

Renormalization-group improved inflationary scalar electrodynamics and $SU(5)$ scenarios confronted with Planck 2013 and BICEP2 results

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(Received 8 August 2014; published 1 October 2014)

The possibility to construct inflationary models for the renormalization-group (RG) improved potentials corresponding to scalar electrodynamics and to $SU(2)$ and $SU(5)$ models is investigated. In all cases, the tree-level potential, which corresponds to the cosmological constant in the Einstein frame, is seen to be nonsuitable for inflation. Rather than adding the Hilbert-Einstein term to the action, quantum corrections to the potential, coming from the RG equation, are included. The inflationary scenario is analyzed with unstable de Sitter solutions that correspond to positive values of the coupling function, only. We show that, for the finite $SU(2)$ model and $SU(2)$ gauge model, there are no de Sitter solutions suitable for inflation, unless exit from it occurs according to some weird, nonstandard scenarios. Inflation is realized both for scalar electrodynamics and for $SU(5)$ RG-improved potentials, and the corresponding values of the coupling function are seen to be positive. It is shown that, for quite reasonable values of the parameters, the inflationary models obtained both from scalar electrodynamics and from the $SU(5)$ RG-improved potentials are in good agreement with the most recent observational data coming from the Planck 2013 and BICEP2 collaborations.

DOI: [10.1103/PhysRevD.90.084001](https://doi.org/10.1103/PhysRevD.90.084001)

PACS numbers: 04.50.Kd, 11.10.Hi, 98.80.-k, 98.80.Cq

I. INTRODUCTION

Precise astronomical data coming from recent observational missions [1–3] (see also [4]) support the existence of an extremely short and intense stage of accelerated expansion in the early universe (inflation), as well as of a long-lasting accelerated phase at present. These results set important restrictions on existing inflationary models [5–20] (see also [21–23] and references therein).

Moreover, these observational data give strong support to the fact that the postinflationary universe was nearly homogeneous, isotropic, and spatially flat, at very large distances or short times. Presently, the evolution of our Universe can be well described in terms of a spatially flat Friedmann-Lemaître-Robertson-Walker background, and cosmological perturbations and models with scalar fields are very well suited to describe an evolution of this kind. It has also been proven that some modified gravity models, as $f(R)$ gravity, can in a sense be considered as generic general relativity models with additional scalar fields. This is the reason why scalar fields play such an essential role in modern cosmology, in particular, in the current description of the evolution of the Universe at a very early epoch

[5–11]. Many inflationary models involve scalar fields nonminimally coupled to the Ricci curvature scalar [12,14–20]. Note, however, that predictions of the simplest inflationary models with minimal couplings to scalar fields, as the $\lambda\phi^4$ model, are actually in sharp disagreement with the Planck 2013 results [2], and that some of these inflationary scenarios had to be improved by adding a tiny nonminimal coupling of the inflaton field to gravity [19,20]. The conditions for a model to be consistent with the BICEP2 result have been examined in many papers already (see, e.g., [24–33]). And it is, in fact, possible to reconstruct models with minimally coupled scalar fields that realize an inflation compatible with the Planck and the BICEP2 results, by using, e.g., the algorithm proposed in [34].

Also a very crucial issue is the possibility to describe inflation using particle physics models [35,36], as the Standard Model of elementary particles [17,18] or some other quantum field theory, as supersymmetric models [37] or nonsupersymmetric grand unified theories (GUTs) [10,38]. This is a fundamental step toward the long-standing and very ambitious program of the unification of physics at all scales.

As a very important step toward this goal, one should not forget to take into account quantum effects of quantum field theories in curved space-time at the inflationary epoch (see [39] for a general introduction). It is well understood that quantum GUTs in curved space-time lead also to curvature

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induced phase transitions (for a complete description, see [39–41]). Note moreover that curvature induced phase transitions, as discussed in [39,40], may be described with better accuracy when one considers this phenomena within renormalization-group (RG) improved effective potentials (see [41]). Indeed, in this case, the summation of all leading logs is done and the corresponding RG-improved effective potential goes far beyond the one-loop approximation. These phase transitions are very important in early-universe cosmology. Specifically, some models of the inflationary universe [9,23] are based on first-order phase transitions, which took place during the reheating phase of the Universe in the grand unification epoch [10]. Also, curved space-time effects in the grand unification epoch cannot be dismissed, but simply considered to be negligible. Quite on the contrary, all these theories should be treated as quantum field theories in curved space-time, as discussed some time ago in [41]. Indeed, it must be properly emphasized that the recent results by the BICEP2 Collaboration [3] point clearly toward the GUT scale, which is a very impressive hint of a probably deep connection of inflation with the GUT epoch and a validation of the arguments in Ref. [41]. As was emphasized there, GUTs corresponding to the very early universe ought to be treated as quantum field theories in curved space-time, in a proper and rigorous way.

Anyhow, in the lack of a clear prescription for how to combine quantum field theory at nonzero temperature and quantum field theory in curved space-time (external temperature and external gravitational field), it is natural to start by addressing just the second part of this problem. The renormalization-group improved effective potential for an arbitrary renormalizable massless gauge theory in curved space-time was discussed in [41], working in the linear curvature approximation, because at least these linear curvature terms ought to be taken into account in the discussion of the effective potential corresponding to GUTs in the early universe. Quantum corrections with account to gravity effects are predicted to be even more important in a chaotic inflationary model [21]. By generalizing the Coleman-Weinberg approach corresponding to the case of the effective potential in flat space-time, the authors found, at a first instance, the explicit form of the RG improved effective potential in curved space for scalar electrodynamics, the finite $SU(2)$ model, the $SU(2)$ gauge model, and the $SU(5)$ GUT model. The possibility of corresponding curvature-induced phase transitions was also investigated.

