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Gravity duals of 5D $N = 2$ SYM theory from $F(4)$ gauged supergravity

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We study gravity duals of the minimal $N = 2$ super Yang-Mills gauge theories in five dimensions using the matter coupled $F(4)$ gauged supergravity in six dimensions. The $F(4)$ gauged supergravity coupled to n vector multiplets contains $4n + 1$ scalar fields, parametrized by $\mathbb{R}^+ \times SO(4,n)/SO(4) \times SO(n)$ coset manifold. Maximally supersymmetric vacua of the gauged supergravity with $SU(2) \times G$ gauge group, with G being an n-dimensional subgroup of $SO(n)$, correspond to five-dimensional superconformal field theories (SCFTs) with $SU(2)_R$ R symmetry and G global symmetry. Deformations of the UV SCFTs for $G = SU(2)$ and $G = U(2) \sim SU(2) \times U(1)$ symmetries that lead to nonconformal $N = 2$ super Yang-Mills with various unbroken global symmetries are studied holographically.

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I. INTRODUCTION

Much insight to strongly coupled gauge theories can be gained from studying their gravity duals via the AdS/CFT correspondence [1] and its generalization to nonconformal field theories [2–4]. One consequence of the AdS/CFT correspondence which has been extensively studied is holographic RG flows. These flows describe deformations of an UV conformal field theory (CFT) to another conformal fixed point or to a nonconformal field theory in the IR. On the gravity side, an RG flow in the dual field theory is described by an asymptotically anti–de Sitter (AdS) solution which becomes AdS space in a certain limit corresponding to the UV CFT. The gravity solutions interpolate between this AdS space and another AdS space in the case of flows to some IR fixed points. For flows to nonconformal field theories, gravity solutions in the IR will take the form of a domain wall [5]. Furthermore, in flows between CFTs, bulk scalar fields take finite constant values at both conformal fixed points while in flows to nonconformal theories, they are usually logarithmically divergent.

The above argument leads to gravity duals of various supersymmetric gauge theories in four dimensions, and many important characteristics of the gauge theories such as gaugino condensates and confinements can be successfully described by gravity solutions of five-dimensional gauged supergravity; see, for example, [6–8]. On the other hand, holographic duals of higher dimensional gauge theories have not much been explored in the literature. In this paper, we will carry out a similar study for $N = 2$ supersymmetric Yang-Mills (SYM) gauge theories in five dimensions using six-dimensional $F(4)$ gauged supergravity. This should provide the five-dimensional analogue of the four-dimensional results in [6–8].

Five-dimensional field theories are interesting in their own right. It has been discovered in [9–11] that five-dimensional gauge theories admit nontrivial fixed points with enhanced global symmetry. The five-dimensional (5D) field theory describes the dynamics of the D4/D8-brane system whose near horizon limit gives rise to AdS_6 geometry [12]. At the fixed points, the $SO(2N_f) \times U(1)$ global symmetry of the gauge theory with $N_f < 8$ flavors is enhanced to E_{N_f+1}. $E_{6,7,8}$ are the usual exceptional groups and other groups are defined by $E_1 = SU(2)$, $E_2 = SU(2) \times U(1)$, $E_3 = SU(3) \times SU(2)$, $E_4 = SU(5)$, and $E_5 = SO(10)$ [9]. This symmetry enhancement in the case of $SU(2)$ gauge theories has also been shown to appear in the superconformal indices [13].

By using AdS_6/CFT_5 correspondence, it has been proposed in [14] that five-dimensional superconformal field theories with global symmetry G should correspond to AdS_6 vacua of the matter coupled $F(4)$ gauged supergravity in the six-dimensional bulk with the $SU(2)_R \times G$ gauge group. The $SU(2)_R$ R symmetry is gauged by three of the four vector fields in the supergravity multiplet, while the G part of the gauge group is gauged by the vectors in the vector multiplets. The dual field theory has been identified with a singleton field theory on the boundary. A number of papers on gauge/gravity correspondence involving 5D gauge theories and the generalization to quiver gauge theories from the ten-dimensional point of view have appeared in [15–17]. RG flows between 5D quiver gauge theories with $N_f = 0$ have been studied recently in [18] in the ten-dimensional context. Holographic RG flows within the framework of $F(4)$ gauged supergravity have also been studied in [19] and [20]. In this paper, we will give another example of flow solutions to 5D nonconformal gauge theories in the framework of six-dimensional gauged supergravity. As in lower dimensions, this should be more convenient to work with than the ten-dimensional computation and could

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provide a useful tool in the holographic study of $N = 2$ 5D SYM.

Furthermore, the study of gravity duals of 5D gauge theories is not only important in AdS_6/CFT_5 correspondence but is also useful in the context of AdS_7/CFT_6 correspondence [21,22]. This originates from the proposal that the less understood $N = (2,0)$ gauge theory in six dimensions could be defined in term of 5D SYM. Furthermore, it has been shown that 5D superconformal field theory (SCFT) could be an IR fixed point of $N = 2^*$ gauge theory in four dimensions [23]. Therefore, having gravity duals of 5D SYM could be very useful in understanding the dynamics of M5-branes and gauge theories in other dimensions as well.

The paper is organized as follows. In Sec. II, we review relevant information about matter coupled $F(4)$ gauged supergravity in six dimensions and formulas used throughout the paper. Holographic RG flows to nonconformal field theories from the UV fixed point identified with the maximally supersymmetric AdS_6 critical points will be given in Secs. III and IV. All of the solutions can be analytically obtained and would be more useful than the numerical solutions given in some other cases. We end the paper by giving some conclusions and comments in Sec. V.

II. MATTER COUPLED $F(4)$ GAUGED SUPERGRAVITY AND THE DUAL $N = 2$ SUPER YANG-MILLS THEORY

We begin with a brief review of the matter coupled $F(4)$ gauged supergravity in six dimensions. The theory is an extension of the pure $F(4)$ gauged supergravity, constructed a long time ago in [24], by coupling n vector multiplets to the $N = (1,1)$ supergravity multiplet. The resulting theory is elegantly constructed by using the superspace approach in [25–27]. In the present work, we will need only supersymmetry transformations of fermions and the bosonic Lagrangian involving the metric and scalars. Most of the notations and conventions are the same as those given in [25] and [26] but with the metric signature $(-+++++)$.

In half-maximal $N = (1,1)$ supersymmetry, the field content of the supergravity multiplet is given by

$$(e^a_\mu, \psi^A_\mu, A^\alpha_\mu, B_{\mu\nu}, \chi^A, \sigma),$$

where e^a_μ, χ^A, and ψ^A_μ denote the graviton, the spin-1/2 field, and the gravitini, respectively. Both χ^A and ψ^A_μ are eight-component pseudo-Majorana spinors with indices A, $B = 1, 2$ referring to the fundamental representation of the $SU(2)_R \sim USp(2)_R$ R symmetry. The remaining fields are given by the dilaton σ, four vectors A^α_μ, $\alpha = 0, 1, 2, 3$, and a two-form field $B_{\mu\nu}$.

