Anomaly mediated gaugino mass and path-integral measure

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In recent years, there has been controversy concerning the anomaly-mediated gaugino mass in the superspace formalism of supergravity. In this paper, we reexamine the gaugino mass term in this formalism by paying particular attention to the symmetry that controls gaugino masses in supergravity. We first discuss super-diffeomorphism invariance of path-integral measures of charged superfields. As we will show, the super-diffeomorphism-invariant measure is not invariant under a super-Weyl transformation, which is the origin of the anomaly-mediated gaugino mass. We show how the anomaly-mediated gaugino mass is expressed as a local operator in a Wilsonian effective action in a super-diffeomorphism-covariant way. We also obtain a gaugino mass term independent of the gauge choice of the fictitious super-Weyl symmetry in the super-Weyl compensator formalism, which reproduces the widely accepted result. We also discuss how to reconcile the gaugino mass term in the local Wilsonian effective action and the gaugino mass term appearing in a nonlocal one-particle irreducible quantum effective action.

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I. INTRODUCTION

After the discovery of the Higgs boson with a mass of about 126 GeV at the LHC [1], the anomaly-mediation mechanism for the gaugino mass generation [2,3] (see also Ref. [4]) is gathering renewed attention. The anomaly-mediated gaugino mass plays a crucial role in constructing a class of high-scale supersymmetry models where sfermion masses are in the hundreds to thousands of TeV range while gaugino masses are within the TeV range proposed in Refs. [2,5] and subsequently in Refs. [6–16]. On top of a successful prediction of the observed Higgs boson mass [17–19], models in this class are free from the so-called cosmological Polonyi problem [20] (see also Ref. [21]), for no singlet supersymmetry-breaking fields are required in the models.[1] This class of models is also free from infamous gravitino problems [24,25]. Besides, the lightest supersymmetric particle is the almost pure wino in most of the parameter space, which is a good candidate for dark matter when it is produced either thermally [26,27] or nonthermally [28,29] (see also Ref. [30]).[2]

As illustrated in Refs. [2,3], the most transparent way to look at the anomaly-mediated gaugino mass is to use the conformal compensator formalism of supergravity [40,41]. In this formalism, only the conformal compensator has a nonvanishing F-term vacuum expectation value (VEV) after supersymmetry breaking in the gravity sector, and hence, the anomaly-mediated gaugino mass can be extracted by looking at how the chiral compensator appears in the gauge kinetic function. (See also Refs. [42–50] for informative discussions on the anomaly-mediated gaugino mass.)

In the last few years, however, there has been controversy [48,51] over how the anomaly-mediated gaugino mass appears in the superspace formalism of supergravity [52,53]. In particular, the author of Ref. [51] examined the gaugino mass in this formalism by introducing a chiral super-Weyl compensator field, C, along the lines of Ref. [54], so that the model possesses a fictitious (but exact) super-Weyl gauge symmetry. Then, by looking at how the super-Weyl compensator C appears in the gauge kinetic function, the author claimed that the anomaly-mediated gaugino mass derived in Refs. [2,3] vanishes. This claim was refuted in a subsequent paper [48], which pointed out that the gravity multiplets also possess nonvanishing F-term VEVs for the gauge choice of the fictitious super-Weyl gauge symmetry in Ref. [51]. Thus, the full anomaly-mediated gaugino masses cannot be extracted just by looking at the C dependence of the gauge kinetic function. Eventually, by arguing that the gaugino mass should be independent of the gauge choice of the super-Weyl gauge symmetry, the gaugino mass in Refs. [2,3] was reproduced in Ref. [48] by taking a gauge in which the F-term VEVs of the gravity multiplets vanish.

In these discussions, there remain unsettled questions. First of all, it is not clear whether the anomaly-mediated gaugino mass can be expressed as a local operator in a Wilsonian effective action in a super-diffeomorphism-covariant way without invoking the super-Weyl symmetry

[1]A simple implementation of the μ term in this class of "without-singlet" models was first done in Ref. [6] by coupling the Higgs doublets to the R-symmetry-breaking sector along the lines of the Casas-Munoz mechanism [22] (or the generalization of the Giudice-Masiero mechanism [23]).

[2]For the current status and future prospects of wino dark matter detection, see Refs. [31–34], and for collider searches see Ref. [35]. See also Refs. [36–39] for related discussions on wino dark matter.

compensator C. Second, the lack of the local term expression without C inevitably seems to mean that there is no consistent expression of the anomaly-mediated gaugino mass as a local operator independent of the gauge choice of the fictitious super-Weyl gauge symmetry.

The main purpose of this paper is to settle these problems. For that purpose, we reexamine the gaugino mass term in the superspace formalism of supergravity without invoking the fictitious super-Weyl gauge symmetry. We pay particular attention to super-diffeomorphism invariance of path-integral measures of charged supermultiplets. As we will show, the super-diffeomorphism-invariant measure is not invariant under an approximate super-Weyl symmetry which forbids the gaugino mass at the classical level. Anomalous breaking of the approximate super-Weyl symmetry (not to be confused with the fictitious super-Weyl symmetry in Ref. [54]) is the origin of the anomaly-mediated gaugino mass in the superspace formalism.

Armed with the super-diffeomorphism invariant measure, we show that the anomaly-mediated gaugino mass can be read off from a local operator in a Wilsonian action when we change the path-integral measure from the super-diffeomorphism-invariant one to the super-Weyl-invariant one. There, we emphasize that the corresponding local operator is not invariant under the super-diffeomorphism. The noninvariance of the local term is required for the super-diffeomorphism invariance of the quantum theory.

Once we learn how the local gaugino mass term arises in the superspace formalism of supergravity, it is straightforward to derive the local term expression of the gaugino mass term which is independent of the gauge choice of the fictitious super-Weyl gauge symmetry in the super-Weyl compensator formalism. We also discuss how to reconcile the anomaly-mediated gaugino mass term in the Wilsonian effective action and the nonlocal expression of the gaugino mass term appearing in the one-particle irreducible (1PI) quantum effective action derived in Ref. [42].

The organization of this paper is as follows. In Sec. II, we discuss the gaugino mass appearing in the supergravity action at the classical level. There, we show that the gaugino mass is highly suppressed at the classical level due to the approximate super-Weyl symmetry which is respected by relevant interactions of gauge and charged matter supermultiplets.[3] In Sec. III, we discuss the super-diffeomorphism-invariant path-integral measure of the charged matter which has a nontrivial but unique dependence on the chiral density of the gravity multiplet. There, we see that the super-diffeomorphism-invariant measure is not invariant under the approximate super-Weyl symmetry. This property is important to understand how the anomaly-mediated gaugino mass term can be expressed as a local term in the Wilsonian effective action in a super-diffeomorphism-covariant way. In Sec. IV, we show the local expression of the gaugino mass term which is independent of the gauge choice of the fictitious super-Weyl gauge symmetry in the super-Weyl compensator formalism in Ref. [54]. We also show how the gaugino mass term is related to the gaugino mass term in the nonlocal 1PI quantum effective action derived in Ref. [42]. We summarize our discussion in Sec. V.

II. APPROXIMATE SUPER-WEYL SYMMETRY IN THE CLASSICAL ACTION

Before discussing the anomaly-mediated gaugino mass, let us first clarify the gaugino mass expected in the local supergravity action at the classical level.[4] In our discussion, we concentrate ourselves in a situation where supersymmetry is dominantly broken by some charged fields under some symmetries or by composite fields. Otherwise direct interactions between the supersymmetry-breaking fields and gauge multiplets lead to the "tree-level" gaugino mass of the order of the gravitino mass, $m_{3/2}$. Under this assumption, the direct interactions between the supersymmetry-breaking fields and the gauge supermultiplets are suppressed at least by a second power of the Planck scale, $M_{\rm PL}$, and hence, resultant gaugino masses from those interactions are negligible. For the same reason, we also assume that no supersymmetry-breaking field obtains a vacuum expectation value of the order of the Planck scale.[5]

Once we assume that the gaugino mass from couplings to the supersymmetry-breaking sector is highly suppressed, remaining sources of the gaugino mass are couplings to the supergravity multiplets. As is well known, however, gaugino masses from tree-level interactions to the supergravity multiplets are also suppressed in spite of the apparent F-term VEVs of $O(m_{3/2})$ in the supergravity multiplets. As we shortly discuss, the absence of $O(m_{3/2})$ gaugino masses from the supergravity multiplets is due to an approximate super-Weyl symmetry, which is the key to understanding the origin of the anomaly-mediated gaugino mass in the next section. For the time being, we restrict ourselves to the gaugino mass generation in a U(1) gauge theory with a pair of vector-like matters. The following discussion can be extended to general non-Abelian gauge theories (see discussions in Sec. V).