By carrying out one-loop calculations in a weak gravitational field it was shown [42,43] that it is necessary to introduce an induced gravity term proportional to $R\phi^2$ in order to renormalize the theory of a scalar field in curved space-time. Here, we consider different RG-improved effective potentials for the tree-level potential $\lambda\phi^4 - \xi\phi^2 R$. These potentials were proposed in [41,44]. In the Einstein frame the tree-level potential corresponds to the cosmological constant and is not suitable for the construction of an inflationary

scenario. We will check the possibility to construct inflationary models using the RG-improved effective potentials and consider inflation based on an unstable de Sitter solution. We will start by checking the existence of such solutions. Then we will examine if the inflationary model with this potential is compatible with the Planck 2013 and BICEP2 data. To do that, we will use conformal transformation and the slow-roll parameters in the Einstein frame.

The paper is organized as follows. In Sec. II, we consider the action with a nonminimally coupled scalar field and the corresponding equations of motion. In Sec. III we summarize the standard theory of Lyapunov's stability, as applied to de Sitter solutions in these models. In Sec. IV, we discuss the general procedure for the construction of RG-improved effective potentials. The existence and stability of de Sitter solutions in scalar electrodynamics is considered in Sec. V. Sections VI and VII are devoted to RG-improved effective potentials for the cases of the finite $SU(2)$ and of the $SU(2)$ models, respectively. Unstable de Sitter solutions for the $SU(5)$ model are dealt with in Sec. VIII. In Sec. IX, cosmological parameters from the inflationary models considered are extracted, and it is shown that, for some specific models, they are compatible with the Planck 13 and BICEP2 results. The last section is devoted to conclusions.

II. MODELS WITH NONMINIMALLY COUPLED SCALAR FIELDS

Different models with the Ricci scalar multiplied by a function of the scalar field are being intensively studied in cosmology [12,16,17,45–54] (see also [55–57] and references therein). Generically, these models are described by

$$S = \int d^4x \sqrt{-g} \left[U(\phi)R - \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi) \right], \quad (1)$$

where $U(\phi)$ and $V(\phi)$ are differentiable functions of the scalar field ϕ , g is the determinant of the metric tensor $g_{\mu\nu}$, and R is the scalar curvature. We will use the signature $(-, +, +, +)$ throughout.

Let us consider a spatially flat Friedmann-Lemaître-Robertson-Walker universe with metric interval

$$ds^2 = -dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2).$$

The Friedmann equations, derived by variation of action (1), have the following form [52]:

$$6UH^2 + 6\dot{U}H = \frac{1}{2}\dot{\phi}^2 + V, \quad (2)$$

$$2U(2\dot{H} + 3H^2) + 4\dot{U}H + 2\ddot{U} = -\frac{1}{2}\dot{\phi}^2 + V, \quad (3)$$

where the Hubble parameter is the logarithmic derivative of the scale factor: $H = \dot{a}/a$ and differentiation with respect to time t is denoted by a dot. Variation of the action (1) with respect to ϕ yields

$$\ddot{\phi} + 3H\dot{\phi} + V' = 6(\dot{H} + 2H^2)U', \quad (4)$$

where the prime denotes derivation with respect to the argument of the functions, that is, the scalar field ϕ . Combining Eqs. (2) and (3), we obtain

$$4UH - 2\dot{U}H + 2\ddot{U} + \dot{\phi}^2 = 0. \quad (5)$$

From Eqs. (2)–(5), one can get the following system of first order differential equations [54]:

$$\begin{aligned} \dot{\phi} &= \psi, \\ \dot{\psi} &= -3H\psi - \frac{[(6U'' + 1)\psi^2 - 4V]U' + 2UV'}{2(3U'^2 + U)}, \\ \dot{H} &= -\frac{2U'' + 1}{4(3U'^2 + U)}\psi^2 + \frac{2U'}{3U'^2 + U}H\psi \\ &\quad - \frac{6U'^2}{3U'^2 + U}H^2 + \frac{U'V'}{2(3U'^2 + U)}. \end{aligned} \quad (6)$$

Note that Eq. (2) is not a consequence of the system (6). On the other hand, if Eq. (2) is satisfied for an initial time, then it follows from the system (6) that Eq. (2) is also satisfied for any value of time. In other words, it turns out that the system (6) is equivalent to the initial system of equations, (2)–(4), if and only if one chooses the initial data so that Eq. (2) is fulfilled.

III. LYAPUNOV STABILITY OF THE DE SITTER SOLUTIONS

We are here considering the possibility of inflationary scenarios in models with RG-improved potentials. Our first goal, therefore, is to find unstable de Sitter solutions. The standard way to explore an inflationary model is to formulate it in the Einstein frame. This is actually very convenient when U is a simple function, for instance, for induced gravity models [46]. However, in our case the Jordan frame is more suitable to perform an analysis of the stability of the de Sitter solutions, because the potential can be expressed in terms of elementary functions in this frame only. We will consider de Sitter solutions that correspond to a constant ϕ only. In other words, we consider a fixed point of Eqs. (6), with the additional condition (2).

Substituting constant values for $H = H_f$ and $\phi = \phi_f$ into Eqs. (2) and (4), we get

$$H_f^2 = \frac{V(\phi_f)}{6U(\phi_f)}, \quad (7)$$

$$V'(\phi_f) = 12H_f^2 U'(\phi_f). \quad (8)$$

Therefore, we come up with the following simple condition:

$$2 \frac{U'(\phi_f)}{U(\phi_f)} = \frac{V'(\phi_f)}{V(\phi_f)}. \quad (9)$$

We consider the stability with respect to homogeneous isotropic perturbations. In other words, we use (6) and analyze the Lyapunov stability of the de Sitter solutions derived from it. For this we apply Lyapunov's theorem [58,59] and study the corresponding linearized system. We expand around the fixed point, in the way

$$\begin{aligned} \phi(t) &= \phi_f + \varepsilon\phi_1(t), \\ \psi(t) &= \varepsilon\psi_1(t), \\ H(t) &= H_f + \varepsilon H_1(t), \end{aligned} \quad (10)$$

where ε is a small parameter. Substituting (10) into (6), to first order in ε we obtain the following linear system [60]:

$$\begin{aligned} \dot{\phi}_1 &= \psi_1, \\ \dot{\psi}_1 &= \frac{V'_f U'_f + 2V_f U''_f - U_f V''_f}{3(U'_f)^2 + U_f} \phi_1 - 3H_f \psi_1, \\ \dot{H}_1 &= \frac{(U'_f V''_f - V'_f U''_f)}{2(3(U'_f)^2 + U_f)} \phi_1 + \frac{2H_f U'_f}{3(U'_f)^2 + U_f} \psi_1 \\ &\quad - \frac{12H_f (U'_f)^2}{3(U'_f)^2 + U_f} H_1. \end{aligned} \quad (11)$$