A vector multiplet has component fields

$$(A_\mu, \lambda_A, \phi^\alpha).$$

Each multiplet will be labeled by an index $I = 1, \ldots, n$. The $4n$ scalars $\phi^{\alpha I}$ are described by a symmetric quaternionic manifold $SO(4,n)/SO(4) \times SO(n)$. The dilaton σ can also be regarded as living in the coset space $\mathbb{R}^+ \sim O(1,1)$. As in [25], it is convenient to decompose the α index into $\alpha = (0, r)$ in which $r = 1, 2, 3$. The $SU(2)_R$ R symmetry is identified with the diagonal subgroup of $SU(2) \times SU(2) \sim SO(4) \subset SO(4) \times SO(n)$. A general compact gauge group is then given by $SU(2) \times G$ with $\dim G = n$.

The $4n$ scalars living in the $SO(4,n)/SO(4) \times SO(n)$ coset can be parametrized by the coset representative $L^\Lambda{}_\Sigma$, $\Lambda, \Sigma = 0, \ldots, 3+n$. Using the index splitting $\alpha = (0, r)$, we can split $L^\Lambda{}_\Sigma$ into $(L^\Lambda{}_\alpha, L^\Lambda{}_I)$ and further to $(L^\Lambda{}_0, L^\Lambda{}_r, L^\Lambda{}_I)$. The vielbein of the $SO(4,n)/SO(4) \times SO(n)$ coset P^I_α can be obtained from the left-invariant 1-form of $SO(4,n)$

$$\Omega^\Lambda{}_\Sigma = (L^{-1})^\Lambda{}_\Pi \nabla L^\Pi{}_\Sigma, \qquad \nabla L^\Lambda{}_\Sigma = dL^\Lambda{}_\Sigma - f_\Gamma{}^\Lambda{}_\Pi A^\Gamma L^\Pi{}_\Sigma, \tag{1}$$

via

$$P^I_\alpha = (P^I{}_0, P^I{}_r) = (\Omega^I{}_0, \Omega^I{}_r). \tag{2}$$

The structure constants of the full gauge group $SU(2)_R \times G$ are denoted by $f^\Lambda{}_{\Pi\Sigma}$, which can be split into ϵ_{rst} and C_{IJK} for $SU(2)_R$ and G, respectively. The direct product structure of the gauge group $SU(2)_R \times G$ leads to two coupling constants, g_1 and g_2, which, in the above equation, are encoded in $f^\Lambda{}_{\Pi\Sigma}$.

In this paper, we are interested in $n = 3, 4$ cases with gauge groups $SU(2)_R \times SU(2)$ and $SU(2)_R \times SU(2) \times U(1)$. To describe $SO(4,n)/SO(4) \times SO(n)$, we introduce basis elements of $(4+n) \times (4+n)$ matrices by

$$(e^{xy})_{zw} = \delta_{xz}\delta_{yw}, \qquad w, x, y, z = 1, \ldots, n+4. \tag{3}$$

The $SO(4)$, $SU(2)_R$, and noncompact generators of $SO(4,n)$ are accordingly given by

$$\begin{aligned} SO(4):&\ J^{\alpha\beta} = e^{\beta+1,\alpha+1} - e^{\alpha+1,\beta+1}, \quad \alpha, \beta = 0, 1, 2, 3, \\ SU(2)_R:&\ J^{rs} = e^{s+1,r+1} - e^{r+1,s+1}, \quad r, s = 1, 2, 3, \\ &\ Y^{\alpha I} = e^{\alpha+1,I+4} + e^{I+4,\alpha+1}, \quad I = 1, \ldots, n. \end{aligned} \tag{4}$$

Gaugings lead to fermionic mass–like terms and the scalar potential in the Lagrangian, as well as some modifications to the supersymmetry transformations at first order in the coupling constants. We will give only information relevant to the study of supersymmetric RG flows and refer the reader to [25] and [26] for more details and complete formulas. The bosonic Lagrangian for the metric and scalar fields is given by [26]

$$\mathcal{L} = \frac{1}{4}eR - e\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{4}eP_{I\alpha\mu}P^{I\alpha\mu} - eV, \qquad (5)$$

where $e = \sqrt{-g}$. The scalar kinetic term is written in term of $P^{I\alpha}_\mu = P^{I\alpha}_i\partial_\mu\phi^i$, $i = 1, \ldots, 4n$. For completeness, we also give the explicit form of the scalar potential

$$V = -e^{2\sigma}\left[\frac{1}{36}A^2 + \frac{1}{4}B^iB_i - \frac{1}{4}(C^I{}_tC_{It} + 4D^I{}_tD_{It})\right] - m^2e^{-6\sigma}N_{00} + me^{-2\sigma}\left[\frac{2}{3}AL_{00} - 2B^iL_{0i}\right], \qquad (6)$$

where N_{00} is the 00 component of the scalar matrix defined by

$$N_{\Lambda\Sigma} = L^0_\Lambda(L^{-1})_{0\Sigma} + L^i_\Lambda(L^{-1})_{i\Sigma} - L^I_\Lambda(L^{-1})_{I\Sigma}. \qquad (7)$$

Various quantities appearing in the scalar potential and in the supersymmetry transformations given below are defined as follows:

$$A = \epsilon^{rst}K_{rst}, \qquad B^i = \epsilon^{ijk}K_{jk0}, \qquad (8)$$

$$C^t_I = \epsilon^{trs}K_{rIs}, \qquad D_{It} = K_{0It}, \qquad (9)$$

where

$$\begin{aligned}
K_{rst} &= g_1\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^mL^n_t + g_2C_{IJK}L^I{}_r(L^{-1})_s{}^JL^K_t,\\
K_{rs0} &= g_1\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^mL^n_0 + g_2C_{IJK}L^I{}_r(L^{-1})_s{}^JL^K_0,\\
K_{rIt} &= g_1\epsilon_{lmn}L^l{}_r(L^{-1})_I{}^mL^n_t + g_2C_{IJK}L^I{}_r(L^{-1})_I{}^JL^K_t,\\
K_{0It} &= g_1\epsilon_{lmn}L^l{}_0(L^{-1})_I{}^mL^n_t + g_2C_{IJK}L^I{}_0(L^{-1})_I{}^JL^K_t.
\end{aligned} \qquad (10)$$