A. Classical supergravity action

In this paper, we follow the notation and the formulation in Ref. [52], except for the notation of complex conjugate

[3]Throughout this paper, relevant interaction terms denote the interaction terms with mass dimensions less than or equal to four.

[4]Here, we assume that the classical action consists of local interactions. If the classical action is allowed to be nonlocal, an arbitrary gaugino mass of $O(m_{3/2})$ can be introduced by using the nonlocal term in Eq. (42) without conflicting with the super-diffeomorphism invariance.

[5]These assumptions also reduce contributions to gaugino masses from the Kähler and sigma-model anomalies [42,47].

(we use †) and for the normalization of gauge supermultiplets for which we adopt the one in Ref. [55]. For a simple model with charged chiral multiplets Q and $\bar{Q}$, and a U(1) gauge multiplet V, the classical supergravity action is given by,

$$\begin{aligned}\mathcal{L} &= M_{\mathrm{Pl}}^2 \int \mathrm{d}^2\Theta 2\mathcal{E}\frac{3}{8}(\mathcal{D}^{\dagger 2} - 8R)\exp\left[-\frac{K}{3M_{\mathrm{Pl}}^2}\right] \\ &\quad + \frac{1}{16g^2}\int \mathrm{d}^2\Theta 2\mathcal{E}W^\alpha W_\alpha + \mathrm{H.c.}, \\ K &= Q^\dagger e^{2V}Q + \bar{Q}^\dagger e^{-2V}\bar{Q} + \cdots, \\ W_\alpha &\equiv -\frac{1}{4}(\mathcal{D}^{\dagger 2} - 8R)(e^{-2V}\mathcal{D}_\alpha e^{2V}),\end{aligned} \tag{1}$$

where Θ^α, $\mathcal{E}$, $\mathcal{D}_\alpha$, R, K, and g are the fermionic coordinate, the chiral density, the covariant derivative, the superspace curvature, the Kähler potential, and the gauge coupling constant, respectively. Here, we have assumed that the chiral multiplets Q and $\bar{Q}$ are massless. By expanding the chiral multiplets, we can extract relevant interactions,

$$\begin{aligned}\mathcal{L}_{\mathrm{kin,matter}} &= -\frac{1}{8}\int \mathrm{d}^2\Theta 2\mathcal{E}(\mathcal{D}^{\dagger 2} - 8R) \\ &\quad \times (Q^\dagger e^{2V}Q + \bar{Q}^\dagger e^{-2V}\bar{Q}) + \mathrm{H.c.},\end{aligned} \tag{2}$$

$$\mathcal{L}_{\mathrm{kin,gauge}} = \frac{1}{16g^2}\int \mathrm{d}^2\Theta 2\mathcal{E}W^\alpha W_\alpha + \mathrm{H.c.}, \tag{3}$$

from which we can extract gauge interactions and kinetic terms. Other interactions are suppressed by the Planck scale.

Now, let us expand W^α, $\mathcal{E}$, and R in terms of component fields:

$$\begin{aligned}W^\alpha &= -2i\lambda^\alpha + \cdots, \\ 2\mathcal{E} &= e(1 - M^*\Theta^2) + \cdots, \\ R &= -\frac{1}{6}M - \frac{1}{9}|M|^2\Theta^2 + \cdots.\end{aligned} \tag{4}$$

Here, λ^α, e, and M are the gaugino, the determinant of the vielbein, and the auxiliary scalar component of the gravity multiplet, respectively. The ellipses denote terms which are irrelevant for our discussion on the gaugino mass. The auxiliary field M is fixed by the equation of motion as

$$M^* = -3m_{3/2}, \tag{5}$$

where we have omitted contributions from the supersymmetry-breaking sector which are negligible under the assumption we have made at the beginning of this section.

Since the chiral density $\mathcal{E}$ has a nonvanishing Θ^2 term, it may seem nontrivial that the gaugino mass of $O(m_{3/2})$ does not appear from the interaction in Eq. (3). In the rest of this section, we show that the absence of the gaugino mass in the classical action is understood by an approximate super-Weyl symmetry.

B. Approximate super-Weyl symmetry

Let us consider the super-Weyl transformation parametrized by a chiral scalar Σ [52],[6]

$$\begin{aligned}\delta_{\mathrm{SW}}\mathcal{E} &= 6\Sigma\mathcal{E} + \frac{\partial}{\partial\Theta^\alpha}(S^\alpha\mathcal{E}), \\ \delta_{\mathrm{SW}}R &= -4\Sigma R - \frac{1}{4}(\mathcal{D}^{\dagger 2} - 8R)\Sigma^\dagger - S^\alpha\frac{\partial}{\partial\Theta^\alpha}R, \\ \delta_{\mathrm{SW}}W^\alpha &= -3\Sigma W^\alpha + \cdots, \\ \delta_{\mathrm{SW}}Q &= w\Sigma Q - S^\alpha\frac{\partial}{\partial\Theta^\alpha}Q, \\ S^\alpha &\equiv \Theta^\alpha(2\Sigma^\dagger - \Sigma)| + \Theta^2\mathcal{D}^\alpha\Sigma|,\end{aligned} \tag{6}$$

where the ellipses denote terms which are irrelevant for our discussion. A parameter w is the Weyl weight of Q.[7] The S^α-dependent terms are inhomogeneous transformations which can be canceled by the super-diffeomorphism [see Eq. (11)]. From Eqs. (4) and (6), the transformation laws of e, M and λ^α are given by

$$\begin{aligned}\delta_{\mathrm{SW}}e &= 4(\Sigma + \Sigma^\dagger)|e, \\ \delta_{\mathrm{SW}}M &= -2(2\Sigma - \Sigma^\dagger)|M + \frac{3}{2}\mathcal{D}^{\dagger 2}\Sigma^\dagger|, \\ \delta_{\mathrm{SW}}\lambda^\alpha &= -3\Sigma|\lambda^\alpha,\end{aligned} \tag{8}$$

where $\mathcal{X}|$ denotes the lowest component of a superfield $\mathcal{X}$.

From the transformation laws of the component fields in Eq. (8), it is clear that the possible origin of the gaugino mass of $O(m_{3/2})$,

$$\int \mathrm{d}^4 x e M^{(*)}\lambda\lambda, \tag{9}$$

is not invariant under the super-Weyl transformation. This shows that the gaugino mass is generated only through terms which break the super-Weyl symmetry.

As we immediately see, the kinetic term of the gauge multiplet in Eq. (3) is invariant under the super-Weyl transformation, and hence, does not contribute to the gaugino mass. Higher-dimensional terms omitted in Eq. (1) are, on the other hand, not invariant under the

[6]In this paper we define an infinitesimal transformation of a superfield $\mathcal{X}$ by $\mathcal{X}' = \mathcal{X} - \delta\mathcal{X}$.

[7]If Q is not a chiral scalar but rather a chiral density with a density weight $\tilde{w}$, the super-Weyl transformation is given by,

$$\delta_{\mathrm{SW}}Q = w\Sigma Q - S^\alpha\frac{\partial}{\partial\Theta^\alpha}Q + \tilde{w}Q\frac{\partial}{\partial\Theta^\alpha}S^\alpha. \tag{7}$$

super-Weyl transformation. Contributions from such terms to the gaugino mass are, however, at the largest of $O(m_{3/2}^2/M_{\rm Pl})$, and hence are negligible. Altogether, we find that there is no gaugino mass of $O(m_{3/2})$ from couplings to the supergravity multiplets due to the approximate super-Weyl symmetry.[8]

For later convenience, let us also note that the terms of massless matter fields in Eq. (2) are also invariant under the super-Weyl symmetry. That is, for $w = -2$, it can be shown that

$$\delta_{\rm SW}((\bar{\mathcal{D}}^2 - 8R)(Q^\dagger Q)) = -6\Sigma(\bar{\mathcal{D}}^2 - 8R)(Q^\dagger Q) - S^\alpha \frac{\partial}{\partial\Theta^\alpha}((\bar{\mathcal{D}}^2 - 8R)(Q^\dagger Q)). \tag{10}$$

From Eqs. (6) and (10), the terms in Eq. (2) are invariant under the super-Weyl transformation.