The following matrix, A , corresponds to (11),

$$A = \begin{vmatrix} 0 & 1 & 0 \\ \frac{V'_f U'_f + 2V_f U''_f - U_f V''_f}{3(U'_f)^2 + U_f} & -3H_f & 0 \\ \frac{U'_f V''_f - V'_f U''_f}{2(3(U'_f)^2 + U_f)} & \frac{2H_f U'_f}{3(U'_f)^2 + U_f} & -\frac{12H_f (U'_f)^2}{3(U'_f)^2 + U_f} \end{vmatrix}. \quad (12)$$

Its associated characteristic equation,

$$\begin{aligned} \det(A - \tilde{\lambda}I) &= \left(\frac{12H_f U'_f}{3(U'_f)^2 + U_f} + \tilde{\lambda} \right) \\ &\quad \times \left(\tilde{\lambda}(3H_f + \tilde{\lambda}) - \frac{V'_f U'_f + 2V_f U''_f - U_f V''_f}{3(U'_f)^2 + U_f} \right) \\ &= 0, \end{aligned} \quad (13)$$

has the following roots:

$$\begin{aligned} \tilde{\lambda}_{\pm} &= -\frac{3H_f}{2} \pm \sqrt{\frac{9H_f^2}{4} + \frac{V'_f U'_f + 2V_f U''_f - U_f V''_f}{3(U'_f)^2 + U_f}}, \\ \tilde{\lambda}_3 &= -\frac{12H_f U'_f}{3(U'_f)^2 + U_f}. \end{aligned} \quad (14)$$

Lyapunov's theorem [58,59] states that in order to prove the stability of a fixed point of a nonlinear system it is sufficient to prove the stability of this fixed point for the corresponding linearized system. Stability of the linear system relies, on its turn, on the real parts of the roots $\tilde{\lambda}_k$ of the characteristic equation (13), which must all be negative. If at least one of them is positive, then the fixed point is unstable.

To describe inflation we are interested in finding unstable de Sitter solutions with $H_f > 0$. Note that the perturbation $H_1(t)$ is not independent, because it is connected with ϕ_1 and ψ_1 due to Eq. (2). So, the de Sitter solution is stable if the real parts of $\tilde{\lambda}_{\pm} < 0$. The real part of $\tilde{\lambda}_-$ is always negative; hence, just $\tilde{\lambda}_+$ defines the stability.

Introducing

$$K_f \equiv \frac{V'_f U'_f + 2V_f U''_f - U_f V''_f}{3(U'_f)^2 + U_f} = \frac{2\left(\frac{U'_f}{U_f}\right)' - \left(\frac{V'_f}{V_f}\right)'}{\frac{3}{4}\left(\frac{V'_f}{V_f}\right)^2 \frac{U_f}{V_f} + \frac{1}{V_f}}, \quad (15)$$

we can then formulate a sufficient stability condition as follows: the de Sitter solution ($H_f > 0$) is stable at $K_f < 0$ and unstable at $K_f > 0$.

IV. RENORMALIZATION-GROUP IMPROVED EFFECTIVE POTENTIAL

The renormalization-group improved effective potential for an arbitrary renormalizable massless gauge theory in curved space-time was discussed in detail in [41]. In this section we will just remind the reader of the basic steps for the construction of the renormalization-group improved effective potential.

The tree-level potential reads as follows [41]:

$$W^{(0)}(\phi) = a\lambda\phi^4 - b\xi\phi^2 R = V_0 - U_0 R, \quad (16)$$

where a and b are positive constants and ξ is the conformal coupling. The potential $W^{(0)}$ includes both the potential V_0 and the function U_0 multiplied by the scalar curvature.

As is known (see [39,41]), the renormalization-group equation for the effective potential in curved space-time has the form

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_{\tilde{g}} \frac{\partial}{\partial \tilde{g}} + \delta \frac{\partial}{\partial \alpha} + \beta_{\xi} \frac{\partial}{\partial \xi} - \gamma \phi \frac{\partial}{\partial \phi}\right) W = 0, \quad (17)$$

where α is the gauge parameter and \tilde{g} is the set of all coupling constants of the theory (Higgs, gauge, and Yukawa ones). The standard flat-space renormalization-group equation [62,63] is modified in curved space-time; for instance, it has an additional term related with the contribution from the nonminimal coupling constant ξ and the corresponding β_{ξ} function [64].

It is natural to split W into two parts, namely

$$W \equiv V - UR \equiv a f_1(p, \phi, \mu) \phi^4 - b f_2(p, \phi, \mu) \phi^2 R, \quad (18)$$

where f_1 and f_2 are some unknown functions, and $p = \{\tilde{g}, \alpha, \xi\}$. Actually, in [41] the authors imposed the additional restriction that not only does the function W satisfy (17) but also the functions V and U satisfy it, separately.

It is easy to see [52] that for

$$V_0(\phi) = C U_0^2(\phi), \quad (19)$$

where C is a constant and the model considered has a de Sitter solution, with an arbitrary constant $\phi = \phi_f$. Therefore, for the tree-level potential $W^{(0)}$, de Sitter solutions do exist, for any value of ϕ_f , and the corresponding Hubble parameter reads

$$H_{0f} = \pm \sqrt{-\frac{a\lambda}{6b\xi}} \phi_f.$$

One aim of this paper is to consider de Sitter solutions in cosmological models with different RG-improved W potentials and the possibility of inflationary scenarios in such models, too.