Finally, the supersymmetry transformations of χ^A, λ^I_A, and ψ^A_μ involving only scalars and the metric are given by

$$\delta\psi_{\mu A} = D_\mu\epsilon_A - \frac{1}{24}(Ae^\sigma + 6me^{-3\sigma}(L^{-1})_{00})\epsilon_{AB}\gamma_\mu\epsilon^B - \frac{1}{8}(B_te^\sigma - 2me^{-3\sigma}(L^{-1})_{t0})\gamma^7\sigma^t_{AB}\gamma_\mu\epsilon^B, \qquad (11)$$

$$\delta\chi_A = \frac{1}{2}\gamma^\mu\partial_\mu\sigma\epsilon_{AB}\epsilon^B + \frac{1}{24}[Ae^\sigma - 18me^{-3\sigma}(L^{-1})_{00}]\epsilon_{AB}\epsilon^B - \frac{1}{8}[B_te^\sigma + 6me^{-3\sigma}(L^{-1})_{t0}]\gamma^7\sigma^t_{AB}\epsilon^B, \qquad (12)$$

$$\delta\lambda^I_A = P^I_{ri}\gamma^\mu\partial_\mu\phi^i\sigma^r{}_{AB}\epsilon^B + P^I_{0i}\gamma^7\gamma^\mu\partial_\mu\phi^i\epsilon_{AB}\epsilon^B - (2i\gamma^7D^I{}_t + C^I{}_t)e^\sigma\sigma^t_{AB}\epsilon^B - 2me^{-3\sigma}(L^{-1})^I{}_0\gamma^7\epsilon_{AB}\epsilon^B, \qquad (13)$$

where $\sigma^{tC}{}_B$ are Pauli matrices and $\epsilon_{AB} = -\epsilon_{BA}$. The space-time gamma matrices γ^a, with a being tangent space indices, satisfy

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, \qquad \eta^{ab} = \mathrm{diag}(-1, 1, 1, 1, 1, 1), \qquad (14)$$

and $\gamma^7 = \gamma^0\gamma^1\gamma^2\gamma^3\gamma^4\gamma^5$.

We now give a short description of the UV SCFT which is identified with the AdS_6 vacuum preserving 16 supercharges. At this vacuum, all scalars vanish, and the full gauge group $SU(2)_R \times G$ is preserved. The bulk fields in the supergravity multiplet are dual to the operators in the energy-momentum tensor supermultiplet in the five-dimensional field theory, while the bulk vector multiplets correspond to the global current supermultiplets. The full spectrum of all supergravity fields can be found in [25] and [26]. $SU(2)_R$ singlet scalars in the adjoint representation of G are dual to operators of dimension four corresponding to the highest components of the global current supermultiplets. These scalars give supersymmetry preserving deformations, as discussed in [14]. On the other hand, the dilaton and $SU(2)_R$ triplet scalars are dual to operators of dimension three and correspond to supersymmetry breaking deformations.

III. RG FLOWS FROM $SU(2)_R \times SU(2)$ SCFT

We begin with the simplest possibility with $n = 3$ and the $SU(2)_R \times SU(2)$ gauge group. The gravity theory consists of 13 scalars parametrized by $O(1,1) \times SO(4,3)/SO(4) \times SO(3)$ coset space. We are interested in $SU(2)_R$ singlet scalars which are given by σ and an additional three scalars from $SO(4,3)/SO(4) \times SO(3)$. The latter correspond to the noncompact generators Y_{11}, Y_{12}, and Y_{13}. The coset representative is accordingly written as

$$L = e^{a_1Y_{11}}e^{a_2Y_{12}}e^{a_3Y_{13}}. \qquad (15)$$

The space-time metric is the standard domain wall ansatz

$$ds^2 = e^{2A(r)}dx^2_{1,4} + dr^2, \qquad (16)$$

in which five-dimensional Poincaré symmetry is manifest. From now on, the six-dimensional space-time indices will be split as (μ, r) with $\mu = 0, \ldots, 4$.

Using (11), (12), and (13), we find the following Bogomol'nyi-Prasad-Sommerfeld (BPS) equations:

$$a_1{}' = -2e^{-3\sigma}m\frac{\sinh a_1}{\cosh a_2\cosh a_3}, \qquad (17)$$

$$a_2{}' = -2e^{-3\sigma}m\frac{\cosh a_1\sinh a_2}{\cosh b_3}, \qquad (18)$$

$$a_3{}' = -2e^{-3\sigma}m\cosh a_1\cosh a_2\sinh a_3, \qquad (19)$$

$$\sigma' = -\frac{1}{2}[e^{\sigma}g_1 - 3e^{-3\sigma}m\cosh a_1\cosh a_2\cosh a_3], \quad (20)$$

$$A' = \frac{1}{2}[e^{\sigma}g_1 + e^{-3\sigma}m\cosh a_1\cosh a_2\cosh a_3], \quad (21)$$

where $'$ denotes $\frac{d}{dr}$ and we have used the projection $\gamma^r\epsilon^A = \epsilon^A$. The presence of γ^7 in $\delta\lambda^I_A$ does not impose any condition on ϵ^A since it appears as an overall factor in all of the BPS equations obtained from $\delta\lambda^I_A = 0$. That the bulk gravity solution preserves eight supercharges is to be expected because the minimal SYM in five dimensions has eight supercharges. The equation for the warp factor $A(r)$ is obtained from $\delta\psi^A_\mu, \mu = 0, 1, 2, 3, 4$. The $\delta\psi^A_r = 0$ equation would give the dependence of the Killing spinors on the r coordinate as in other cases. We now look at solutions of interest.

A. Flow to $SU(2)_R \times U(1)$ SYM

We first study the solution that breaks the $SU(2)$ global symmetry to $U(1)$. This corresponds to turning on only a_3 and σ. The latter is of course a singlet of the full gauge group $SU(2)_R \times SU(2)$. With $a_1 = a_2 = 0$, Eqs. (17) and (18) are trivially satisfied, and Eqs. (19), (20), and (21) become

$$a_3{}' = -2e^{-3\sigma}m\sinh a_3, \quad (22)$$

$$\sigma' = \frac{1}{2}(-g_1e^{\sigma} + 3e^{-3\sigma}m\cosh a_3), \quad (23)$$

$$A' = \frac{1}{2}(g_1e^{\sigma} + e^{-3\sigma}m\cosh a_3). \quad (24)$$

We can solve Eq. (22) by introducing a new radial coordinate $\tilde{r}$ such that $\frac{d\tilde{r}}{dr} = e^{-3\sigma}$. We then find the solution for a_3,

$$a_3 = \pm\ln\left[\frac{1 + e^{-2m\tilde{r}+C_1}}{1 - e^{-2m\tilde{r}+C_1}}\right]. \quad (25)$$

This form is very similar to the solution studied in [6] for the four-dimensional (4D) SYM. C_1 is an integration constant. There are two possibilities for the two signs. Combining Eqs. (22) and (23) gives an equation for $\frac{d\sigma}{da_3}$,

$$\frac{d\sigma}{da_3} = \frac{1}{4m}(e^{4\sigma}g_1\mathrm{csch}a_3 - 3m\coth a_3), \quad (26)$$

whose solution is given by

$$\sigma = -\frac{1}{4}\ln\left[\frac{g_1(3\cosh a_3 - \cosh(3a_3) + 18C_2\sinh^3 a_3)}{6m}\right], \quad (27)$$

with C_2 being another integration constant.