Finally, let us stress that interaction terms of the gauge supermultiplets which are unsuppressed by the Planck scale are uniquely determined to be of the form of Eqs. (2) and (3) by the super-diffeomorphism invariance and by the gauge invariance. Thus, one may regard the approximate super-Weyl symmetry as an accidental one. Due to this accidental symmetry, the gaugino mass of $O(m_{3/2})$ is suppressed at the classical level.

III. ANOMALY OF THE SUPER-WEYL SYMMETRY AND THE GAUGINO MASS

In the last section, we have shown that no gaugino mass of $O(m_{3/2})$ is generated through couplings to the supergravity multiplets even after supersymmetry breaking due to the approximate super-Weyl symmetry. However, the approximate super-Weyl symmetry is in general broken by quantum effects. In this section, we investigate effects of quantum violation of the approximate super-Weyl symmetry by Fujikawa's method [56] in a Wilsonian effective action.

A. Wilsonian effective action

To discuss quantum effects on the super-Weyl symmetry, we take the local classical action in the previous section [Eq. (1)] as the Wilsonian effective action with a cutoff at the Planck scale. Here, let us remind ourselves that effective quantum field theories suffer from ultraviolet divergences, and hence, they are well defined only after the divergences are properly regularized. In our arguments, we presume an ultraviolet regularization such that the "tree-level" action at the cutoff scale is manifestly invariant under the super-diffeomorphism and the gauge transformations. We refer to this super-diffeomorphism-invariant tree-level action at the cutoff scale as the Wilsonian effective action.[9]

The Wilsonian effective action, in general, includes higher-dimensional interactions than those in Eq. (1) suppressed by the cutoff scale. As we have discussed, however, contributions from those terms to the gaugino mass are highly suppressed by the cutoff scale and hence negligible. One concern is whether nonlocal interaction terms appear in the Wilsonian effective action at the cutoff scale, which could lead to the gaugino mass of $O(m_{3/2})$. In our argument, we presume that such nonlocal interactions do not show up in the Wilsonian effective action, which is reasonable since we are dealing with effective field theories after integrating out ultraviolet modes.

B. Super-diffeomorphism invariance

In the above definition of the super-diffeomorphism-invariant theory, there is a missing ingredient: the measure of the path integral. As elucidated in Ref. [56], the path-integral measure plays a crucial role in discussing quantum violations of symmetries. Moreover, the definition of the "tree-level" interactions in the Wilsonian effective action depends on the choice of the path-integral measure, which we will encounter shortly. To clarify these issues, let us first discuss which path-integral measure we should use in conjunction with the "tree-level" Wilsonian action.

Under the infinitesimal (chiral) super-diffeomorphism transformation, Q and $\mathcal{E}$ transform as

$$\begin{aligned} Q &\to Q' = Q - \eta^M(x,\Theta)\partial_M Q, \\ \mathcal{E} &\to \mathcal{E}' = \mathcal{E} - \eta^M(x,\Theta)\partial_M\mathcal{E} - (-)^M(\partial_M\eta^M(x,\Theta))\mathcal{E}, \end{aligned} \tag{11}$$

where $M = (m, \alpha)$ denotes the indices of the chiral super coordinate (x^m, Θ^α), $\eta^M(x,\Theta)$ parametrizes the super-diffeomorphism, and $(-)^M = (1, -1)$ for $M = (m, \alpha)$. As is shown in Appendix A, path-integral measures of chiral fields are not invariant under the super-diffeomorphism due to the anomaly of the gauge interactions, i.e.

$$[DQ] \to [DQ'] \neq [DQ], \quad [D\bar{Q}] \to [D\bar{Q}'] \neq [D\bar{Q}]. \tag{12}$$

Instead, anomaly-free measures are given by

$$[D(2\mathcal{E})^{1/2}Q], \qquad [D(2\mathcal{E})^{1/2}\bar{Q}]. \tag{13}$$

For a later purpose, we define weighted chiral fields $Q_{\rm diff} = (2\mathcal{E})^{1/2}Q$ ($\bar{Q}_{\rm diff} = (2\mathcal{E})^{1/2}\bar{Q}$) which are no longer chiral

[8]The term in Eq. (9) is invariant under the R symmetry and the dilatational symmetry, which are parts of the super-Weyl symmetry. Thus, the gaugino mass from the couplings to the supergravity multiplets cannot be forbidden by the R symmetry or the dilataional symmetry.

[9]Although we fix the cutoff scale to the Planck scale for a while, the following discussion is essentially unchanged as long as the cutoff scale is far larger than the gravitino mass. We also discuss effects of the change of the cutoff scale later.

scalar fields but rather chiral density fields with density weights $1/2$.

In our discussion, we take the super-diffeomorphism-invariant Wilsonian effective action. Therefore, in order to obtain a super-Diffeomporphism-invariant quantum theory, we inevitably use the super-diffeomorphism-invariant path-integral measure in Eq. (13). If we use different measures, instead, we need to add appropriate super-diffeomorphism-variant counterterms to the tree-level Wilsonian action so that the super-diffeomorphism is restored in the quantum theory.

C. Anomalous breaking of the super-Weyl symmetry

Once we choose appropriate path-integral measures for the charged fields, we can discuss quantum violation of the super-Weyl symmetry. Here, since we are interested in the gaugino mass, we only look at the breaking of the super-Weyl symmetry by the anomaly of the corresponding gauge interaction.

Before proceeding further, let us comment on a technical point. As in Eq. (6), the super-Weyl transformation is accompanied by a super-diffeomorphism parametrized by S^α, so that the super-Weyl symmetry can be expressed in terms of the component fields defined in the chiral superspace spanned by (x,Θ). The accompanied super-diffeomorphism, however, makes it complicated to discuss the quantum violation of the super-Weyl symmetry. To avoid such a complication, we only consider a subset of the super-Weyl transformation where Σ has only an F-term, i.e.

$$\Sigma(x,\Theta) = f(x)\Theta^2. \tag{14}$$

Here, f is an arbitrary function of the space-time. Under this restricted super-Weyl transformation, we find $S^\alpha = 0$, and hence, no super-diffeomorphism is accompanied. We refer to this type of super-Weyl transformation as a "F-type" super-Weyl transformation. It should be noted that the F-type super-Weyl transformation is sufficient to forbid the gaugino mass from the term in Eq. (3) in the discussion in Sec. II. In the following, we concentrate on the anomalous breaking of the F-type super-Weyl symmetry.

Now let us examine the invariance of the path-integral measures in Eq. (13) under the F-type super-Weyl transformation. Under the transformation, $Q_{\rm diff}$ and $\bar{Q}_{\rm diff}$ are not invariant but transform by

$$\begin{aligned} Q_{\rm diff} &= (2\mathcal{E})^{1/2}Q \to Q'_{\rm diff} = e^{-\Sigma}Q_{\rm diff},\\ \bar{Q}_{\rm diff} &= (2\mathcal{E})^{1/2}\bar{Q} \to \bar{Q}'_{\rm diff} = e^{-\Sigma}\bar{Q}_{\rm diff}. \end{aligned}\tag{15}$$

Here, we have used the fact that the super-Weyl weight of the massless chiral fields are -2 so that Eq. (2) is invariant under the super-Weyl symmetry. Thus, due to the Konishi-Shizuya anomaly [57], we find that the super-diffeomorphism-invariant measure is not invariant under the F-type super-Weyl transformation. Instead, the F-type super-Weyl-invariant measures are given by

$$[DQ_{\rm SW}] \equiv [D(2\mathcal{E})^{1/3}Q] = [D(2\mathcal{E})^{-1/6}Q_{\rm diff}], \tag{16}$$

$$[D\bar{Q}_{\rm SW}] \equiv [D(2\mathcal{E})^{1/3}\bar{Q}] = [D(2\mathcal{E})^{-1/6}\bar{Q}_{\rm diff}], \tag{17}$$

where $Q_{\rm SW}$ and $\bar{Q}_{\rm SW}$ are invariant under the F-type super-Weyl transformation. Here, the weighted chiral superfields $Q_{\rm SW}$ and $\bar{Q}_{\rm SW}$ have density weights $1/3$.

