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Quark-antiquark bound state in momentum-helicity representation

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In this paper we have extended a three-dimensional approach for describing quark-antiquark bound states based on a momentum-helicity representation. To this end we have formulated the relativistic form of the Lippmann-Schwinger equation in the momentum-helicity space which leads to integral equations with one variable. Then we have solved these integral equations by inserting a spin-dependent quark-antiquark potential model numerically. Finally we have obtained the mass spectrum of light mesons and we have compared these results with the results which are obtained in the partial wave representation.

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I. INTRODUCTION

Recently the three-dimensional (3D) approach for nucleon-nucleon (NN) scattering and the deuteron state on the momentum-helicity basis state have been developed. This approach is based on a helicity representation with respect to the total spin of the two-nucleon system. The important advantage of a three-dimensional approach is that a sometimes tedious partial wave expansion of a complex NN force is no longer needed. Instead one introduces a helicity representation of the NN force, which is perfectly adapted to the set of six operators completely describing the most general NN force compatible with general invariance principles. Thus, for any NN force given in operator form, this scheme is applicable. In this work we have extended this approach to studying quark-antiquark bound states [1,2].

To this end we have considered the relativistic quark-antiquark interaction in terms of a linear confinement term, a Coulomb term, and various spin-dependent pieces. This potential is not a simple nonrelativistic reduction of an effective one gluon exchange potential since the coefficients of the various terms in the potential will be constrained by phenomenological considerations alone. The various terms making up the potential are used because they are invariant under rotations, space reflection, and time reversal [3].

This article is organized as follows. In Sec. II a new formalism based on the momentum-helicity basis state is presented for describing the meson bound states. In Sec. III we evaluate the matrix elements of the spin-dependent quark-antiquark potential in the momentum-helicity space. In Sec. IV we give the numerical calculations and results, and in Sec. V a dissection and outlook are presented.

II. FORMULATION OF QUARK-ANTIQUARK BOUND STATE BASED ON HELICITIES

The momentum-helicity basis state is defined as [1]

$$|\mathbf{p};\hat{\mathbf{p}}S\Lambda\rangle \equiv |\mathbf{p}\rangle|\hat{\mathbf{p}}S\Lambda\rangle, \tag{1}$$

where $\mathbf{p}$ is the relative momentum of the quark and antiquark, S is the total spin, and Λ is the spin projection along the relative momentum. This basis state is the eigenstate of the helicity operator $\mathbf{S}\cdot\hat{\mathbf{p}}$:

$$\mathbf{S}\cdot\hat{\mathbf{p}}|\mathbf{p};\hat{\mathbf{p}}S\Lambda\rangle = \Lambda|\mathbf{p};\hat{\mathbf{p}}S\Lambda\rangle. \tag{2}$$

The orthogonality and completeness relations for this state are defined as

$$\langle\mathbf{p}';\hat{\mathbf{p}}'S'\Lambda'|\mathbf{p};\hat{\mathbf{p}}S\Lambda\rangle = \delta(\mathbf{p}'-\mathbf{p})\delta_{S'S}\delta_{\Lambda'\Lambda}, \tag{3}$$

$$\sum_{S\Lambda}\int d\mathbf{p}|\mathbf{p};\hat{\mathbf{p}}S\Lambda\rangle\langle\mathbf{p};\hat{\mathbf{p}}S\Lambda| = 1. \tag{4}$$

The relativistic form of the Schrödinger equation for the two-body bound state is

$$H|\Phi_j^{M_j}\rangle = E|\Phi_j^{M_j}\rangle, \tag{5}$$

where the Hamiltonian H is the sum of a relativistic kinetic energy operator T and a relativistic potential operator V. Furthermore $|\Phi_j^{M_j}\rangle$ is the quark-antiquark bound state with the total angular momentum j, and E represent the mass of two-body system. The kinetic energy operator has the form