V. EFFECTIVE POTENTIALS FOR SCALAR ELECTRODYNAMICS

Let us now consider the de Sitter solution for the case of the following effective potentials for scalar electrodynamics:

$$V = \frac{\lambda\phi^4}{4!} + \frac{3e^4\phi^4}{(8\pi)^2} \ln \frac{\phi^2}{\mu^2}, \quad U = \frac{\xi\phi^2}{2} + \frac{e^2\phi^2}{(8\pi)^2} \ln \frac{\phi^2}{\mu^2}, \quad (20)$$

where e is a constant. Using

$$\frac{U'}{U} = \frac{2}{\phi} \left(1 + \frac{e^2}{32\xi + e^2 \ln \frac{\phi^2}{\mu^2}}\right),$$

$$\frac{V'}{V} = \frac{2}{\phi} \left(2 + \frac{9e^4}{8\lambda\pi^2 + 9e^4 \ln \frac{\phi^2}{\mu^2}}\right),$$

and the condition (9), we get

$$\phi_f = \pm \mu \exp \left[\frac{8\pi^2}{9e^4} (18e^2\xi - \lambda) \right]. \quad (21)$$

From (7), we obtain

$$H_f^2 = \frac{\mu^2 e^2}{4} \exp \left[\frac{16\pi^2}{9e^4} (18\xi e^2 - \lambda) \right] = \frac{e^2 \phi_f^2}{4}. \quad (22)$$

For the de Sitter solutions obtained, we then get

$$V_f = \frac{36e^2\xi - \lambda}{24} \phi_f^4, \quad U_f = \frac{36e^2\xi - \lambda}{36e^2} \phi_f^2. \quad (23)$$

Let us now consider the stability of the above solutions. Using

$$2 \left(\frac{U'}{U} \right)' \Big|_{\phi=\phi_f} - \left(\frac{V'}{V} \right)' \Big|_{\phi=\phi_f} = \frac{81e^8}{32\pi^4 (36e^2\xi - \lambda)^2 \phi_f^2} > 0,$$

and (15), we get that $K_f > 0$ at $U_f > 0$. This means that $\lambda_+ > 0$ and that the corresponding de Sitter solution is unstable. Thus, we get in the end an unstable de Sitter solution with $H_f > 0$ and $U_f > 0$. Note that the variation of the Hubble parameter is considered as a function of the variations of both the scalar field and its first derivative.

VI. FINITE $SU(2)$ MODELS

A number of GUTs turn out to yield finite models. Some of them, as for instance the finite supersymmetric $SU(5)$ GUT [65], may lead to reasonable phenomenological consequences and deserve attention as realistic models of grand unification. Asymptotically finite GUTs, which are generalizations of the concept of a finite theory, have been proposed in [66]. In these theories, the zero charge problem is absent, both in the UV and in the IR limits, since in these limits the effective coupling constants tend to some constant values (corresponding to finite phases).

When we consider flat space-time, there is not much sense in discussing quantum corrections to the classical potential, in a massless finite or massless asymptotically finite GUT, since they are either simply absent or highly suppressed asymptotically. However, when we study finite theories in curved space-time [67] (for a general review, see [39]) the situation changes drastically [44].

In the following, we will study de Sitter solutions in cosmological models with renormalization-group improved effective $SU(2)$ potentials for the two finite theories in curved space-time constructed in [44]. In those models the coupling parameter corresponding to the nonminimal scalar-gravitational interaction ξ depends on ϑ , where $\vartheta = \frac{1}{2} \ln(\phi^2/\mu^2)$.

The general structure of the one-loop effective coupling constant $\xi(\vartheta)$ for “finite” theories in curved space-time has been obtained in [67]

$$\xi(\vartheta) = \frac{1}{6} + \left(\xi_0 - \frac{1}{6} \right) \exp(Cg^2\vartheta), \quad (24)$$

with constant ξ_0 and $C \neq 0$.

In particular, for the $SU(2)$ finite gauge model [68], it was obtained that $C = 6$ or $C \approx 28$ [67]. Hence, in such theories we have $|\xi(\vartheta)| \rightarrow \infty$ (nonasymptotical conformal invariance) in the UV limit ($t \rightarrow \infty$). In the models that have $C < 0$ one gets $\xi(\vartheta) \rightarrow 1/6$ (asymptotical conformal invariance).

The tree-level potential is taken to be of the form (16) and the RG-improved potential reads as follows (see [44] for details):

$$W = a\lambda(\vartheta)f^4(\vartheta)\phi^4 - b\xi(\vartheta)f^2(\vartheta)\phi^2R. \quad (25)$$

Notice that this potential is actually obtained in the linear curvature approximation, which is good enough for GUTs corresponding to the curved space-time corresponding to the early universe [44].

The function $f(\vartheta)$ is defined as [44]

$$f(\vartheta) = \exp \left[- \int_0^\vartheta d\vartheta' \bar{\gamma}(\bar{g}(\vartheta'), \alpha(\vartheta')) \right]. \quad (26)$$

Therefore, the form of the RG-improved effective potential is determined by the $\bar{\gamma}$ function of the scalar field in (26). At the one-loop level, $\bar{\gamma}(\vartheta) \sim a_1 g^2(\vartheta) + a_2 h^2(\vartheta)$, where a_1 and a_2 are constants, with values that depend on the choice of gauge and on other features of the theory. As $h^2 = \kappa_1 g^2$, it turns out that $\bar{\gamma}(\vartheta) \sim (a_1 + \kappa_1 a_2) g^2(\vartheta)$. Through the choice of the gauge parameter, one can obtain different values for $\bar{\gamma}$. To reach as much “finiteness” in our theory as possible, we can choose a gauge such that the one-loop $\bar{\gamma}$ function is equal to zero. This choice is always possible; moreover, in supersymmetric finite theories it does appear in a very natural way (especially if the superfield technique is used).

After having done all this, it turns out that the RG-improved effective potential (in the linear-curvature and leading-log approximation) for a finite theory in curved space-time is given by

$$W = a\kappa_1 g^2 \phi^4 - b\xi(\vartheta)\phi^2R. \quad (27)$$

Straightforward calculations show that a de Sitter solution exists when $C = 0$ only. It corresponds to $\xi(\vartheta) = \xi_0$, and it is not interesting, because it leads to $W^{(0)}(\phi)$, given by (16).