It should be noted that the component fields of $Q_{\rm SW}$ ($\bar{Q}_{\rm SW}$) defined by

$$Q_{\rm SW} = e^{1/3}[A_{Q_{\rm SW}} + \sqrt{2}\Theta\chi_{Q_{\rm SW}} + \Theta^2 F_{Q_{\rm SW}}], \tag{18}$$

have the canonical kinetic terms at the leading order which decouple from the supergravity multiplets in the flat limit. That is, for a generic chiral scalar superfield, $X = A + \sqrt{2}\Theta\chi + \Theta^2 F$, the chiral projection of its complex conjugate is given by

$$\begin{aligned}(\mathcal{D}^2 - 8R)X^\dagger = {}& -4F^* + \frac{4}{3}MA^* + \Theta^\alpha[-4i\sqrt{2}\sigma^m\partial_m\chi^\dagger]\\ &+\Theta^2\left[-4\partial^2A^* - \frac{8}{3}M^*F^* + \frac{8}{9}A^*|M|^2\right]\\ &+\cdots,\end{aligned}\tag{19}$$

where the ellipses denote higher-dimensional terms. Then, by remembering that the component fields of $Q_{\rm SW}$ are related to those of Q via

$$Q = \left(1 + \frac{1}{3}M^*\Theta^2\right)(A_{Q_{\rm SW}} + \sqrt{2}\Theta\chi_{Q_{\rm SW}} + \Theta^2 F_{Q_{\rm SW}}) + \cdots, \tag{20}$$

we find that the kinetic terms of the component fields of $Q_{\rm SW}$ are canonical and decouple from M.[10] Therefore, it is appropriate to identify the component fields of $Q_{\rm SW}$ as the component fields of the corresponding chiral field in the rigid supersymmetry,[11]

$$Q_{\text{rigid supersymmetry}} = A_{Q_{\rm SW}} + \sqrt{2}\theta\chi_{Q_{\rm SW}} + \theta^2 F_{Q_{\rm SW}}, \tag{21}$$

with θ being the fermionic coordinate of the rigid superspace.

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[10] In terms of the component fields of Q, M does not decouple from the kinetic term and mixes with the scalar fields via, $M^*F^*_QA_Q$ as well as $|A_Q|^2|M|^2$ terms.

[11] Here, we have neglected higher-dimensional terms. If we take them into account, we need to perform a Kähler-Weyl transformation to achieve the canonical normalization in the Einstein frame.

D. Gaugino mass in the Wilsonian effective action

As we have discussed in the previous section, the gaugino mass vanishes if the F-type super-Weyl symmetry is preserved, and it is generated only through violations of the F-type super-Weyl symmetry. As relevant terms of the gauge supermultiplet preserve the super-Weyl symmetry, the gaugino mass appearing in the super-diffeomorphism-invariant "tree-level" Wilsonian action is highly suppressed.

The approximate F-type super-Weyl symmetry is, however, anomalously broken by the super-diffeomorphism-invariant measure $[DQ_{\rm diff}]$. To read off the gaugino mass from this violation, it is transparent to change the path-integral measure to the F-type super-Weyl-invariant measure, $[DQ_{\rm SW}]$, so that the super-Weyl variance is apparent in the corrected "tree-level" Wilsonian action. In fact, the change of the measures from $[DQ_{\rm diff}]$ to $[DQ_{\rm SW}]$ is accompanied by the Konishi-Shizuya anomaly [57],[12]

$$
\begin{aligned}
&[DQ_{\rm diff}][D\bar{Q}_{\rm diff}][DQ^\dagger_{\rm diff}][D\bar{Q}^\dagger_{\rm diff}] \\
&\quad = [DQ_{\rm SW}][D\bar{Q}_{\rm SW}][DQ^\dagger_{\rm SW}][D\bar{Q}^\dagger_{\rm SW}] \times \exp[i\Delta S], \\
&\Delta S = \frac{1}{16}\frac{1}{2\pi^2} \times \int d^4x d^2\Theta 2\mathcal{E} \ln(2\mathcal{E})^{1/6} W^\alpha W_\alpha + \text{H.c.} \qquad (22)
\end{aligned}
$$

Accordingly, the "tree-level" Wilsonian effective action which should be taken in conjunction with $[DQ_{\rm SW}]$ is given by,

$$
S = S_{\rm SD} + \Delta S. \qquad (23)
$$

Here, $S_{\rm SD}$ denotes the super-diffeomorphism-invariant local Wilsonian effective action discussed above. Without surprise, ΔS is not invariant under the super-diffeomorphism, which cancels the anomalous breaking of the super-diffeomorphism invariance by $[DQ_{\rm SW}]$. We summarize properties of the measures in Table I.[13]

Armed with a correct "tree-level" Wilsonian action along with the super-Weyl-invariant measure, we can now read off the gaugino mass directly from the local term in the action, ΔS, which leads to

TABLE I. Properties of two path-integral measures. Here, SD and SW denote the super-diffeomorphism and the F-type super-Weyl invariances, respectively. The cancel lines denote noninvariances.

	Measure	Action	Gaugino mass
$[DQ_{\rm diff}]$	SD, ~~SW~~	SD, SW	Hidden in the measure
$[DQ_{\rm SW}]$	~~SD~~, SW	~~SD~~, ~~SW~~	Apparent in the action

$$
\begin{aligned}
m_\lambda/g^2 &= -\frac{1}{2}\frac{1}{2\pi^2}\ln(2\mathcal{E})^{1/6}|_{\Theta^2} \\
&= \frac{1}{24\pi^2}M^* = -\frac{1}{16\pi^2}\times 2m_{3/2}, \qquad (24)
\end{aligned}
$$

where $\mathcal{X}|_{\Theta^2}$ denotes the Θ^2 component of a superfield $\mathcal{X}$. This gaugino mass reproduces the anomaly-mediated gaugino mass given in Refs. [2,3]. In this way, we find that the anomaly-mediated gaugino mass can be read off from the non-super-diffeomorphism-invariant term ΔS in the superspace formalism of supergravity.[14]

E. Radiative corrections from path integration

So far, we have fixed the Wilsonian scale to $M_{\rm PL}$ and have not performed any path integration. Here, let us discuss effects of the path integration. After integrating out modes above a scale $\Lambda(< M_{\rm PL})$, the Wilsonian effective action at Λ is again given by the form of Eq. (23), with renormalized coefficients and higher-dimensional operators suppressed not only by $M_{\rm Pl}$ but also by Λ. Due to the presence of cutoff scales, the super-Weyl symmetry in the Wilsonian action at the scale Λ is hardly preserved. As we have discussed, however, the relevant terms of the matter and the gauge supermultiplets have an accidental approximate super-Weyl symmetry due to the super-diffeomorphism invariance. Therefore, radiative corrections do not generate the gaugino mass term beyond the one in Eq. (24) up to Λ or $M_{\rm Pl}$ suppressed corrections.

It should also be noted that, among various corrections, the ones from diagrams which involve Planck-suppressed interactions lead to higher-dimensional operators suppressed at least by a single power of $M_{\rm PL}$ in the effective action at Λ.[15] Effects leading to lower-dimensional operators through ultraviolet divergences are renormalized by the shifts of the corresponding operators [61]. Visible effects of higher-dimensional operators only show up

[12]The identity in Eq. (22) is not quite correct. In general, ΔS involves higher-dimensional terms suppressed by the cutoff of the Wilsonian effective action. However, such higher-dimensional terms are negligible.

[13]Throughout this paper, we presume the regularization scheme of the path-integral measure which reproduces the Konishi-Shizuya anomaly in the form in Eq. (22). In the dimensional regularization/reduction, on the other hand, the change of the path-integral measures is not accompanied by the rescaling anomaly, while the approximate super-Weyl symmetry is explicitly broken by the relevant interactions which eventually leads to a consistent gaugino mass [43].

[14]In this paper, we concentrate on the anomaly-mediated gaugino mass at the one-loop level.

[15]If there are ultraviolet divergences which are canceled only by nonlocal terms, $M_{\rm PL}$ suppressed interactions could lead to higher-dimensional operators suppressed not by $M_{\rm PL}$ but only by Λ at the cutoff scale Λ. The Bogoliubov-Parasiuk-Hepp-Zimmermann prescription [58–60] shows that ultraviolet divergences in general can be renormalized away by local terms.

through higher-dimensional operators even in the effective action at Λ.

Concretely, radiative corrections from loop diagrams involving gravity supermultiplets (in particular gravitinos with small momenta) may lead to higher-dimensional operators such as $|M|^n M^* \lambda\lambda$ $(n \geq 0)$ suppressed only by $M_{\rm PL}^2 \Lambda^{n-2}$. Such diagrams involving the gravitinos however damp for $\Lambda \ll m_{3/2}$. Therefore, their contribution to the gaugino mass is at most $O(m_{3/2}^3/M_{\rm PL}^2)$.