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$$T = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}, \tag{6}$$

where m_1 and m_2 are the masses of the quark and antiquark. In what follows, we assume $m \equiv m_1 = m_2$. Equation (5) can be rewritten as the relativistic form of the homogenous Lippmann-Schwinger equation:

$$|\Phi_j^{M_j}\rangle = \frac{1}{E - 2\sqrt{m^2 + p^2}} V |\Phi_j^{M_j}\rangle. \tag{7}$$

By inserting the completeness relation into this equation, we have

$$\langle \mathbf{p}; \hat{\mathbf{p}} S\Lambda | \Phi_j^{M_j}\rangle = \frac{1}{E - 2\sqrt{m^2 + p^2}} \times \sum_{\Lambda'} \int d\mathbf{p}' \langle \mathbf{p}; \hat{\mathbf{p}} S\Lambda | V | \mathbf{p}'; \hat{\mathbf{p}}' S\Lambda'\rangle \times \langle \mathbf{p}'; \hat{\mathbf{p}}' S'\Lambda' | \Phi_j^{M_j}\rangle. \tag{8}$$

Now, we extract the angular dependency of the wave function as follows:

$$\begin{aligned}\langle \mathbf{p}; \hat{\mathbf{p}} S\Lambda | \Phi_j^{M_j}\rangle &= \langle p\hat{\mathbf{z}}; \hat{\mathbf{z}} S\Lambda | R^\dagger(\hat{\mathbf{p}}) | \Phi_j^{M_j}\rangle \\ &= \langle p\hat{\mathbf{z}}; \hat{\mathbf{z}} S\Lambda | e^{iJ_y\theta} e^{iJ_z\varphi} | \Phi_j^{M_j}\rangle \\ &= e^{iM_j\varphi} \langle p\hat{\mathbf{z}}; \hat{\mathbf{z}} S\Lambda | e^{iJ_y\theta} | \Phi_j^{M_j}\rangle \\ &= e^{iM_j\varphi} d^j_{M_j\Lambda}(\theta) \Phi^\Lambda_{Sj}(p). \end{aligned} \tag{9}$$

In this stage, we define a suitable coordinate system so that the vector $\mathbf{p}$ is along the z axis. With this consideration Eq. (8) leads to

$$\Phi^{M_j}_{Sj}(p) = \frac{1}{E - 2\sqrt{m^2 + p^2}} \times \sum_{\Lambda'} \int d\mathbf{p}' \langle p\mathbf{z}; \hat{\mathbf{z}} S\Lambda | V | \mathbf{p}'; \hat{\mathbf{p}}' S\Lambda'\rangle \times e^{iM_j\varphi'} d^j_{M_j\Lambda'}(\theta') \Phi^{\Lambda'}_{Sj}(p'), \tag{10}$$

where we use $d^j_{mm'}(0) = \delta_{mm'}$. The azimuthal dependence of the potential as we have shown in the next section can be factored out:

$$\langle p\mathbf{z}; \hat{\mathbf{z}} S\Lambda | V | \mathbf{p}'; \hat{\mathbf{p}}' S\Lambda'\rangle = e^{-i\Lambda\varphi'} V^S_{\Lambda\Lambda'}(p, p', \theta'). \tag{11}$$

Thus Eq. (10) can be rewritten as

$$\Phi^{M_j}_{Sj}(p) = \frac{1}{E - 2\sqrt{m^2 + p^2}} \times \sum_{\Lambda'} \int_0^\infty dp' p'^2 \int_0^\pi d\theta' V^S_{\Lambda\Lambda'}(p, p', \theta') \times d^j_{M_j\Lambda'}(\theta') \Phi^{\Lambda'}_{Sj}(p') \int_0^{2\pi} d\varphi' e^{i(M_j - \Lambda)\varphi'}. \tag{12}$$

