at the present time η_0 and in the direction $\hat{\mathbf{n}}$ by summing over all their Fourier modes K, analogously to the normal approach [11,12,19],

$$\Delta_{E,B}^{(S)}(\hat{\mathbf{n}}) = \int d^3\mathbf{K}\xi(\mathbf{K})e^{\mp 2i\phi_{K,n}}\Delta_{E,B}^{(S)}(\eta_0,K,\mu), \quad (31)$$

where $\phi_{K,n}$ is the angle needed to rotate the $\mathbf{K}$ and $\hat{\mathbf{n}}$ dependent basis to a fixed frame in the sky. The random variable $\xi(\mathbf{K})$ used to characterize the initial amplitude of the mode satisfies (see for example [11,12,19])

$$\langle\xi^*(\mathbf{K}_1)\xi(\mathbf{K}_2)\rangle = P_S(\mathbf{K})\delta(\mathbf{K}_1 - \mathbf{K}_2), \quad (32)$$

where $P_S(K)$ is the initial power spectrum of the scalar mode perturbation.

As a result, by integrating Eqs. (31) and (32) over the initial power spectrum of the metric perturbation, we obtain the power spectrum for E and B modes

$$C^{EE,BB}_{\ell(S)} = \frac{1}{2\ell+1}\frac{(\ell-2)!}{(\ell+2)!}\int d^3KP_S(K) \times\left|\sum_m \int d\Omega Y^*_{lm}\Delta^{(S)}_{E,B}(\eta_0,K,\mu)\right|^2. \quad (33)$$

Using identities $\partial^2_\mu(1-\mu^2)^2e^{ix\mu} \equiv (1+\partial^2_x)x^2e^{ix\mu}$ and $\int d\Omega Y^*_{\ell m}e^{ix\mu} = (i)^\ell\sqrt{4\pi(2\ell+1)}j_\ell(x)\delta_{m0}$, we obtain the polarized CMB power spectrum in multipole moments ℓ,

$$C^{EE}_{\ell(S)} = (4\pi)^2\frac{(\ell+2)!}{(\ell-2)!}\int d^3KP_S(K) \times\left|\frac{3}{4}\int_0^{\eta_0} d\eta g(\eta)\Pi(\eta,K)\frac{j_\ell}{x^2}\cos\kappa(\eta)\right|^2, \quad (34)$$

$$C^{BB}_{\ell(S)} = (4\pi)^2\frac{(\ell+2)!}{(\ell-2)!}\int d^3KP_S(K) \times\left|\frac{3}{4}\int_0^{\eta_0} d\eta g(\eta)\Pi(\eta,K)\frac{j_\ell}{x^2}\sin\kappa(\eta)\right|^2, \quad (35)$$

where $j_\ell(x)$ is a spherical Bessel function of rank ℓ. The evolution equations derived for photon and neutrino perturbations can be solved numerically once the initial perturbations are specified. By starting the integration at early times when a given K-mode is still outside the horizon $K\tau \ll 1$ and implementing a very basic isocurvature and adiabatic initial condition given by [14], the above quantities (34) and (35) can be estimated. In Fig. 1, we plot the numerical value $C^{BB}_{\ell(S)}$ of Eq. (35) together with the BICEP2 result. It is shown that the contribution of photon-neutrino (CNB) scattering to the B mode is negligible for small ℓ; however, it is significantly large for $50 < \ell < 200$.

To order to better understand our results (34) and (35), we approximately write $C^{EE}_{\ell(S)}$ and $C^{BB}_{\ell(S)}$ as follows:

$$C^{EE}_{\ell(S)} \approx \bar{C}^{EE}_{\ell(S)}(\cos^2\bar{\kappa}), \qquad C^{BB}_{\ell(S)} \approx \bar{C}^{EE}_{\ell(S)}(\sin^2\bar{\kappa}), \quad (36)$$

where

$$\bar{C}^{EE}_{\ell(S)} = (4\pi)^2\frac{(l+2)!}{(l-2)!}\int d^3KP_S(K) \times\left|\frac{3}{4}\int_0^{\eta_0} d\eta g(\eta)\Pi(\eta,K)\frac{j_\ell}{x^2}\right|^2 \quad (37)$$

is the power spectrum of the E-mode polarization contributed from the Compton scattering in the case of scalar perturbation [11]. In Eq. (36), the mean value $\bar{\kappa}$ of Eq. (24)

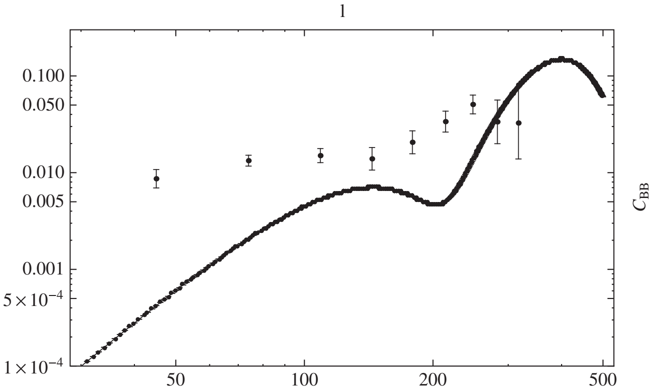

FIG. 1. The solid line represents $C_{BB} \equiv \ell(\ell+1)C^{BB}_{\ell(S)}/2\pi[\mu K^2]$ due to the primordial scalar perturbations and photon-neutrino (CNB) scattering. The experiment BICEP2 results (dots with their error bars) are plotted.