From these arguments, we see that higher-dimensional operators which are not suppressed by $M_{\rm PL}$ but only by Λ in the Wilsonian effective action at the cutoff scale Λ are generated only from relevant interactions of the matter and gauge supermultiplets. Such effects can be properly taken care of within the renormaizable effective theory of the matter and the gauge supermultiplets with softly broken supersymmetry.

Let us emphasize again that the super-diffeomorphism violation is not arbitrary in the Wilsonian effective action at Λ, although the super-diffeomorphism invariance is broken by $[DQ_{\rm SW}]$. The super-diffeomorphism violation in the Wilsonian action is uniquely given by ΔS at each Wilsonian scale, so that the super-diffeomorphism is preserved in the quantum theory. Thus, the accidental approximate super-Weyl symmetry which is the outcome of the super-diffeomorphism invariance is justified even after performing path integration.

Putting it all together, we find that the anomaly-mediated gaugino mass can be extracted from the non-super-diffeomorphism-invariant local term in the Wilsonian effective action at the scale $\Lambda \gg m_{3/2}$ in the superspace formalism of supergravity. Radiative corrections to the gaugino mass operator are dominantly given by relevant interactions of the matter and the gauge supermultiplets. Therefore, once we extract a gaugino mass at some high cutoff scale, we can use the gaugino mass as the boundary condition of the renormalization group equation at Λ in the low-energy effective renormalizable supersymmetric theory with soft supersymmetry breaking.

F. Decoupling effects of massive matter

Before closing this section, let us consider the contribution to the gaugino mass from charged matter multiplets with a supersymmetric mass m far larger than $m_{3/2}$,

$$\mathcal{L}_{\rm mass} = \int d^2\Theta 2\mathcal{E} m Q\bar{Q} + \text{H.c.} \tag{25}$$

If the cutoff scale of the Wilsonian effective action is far above m, the mass m is negligible in comparison with the kinetic term and hence the above discussion holds. When the cutoff scale is below m, the mass term dominates over the kinetic term. In that situation, the approximate super-Weyl symmetry is such that the mass term is invariant.[16] This observation leads to the Weyl weights of -3 for Q and $\bar{Q}$, i.e. $\delta_{\rm SW,massive} Q = -3\Sigma Q + \cdots$, and hence, the super-Weyl-invariant measures of the massive matter are given by

$$\begin{aligned}[DQ_{\rm SW,massive}] &\equiv [D(2\mathcal{E})^{1/2} Q],\\ [D\bar{Q}_{\rm SW,massive}] &\equiv [D(2\mathcal{E})^{1/2} \bar{Q}],\end{aligned} \tag{26}$$

which coincide with the super-diffeomorphism-invariant measures in Eq. (13). Thus, below the scale m, the approximate super-Weyl symmetry is well described by the super-diffeomorphism-invariant Wilsonian effective action, i.e. $\Delta S = 0$, and hence, no anomaly-mediated gaugino mass term appears up to $O(m_{3/2}^2/m)$ contributions. This argument reconfirms the insensitivity of the anomaly-mediated gaugino mass to ultraviolet physics [2].

If m is close to $m_{3/2}$, the decoupling does not hold in general. The Wilsonian effective action below the mass threshold of Q and $\bar{Q}$ includes terms suppressed only by m, which might make a contribution to the gaugino mass as large as $m_{3/2}^2/m$. Integration of Q and $\bar{Q}$ should be performed explicitly, as is the case with the Higgsino threshold correction in the minimal supersymmetric standard model [2].

IV. FICTITIOUS SUPER-WEYL GAUGE-SYMMETRIC FORMULATION

In the discussion in Refs. [48,51], the origin of the gaugino mass has been discussed in the superspace formalism of supergravity with the help of a fictitious (and exact) super-Weyl gauge symmetry by introducing a chiral super-Weyl compensator field, C, along the lines of Ref. [54]. We call this super-Weyl symmetry the fictitious super-Weyl gauge symmetry throughout the paper to distinguish it from the approximate super-Weyl symmetry we have discussed so far. One of the keys to settling the puzzle in the discussion in Refs. [48,51] is how to write down the anomaly-mediated gaugino mass term of the fictitious super-Weyl gauge symmetry in a gauge-independent way. In this section, we show how to write down the gauge-independent gaugino mass term, where the knowledge of the super-diffeomorphism-invariant path-integral measure plays a crucial role.

A. Fictitious super-Weyl gauge symmetry

The fictitious (and exact) super-Weyl gauge symmetry is introduced to the action in Eq. (1) by performing a finite super-Weyl transformation in Eq. (6) with $\Sigma = \ln C/2$ and $w = 0$ [54]. The resulting classical acton is given by

[16] In the Pauli-Villars regularization, the anomaly-mediated gaugino mass is understood by the difference of super-Weyl-invariant measures between massive Pauli-Villars fields and massless matter fields (see Appendix B).

$$\mathcal{L} = M_{\rm Pl}^2 \int \mathrm{d}^2\Theta 2\mathcal{E}' \frac{3}{8}(\mathcal{D}'^{\dagger 2} - 8R')CC^\dagger \exp\left[-\frac{K'}{3M_{\rm Pl}^2}\right] + \frac{1}{16g^2}\int \mathrm{d}^2\Theta 2\mathcal{E}' W'^\alpha W'_\alpha + \text{H.c.}, \tag{27}$$

where primes denote fields after the transformation. Now, the action is exactly invariant under the super-Weyl symmetry in Eq. (6) in terms of $\mathcal{E}'$, W'^α, Q' and $\bar{Q}'$ with $w = 0$, while giving a Weyl weight -2 to the "super-Weyl compensator" C,

$$\delta_{\rm SW,fic} C = -2\Sigma C - S^\alpha \frac{\partial}{\partial \Theta^\alpha} C. \tag{28}$$

It should be noted that the compensator C is a gauge degree of freedom, which can be completely eliminated by performing the fictitious super-Weyl transformation. In other words, one may take any C so that the calculation that one performs is as simple as possible.[17] In particular, in the presence of the compensator, the equation of motion of M' is changed from Eq. (5) to

$$F^C - \frac{1}{3}M'^* = m_{3/2}, \tag{29}$$

where we have taken $C = 1 + F^C\Theta^2$. Thus, for example, it is convenient to take the gauge where $M' = 0$, which was taken in Ref. [48] up to higher-dimensional terms (see also Ref. [62]).

B. Gaugino mass

As we have discussed in the previous section, the super-Weyl transformation performed to introduce C is anomalous where the measure is transformed from $[DQ_{\rm dff}]$ to $[DQ'_{\rm dff}]$.[18] The transformation invokes the following term in the Wilsonian effective action:

$$\Delta S'_C = +\frac{1}{16}\frac{3}{4\pi^2}\int \mathrm{d}^4x \mathrm{d}^2\Theta 2\mathcal{E}' \ln C W'^\alpha W'_\alpha + \text{H.c.} \tag{30}$$

This term can also be derived from the condition that the fictitious super-Weyl symmetry is free from the gauge anomaly [54]. Further, let us eliminate C from the kinetic term of the matter fields by the redefinitions, $Q'' \equiv Q'C$ and $\bar{Q}'' \equiv \bar{Q}'C$. After the redefinitions, the integration of the matter fields does not generate the gaugino mass proportional to F^C at the one-loop level, so that the gaugino mass is directly read off from the Wilsonian effective action. By combining the counterterms of the anomalies to reach $Q''_{\rm diff} = (2\mathcal{E}')^{1/2}Q'C$ and $\bar{Q}''_{\rm diff} = (2\mathcal{E}')^{1/2}\bar{Q}'C$, we eventually obtain

$$\Delta S_C = \frac{1}{16}\frac{1}{4\pi^2} \times \int \mathrm{d}^4x \mathrm{d}^2\Theta 2\mathcal{E}' \ln C W'^\alpha W'_\alpha + \text{H.c.}, \tag{31}$$

where the corresponding path-integral measures are given by $[DQ''_{\rm dff}]$ and $[D\bar{Q}''_{\rm dff}]$.

In Ref. [51], it was claimed that there is no anomaly-mediated gaugino mass (derived in Refs. [2,3]) by taking a gauge with $F^C = 0$. On the other hand, in Ref. [48], taking another gauge with $M' = 0$, the anomaly-mediated gaugino mass was reproduced. These arguments pose a puzzle, as the gaugino mass should not depend on the gauge choice of F^C.