Another possibility is to keep the gauge arbitrary; then we cannot demand that $\bar{\gamma}$ vanishes. We get in this case

$$W = a\kappa_1 g^2 f^4(\vartheta)\phi^4 - b\xi(\vartheta)f^2(\vartheta)\phi^2R \quad (28)$$

and $\bar{\gamma} = C_1 g^2$, C_1 being some constant that depends on the gauge parameter and on the features of the theory, $f(\vartheta) = \exp(-C_1 g^2 \vartheta)$, and $\xi(\vartheta)$ is given by (24).

From

$$V = a\kappa_1 g^2 f^4(\vartheta)\phi^4, \quad U = b\xi(\vartheta)f^2(\vartheta)\phi^2, \quad (29)$$

where

$$f(\vartheta) = \left(\frac{\phi}{\mu}\right)^{-C_1 g^2}, \quad \xi(\vartheta) = \frac{1}{6} + \left(\xi_0 - \frac{1}{6}\right) \left(\frac{\phi}{\mu}\right)^{C_1 g^2},$$

we get

$$V = a k_1 g^2 \left(\frac{\phi}{\mu}\right)^{-4C_1 g^2} \phi^4, \\ U = b \left(\frac{1}{6} + \left(\xi_0 - \frac{1}{6}\right) \left(\frac{\phi}{\mu}\right)^{C_1 g^2}\right) \left(\frac{\phi}{\mu}\right)^{-2C_1 g^2} \phi^2, \quad (30)$$

and

$$\frac{U'}{U} = \frac{2(1 - C_1 g^2)}{\phi} + \frac{(6\xi_0 - 1)C_1 g^2 \left(\frac{\phi^2}{\mu^2}\right)^{C_1 g^2/2}}{[1 + (6\xi_0 - 1)\left(\frac{\phi^2}{\mu^2}\right)^{C_1 g^2/2}]\phi}, \\ \frac{V'}{V} = \frac{4(1 - C_1 g^2)}{\phi}. \quad (31)$$

Now, we use the de Sitter condition (9) and conclude that this equation has as its only solution $C = 0$.

Summing up, in both cases there is no de Sitter solution for a nonconstant ξ . In the case of a constant ξ , we get a model with a power-law potential V and power-law coupling function U , which satisfies the condition (19). Thus, if the signs of the constants are such that

$$H_f^2 = \frac{k_1 g^2}{6b\xi_0} \phi_f^2 \left(\frac{\phi_f^2}{\mu^2}\right)^{-C_1 g^2} > 0, \quad (32)$$

then de Sitter solutions do exist for any constant value of ϕ_f . Note that, after conformal transformation to the Einstein frame, one gets a model with a minimally coupled scalar field, whose potential is a constant. Namely, in the case $C = 0$ we get $\xi = \xi_0$ and, using (29),

$$V_E = \frac{a\kappa_1 g^2}{4\xi_0^2 \kappa^4 b^2}.$$

See formula (50) in Sec. IX. Note that the above property holds in the presence of quantum corrections as we take them into account via the effective potential.

VII. THE $SU(2)$ GAUGE MODEL

Let us consider the cosmological model with

$$V = \frac{k_1 \phi^4 f^4(\vartheta) g^2(\vartheta)}{24}, \\ U = \frac{\phi^2 f^2(\vartheta)}{12} [1 + (6\xi - 1)\tilde{\Theta}^{-k_3}], \quad (33)$$

where we use $k_3 = (12 - 5k_1/3 - 8k_2)/\tilde{a}^2$ to simplify notations. k_i and \tilde{a} are constant. Also,

$$\tilde{\Theta} = 1 + \frac{\tilde{a}^2 g^2 \vartheta}{(4\pi)^2}, \quad f(\vartheta) = \tilde{\Theta}^{(6-4k_2)/\tilde{a}^2}, \quad g^2(\vartheta) = \frac{g^2}{\tilde{\Theta}}.$$

By straightforward calculation, we obtain

$$\frac{V'}{V} = \frac{4}{\phi} + \left\{ \frac{4(6-4k_2)}{\tilde{a}^2} - 1 \right\} \frac{1}{\tilde{\Theta}} \frac{d\tilde{\Theta}}{d\phi}, \\ \frac{U'}{U} = \frac{2}{\phi} + \left\{ \frac{2(6-4k_2)}{\tilde{a}^2} - \frac{(6\xi-1)k_3}{\tilde{\Theta}^{k_3} + 6\xi - 1} \right\} \frac{1}{\tilde{\Theta}} \frac{d\tilde{\Theta}}{d\phi}.$$

Then, using (9) and the conditions $\tilde{\Theta}_f^{k_3} \neq -6\xi + 1$, $\frac{d\tilde{\Theta}}{d\phi} \neq 0$, we get

$$\tilde{\Theta}_f^{k_3} = (2k_3 - 1)(6\xi - 1). \quad (34)$$

Therefore, there is no de Sitter solution for $\xi = 1/6$. For other values of ξ , we get

$$\tilde{\Theta}_f = [(6\xi - 1)(2k_3 - 1)]^{1/k_3} \quad \text{and} \\ \phi_f = \pm \mu \exp\left(\frac{(4\pi)^2}{\tilde{a}^2 g^2} [[(6\xi - 1)(2k_3 - 1)]^{1/k_3} - 1]\right). \quad (35)$$

Note that $\tilde{\Theta}_f \neq 0$; hence $k_3 \neq 1/2$. Using (7), we calculate the Hubble parameter H for the de Sitter solution

$$H_f^2 = \frac{g^2 k_1 (2k_3 - 1)}{24k_3} \phi_f^2 \tilde{\Theta}_f^{\frac{2(6-4k_2)}{\tilde{a}^2} - 1}. \quad (36)$$

In [69] the authors show that asymptotically free models exist only when $f^2(\vartheta) \sim g^4(\vartheta)$, which corresponds to $\tilde{a}^2 = 4k_2 - 6$. For this choice of the parameter \tilde{a}^2 , we obtain

$$V_f = \frac{g^2 k_1 \phi_f^4}{24} \tilde{\Theta}_f^{-5}, \\ U_f = \frac{\phi_f^2}{12} \tilde{\Theta}_f^{-2} [1 + (6\xi - 1)\tilde{\Theta}_f^{-k_3}] = \frac{\phi_f^2 k_3}{6(2k_3 - 1)\tilde{\Theta}_f^2}.$$