Integration on the azimuthal angle φ' yields

$$\Phi^{M_j}_{Sj}(p) = \frac{2\pi}{E - 2\sqrt{m^2 + p^2}} \sum_{\Lambda'} \int_0^\infty dp' p'^2 \int_0^\pi d\theta' \times V^S_{M_j\Lambda'}(p, p', \theta') d^j_{M_j\Lambda'}(\theta') \Phi^{\Lambda'}_{Sj}(p'). \tag{13}$$

Finally by defining

$$V^S_{M_j\Lambda'}(p, p') \equiv \int_0^\pi d\theta' V^S_{M_j\Lambda'}(p, p', \theta') d^j_{M_j\Lambda'}(\theta'), \tag{14}$$

Eq. (13) is reduced to an equation in one variable as

$$\Phi^{M_j}_{Sj}(p) = \frac{2\pi}{E - 2\sqrt{m^2 + p^2}} \times \sum_{\Lambda'} \int_0^\infty dp' p'^2 V^S_{M_j\Lambda'}(p, p') \Phi^{\Lambda'}_{Sj}(p'). \tag{15}$$

It is well known that the total spin states of two quarks are singlet $(S = 0)$ and triplet $(S = 1)$ states. For the singlet case and arbitrary j, Eq. (15) leads to one equation as

$$\Phi^{M_j}_{0j}(p) = \frac{2\pi}{E - 2\sqrt{m^2 + p^2}} \int_0^\infty dp' p'^2 V^0_{M_j0}(p, p') \Phi^0_{0j}(p'). \tag{16}$$

Also for the triplet case and $j = 0$, Eq. (15) leads to one equation as

$$\Phi^0_{1j}(p) = \frac{2\pi}{E - 2\sqrt{m^2 + p^2}} \int_0^\infty dp' p'^2 V^1_{00}(p, p') \Phi^0_{1j}(p'). \tag{17}$$

For the triplet case and $j > 0$ it is more complicated. For example, for $j = 1$, Eq. (15) leads to one equation for the P channel and leads to two coupled equations for the S and D channels as follows:

$$\Psi_{111}(p) = \frac{2\pi}{E - 2\sqrt{m^2 + p^2}} \int_0^\infty dp' p'^2 [V^1_{11}(p, p') - V^1_{-11}(p, p')] \Psi_{111}(p'), \tag{18}$$

$$\Psi_{011}(p) = \frac{2\pi}{E-2\sqrt{m^2+p^2}}\frac{1}{3}\int_0^\infty dp' p'^2 \{[2V^1_{11}(p,p') + 2V^1_{01}(p,p') + V^1_{00}(p,p') + 2V^1_{10}(p,p') + 2V^1_{-11}(p,p')]\Psi_{011}(p') + \sqrt{2}[V^1_{11}(p,p') + V^1_{01}(p,p') - V^1_{00}(p,p') - 2V^1_{10}(p,p') + V^1_{-11}(p,p')]\}\Psi_{211}(p'), \tag{19}$$

$$\Psi_{211}(p) = \frac{2\pi}{E-2\sqrt{m^2+p^2}}\frac{1}{3}\int_0^\infty dp' p'^2 \{\sqrt{2}[V^1_{11}(p,p') - 2V^1_{01}(p,p') - V^1_{00}(p,p') + V^1_{10}(p,p') + V^1_{-11}(p,p')]\Psi_{011}(p') + [V^1_{11}(p,p') - 2V^1_{01}(p,p') + 2V^1_{00}(p,p') - 2V^1_{10}(p,p') + V^1_{-11}(p,p')]\}\Psi_{211}(p'), \tag{20}$$

where for the derivation of Eqs. (18), (19), and (20), we have used Eq. (A4) and the symmetry relation for helicity components of the potential as follows:

$$V^S_{-\Lambda\Lambda'}(p,p') = V^S_{\Lambda,-\Lambda'}(p,p'). \tag{21}$$

III. QUARK-ANTIQUARK POTENTIALS IN MOMENTUM-HELICITIY REPRESENTATION

The general form of the potential that we have used in our calculations is the sum of five terms [3,4],