This puzzle is solved by remembering the discussion in Sec. III. There, in order to read off the gaugino mass from the Wilsonian effective action, we used the canonical measure $[DQ_{\rm SW}] \equiv [D(2\mathcal{E})^{1/3}Q]$. Similarly, we should use this measure again,

$$[DQ_c] \equiv [D(2\mathcal{E}')^{1/3}CQ'] = [D(2\mathcal{E}')^{-1/6}Q''_{\rm diff}], \tag{32}$$

which is again invariant under the "approximate" super-Weyl symmetry. The kinetic term of Q_c is free from the mixings to both M' and F^C, and hence, canonical. Eventually, by translating the measure from $[DQ''_{\rm diff}]$ to $[DQ_c]$, the Wilsonian effective action obtains a correction ΔS, which add up with ΔS_C,[19]

$$\Delta S + \Delta S_C = \frac{1}{16}\frac{1}{4\pi^2} \times \int \mathrm{d}^4x \mathrm{d}^2\Theta 2\mathcal{E}'(\ln(2\mathcal{E}')^{1/3} + \ln C)W'^\alpha W'_\alpha + \text{H.c.} \tag{34}$$

This expression is manifestly invariant under the fictitious super-Weyl transformation. Again the counterterm is not invariant under the super-diffeomorphism, which is inevitable in order to cancel the anomaly of the super-diffeomorphism due to $[DQ_c]$. From this expression, we obtain the anomaly-mediated gaugino mass

$$m_\lambda/g^2 = -\frac{1}{2}\frac{1}{4\pi^2}(\ln(2\mathcal{E}')^{1/3} + \ln C)|_{\Theta^2} = -\frac{1}{8\pi^2}\left(F^C - \frac{1}{3}M'^*\right) = -\frac{1}{16\pi^2} \times 2m_{3/2}, \tag{35}$$

which is independent of the gauge choice of F^C.

In our argument, the super-diffeomorphism-variant counterterm ΔS is the key to obtaining the manifestly invariant expression of the anomaly-mediated gaugino mass under the fictitious super-Weyl gauge symmetry. It should also be stressed that the combination,

[17] The singular transformation leading to $C = 0$ should be avoided.

[18] The weighted chiral field $Q_{\rm diff}$ has a Weyl weight 3 for $w = 0$.

[19] One may obtain the following counterterm directly from the relation:

$$[DQ_c] = [D(2\mathcal{E})^{-1/6}C^{-1/2}Q_{\rm diff}]. \tag{33}$$

$$\int \mathrm{d}^4x\mathrm{d}^2\Theta 2\mathcal{E}'(\ln(2\mathcal{E}')^{1/3} + \ln C)W'^{\alpha}W'_{\alpha} + \mathrm{H.c.}, \tag{36}$$

is invariant under the fictitious super-Weyl symmetry. Thus, the mere knowledge of the anomaly of the fictitious super-Weyl gauge symmetry cannot determine the overall coefficient of Eq. (34), and it is crucial to start with the super-diffeomorphism-invariant measure to obtain Eq. (34).[20]

C. Relation to the 1PI quantum effective action (I)

As is clear from Eq. (35), the gaugino mass is simply read off from the counterterm in the Wilsonian effective action, ΔS_C, by taking the gauge with $M' = 0$ and $F^C = m_{3/2}$. In the 1PI quantum effective action, on the other hand, it should also be possible to write down the gaugino mass term without using the compensator C. To see how the gaugino mass appears in the 1PI action, let us consider a finite super-Weyl transformation of R,

$$R' = -\frac{1}{8}e^{4\Sigma}(\mathcal{D}^{\dagger 2} - 8R)e^{-2\Sigma^\dagger} + \cdots. \tag{37}$$

Here, ellipses denote terms which are irrelevant for the transformation of the lowest component of R. Then, by taking Σ such that

$$(\mathcal{D}^2 - 8R^\dagger)e^{-2\Sigma} = 0, \tag{38}$$

we can eliminate the lowest component of R. The solution of Eq. (38) is given by [53,63]

$$e^{-2\Sigma} \equiv \Omega = 1 + \frac{1}{2\Box_+}(\mathcal{D}^{\dagger 2} - 8R)R^\dagger,$$
$$\Box_+ \equiv \frac{1}{16}(\mathcal{D}^{\dagger 2} - 8R)(\mathcal{D}^2 - 8R^\dagger). \tag{39}$$

Thus, by setting $C = \Omega^{-1}$, we can achieve the desirable gauge choice of the fictitious super-Weyl gauge symmetry where $M' = 0$. It should be noted that the apparent nonlocal expression of Ω does not cause problems since the chiral field Ω is reduced to a local field expression,

$$\Omega \simeq 1 + \frac{1}{3}M^*\Theta^2, \tag{40}$$

in the flat limit. Thus, as long as we are interested in the flat limit, Ω can be treated as a local field.

In this gauge, ΔS_C is now expressed by,

$$\Delta S_{C=\Omega^{-1}} = \frac{1}{16}\frac{1}{4\pi^2} \times \int \mathrm{d}^4x\mathrm{d}^2\Theta 2\mathcal{E}' \ln\Omega^{-1} W'^{\alpha}W'_{\alpha} + \mathrm{H.c.} \tag{41}$$

By expanding this expression around $\Omega = 1$, we obtain

$$\Delta S_{C=\Omega^{-1}} \simeq -\frac{1}{16}\frac{1}{8\pi^2}\int \mathrm{d}^4x\mathrm{d}^2\Theta 2\mathcal{E}\frac{1}{\Box_+}(\mathcal{D}^{\dagger 2} - 8R) \times R^\dagger W^\alpha W_\alpha + \mathrm{H.c.}, \tag{42}$$

at the leading order. Here, we have reverted $\mathcal{E}'$ and W'^{α} to $\mathcal{E}$ and W^α. Since this term is expressed in terms of the gravity multiplet and is independent of C, this provides an appropriate expression for the super-Weyl variance in the 1PI effective action. In fact, the final expression reproduces the 1PI quantum effective action given in Ref. [42].[21] By substituting Eq. (40), we again obtain the anomaly-mediated gaugino mass from Refs. [2,3].

D. Relation with 1PI quantum effective action (II)

The chiral field Ω is also useful for discussing the 1PI quantum effective action along the lines of Sec. III, where we did not introduced the super-Weyl compensator C. There, instead, we relied on the F-type super-Weyl-invariant but super-diffeomorphism-variant measure to read off the gaugino mass from the Wilsonian effective action. The 1PI quantum effective action, however, must be invariant under the super-diffeomorphism by itself. Thus, ΔS should be replaced by a super-diffeomorphism-invariant expression in the 1PI quantum effective action.

To find an appropriate expression, let us remember that the chiral field Ω transforms,

$$\delta_{\mathrm{SW}}\Omega = -2\Sigma\Omega - S^\alpha\frac{\partial}{\partial\Theta^\alpha}\Omega, \tag{43}$$

under the super-Weyl transformation. From this property, we can construct a measure

$$[DQ_{\mathrm{SW,diff}}] \equiv [D\Omega^{1/2}(2\mathcal{E})^{1/2}Q] = [D\Omega^{1/2}Q_{\mathrm{diff}}], \tag{44}$$

which is invariant under both the F-type super-Weyl and the super-diffeomorphism transformations.[22] Thus, in a similar way to Sec. III, the Wilsonian effective action receives a correction by changing the measure from $[DQ_{\mathrm{diff}}]$ to $[DQ_{\mathrm{SW,diff}}]$,

[20]Correspondingly, in the 1PI effective action, the fictitious super-Weyl gauge invariance alone cannot determine the gaugino mass term up to the contribution from Eq. (36) with $\ln(2\mathcal{E}')^{1/3}$ replaced by $\ln\Omega^{-1}$, where the chiral field Ω is defined in the following.

[21]The apparent difference by a factor of 4 between our result and that in Ref. [42] is due to the difference of the normalization of the gauge multiplet.