Note that $k_3 = -2 - 5k_1/(3(4k_2 - 6)) = -2 - 5k_1/(3\tilde{a}^2)$. Therefore,

$$H_f^2 = -\frac{g^2 k_1 (1 - 2k_3)}{24k_3} \phi_f^2 \tilde{\Theta}_f^{-3} \\ = \frac{5k_1 (2k_1 + 3\tilde{a}^2)}{24(5k_1 + 6\tilde{a}^2)} g^2 \phi_f^2 [(2k_3 - 1)(6\xi - 1)]^{-3/k_3}. \quad (37)$$

From here, we get that $H_f^2 > 0$ provided either $-3\tilde{a}^2/2 < k_1 < -6\tilde{a}^2/5$ that is equivalent to $0 < k_3 < 1/2$ or $0 < k_1 < \infty$ that corresponds to $-\infty < k_3 < 0$.

Let us consider the stability of the de Sitter solutions in the case $V_f > 0$ and $U_f > 0$. In this case, the sign of K_f coincides with the sign of

$$2\left(\frac{U'}{U}\right)'|_{\phi=\phi_f} - \left(\frac{V'}{V}\right)'|_{\phi=\phi_f} = \frac{\tilde{a}^4 g^4 (2k_3 - 1)}{512\pi^4 \phi_f^2 \tilde{\Theta}_f^2}.$$

The de Sitter solutions exist only for $k_3 < 1/2$. The additional condition $U_f > 0$ gives $k_3 < 0$. We can see that $K_f < 0$ at any $k_3 < 0$. Thus, for this model with $U_f > 0$ we have stable de Sitter solutions only.

VIII. THE $SU(5)$ RG-IMPROVED POTENTIAL

Now we study the RG-improved potential for the $SU(5)$ GUT [70]. In flat space this theory has been used for the discussion of inflationary cosmology [9,21]. We assume that the breaking $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ has taken place.

The RG-improved effective potential has the following form:

$$V = \frac{3375}{512} \left(g^2 - \frac{g^2}{f_5^{16/9}} \right) \phi^4 f_5^4,$$

$$U = \frac{15}{4} \left[\frac{1}{6} + \left(\xi - \frac{1}{6} \right) \tilde{\Theta}^{-9/8} \right] \phi^2 f_5^2, \quad (38)$$

where $\tilde{\Theta} = 1 + \frac{5g^2\theta}{3\pi^2}$, $f_5 = \tilde{\Theta}^{9/16}$, and g is a nonzero constant.

To get the de Sitter solution, we use Eq. (9), which for the model at hand reads

$$\tilde{\Theta}' \frac{(9\tilde{\Theta} - 5)(6\xi - 1) + 4\tilde{\Theta}^{9/8}}{\tilde{\Theta}(\tilde{\Theta} - 1)(\tilde{\Theta}^{9/8} + 6\xi - 1)} = 0. \quad (39)$$

Thus, $\tilde{\Theta} \neq 0$ and $\tilde{\Theta} \neq 1$. Note that

$$\tilde{\Theta}' = \frac{5g^2}{3\pi\phi} \neq 0. \quad (40)$$

It is easy to see that, for $\xi = 1/6$, there is no de Sitter solution. For other values of ξ , the de Sitter solutions are defined by

$$H_{ds}^2 = \frac{225\phi_f^2 \tilde{\Theta}_f^{5/4} g^2 (\tilde{\Theta}_f - 1)}{128(\tilde{\Theta}_f^{9/8} + 6\xi - 1)}. \quad (41)$$

The number $\tilde{\Theta}_f$ is a root of Eq. (39), which can be rewritten as follows:

$$\xi = \frac{1}{6} - \frac{2\tilde{\Theta}_f^{9/8}}{3(9\tilde{\Theta}_f - 5)}. \quad (42)$$

We can eliminate ξ and express H_{ds}^2 as

$$H_{ds}^2 = \frac{225g^2\phi_f^2(\tilde{\Theta}_f - 1)\tilde{\Theta}_f^{5/4}}{128(\tilde{\Theta}_f^{9/8} - \frac{(-9\tilde{\Theta}_f + 5 + 4\tilde{\Theta}_f^{9/8})}{(9\tilde{\Theta}_f - 5)} - 1)}$$

$$= \frac{25}{128} \tilde{\Theta}_f^{1/8} (9\tilde{\Theta}_f - 5) \phi_f^2 g^2. \quad (43)$$

Therefore, the Hubble parameter H is real if and only if $\tilde{\Theta}_f \geq 5/9$, and this is possible for $\xi < 1/6$ only. Using (42), we get

$$V_f = \frac{3375}{512} g^2 (1 - \tilde{\Theta}_f^{-1}) \phi_f^4 \tilde{\Theta}_f^{9/4} \quad \text{and}$$

$$U_f = \frac{15}{4} \left(\frac{1}{6} - \frac{2}{3(9\tilde{\Theta}_f - 5)} \right) \phi_f^2 \tilde{\Theta}_f^{9/8}. \quad (44)$$

We see that $U_f < 0$ at $5/9 < \tilde{\Theta}_f < 1$ and $U_f > 0$ for $1 < \tilde{\Theta}_f$. Let us consider the stability of the de Sitter solutions here obtained. For $1 < \tilde{\Theta}_f$, $U_f > 0$ and $V_f > 0$, so the denominator of K_f calculated by (15) is positive, and thus the sign of K_f coincides with the sign of its numerator:

$$2\left(\frac{U'}{U}\right)'|_{\phi=\phi_f} - \left(\frac{V'}{V}\right)'|_{\phi=\phi_f} = \frac{(5 - \tilde{\Theta}_f)(\tilde{\Theta}_f')^2}{8(\tilde{\Theta}_f - 1)^2 \tilde{\Theta}_f^2}. \quad (45)$$

Note that we used the conditions $\tilde{\Theta}_f \neq 1$. We come to the conclusion that $K_f > 0$, for $1 < \tilde{\Theta}_f < 5$, and $K_f < 0$, for $5 < \tilde{\Theta}_f$.