After changing to the $\tilde{r}$ coordinate and using the a_3 solution, we find that the combination of (24) and (23) becomes, with $'$ now being $\frac{d}{d\tilde{r}}$,

$$A' + \sigma' = \frac{2m(e^{4m\tilde{r}} + e^{2C_1})}{e^{2C_1} - e^{4m\tilde{r}}}. \quad (28)$$

The solution to this equation can be readily found to be

$$A = 2m\tilde{r} + \ln(1 - e^{C_1-2m\tilde{r}}) + \ln(1 + e^{C_1-2m\tilde{r}}) - \sigma, \quad (29)$$

where we have neglected the additive integration constant to A by absorbing it into the rescaling of the x^μ coordinates. To identify the maximally supersymmetric vacuum at $\sigma = a_3 = 0$ with the $N = 2$ SCFT, we have to set $g_1 = 3m$. In the above solutions, we have not done this in order to keep the solutions in a generic form. Note also that if we try to truncate σ out by setting $\sigma = 0$, Eq. (23) will imply $a_3 = 0$. Therefore, to obtain a nontrivial solution, we must keep σ nonvanishing.

An RG flow to a nonconformal field theory with only the dilaton σ in pure $F(4)$ gauged supergravity has been studied in [19]. The resulting solution is interpreted as the analogue of the Coulomb branch flow. We now have a more general flow solution in the case of matter coupled $F(4)$ gauged supergravity. As $r \to \infty$, σ, $a_3 \to 0$, we see that $\tilde{r} \sim r \to \infty$. In this limit, we obtain the maximally supersymmetric AdS_6 background with $A \sim 2mr = \frac{r}{L}$, where the AdS_6 radius in the UV is given by $L = \frac{1}{2m}$. According to the AdS/CFT correspondence, this is identified with the UV SCFT with $SU(2)_R \times SU(2)$ SCFT in five dimensions. From the above solutions, the behavior of σ and a_3 near the UV point with $g_1 = 3m$ is readily seen to be

$$a_3 \sim e^{-2mr} = e^{-\frac{r}{L}}, \qquad \sigma \sim a_3^3 \sim e^{-6mr} = e^{-\frac{3r}{L}}. \quad (30)$$

We see that a_3 corresponds to a deformation by a relevant operator of dimension $\Delta = 4$ while σ describes a deformation by a vacuum expectation value of operator of dimension $\Delta = 3$.

There is an issue of singularities in the IR which are typical in flows to nonconformal field theories. Physical and unphysical singularities can be classified by using the criterion given in [28]. From the solution, we see that a_3 is singular when $\tilde{r} \to \frac{C_1}{2m}$. We now consider the case with $a_3 > 0$ and $a_3 < 0$ separately. For $a_3 > 0$, we find $a_3 = -\ln(2m\tilde{r} - C_1) + \ln 2$, as $2m\tilde{r} \sim C_1$ and

$$\begin{aligned}\sigma &= \frac{3}{4}\ln(2m\tilde{r} - C_1)\\ &\quad - \frac{1}{4}\ln[9C_2 - 2 + 3(2m\tilde{r} - C_1)^2(9C_2 - 2)\\ &\quad + 3(9C_2 + 2)(2m\tilde{r} - C_1)^4 + \cdots].\end{aligned} \quad (31)$$

The warp factor A near $\tilde{r} \to \frac{C_1}{2m}$ is given by

$$A = \ln(2m\tilde{r} - C_1) - \sigma. \quad (32)$$

For $C_2 = \frac{2}{9}$, we find that

$$\sigma \sim -\frac{1}{4}\ln(2m\tilde{r} - C_1), \qquad A \sim \frac{5}{4}\ln(2m\tilde{r} - C_1). \quad (33)$$

We can find the relation between $\tilde{r}$ and r in this limit by using $\frac{d\tilde{r}}{dr} = e^{-3\sigma}$. The relation is given by

$$2mr - C = 4(2m\tilde{r} - C_1)^4, \quad (34)$$

where C is a new integration constant. The metric becomes

$$ds^2 = (2mr - C)^{10} dx_{1,4}^2 + dr^2, \quad (35)$$

where we have absorbed the multiplicative constant to the scaling of x^μ coordinates. According to the domain-wall/quantum field theory correspondence, this background is dual to a nonconformal SYM theory in five dimensions.

To determine whether the singularity in the solution is acceptable or not, we check the scalar potential on the solution. With $a_1 = a_2 = 0$ and $g_1 = 3m$, the potential is given by

$$V = e^{-6\sigma} m^2 [\cosh(2a_3) - 12 e^{4\sigma}\cosh a_3 - 9e^{8\sigma}]. \quad (36)$$

It can be verified that $V \to -\infty$ as $a_3, \sigma \to \infty$. The singularity is then physical according to the criterion of [28]. For $a_3 < 0$, it can be easily checked that the singularity is acceptable for the choice $C_2 = -\frac{2}{9}$ which leads to

$$a_3 \sim \ln(2m\tilde{r} - C_1), \qquad \sigma - \frac{1}{4}\ln(2m\tilde{r} - C_1),$$
$$ds^2 = (2mr - C)^{10} dx_{1,4}^2 + dr^2. \quad (37)$$

On the other hand, if $C_2 \neq \pm\frac{2}{9}$ for $a_3 \sim \pm\ln(2m\tilde{r} - C_1)$, respectively, the solution is asymptotic to

$$a_3 \sim \pm\ln(2m\tilde{r} - C_1), \qquad \sigma \sim \frac{3}{4}\ln(2m\tilde{r} - C_1),$$
$$ds^2 = (2mr - C)^{\frac{2}{13}} dx_{1,4}^2 + dr^2, \quad (38)$$

where we have used the relation $(2m\tilde{r} - C_1)^{\frac{13}{4}} = \frac{13}{4}(2mr - C)$, near $\tilde{r} \sim \frac{C_1}{2m}$, with a constant C. The singularity in this case is, however, not acceptable since $V \to \infty$.

It is useful to comment on the IR singularities. Following the discussion in [5], the criterion of [28] is related to the fact that the divergence in a vacuum expectation value of an operator O dual to a canonical scalar ϕ is excluded. In the IR, the scalar bulk action is given by $S \sim \int e^{5A}(\partial\phi)^2$ since the potential is irrelevant due to the divergence of the scalar. The expectation value of O is then given by

$$\langle O \rangle \sim \frac{\delta S}{\delta \phi} \sim e^{5A}\partial_r \phi \sim (r - r_0)^{5\kappa - 1}, \quad (39)$$

where we have used the asymptotic behavior $\phi \sim \phi_0 \ln(r - r_0)$ and $A \sim \kappa \ln(r - r_0)$. The singularity occurs at $r = r_0$. We see that $\langle O \rangle$ diverges when $\kappa < \frac{1}{5}$. In the present case, the physical flow has $\kappa = 5$ while the unphysical one has $\kappa = \frac{1}{13}$. This is consistent with the finiteness of the expectation value of the dual operator.

B. Flow to $SU(2)_R$ SYM

If the other scalars, a_1 and a_2, are nonvanishing, the solution will break the $SU(2)$ global symmetry completely. It is now more difficult to solve all five BPS equations, but it turns out that these equations admit analytic solutions.