[22]The component fields of $Q_{\mathrm{SW,diff}}$ defined by, $Q_{\mathrm{SW,diff}} = e^{1/2}[A_{Q_{\mathrm{SW,diff}}} + \sqrt{2}\Theta\chi_{Q_{\mathrm{SW,diff}}} + \Theta^2 F_{Q_{\mathrm{SW,diff}}}]$, have the same canonical kinetic term with those of Q_{SW} in Eq. (18).

$$[DQ_{\mathrm{diff}}][D\bar{Q}_{\mathrm{diff}}][DQ^{\dagger}_{\mathrm{diff}}][D\bar{Q}^{\dagger}_{\mathrm{diff}}] = [DQ_{\mathrm{SW,diff}}][D\bar{Q}_{\mathrm{SW,diff}}][DQ^{\dagger}_{\mathrm{SW,diff}}][D\bar{Q}^{\dagger}_{\mathrm{SW,diff}}] \times \exp[i\Delta S_{\mathrm{diff}}],$$
$$\Delta S_{\mathrm{diff}} = \frac{1}{16}\frac{1}{4\pi^2} \times \int \mathrm{d}^4x\mathrm{d}^2\Theta 2\mathcal{E} \ln \Omega^{-1} W^{\alpha} W_{\alpha} + \mathrm{H.c.} \tag{45}$$

Unlike ΔS, ΔS_{diff} is invariant under the super-diffeomorphism. Thus, ΔS_{diff} is an appropriate expression of the super-Weyl breaking in the 1PI quantum effective action. Again, this expression reproduces the super-Weyl-breaking term in the 1PI effective action in Ref. [42].

V. SUMMARY

In this paper, we have reexamined the anomaly-mediated gaugino mass term in the superspace formalism of supergravity. The absence of the gaugino mass term of $O(m_{3/2})$ in the classical supergravity action is understood by an approximate super-Weyl symmetry of the super-diffeomorphism-invariant local classical action. Then, we find that the anomaly-mediated gaugino mass originates from the anomalous breaking of the approximate super-Weyl symmetry caused by the super-diffeomorphism-invariant measure of the charged field. By changing the path-integral measure from the super-diffeomorphism-invariant one to the super-Weyl-invariant one, we have shown that the gaugino mass term can be read off from the local counterterm in the Wilsonian action. It should be stressed that the counterterm is not invariant under the super-diffeomorphism, which is required for the super-diffeomorphism invariance of the quantum theory. As is clear from our discussion, the path-integral measure plays a crucial role in determining the gaugino mass term. This observation fills a gap in the literature on the anomaly-mediated gaugino mass.

We have also discussed the gaugino mass in the formulation with a fictitious super-Weyl gauge symmetry. There, the action is made invariant under a fictitious super-Weyl gauge symmetry by introducing a chiral compensator C. Since C is a gauge degree of freedom, the gaugino mass should be independent of the choice of the value of C. A gauge-independent expression of the local gaugino mass term was, however, not known in the literature, which is one of the origins of the controversy in Refs. [48,51]. In our discussion, we have shown how the gauge-independent expression is obtained with the aid of the super-diffeomorphism-invariant measure. We have also discussed how to reconcile the gaugino mass term appearing in the nonlocal 1PI effective action given in Ref. [42] and the one in the local effective Wilsonian action.

In our discussion, we have concentrated on the anomaly-mediated gaugino mass at the one-loop level. At the one-loop level, the violation of the F-type super-Weyl symmetry in the gauge kinetic function, which is the origin of the gaugino mass, is extracted by calculating the anomalous Jacobian associated with the change of the measure from the super-diffeomorphism-invariant one to the F-type super-Weyl-invariant one [see Eq. (22)]. This corresponds to the fact that a one-loop beta function of a gauge theory is extracted by calculating the anomalous Jacobian associated with the Weyl transformation [64]. Note that the approximate super-Weyl symmetry is also broken by anomalous dimensions of the matter and gauge multiplets, which contributes to the gaugino mass at the two-loop level and higher. It is difficult to extract these contributions in the Wilsonian effective action. It would be easier to discuss the violation of the super-Weyl symmetry in the 1PI effective action.

As a final remark, let us sketch the gaugino mass in a non-Abelian gauge theory. In the non-Abelian gauge theory, the path-integral measure of the gauge multiplet should be taken into account. The super-diffeomorphism-invariant measure and the F-type super-Weyl-invariant measure are given by

$$[DV_{\mathrm{diff}}] = [DE^{1/2}V],$$
$$[DV_{\mathrm{SW}}] = [D(2\mathcal{E})^{-1/6}(2\mathcal{E}^{\dagger})^{-1/6}V_{\mathrm{diff}}], \tag{46}$$

where E is the determinant of the supersymmetric vielbein in a real superspace. The super-Weyl transformation law of E is given by

$$\delta_{\mathrm{SW}}E = 2(\Sigma + \Sigma^{\dagger})E + \cdots, \tag{47}$$

where the ellipses denote inhomogeneous terms which can be canceled by the super-diffeomorphism. Here, we collectively represent the gauge multiplet and the ghost multiplets by V, and V_{diff} and V_{SW} are also represented accordingly.

The translation from $[DV_{\mathrm{diff}}]$ to $[DV_{\mathrm{SW}}]$ is easily performed in the following way. Let us introduce a chiral compensator C as in Sec. IV, which defines E' via,

$$E = CC^{\dagger}E'. \tag{48}$$

By remembering that the super-Weyl transformation is anomalous, the gauge kinetic function receives a counterterm depending on C as [54],

$$[DE^{1/2}V] = [DE'^{1/2}V] \times e^{i\Delta S^V_C},$$
$$\Delta S^V_C = -\frac{1}{16}\frac{3T_G}{8\pi^2} \times \int \mathrm{d}^4x\mathrm{d}^2\Theta 2\mathcal{E}' \ln C W'^{\alpha} W'_{\alpha} + \mathrm{H.c.}, \tag{49}$$

where T_G is the Dynkin index of the adjoint representation. It should be noted that ΔS^V_C includes the rescaling anomaly form the ghost multiplets. Then, by comparing Eqs. (47), (48) and (49), we find that the counterterm appearing in the translation from $[DV_{\rm diff}]$ to $[DV_{\rm SW}]$ is given by replacing C to $(2\mathcal{E})^{1/3}$,[23] which leads to

$$[DV_{\rm diff}] = [DV_{\rm SW}] \times e^{i\Delta S_C^V}, \quad C = (2\mathcal{E})^{1/3}. \tag{50}$$

By putting Eqs. (22) and (50) together, we obtain

$$\prod_R [DQ^R_{\rm diff}][DQ^{R\dagger}_{\rm diff}][DV_{\rm diff}] = \prod_i [DQ^R_{\rm SW}][DQ^{R\dagger}_{\rm SW}][DV_{\rm SW}] \times e^{i\Delta S},$$
$$\Delta S = -\frac{1}{16}\frac{3T_G - \sum T_R}{8\pi^2} \times \int d^4x d^2\Theta 2\mathcal{E} \ln{(2\mathcal{E})^{1/3}} W^\alpha W_\alpha + \text{H.c.}, \tag{51}$$

where T_R is the total Dynkin index of matter fields Q^R. As a result, we find an expression for the gaugino mass,

$$m_\lambda/g^2 = \frac{1}{2}\frac{3T_G - T_R}{8\pi^2}\ln(2\mathcal{E})^{1/3}|_{\Theta^2} = \frac{3T_G - T_R}{16\pi^2} \times m_{3/2}, \tag{52}$$

which reproduces the anomaly-mediated gaugino mass found in Refs. [2,3]. We may also obtain the manifestly gauge-independent expression in the fictitious super-Weyl symmetry for the non-Abelian gauge theory by using Eq. (50) along the lines of Sec. IV.

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APPENDIX A: SUPER-DIFFEOMORPHISM-INVARIANT MEASURE

In this appendix, we show that the measure given in Eq. (13) is invariant under the super-diffeomorphism. Under the tranformation given in Eq. (11), the variable $Q_{\rm diff}$ transforms as

$$Q_{\rm diff} \to Q'_{\rm diff} = Q_{\rm diff} - \eta^M(x,\Theta)\partial_M Q_{\rm diff} - \frac{1}{2}(-)^M(\partial_M \eta^M(x,\Theta))Q_{\rm diff}. \tag{A1}$$

[23] The expression for the rescaling anomaly does not depend on whether the rescaling factor is a chiral superfield or a chiral density superfield.

Then, the path-integral measure $[DQ_{\rm diff}]$ transforms,

$$[DQ'_{\rm diff}] = [DQ_{\rm diff}] \times \exp{[\text{sTr}\mathcal{O}(z', z)]}, \tag{A2}$$

$$\mathcal{O}(z', z) \equiv -\left[\eta^M\partial_M + \frac{1}{2}(-1)^M(\partial_M\eta^M)\right]\delta^6(z'-z), \tag{A3}$$

where we have collectively represented x and Θ by z. Formally, the supertrace sTr is expressed by

$$\text{sTr}\mathcal{O}(z', z) = \int d^6z d^6z' \delta^6(z'-z)\mathcal{O}(z', z). \tag{A4}$$

A naive conclusion is that the supertrace vanishes due to the saturation of Grassmann variables Θ and Θ' from the delta functions in Eqs. (A3) and (A4). However, since there is also a factor of $\delta^4(x'-x)$, which is well defined only after integrating over x or x', one should carefully investigate the integration.