Using (40), we get

$$K_f = \frac{9375g^6(5 - \tilde{\Theta}_f)\tilde{\Theta}_f^{1/8}\phi_f^2(9\tilde{\Theta}_f - 5)}{32[128\tilde{\Theta}_f^{7/8}\pi^4(\tilde{\Theta}_f - 1)(9\tilde{\Theta}_f - 5) + 30375(\frac{16}{15}\tilde{\Theta}_f\pi^2(\tilde{\Theta}_f - 1) + \frac{g^2}{9}(9\tilde{\Theta}_f - 5))^2]}. \quad (46)$$

Let us now return to the interval $5/9 < \tilde{\Theta}_f < 1$. In this interval the numerator of K_f is positive. The sign of K_f is determined by the sign of its denominator

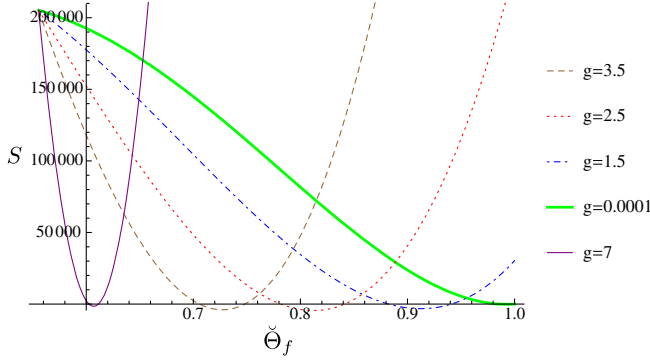


FIG. 1 (color online). The function $S(\check{\Theta}_f)$ for different values of g .

$$S = 128\check{\Theta}_f^{\frac{7}{8}}\pi^4(\check{\Theta}_f - 1)(9\check{\Theta}_f - 5) + 30375 \left(\frac{16}{15}\check{\Theta}_f\pi^2(\check{\Theta}_f - 1) + \frac{g^2}{9}(9\check{\Theta}_f - 5) \right)^2. \quad (47)$$

The denominator of K_f (the function S) is plotted in Fig. 1 for different values of g . One can see that the sign of this denominator depends on the value of $\check{\Theta}_f$. So, for $U_f < 0$ unstable de Sitter solutions can exist.

IX. INFLATIONARY MODEL CONSISTENT WITH OBSERVATIONAL RESULTS

A. Parameters of an inflationary model

Our goal is to construct an inflationary model using the RG-improved potentials and to examine if the inflationary model with this potential is compatible with the Planck 13 and BICEP2 data.

Much of the formalism developed for calculating the parameters of inflation, for example, the primordial spectral index n_s , assume general relativity models with minimally coupled scalar fields. The standard way to use this formalism is to perform a conformal transformation and to consider the model in the Einstein frame (see, for example, Ref. [18]). It has been shown [46] that in the case of quasi-de Sitter expansion there is no difference between spectral indexes calculated either in the Jordan frame directly or in the Einstein frame after conformal transformation.

Let us make the conformal transformation of the metric

$$\tilde{g}_{\mu\nu} = 2\kappa^2 U(\phi) g_{\mu\nu},$$

where quantities in the new frame are marked with a tilde, and the quantity $\kappa^2 = 8\pi M_{pl}^{-2}$, where M_{pl} is the Planck mass. We also introduce a new scalar field ϕ , such that

$$\frac{d\phi}{d\check{\phi}} = \frac{\sqrt{U + 3U'^2}}{\sqrt{2}\kappa U} \Rightarrow \phi = \frac{1}{\sqrt{2}\kappa} \int \frac{\sqrt{U + 3U'^2}}{U} d\check{\phi}. \quad (48)$$

We thus get a model for a minimally coupled scalar field, described by the following action:

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2\kappa^2} R(\tilde{g}) - \frac{1}{2} \tilde{g}^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu} + V_E(\varphi) \right], \quad (49)$$

where

$$V_E(\varphi) = \frac{V(\check{\phi}(\varphi))}{4\kappa^4 U^2(\check{\phi}(\varphi))}. \quad (50)$$

Inflationary universe models are based upon the possibility of a slow evolution of some scalar field φ in the potential $V(\varphi)$. The slow-roll approximation, which neglects the most slowly changing terms in the equations of motion, is used. To calculate parameters of inflation that can be tested via observations, we use the slow-roll approximation parameters.

As known [13,71] (see also [34,46]), the slow-roll parameters ϵ , η , and ζ are connected with the potential in the Einstein frame as follows [72]:

$$\begin{aligned} \epsilon &\equiv \frac{1}{2\kappa^2} \left(\frac{V'_{E,\varphi}(\varphi)}{V_E(\varphi)} \right)^2, & \eta &\equiv \frac{1}{\kappa^2} \frac{V''_{E,\varphi}(\varphi)}{V_E(\varphi)}, \\ \zeta^2 &\equiv \frac{1}{\kappa^4} \frac{V'_E(\varphi) V'''_{E,\varphi}(\varphi)}{V_E(\varphi)^2}. \end{aligned} \quad (51)$$

Note that the prime denotes the derivative with respect to the argument of the functions, that is φ , so $V'_{E,\varphi}(\varphi) \equiv \frac{dV_E(\varphi)}{d\varphi}$. We add the additional subscript $_{,\varphi}$ to denote derivatives with respect to φ . During inflation, each of these parameters should remain to be less than one.