$$V = V_L + V_C + V_{\rm SS} + V_{\rm LS} + V_T, \tag{22}$$

with the following representations in the momentum space:

$$\begin{aligned}
\langle \mathbf{p}|V_L|\mathbf{p}'\rangle &= C_L\sqrt{\sigma}\Big[\delta(\mathbf{q})r_c + \frac{1}{2\pi^2 q^4}(2\cos(qr_c) - 2 + qr_c\sin(qr_c))\Big],\\
\langle \mathbf{p}|V_C|\mathbf{p}'\rangle &= f_c\alpha_s e^{-\lambda^2 q^2} C_C \times\left[\frac{\delta(\mathbf{q})}{r_c} + \frac{1}{2\pi^2 q^2}\left(1 - \frac{\sin(qr_c)}{qr_c}\right)\right],\\
\langle \mathbf{p}|V_{\rm SS}|\mathbf{p}'\rangle &= -f_c\alpha_s e^{-\lambda^2 q^2}\frac{C_{\rm SS}}{12\pi^2 m^2}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2),\\
\langle \mathbf{p}|V_{\rm LS}|\mathbf{p}'\rangle &= f_c\alpha_s e^{-\lambda^2 q^2}\frac{3C_{\rm LS}}{8\pi^2 m^2 q^2} i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{p}\times\mathbf{p}',\\
\langle \mathbf{p}|V_T|\mathbf{p}'\rangle &= f_c\alpha_s e^{-\lambda^2 q^2}\frac{C_T}{24\pi^2 m^2}\times[3(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{q}}) + (\boldsymbol{\sigma}_2\cdot\hat{\mathbf{q}}) - (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)].
\end{aligned} \tag{23}$$

These terms represent the linear confining, Coulomb, spin-spin, spin-orbit, and tensor interactions, respectively. $\sqrt{\sigma}$ is the string tension, α_s is the strong-interaction fine-structure constant, f_c is the color factor which is $-4/3$ for quark-antiquark and $-2/3$ for quark-quark, $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the Pauli matrices, and $\mathbf{q} = \mathbf{p}' - \mathbf{p}$ is the momentum transfer. For the linear confining and Coulomb parts of the potential, we have used a Fourier transformation of the regularized form of them [4]. Multiplying factors $C_L, C_C, C_{\rm SS}, C_{\rm LS}$, and C_T, which represent the strength of every term, can be varied to fit the meson data. To write this potential in terms of the momentum-helicity basis state, we have used the following operators [1]:

$$\begin{aligned}
\Omega_1 &= 1,\\
\Omega_2 &= \mathbf{S}^2,\\
\Omega_3 &= (\mathbf{S}\cdot\mathbf{p}')(\mathbf{S}\cdot\mathbf{p}'),\\
\Omega_4 &= (\mathbf{S}\cdot\mathbf{p}')(\mathbf{S}\cdot\mathbf{p}),\\
\Omega_5 &= (\mathbf{S}\cdot\mathbf{p}')^2(\mathbf{S}\cdot\mathbf{p})^2,\\
\Omega_6 &= (\mathbf{S}\cdot\mathbf{p})(\mathbf{S}\cdot\mathbf{p}).
\end{aligned} \tag{24}$$