To obtain the solution, we consider A, σ, a_1, and a_2 as functions of a_3. Combining Eqs. (18) and (19), we find

$$\frac{da_2}{da_3} = \frac{\tanh a_2}{\sinh a_3 \cosh a_3}. \quad (40)$$

This is easily solved by

$$a_2 = \ln\left[\frac{e^{2a_3 + C_1} - e^{C_1} + \sqrt{(1 + e^{2a_3})^2 + e^{2C_1}(e^{2a_3} - 1)}}{1 + e^{2a_3}}\right]$$
$$= \sinh^{-1}(e^{C_1}\tanh a_3). \quad (41)$$

Similarly, by solving Eqs. (17) and (19), we obtain

$$a_1 = \sinh^{-1}\frac{e^{C_2}\sinh a_3}{\sqrt{1 - e^{2C_1} + (1 + e^{2C_1})\cosh(2a_3)}}. \quad (42)$$

Using the a_1 and a_2 solutions and the new radial coordinate $\tilde{r}$, we find the solution for a_3:

$$a_3 = \pm\frac{1}{2}\cosh^{-1}\left[\frac{e^{2C_2} + 2e^{2C_1} - 2 + 4\tanh^2(2m\tilde{r} - C_3)}{2 + 2e^{2C_1} + e^{2C_2}}\right]. \quad (43)$$

We can similarly solve for σ as a function of a_3. The solution is given by

$$\sigma = \frac{1}{4}\ln[3m(\tilde{A}^2 + \tilde{B}^2)^2 \mathrm{csch}^6 a_3 (36\tilde{A}^2 C_4 (\tilde{A}^2 + \tilde{B}^2)^2 \sinh^3 a_3 (\tilde{A}^2\cosh(2a_3) + \tilde{B}^2)$$
$$-2(3\tilde{A}^2 + \tilde{B}^2 - 2\tilde{A}^2\cosh(2a_3))(\tilde{A}^2\cosh(2a_3) + \tilde{B}^2)^{3/2})] - \frac{1}{4}\ln[1296\tilde{A}^4 C_4^2 g_1 (\tilde{A}^2 + \tilde{B}^2)^4 (\tilde{A}^2\cosh(2a_3) + \tilde{B}^2)$$
$$-4g_1 \mathrm{csch}^6 a_3 (\tilde{A}^4\cosh(4a_3) + \tilde{A}^4 + \tilde{A}^2(\tilde{B}^2 - 3\tilde{A}^2)\cosh(2a_3) - 3\tilde{A}^2\tilde{B}^2 - \tilde{B}^4)^2]. \quad (44)$$

We have defined two new constants, $\tilde{A}=\sqrt{2+2e^{2C_1}+e^{2C_2}}$ and $\tilde{B}=\sqrt{2-2e^{2C_1}-e^{2C_2}}$, for convenience.

Finally, adding (20) to (21) and changing the variable from r to a_3, we find a simple equation for A:

$$\frac{dA}{da_3}+\frac{d\sigma}{da_3}=-\coth a_3, \tag{45}$$

whose solution is

$$A=-\sigma-\ln(\sinh a_3). \tag{46}$$

Near the UV point, we find $r\sim\tilde{r}\to\infty$, $a_1\sim a_2\sim a_3\sim e^{-\frac{r}{L}}$, and $\sigma\sim e^{-\frac{3r}{L}}$. The solution for A then gives $A\sim 2mr=\frac{r}{L}$. The flow is again driven by turning on operators of dimension four corresponding to $a_{1,2,3}$ and a vacuum expectation value (VEV) of a dimension three operator dual to σ.

It can be checked by expanding (43) that $a_3\to\pm\infty$ as $2m\tilde{r}\to\tilde{C}$, where we have collectively denoted all constant terms from the expansion by $\tilde{C}$. The behavior of a_3 near this point is $a_3\sim\pm\ln(2m\tilde{r}-\tilde{C})$. Although a_3 blows up when $2m\tilde{r}\sim\tilde{C}$, a_1 and a_2 remain finite, with $a_2\sim\sinh^{-1}e^{C_1}$ and $a_1\sim\sinh^{-1}\frac{e^{C_2}}{\sqrt{2+2e^{2C_1}}}$. Similar to the previous case, the criterion of [28] requires $C_4=\pm\frac{2\sqrt{2}\tilde{A}}{9(\tilde{A}^2+\tilde{B}^2)^2}$ for the singularity to be physical. This is true for both $a_3<0$ and $a_3>0$. We find that

$$a_3\sim\pm\ln(2m\tilde{r}-\tilde{C}),\qquad \sigma\sim-\frac{1}{4}\ln(2m\tilde{r}-\tilde{C}),$$
$$ds^2=(2mr-C)^{10}dx_{4,1}^2+dr^2. \tag{47}$$

It can be readily verified that there always exist the values of C_1 and C_2 at which this behavior gives $V\to-\infty$.

For $C_4\neq\pm\frac{2\sqrt{2}\tilde{A}}{9(\tilde{A}^2+\tilde{B}^2)^2}$, the solution near $2m\tilde{r}\sim\tilde{C}$ becomes

$$a_3\sim\pm\ln(2m\tilde{r}-\tilde{C}),\qquad \sigma\sim-\frac{3}{4}a_3=\frac{3}{4}\ln(2m\tilde{r}-\tilde{C}),$$
$$ds^2=(2mr-C)^{\frac{2}{13}}dx_{1,4}^2+dr^2. \tag{48}$$

This solution is not physical, as it can be checked that $V\to\infty$ for all values of C_1 and C_2.

C. Flow to $SU(2)_{\rm diag}$ SYM

In this subsection, we will look at an RG flow with $SU(2)_{\rm diag}\sim(SU(2)_R\times SU(2))_{\rm diag}$ singlet scalars. Some nonsupersymmetric AdS_6 vacua and holographic RG flows interpolating between these critical points and the maximally supersymmetric AdS_6 have been studied in [20]. In this work, we will give a supersymmetric flow to a nonconformal field theory.

There is only one singlet scalar under $SU(2)_{\rm diag}$ from $\frac{SO(4,3)}{SO(4)\times SO(3)}$; see the details in [20]. The coset representative can be written as

$$L=e^{a(Y_{21}+Y_{32}+Y_{43})}. \tag{49}$$

The supersymmetry transformations of ψ^A_μ, χ^A, and λ^I_A give the following BPS equations:

$$a'=-e^{\sigma}\sinh(2a)(g_1\cosh a-g_2\sinh a), \tag{50}$$

$$\sigma'=\frac{1}{2}e^{-3\sigma}[3m+e^{4\sigma}(g_2\sinh^3 a-g_1\cosh^3 a)], \tag{51}$$

$$A'=\frac{1}{2}e^{-3\sigma}[m+e^{4\sigma}(g_1\cosh^3 a-g_2\sinh^3 a)]. \tag{52}$$

Note that for nonsinglet scalars of $SU(2)_R$, the $SU(2)$ coupling g_2 appears.