is an average from the last scattering time (redshift $z_l \approx 10^3$) to the present time ($z_0 = 0$). Using the matter dominated Friedmann equation $H^2/H_0^2 = \Omega^0_M(1+z)^3 + \Omega^0_\Lambda$, $H_0 \approx 74$ km/s/Mpc, $\Omega^0_M \approx 0.27, \Omega^0_\Lambda \approx 0.73$, and $ad\eta = -dz/H(1+z)$, as well as the conservation of the total neutrino number $n_\nu = n^0_\nu(1+z)^3$, we obtain

$$\kappa(z) = \int_\eta^{\eta_0} ad\eta\dot{\kappa}_\pm = \frac{\sqrt{2}}{12\pi}\alpha G^F n^0_\nu\int_z^{z_l} dz'\frac{(1+z')^2}{H(z')} = \frac{\sqrt{2}}{12\pi}\alpha G^F n^0_\nu\frac{2H(z')}{3\Omega^0_M H_0^2}\Big|_z^{z_l}$$
$$\bar{\kappa} \equiv \frac{1}{z_1 - z_0}\int_{z_1}^{z_0} dz\kappa(z) \approx 0.16, \quad (38)$$

where the present number density of all flavor neutrinos and antineutrinos $n^0_\nu = \sum(n^0_\nu + n^0_{\bar{\nu}}) \approx 340\ \mathrm{cm}^{-3}$. Actually, the $\bar{\kappa}$ is the mean opacity of CMB photons against the photon-neutrino (CNB) scattering.

To end this section, we would like to point out that the B-mode power spectrum $C^{BB}_{\ell(S)}$ of Eq. (35) is attributed only to the scalar perturbations and photon-neutrino (CNB) scattering. This result implies that the B-mode power spectrum could be contributed by other mechanisms with scalar perturbations, in addition to the contribution from the primordial tensor perturbations [11]. Therefore, this is crucial how to interpret the measurement of r parameter that is the ratio of the B-mode power spectrum and the E-mode power spectrum.

VI. SUMMARY

Suppose that the total contribution to the B-mode polarization of CMB photons comes from the primordial tensor perturbations (T); one obtains the B-mode power spectrum [11]

$$\bar{C}^{BB}_{\ell(T)} = (4\pi)^2 \int d^3K P_T(K) \times \left| \frac{3}{4} \int_0^{\eta_0} d\eta g(\eta) S^T(\eta, K) \left(2j'_\ell + \frac{j_\ell}{x} \right) \right|^2, \quad (39)$$

$$S^T(\eta, K) = \left[\frac{1}{10} \Delta_I^{0(T)} + \frac{1}{7} \Delta_I^{2(T)} + \frac{3}{70} \Delta_I^{4(T)} - \frac{3}{5} \Delta_P^{0(T)} + \frac{6}{7} \Delta_P^{2(T)} - \frac{3}{70} \Delta_P^{4(T)} \right], \quad (40)$$

where $P_T(K)$ is the initial power spectrum of primordial tensor perturbations. Based on this assumption, one can approximately obtain the r parameter $r = P_T/P_S \propto C^{BB}_{\ell(T)}/\bar{C}^{EE}_{\ell(S)}$. Taking into account contribution (34) or (36) of photon-neutrino (CNB) scattering and assuming the total observed B-mode power spectrum $C^{BB}_{\ell(ob)}$ given by $C^{BB}_{\ell(ob)} = C^{BB}_{\ell(T)} + C^{BB}_{\ell(S)}$, we have

$$r = P_T/P_S \propto (C^{BB}_{\ell(ob)} - C^{BB}_{\ell(S)})/\bar{C}^{EE}_{\ell(S)} \approx C^{BB}_{\ell(T)}/\bar{C}^{EE}_{\ell(S)} - \sin^2\bar{\kappa}, \quad (41)$$

where $\sin^2\bar{\kappa} \sim \bar{\kappa}^2 \simeq 0.025$. This implies that the measured r parameter would not be completely originated from primordial tensor perturbations. In addition, there might be other contributions from either some astrophysical effects [6,8] or some microscopic effects, for example the CMB photon-photon scatterings [23,24]. Therefore, it is important to study possibly significant contributions to the B-mode power spectrum of polarized CMB photons so that one can better understand the contribution of primordial tensor perturbations to the r parameter experimentally measured, $r = 0.2$ as reported by BICEP2.

In summary, we have studied the quantum Boltzmann equation governing the time evolution of the density matrix (Stokes parameters) of polarized CMB photons by considering both the Compton and photon-neutrino (CNB) scattering in the background of primordial scalar perturbations. It is shown that in this case the B-mode spectrum of polarized CMB photons can also be generated without primordial tensor perturbations. We quantitatively calculate the generated B-mode spectrum which is related to the mean opacity $\bar{\kappa}$ (38) of CMB photons scattering with neutrinos (CNB). On the other hand, we compare our result with the B-mode spectrum generated by the Compton scattering in the background of primordial tensor perturbations, which seems to be dominated. We generally discuss the possible implication of our result on the interpretation of the BICEP2 measurement $r = 0.2$ in terms of primordial tensor perturbations.

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