The matrix elements of these operators are easier to evaluate as

$$\begin{aligned}
\langle\hat{\mathbf{p}}S\Lambda|\Omega_1|\hat{\mathbf{p}}'S\Lambda'\rangle &= \langle\hat{\mathbf{p}}S\Lambda|\hat{\mathbf{p}}'S\Lambda'\rangle,\\
\langle\hat{\mathbf{p}}S\Lambda|\Omega_2|\hat{\mathbf{p}}'S\Lambda'\rangle &= S(S+1)\langle\hat{\mathbf{p}}S\Lambda|\hat{\mathbf{p}}'S\Lambda'\rangle,\\
\langle\hat{\mathbf{p}}S\Lambda|\Omega_3|\hat{\mathbf{p}}'S\Lambda'\rangle &= \Lambda'^2\langle\hat{\mathbf{p}}S\Lambda|\hat{\mathbf{p}}'S\Lambda'\rangle,\\
\langle\hat{\mathbf{p}}S\Lambda|\Omega_4|\hat{\mathbf{p}}'S\Lambda'\rangle &= \Lambda\Lambda'\langle\hat{\mathbf{p}}S\Lambda|\hat{\mathbf{p}}'S\Lambda'\rangle,\\
\langle\hat{\mathbf{p}}S\Lambda|\Omega_3|\hat{\mathbf{p}}'S\Lambda'\rangle &= \Lambda^2\Lambda'^2\langle\hat{\mathbf{p}}S\Lambda|\hat{\mathbf{p}}'S\Lambda'\rangle,\\
\langle\hat{\mathbf{p}}S\Lambda|\Omega_6|\hat{\mathbf{p}}'S\Lambda'\rangle &= \Lambda^2\langle\hat{\mathbf{p}}S\Lambda|\hat{\mathbf{p}}'S\Lambda'\rangle.
\end{aligned} \tag{25}$$

Since the momentum-helicity basis state is the eigenstate of these operators, the potential terms will have a simple form in this space. We can easily show through a simple calculation that

$$\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 = 2\Omega_2 - 3\Omega_1, \tag{26}$$

$$\frac{i}{q}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{p}\times\mathbf{p}' = \frac{pp'}{4q}\left[\gamma\Omega_2 - 2\Omega_4 - \frac{1}{\gamma}(\Omega_2 - 2\Omega_3 - 2\Omega_6 + 2\Omega_5)\right], \tag{27}$$

$$3(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{q}})-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) = -\frac{1}{q^2}\left[6pp'\Omega_4+2p'^2(\Omega_2-3\Omega_3)+2p^2(\Omega_2-3\Omega_6)-pp'\gamma\Omega_2-3pp'\frac{1}{\gamma}(\Omega_2-2\Omega_3-2\Omega_6+2\Omega_5)\right], \tag{28}$$

where $\gamma=\hat{\mathbf{p}}'\cdot\hat{\mathbf{p}}$. Therefore, the final form of the potential in the momentum-helicity space is obtained as

$$\begin{aligned} V^S_{\Lambda\Lambda'}(\mathbf{p},\mathbf{p}') &= C_L\sqrt{\sigma}\langle\hat{\mathbf{p}}S\Lambda|\hat{\mathbf{p}}'S\Lambda'\rangle\left[\delta(\mathbf{q})r_c+\frac{1}{2\pi^2q^4}(2\cos(qr_c)-2+qr_c\sin(qr_c))\right] \\ &+ f_c\alpha_s e^{-\lambda^2q^2}\langle\hat{\mathbf{p}}S\Lambda|\hat{\mathbf{p}}'S\Lambda'\rangle\left\{\frac{C_C}{r_c}\delta(\mathbf{q})+\frac{C_C}{2\pi^2q^2}\left(1-\frac{\sin(qr_c)}{qr_c}\right)-\frac{C_{SS}}{12\pi^2m^2}(2S(S+1)-3)\right. \\ &+\frac{3C_{LS}}{8\pi^2m^2}\frac{pp'}{q^2}\left[\gamma S(S+1)-2\Lambda\Lambda'-\frac{1}{\gamma}(S(S+1)-2\Lambda'^2-2\Lambda^2-2\Lambda'^2\Lambda^2)\right] \\ &-\frac{C_T}{24\pi^2m^2q^2}\left[6pp'\Lambda\Lambda'+2p'^2(S(S+1)-3\Lambda'^2)+2p^2(S(S+1)-3\Lambda^2)-pp'\gamma S(S+1)\right. \\ &\left.\left.-3\frac{pp'}{\gamma}(S(S+1)-2\Lambda'^2-2\Lambda^2-2\Lambda'^2\Lambda^2)\right]\right\}. \end{aligned} \tag{29}$$