In order to solve the above equations, we will treat σ and A as functions of a:

$$\frac{d\sigma}{da}=\frac{3me^{-4\sigma}-g_1\cosh^3 a+g_2\sinh^3 a}{2\sinh(2a)(g_1\cosh a-g_2\sinh a)}, \tag{53}$$

which can be solved by

$$\sigma=\frac{1}{4}\ln\left[\frac{6m\cosh(2a)+C_1\sinh(2a)}{2g_1\cosh a-2g_2\sinh a}\right]. \tag{54}$$

We can check that as $a\to0$ and $g_1=3m$, $\sigma\to0$ as expected for the UV point. This is the case for any value of C_1. To solve for a from Eq. (50), it is convenient to define a new coordinate $\tilde{r}$ via $e^{\sigma}=\frac{d\tilde{r}}{dr}$. In this case only is $\tilde{r}$ defined by $e^{\sigma}=\frac{d\tilde{r}}{dr}$. In all other cases, we have $e^{-3\sigma}=\frac{d\tilde{r}}{dr}$.

With this new variable, we can solve for $\tilde{r}$ as a function of a. The resulting solution is given by

$$2g_1g_2\tilde{r}=g_2\ln\coth\frac{a}{2}-2g_1\tan^{-1}\left[\tanh\frac{a}{2}\right]+2\sqrt{g_1^2-g_2^2}\tan^{-1}\left[\frac{g_1\tanh\frac{a}{2}-g_2}{\sqrt{g_1^2-g_2^2}}\right], \tag{55}$$

where we have neglected the additive integration constant.

Taking the combination (51) $-3\times$ (52) with (50), we can rewrite the equation for A as

$$\frac{d\sigma}{da}-3\frac{dA}{da}=\frac{g_1\sinh a+g_2(1-\cosh a)}{g_1\cosh a-g_2\sinh a}. \tag{56}$$

The solution is readily obtained to be

$$A=\frac{1}{3}[\sigma+\ln\sinh(2a)+\ln(g_1\cosh a-g_2\sinh a)]. \tag{57}$$

From the above solutions, we can find the behavior of a, σ, and A near the UV point, $a = \sigma = 0$. In this limit, $\tilde{r} \sim r \to \infty$, we find $a \sim \sigma \sim e^{-6mr} = e^{-\frac{3r}{L}}$ and $A \sim 2mr = \frac{r}{L}$. This indicates that the flow is driven by vacuum expectation values of operators of dimension three. This is to be expected since it has been pointed out in [20] that the flow driven by turning on the operators dual to σ and a corresponds to a nonsupersymmetric flow to a nonsupersymmetric IR fixed point. In the IR, there are a number of possibilities, depending on the values of g_2 and the integration constant C_1, since these lead to different IR behaviors of a and σ.

We begin with the $g_2 = g_1$ case and consider the solution for large $|a|$. For $a < 0$, we find by expanding the solution in (55) that a diverges as $a \sim \frac{1}{3}\ln(g_1\tilde{r} - \tilde{C})$. As in the previous case, we have collectively denoted all of the constants by $\tilde{C}$. When $C_1 = 6m$, the solutions for σ and A become

$$\sigma \sim \frac{1}{4}\ln(g_1\tilde{r} - \tilde{C}), \qquad A \sim \frac{7}{36}\ln(g_1\tilde{r} - \tilde{C}),$$
$$ds^2 = (3mr - C)^{\frac{14}{27}} dx^2_{1,4} + dr^2. \tag{58}$$

This leads to $V \to -\infty$, which is acceptable.

For $C_1 \neq 6m$, we find different behavior:

$$\sigma \sim -\frac{1}{12}\ln(g_1\tilde{r} - \tilde{C}), \qquad A \sim \frac{1}{12}\ln(g_1\tilde{r} - \tilde{C}),$$
$$ds^2 = (g_1 r - C)^{\frac{2}{13}} dx^2_{1,4} + dr^2, \tag{59}$$

which gives $V \to \infty$, as expected since in this case $\kappa < \frac{2}{5}$.

For $a > 0$, we find that $a \sim -\ln(g_1\tilde{r} - \tilde{C})$. There are two possibilities for $C_1 = -6m$ and $C_1 \neq -6m$ which give, respectively,

$$\sigma \sim \frac{1}{4}\ln(g_1\tilde{r} - \tilde{C}), \qquad A \sim \frac{13}{12}\ln(g_1\tilde{r} - \tilde{C}),$$
$$ds^2 = (g_1 r - C)^{\frac{26}{9}} dx^2_{1,4} + dr^2, \tag{60}$$

and

$$\sigma \sim -\frac{3}{4}\ln(g_1\tilde{r} - \tilde{C}), \qquad A \sim \frac{3}{4}\ln(g_1\tilde{r} - \tilde{C}),$$
$$ds^2 = (g_1 r - C)^{\frac{6}{7}} dx^2_{1,4} + dr^2. \tag{61}$$

Both of them give $V \to -\infty$. We then conclude that for $g_2 = g_1$, all flows with $a > 0$ are physical, but flows with $a < 0$ are physical only for $C_1 = 6m$.

We now move to the $g_1 \neq g_2$ case and quickly look at the $a > 0$ and $a < 0$ flows separately. With $a > 0$, the solution becomes

$$a \sim -\frac{1}{3}\ln[(g_1 - g_2)\tilde{r} - \tilde{C}],$$
$$\sigma \sim -\frac{1}{12}\ln[(g_1 - g_2)\tilde{r} - \tilde{C}],$$
$$ds^2 = [(g_1 - g_2)\tilde{r} - \tilde{C}]^{\frac{2}{13}} dx^2_{1,4} + dr^2, \tag{62}$$

for $C_1 \neq -6m$, and

$$a \sim -\frac{1}{3}\ln[(g_1 - g_2)\tilde{r} - \tilde{C}], \qquad \sigma \sim \frac{1}{4}\ln[(g_1 - g_2)\tilde{r} - \tilde{C}],$$
$$ds^2 = [(g_1 - g_2)\tilde{r} - \tilde{C}]^{\frac{14}{27}} dx^2_{1,4} + dr^2, \tag{63}$$

for $C_1 = -6m$. The former is unphysical, but the latter is physical provided that $-(5 + 4\sqrt{2})m < g_2 < (4\sqrt{2} - 5)m$.

Finally, for $a < 0$, we find the IR behavior

$$a \sim \frac{1}{3}\ln[(g_1 + g_2)\tilde{r} - \tilde{C}], \quad \sigma \sim -\frac{1}{12}\ln[(g_1 + g_2)\tilde{r} - \tilde{C}],$$
$$ds^2 = [(g_1 + g_2)\tilde{r} - \tilde{C}]^{\frac{2}{13}} dx^2_{1,4} + dr^2, \tag{64}$$

for $C_1 \neq 6m$, and

$$a \sim \frac{1}{3}\ln[(g_1 + g_2)\tilde{r} - \tilde{C}], \qquad \sigma \sim \frac{1}{4}\ln[(g_1 + g_2)\tilde{r} - \tilde{C}],$$
$$ds^2 = [(g_1 + g_2)\tilde{r} - \tilde{C}]^{\frac{14}{27}} dx^2_{1,4} + dr^2, \tag{65}$$

for $C_1 = 6m$. Similar to the previous case, only the second possibility is physical, provided that $(5 - 4\sqrt{2})m < g_2 < (5 + 4\sqrt{2})m$. In summary, for $g_2 \neq g_1$, flows with $a > 0$ and $a < 0$ are physical for $C_1 = -6m$ and $C_1 = 6m$, respectively, for some appropriate values of g_2.