To examine the integration, let us expand the delta function by plane waves,

$$\delta^6(z'-z) = \int \frac{d^4k}{(2\pi)^4} d^2\tau \Psi_{-k,-\tau}(z')\Psi_{k,\tau}(z), \tag{A5}$$

$$\Psi_{k,\tau}(z) \equiv \exp(ikx + 2i\tau\Theta). \tag{A6}$$

By substituting this expression into Eq. (A4), the above supertrace is expressed by,

$$\text{sTr}\mathcal{O}(z', z) = -\int d^6z \int \frac{d^4k}{(2\pi)^4} d^2\tau \Psi_{-k,-\tau}(z) \times \left[\eta^M\partial_M + \frac{1}{2}(-)^M(\partial_M\eta^M)\right]\Psi_{k,\tau}(z). \tag{A7}$$

Now, let us notice an identity,

$$\int d^6z \Psi_{k,\eta}(z)\left[\eta^M\partial_M + \frac{1}{2}(-)^M(\partial_M\eta^M)\right]\Psi_{k,\eta}(z) = \frac{1}{2}(-)^M \int d^6z \partial_M[\Psi_{k,\eta}(z)\eta^M\Psi_{k,\eta}(z)] = 0, \tag{A8}$$

where we have used the property that an integration of a total derivative vanishes. By using this identity several times, the supertrace can be rearranged as

$$\begin{aligned}\mathrm{sTr}\mathcal{O}(z',z)&=-\frac{1}{2}\int \mathrm{d}^6 z\int\frac{\mathrm{d}^4 k}{(2\pi)^4}d^2\tau(\Psi_{k,\tau}(z)+\Psi_{-k,-\tau}(z))\\&\quad\times\left[\eta^M\partial_M+\frac{1}{2}(-)^M(\partial_M\eta^M)\right](\Psi_{k,\tau}(z)+\Psi_{-k,-\tau}(z))\\&=-\frac{1}{4}(-)^M\int \mathrm{d}^6 z\int\frac{\mathrm{d}^4 k}{(2\pi)^4}d^2\tau\partial_M[(\Psi_{k,\tau}(z)\\&\quad+\Psi_{-k,-\tau}(z))\eta^M(\Psi_{k,\tau}(z)+\Psi_{-k,-\tau}(z))]\\&=0.\end{aligned}\tag{A9}$$

This shows that the measure given in Eq. (13) is actually invariant under the super-diffeomorphism. It should be noted that the transformation law in Eq. (A1) is crucial when using Eq. (A8), and hence, the super-diffeomorphism invariance does not hold for measures with different weights $[D(2\mathcal{E})^n Q]$ $(n\neq 1/2)$. In fact, the super-diffeomorphism transformation of $[D(2\mathcal{E})^n Q]$ $(n\neq 1/2)$ is accompanied by the Konishi-Sizuya anomaly [57]. This argument provides a superfield expression for the arguments in Ref. [65].

There is a quicker route to show the super-diffeomorphisim invariance of $[DQ_{\mathrm{diff}}]$ from the very definition of the path-integral measure [65]. Let us a consider a superfield $\tilde{Q}(x,\Theta)$ defined in a chiral superspace. The path-integral measure $[D\tilde{Q}]$ is defined by a Gaussian integration,

$$\int[D\tilde{Q}]\exp\left[\frac{i}{2}\int \mathrm{d}^6 z\Theta\tilde{Q}\,\tilde{Q}\right]=N,\tag{A10}$$

where N is a normalization constant. It should be noted that we have not specified the transformation law of $\tilde{Q}$ under the super-diffeomorphism at this point.

Next, let us introduce $Q\equiv(2\mathcal{E})^{-1/2}\tilde{Q}$, and choose the transformation property of $\tilde{Q}$ as a chiral density multiplet with density weights $1/2$, where Q is a chiral scalar multiplet. Then, from Eq. (A10), we obtain

$$\int[D(2\mathcal{E})^{1/2}Q]\exp\left[\frac{i}{2}\int \mathrm{d}^6 z2\mathcal{E}QQ\right]=N.\tag{A11}$$

Now, since $\int \mathrm{d}^6 z\mathcal{E}QQ$ is invariant under the super-diffeomorphism since Q is the chiral scalar multiplet, the path-integral measure is as well $[D(2\mathcal{E}^{1/2})Q]$.

In fact, under the super-diffeomorphism,

$$\mathcal{E}'=\mathcal{E}-\delta_{\mathrm{SD}}\mathcal{E},\qquad Q'=Q-\delta_{\mathrm{SD}}Q,\tag{A12}$$

we have the following identities:

$$\begin{aligned}N&=\int[D(2\mathcal{E})^{1/2}Q]\exp\left[\frac{i}{2}\int \mathrm{d}^6 z2\mathcal{E}QQ\right],\\&=\int[D(2\mathcal{E}')^{1/2}Q']\exp\left[\frac{i}{2}\int \mathrm{d}^6 z2\mathcal{E}'Q'Q'\right],\\&=\int[D(2\mathcal{E}')^{1/2}Q']\exp\left[\frac{i}{2}\int \mathrm{d}^6 z2\mathcal{E}QQ\right].\end{aligned}\tag{A13}$$

Here, the second equality is just a change of variable. We have used the super-diffeomorphism invariance of the exponent in the third equality. Thus, from these identities, we find that

$$[D(2\mathcal{E}')^{1/2}Q']=D[(2\mathcal{E})^{1/2}Q],\tag{A14}$$

which again shows the super-diffeomorphism invariance of the measure $[DQ_{\mathrm{diff}}]$. In the same token, we can derive the super-diffeomorphism invariance of the measure of a scalar multiplet V in a real superspace,

$$[DV_{\mathrm{diff}}]=[DE^{1/2}V],\tag{A15}$$

which we briefly mentioned in Sec. V.

APPENDIX B: GAUGINO MASS IN PAULI-VILLARS REGULARIZATION

In this appendix, we show how our method of extracting the gaugino mass works in the Pauli-Villar regularization [66]. In the Pauli-Villar regularization scheme, we introduce Pauli-Villars fields—a pair of fermonic chiral scalar multiplets P and $\bar{P}$ with a unit charge—and give them a supersymmetric mass term Λ which corresponds to the cutoff scale:

$$\mathcal{L}=\int \mathrm{d}^2\Theta 2\mathcal{E}\Lambda P\bar{P}+\mathrm{H.c.}\tag{B1}$$

As we discussed in Sec. III, it is convenient to use the F-type super-Weyl-invariant measure, $[DQ_{\mathrm{SW}}]$, to extract the gaugino mass from the Wilsonian action. If we also take the measure of the Pauli-Villars fields to be $[DP_{\mathrm{SW}}]$, however, the counterterms appearing when we change the measures are canceled due to the opposite statistic of the Pauli-Villars fields. Thus, in this case, the F-type super-Weyl-invariant measure does not invoke the counterterm in Eq. (22), ΔS.

In the absence of ΔS, what is the origin of the gaugino mass? As we discuss in the main text, the gaugino mass is generated only from violations of the approximate F-type super-Weyl symmetry. For a energy scale well below Λ, the approximate F-type super-Weyl symmetry is explicitly broken by the mass term of the Pauli-Villars fields. Thus, the integration of the Pauli-Villars fields generates the gaugino mass, as was discussed in Ref. [2].[24]

[24]More explicitly, the masses of the fermions and the scalars in the Paulli-Villars multiplets are split by the coupling to M through $\int \mathrm{d}^2\Theta(2\mathcal{E})^{1/3}\Lambda P_{\mathrm{SW}}\bar{P}_{\mathrm{SW}}$.

We can also extract the gaugino mass without explicitly performing the integration of the Pauli-Villars fields. Well below the mass scale Λ, a good approximate super-Weyl symmetry is the one which is consistent with the mass term of the Pauli-Villars fields. Thus, the appropriate measure to read off the gaugino mass from the action is the combination of $[DQ_{\rm SW}]$ and $[DP_{\rm diff}]$. With these measures, the counterterm is again given by ΔS in Eq. (22), from which we can directly read off the anomaly-mediated gaugino mass.

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