The overlap of the momentum-helicity basis states becomes [1]

$$\langle\hat{\mathbf{p}}S\Lambda|\hat{\mathbf{p}}'S\Lambda'\rangle=\sum_{N=-S}^{S}e^{iN(\varphi-\varphi')}d^S_{N\Lambda}(\theta)d^S_{N\Lambda'}(\theta'). \tag{30}$$

For the special case where the vector $\mathbf{p}$ is along the z direction, one obtains the simple form as

$$\langle\hat{\mathbf{z}}S\Lambda|\hat{\mathbf{p}}'S\Lambda'\rangle=e^{-i\Lambda\varphi'}d^S_{\Lambda\Lambda'}(\theta'). \tag{31}$$

IV. NUMERICAL RESULTS

In our numerical calculations we have used the Gaussian quadrature grid points to discretize the momentum and the angle variables. The integration interval for the momentum is covered by two different hyperbolic and linear mappings of the Gauss-Legendre points from the interval $[-1,+1]$ to the intervals $[0,p_2]$ $[p_2,p_{\max}]$, respectively, as

$$p=\frac{1+x}{\frac{1}{p_1}+(\frac{2}{p_2}-\frac{1}{p_1})x},\qquad p=\frac{p_{\max}-p_2}{2}x+\frac{p_{\max}+p_2}{2}. \tag{32}$$

The p' integration is cut off at a value of $p_{\max}=10$ GeV. The typical values for p_1 and p_2 are 1 and 3 GeV, respectively. In our calculations we chose 60 grid points for the momentum variables in the interval $[0,p_2]$, and 80 grid points for the momentum variables in the interval $[p_2,p_{\max}]$. Also, 140 grid points for the spherical angle variable are sufficient. The parameters of the potentials we have used in our calculations are shown in Table I.

The results of light meson mass spectra are shown in Table II. They are compared with the results which are obtained in the partial wave representation and experimental data (Exp.). As we see, the results obtained from both representations are in good agreement, with high accuracy.

TABLE I. Parameters of the model [3].

$\sqrt{\sigma}$ (GeV^2)	0.197
λ (GeV^{-1})	0.645
m (GeV)	0.258
r_c (fm)	10
α_s	0.375
C_L	0.6704
C_C	3.3824
C_{SS}	0.5243
C_{LS}	0.1515
C_T	0.6318

TABLE II. Comparison of the light meson mass spectrum obtained by the momentum-helicity representation and the partial wave representation.

$n\mathrm{LS}J^{\pi}$	State	Exp. [3]	Reference [3]	E (MeV)
0000^-	π	138	140.1	140.00
0011^-	ρ	768.3	775.7	775.75
0101^+	b_1	1233	1174.6	1174.63
0110^+	a_0	983.3	973.7	973.77
0111^+	a_1	1260	1298.2	1298.21
1000^-	$\pi(1300)$	1300	1188.9	1188.93
1011^-	$\rho(1450)$	1450	1472.7	1472.72
0202^-	π_2	1665	1661.9	1661.89
2011^-	$\rho(1700)$	1700	1960.5	1960.45

V. DISCUSSION AND OUTLOOK

During the past few years, several models and methodological approaches have been developed for the studying of light and heavy mesons. Some of them were based on the solving of the relativistic and nonrelativistic forms of the Schrödinger or Lippmann-Schwinger (LS) equations for light and heavy mesons, respectively [3–10]. As we know, the Bethe-Salpeter (BS) equation as a successor of the relativistic quantum mechanical equation has been used for the calculations of quark-antiquark systems [11]. These calculations have been carried out by spinless and spin-dependent forms of the BS equation [12–14].