IV. RG FLOWS FROM $SU(2)_R \times U(2)$ SCFT

To give more examples, we consider $F(4)$ gauged supergravity coupled to four vector multiplets with the $SU(2)_R \times SU(2) \times U(1)$ gauge group. There are 16 scalars parametrized by the $SO(4,4)/SO(4) \times SO(4)$ coset. We will focus on $SU(2)_R$ singlet scalars which are the highest components of the global symmetry multiplet and correspond to supersymmetry preserving deformations. Together with the dilaton σ, there are five $SU(2)_R$ singlet scalars. The coset representative can be written as

$$L = e^{a_1 Y_{11}} e^{a_2 Y_{12}} e^{a_3 Y_{13}} e^{a_4 Y_{14}}. \tag{66}$$

Using the projector $\gamma^r \epsilon^A = \epsilon^A$, we can derive the following BPS equations:

$$a_1{}' = -\frac{2me^{-3\sigma}\sinh a_1}{\cosh a_2 \cosh a_3 \cosh a_4}, \tag{67}$$

$$a_2{}' = -\frac{2me^{-3\sigma}\sinh a_2 \cosh a_1}{\cosh a_3 \cosh a_4}, \tag{68}$$

$$a_3{}' = -\frac{2me^{-3\sigma}\cosh a_1 \cosh a_2 \sinh a_3}{\cosh a_4}, \tag{69}$$

$$a_4{}' = -2me^{-3\sigma}\cosh a_1 \cosh a_2 \cosh a_3 \sinh a_4, \tag{70}$$

$$\sigma' = \frac{1}{2}[3me^{-3\sigma}\cosh a_1 \cosh a_2 \cosh a_3 \cosh a_4 - g_1 e^{\sigma}], \tag{71}$$

$$A' = \frac{1}{2}[me^{-3\sigma}\cosh a_1 \cosh a_2 \cosh a_3 \cosh a_4 + g_1 e^{\sigma}]. \tag{72}$$

We are interested in the RG flows with the symmetry breaking patterns $U(2) \to SU(2)$, $U(2) \to U(1) \times U(1)$, and $U(2) \to U(1)$ and the completely broken $U(2)$. The procedure is essentially the same as in the previous section, so we will neglect some details and simply give the solutions.

A. Flow to $SU(2)_R \times SU(2)$ SYM

In order to preserve $SU(2) \subset SU(2) \times U(1)$, only a_4 is allowed to be nonvanishing. The above equations reduce to three simple equations:

$$a_4{}' = -2me^{-3\sigma}\sinh a_4, \tag{73}$$

$$\sigma\prime = \frac{1}{2}(3me^{-3\sigma}\cosh a_4 - g_1 e^{\sigma}), \tag{74}$$

$$A' = \frac{1}{2}(me^{-3\sigma}\cosh a_4 + g_1 e^{\sigma}). \tag{75}$$

By introducing a new radial coordinate $\tilde{r}$ via $\frac{d\tilde{r}}{dr} = e^{-3\sigma}$ as in the previous section, we find the solutions

$$\begin{aligned} a_4 &= \pm \ln\left[\frac{1+e^{-2m\tilde{r}+C_1}}{1-e^{-2m\tilde{r}+C_1}}\right], \\ \sigma &= -\frac{1}{4}\ln\left[\frac{g_1(3\cosh a_4 - \cosh(3a_4) + 18C_2\sinh^3 a_4)}{6m}\right], \\ A &= 2m\tilde{r} + \ln(1-e^{C_1-2m\tilde{r}}) + \ln(1+e^{C_1-2m\tilde{r}}) - \sigma. \end{aligned} \tag{76}$$

Near the UV point, a_4, σ, and A behave as

$$a_4 \sim e^{-2mr}, \qquad \sigma \sim e^{-6mr}, \qquad A \sim 2mr. \tag{77}$$

Similar to the previous solutions, we find that the IR singularity at $\tilde{r} \sim \frac{C_1}{2m}$ is physical for $a_4 \sim \pm \ln(2m\tilde{r} - C_1)$ if we choose $C_2 = \pm\frac{2}{9}$. In both cases, the IR metric is given by

$$ds^2 = (2mr - C)^{10} dx_{1,4}^2 + dr^2. \tag{78}$$

Other choices of C_2 lead to unacceptable singularities.

B. Flow to $SU(2)_R \times U(1) \times U(1)$ SYM

In this subsection, we will give the solution for the flow to SYM with $SU(2)_R \times U(1)^2$ symmetry. To find this solution, we set $a_1 = a_2 = a_4 = 0$. The BPS equations, which are similar to those in the previous subsection, give the following solutions, in terms of the $\tilde{r}$ coordinate:

$$\begin{aligned} a_3 &= \pm \ln\left[\frac{1+e^{-2m\tilde{r}+C_1}}{1-e^{-2m\tilde{r}+C_1}}\right], \\ \sigma &= -\frac{1}{4}\ln\left[\frac{g_1(3\cosh a_3 - \cosh(3a_3) + 18C_2\sinh^3 a_3)}{6m}\right], \\ A &= 2m\tilde{r} + \ln(1-e^{C_1-2m\tilde{r}}) + \ln(1+e^{C_1-2m\tilde{r}}) - \sigma. \end{aligned} \tag{79}$$

Near the UV point, we find $a_3 \sim e^{-2mr}$, $\sigma \sim e^{-6mr}$, and $A \sim 2mr$. In the IR, $\tilde{r} \to \frac{C_1}{m}$, the physical solution with $C_2 = \pm\frac{2}{9}$ is given by

$$\begin{aligned} &a_4 \sim \pm \ln(2m\tilde{r} - C_1), \qquad \sigma \sim -\frac{1}{4}\ln(2m\tilde{r} - C_1), \\ &ds^2 = (2mr - C)^{10} dx_{1,4}^2 + dr^2. \end{aligned} \tag{80}$$

C. Flow to $SU(2)_R \times U(1)$ SYM

We then consider the flow that breaks $SU(2) \times U(1)$ global symmetry to $U(1)$. In this case, we turn on both a_3 and a_4. This leads to more complicated equations due to the coupling between a_4 and a_3. We will regard a_4 as a new variable and find that the solutions for a_3, σ, and A are given by

$$\begin{aligned} a_3 &= \sinh^{-1}[e^{C_1}\tanh a_4], \\ \sigma &= -\frac{1}{4}\ln\left[\frac{g_1}{6\sqrt{2}m}[72C_2\sinh^3 a_4(1+e^{2C_1}) \right. \\ &\quad - 2\cosh a_4[(1+e^{2C_1})\cosh(2a_4) \\ &\quad \left. - e^{2C_1} - 2]\sqrt{2+2e^{2C_1}\tanh^2 a_4}]\right], \\ A &= -\sigma - \ln\sinh a_4. \end{aligned} \tag{81}$$