Recently a 3D approach has been developed for representation of few-body equations in the momentum space. This approach, because of some advantages, was introduced as a successor of the partial wave (PW) representation [15–24]. We have used this approach for estimation of heavy triply baryon masses by solving the Faddeev equation [25]. As we mentioned in the Introduction, the 3D approach based on momentum-helicity representation has been developed for realistic interactions. One practical advantage of working with helicity states is that states are the eigenstates of the helicity operator appearing in the quark-antiquark potentials. Another advantage is found when the formulation is extended to a relativistic scheme. Using the helicity representation is less complicated than using the spin representation with a fixed quantization axis.

In this paper we have extended the 3D approach based on momentum-helicity basis states from nuclear into particle physics problems. The properties of heavy-flavor baryons have recently received much attention. Several methods have been used to investigate heavy-flavor baryons based on relativistic and nonrelativistic schemes [26–29]. This work is the first step toward studying single, double, and triple heavy-flavor baryons in the framework of the nonrelativistic quark model by formulation of the Faddeev equation in the 3D momentum-helicity representation. Furthermore, we can apply this formalism straightforwardly for investigation of heavy-pentaquark systems, which can be considered as two-body (heavy meson, baryon) systems with meson-nucleon interactions. These works are other tasks which are under way. Formulation of the BS equation in the momentum-helicity representation is a major task that can be done.

In this paper as a test of our formalism we have applied this approach for representation of the relativistic form of the LS equation in the momentum-helicity space. Thus, we have applied a relativistic potential in operator forms in the momentum-helicity representation, and we have calculated the mass spectra of light mesons and compared them with their PW results. It is clear that by replacing the relativistic form of propagator $(E-2\sqrt{m^2+p^2})^{-1}$ with its nonrelativistic form $(E-\frac{p^2}{m})^{-1}$ in the LS equation and applying a nonrelativistic potential $(C_L=C_C=C_{\rm SS}=C_{\rm LS}=C_T=1)$, we can apply this formalism toward calculation of mass spectra of heavy mesons.

APPENDIX: CONNECTION BETWEEN MOMENTUM-HELICITY REPRESENTATION OF WAVE FUNCTION AND ITS PARTIAL WAVE REPRESENTATION

By inserting the complexness relation of the partial wave representation in the momentum-helicity representation of the wave function, we have

$$\langle \mathbf{p};\hat{\mathbf{p}}S\Lambda|\Phi_j^{M_j}\rangle = \sum_{lS'm}\langle \mathbf{p};\hat{\mathbf{p}}S\Lambda|p(lS')jm\rangle\langle p(lS')jm|\Phi_j^{M_j}\rangle. \tag{A1}$$

The scalar product of the partial wave and the momentum-helicity basis states is given as [2]

$$\langle \mathbf{p};\hat{\mathbf{p}}S\Lambda|p(lS')jm\rangle = \sqrt{\frac{2l+1}{4\pi}}\delta_{SS'}e^{i\Lambda\varphi}d^j_{m\Lambda}(\theta)C(lsj;0\Lambda\Lambda). \tag{A2}$$

Inserting this relation into Eq. (A1) yields

$$\langle \mathbf{p};\hat{\mathbf{p}}S\Lambda|\Phi_j^{M_j}\rangle = e^{im\varphi}d^j_{M_j\Lambda}(\theta)\sum_l\sqrt{\frac{2l+1}{4\pi}}C(lsj;0\Lambda\Lambda)\Psi_{lSj}(p). \tag{A3}$$

By comparison with Eq. (9), this equation can be rewritten as

$$\Phi^{\Lambda}_{jS}(p) = \sum_l\sqrt{\frac{2l+1}{4\pi}}C(lsj;0\Lambda\Lambda)\Psi_{lSj}(p). \tag{A4}$$

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