The solution of a_4 in terms of $\tilde{r}$ is given by

$$\tilde{r} = \frac{1}{2m}\tanh^{-1}\sqrt{\frac{1+\cosh(2a_4) + 2e^{2C_1}\sinh^2 a_4}{2}}. \tag{82}$$

At the UV point, we find the expected behavior $a_{3,4} \sim e^{-2mr}$, $\sigma \sim e^{-6mr}$, and $A \sim 2mr$. In the IR, we consider the behavior of the solutions as $a_4 \to \infty$. In this limit, the a_4 solution becomes $a_4 \sim -\ln(2m\tilde{r} - \tilde{C})$ for some constant $\tilde{C}$. We find that the requirement for the IR singularity to be acceptable is given by $C_2 = \frac{1}{9}\sqrt{\frac{1+e^{2C_1}}{2}}$. The behavior of a_3, σ, and A is given by

$$a_3 \sim \sinh^{-1} e^{C_1}, \qquad \sigma \sim -\frac{1}{4}\ln(2m\tilde{r} - \tilde{C}),$$
$$A \sim \frac{5}{4}\ln(2m\tilde{r} - \tilde{C}). \tag{83}$$

With the relation $2mr - C = 4(2m\tilde{r} - \tilde{C})^{\frac{1}{4}}$, the metric in the IR then takes the form of a domain wall

$$ds^2 = (2mr - C)^{10} dx_{1,4}^2 + dr^2. \tag{84}$$

D. Flow to $SU(2)_R$ SYM

We now quickly look at the flow breaking the $U(2)$ symmetry completely. Finding the solution in this case amounts to solving all of the six BPS equations. This, however, turns out not to be difficult. The resulting solutions for a_i, σ, and A are given by

$$\begin{aligned}
a_3 &= \sinh^{-1}(e^{C_1}\tanh a_4),\\
a_2 &= \sinh^{-1}\frac{e^{C_2}\sinh a_4}{\sqrt{1 - e^{2C_1} + (1 + e^{2C_1})\cosh(2a_4)}},\\
a_1 &= \sinh^{-1}\frac{e^{C_3}\sinh a_4}{\sqrt{2 - 2e^{2C_1} - e^{2C_2} + (2 + 2e^{2C_1} + e^{2C_2})\cosh(2a_4)}},\\
\sigma &= \frac{1}{4}\ln\left[96\sqrt{2}m\sqrt{4 + \alpha^2 - \alpha^2\mathrm{sech}^2 a_4}\right] - \frac{1}{4}\ln\Big[g_1(2304(\alpha^2 + 4)C_4\sinh^3 a_4\sqrt{\alpha^2 + 4 - \alpha^2\mathrm{sech}^2 a_4}\\
&\quad - \sqrt{2}\,\mathrm{sech}\, a_4(3\alpha^4 + (\alpha^2 + 4)^2\cosh(4a_4) + 16\alpha^2 - 4(\alpha^4 + 6\alpha^2 + 8)\cosh(2a_4) - 48))\Big],\\
A &= -\sigma - \ln\sinh a_4,\\
a_4 &= \frac{1}{2}\cosh^{-1}\left[\frac{8\tanh^2(2m\tilde{r} - C_5) + \alpha^2 - 4}{\alpha^2 + 4}\right],
\end{aligned} \tag{85}$$

where $\alpha = \sqrt{4e^{2C_1} + 2e^{2C_2} + e^{2C_3}}$. At the UV fixed point, the solutions become

$$a_{1,2,3,4} \sim e^{-2mr}, \qquad \sigma \sim e^{-6mr}, \qquad A \sim 2mr. \tag{86}$$

In the IR, we have to set $C_4 = \frac{1}{144}\sqrt{\frac{4+\alpha^2}{2}}$ in order to obtain a physical solution. The solution is then given by

$$\begin{aligned}
&a_4 \sim -\ln(2m\tilde{r} - \tilde{C}), \qquad a_3 \sim \sinh^{-1} e^{C_1},\\
&a_2 \sim \sinh^{-1}\frac{e^{C_2}}{\sqrt{2 + 2e^{2C_1}}},\\
&a_1 \sim \sinh^{-1}\frac{e^{C_3}}{\sqrt{4 + 4e^{2C_1} + 2e^{2C_2}}},\\
&\sigma \sim -\frac{1}{4}\ln(2m\tilde{r} - \tilde{C}), \qquad A \sim \frac{5}{4}\ln(2m\tilde{r} - \tilde{C}),\\
&ds^2 = (2mr - C)^{10} dx_{1,4}^2 + dr^2.
\end{aligned} \tag{87}$$

All of the flows given above are driven by turning on operators of dimension four and a VEV of a dimension three operator.

V. CONCLUSIONS

We have studied various holographic RG flows from matter coupled $F(4)$ gauged supergravity. These flows describe deformations of the UV $N = 2$ SCFTs with $SU(2)$ and $SU(2) \times U(1)$ global symmetries in five dimensions to nonconformal $N = 2$ SYM theories in the IR. We have explored various symmetry breaking patterns and interpreted the solutions as RG flows driven by turning on operators of dimension four in a vacuum with nonzero VEV of a dimension three operator dual to the six-dimensional dilaton, except for the flow to the $SU(2)_{\mathrm{diag}}$ SYM, which is driven by vacuum expectation values of dimension three operators. We have also identified physical flows which have acceptable IR singularities from the resulting solutions. Therefore, these solutions might be useful in the study of strongly coupled $N = 2$ SYM in five dimensions. However, the identification of the dual five-dimensional SYM corresponding to these solutions in the IR is not clear. Accordingly, the precise physical interpretation of these solutions needs to be clarified.

It is interesting to holographically compute various characteristics of the 5D gauge theories such as the Wilson loops, as done in [29]. It could be useful to do this computation with the six-dimensional solutions given here, similar to the four-dimensional gauge theories studied in [6,7]. The solutions found in this paper would hopefully be useful in this aspect and for other holographic calculations. It will very interesting (if possible) to find a gravity solution describing the enhancement of the global symmetry $SO(2N_f) \times U(1)$ to the E_{N_f+1} fixed point in five dimensions. In this aspect, the six-dimensional framework considered here may not be able to accommodate this solution since the symmetry enhancement is not seen at the classical supergravity level, as remarked in [17].

It is not presently known how to embed the six-dimensional $F(4)$ gauged supergravity coupled to n vector multiplets to 10 or 11 dimensions, although the pure $F(4)$ gauged supergravity and the theory coupled to 20 vector multiplets are known to originate from massive type IIA compactification on warped S^4 and $K3$, respectively [30,31]. The embedding of $F(4)$ gauged supergravity in type IIB theory via the non-Abelian T duality has been proposed recently in [32]. This might also provide another mean to embed the six-dimensional gauged supergravity in higher dimensions. It would be interesting to find such an embedding, which in turn can be used to uplift the solutions found here and in [20] to ten dimensions. This could provide some insight to the dynamics of D4/D8-brane system. We hope to come back to these issues in future works.

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