It is suitable to calculate the slow-roll parameters as functions of the initial scalar field ϕ . It is easy to see [18] that

$$\begin{aligned} \epsilon(\phi) &= \frac{1}{2\kappa^2} \left(\frac{V'_E}{V_E} \right)^2 \left(\frac{d\phi}{d\check{\phi}} \right)^{-2}, \\ \eta(\phi) &= \frac{1}{\kappa^2} \left[\frac{V''_E}{V_E} \left(\frac{d\phi}{d\check{\phi}} \right)^{-2} - \frac{V'_E}{V_E} \left(\frac{d\phi}{d\check{\phi}} \right)^{-3} \frac{d^2\phi}{d\check{\phi}^2} \right], \end{aligned} \quad (52)$$

where the prime denotes a new derivative with respect to ϕ . We get

$$\epsilon(\phi) = \frac{1}{2\kappa^2} \frac{(V'_E)^2}{V_E^2 Q}, \quad \eta(\phi) = \frac{1}{\kappa^2 V_E Q} \left[V''_E - \frac{V'_E Q'}{2Q} \right],$$

$$\text{where } Q = \frac{U + 3U'^2}{2\kappa^2 U^2}. \quad (53)$$

Similar calculations yield

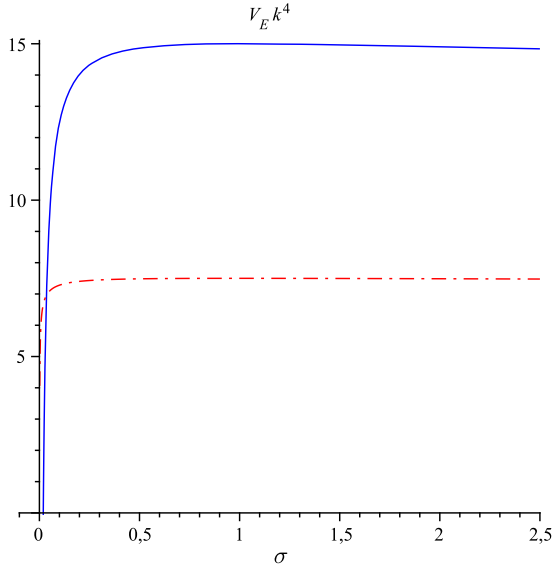


FIG. 2 (color online). The potential $V_E(\sigma)$ multiplied by κ^4 in the scalar electrodynamics model. In the picture $\lambda = 18e^2\xi$. This means that $\sigma_f = 1$. The parameter values are $e = 10$, $\xi = 10$ (red dashed line) and $e = 10$, $\xi = 5$ (blue solid line). Note that the potential V_E with such choice of λ depends on the combination ξ/e^2 only.

$$\zeta^2 = \frac{V'_E}{\kappa^4 V_E^2 Q^2} \left[V_E''' - \frac{3V_E'' Q'}{2Q} - \frac{V_E' Q''}{2Q} + \frac{V_E' (Q')^2}{Q^2} \right]. \quad (54)$$

The number of e -foldings of a slow-roll inflation is given by the following integral [18]:

$$\begin{aligned} N_e(\phi) &= \kappa^2 \int_{\phi_{\text{end}}}^{\phi} \left| \frac{V_E(\tilde{\varphi})}{V'_{E,\varphi}(\tilde{\varphi})} \right| d\tilde{\varphi} = \kappa^2 \int_{\phi_{\text{end}}}^{\phi} \left| \frac{V_E}{V'_E} \right| Q d\tilde{\varphi} \\ &= \frac{\kappa}{\sqrt{2}} \int_{\phi_{\text{end}}}^{\phi} \left(\frac{d\varphi}{d\tilde{\varphi}} \right) \frac{d\tilde{\varphi}}{\sqrt{\epsilon(\tilde{\varphi})}}, \end{aligned} \quad (55)$$

where ϕ_{end} is the value of the field at the end of inflation, defined by $\epsilon = 1$. The number of e -foldings must be matched with the appropriate normalization of the data set and the cosmic history, a typical value being $50 \leq N_e \leq 65$.

The tensor-to-scalar ratio r , the scalar spectral index of the primordial curvature fluctuations n_s , and the associated

TABLE I. Parameter values for the scalar electrodynamics inflationary scenario.

ξ	e	$\sigma_{\text{end}} (\epsilon = 1)$	N_e	σ_N	n_s	r	α_s
5	10	0.02415237908	50	0.20044	0.962	0.013	0.0013
5	10	0.02415237908	55	0.21544	0.965	0.011	0.0011
5	10	0.02415237908	60	0.23023	0.967	0.009	0.0009
5	10	0.02415237908	65	0.24485	0.969	0.008	0.0008
10	10	0.0004788703192	50	0.007189	0.967	0.028	0.0036
10	10	0.0004788703192	55	0.007974	0.965	0.024	0.0032
10	10	0.0004788703192	60	0.008848	0.967	0.022	0.0026
10	10	0.0004788703192	65	0.009756	0.974	0.019	0.0025

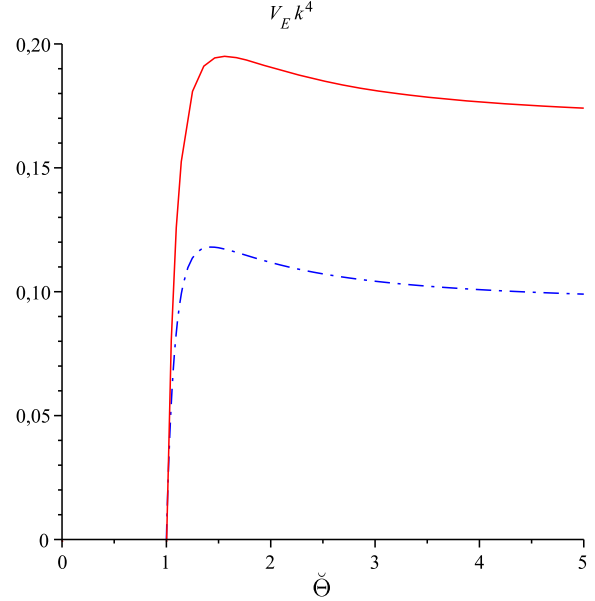


FIG. 3 (color online). The potential $V_E(\check{\Theta})$ multiplied by κ^4 in the $SU(5)$ model at $\xi = 0.04$, $g = 0.15$ (blue dashed line) and at $\xi = 0.045$, $g = 0.2$ (red solid line).

running of the spectral index α_s , are given, to very good approximation, by

$$\begin{aligned} r &= 16\epsilon, & n_s - 1 &\simeq -6\epsilon + 2\eta, \\ \alpha_s &\equiv \frac{dn_s}{d \ln k} \simeq 16\epsilon\eta - 24\epsilon^2 - 2\zeta^2. \end{aligned} \quad (56)$$

Planck 2013 temperature anisotropy measurements [2] combined with the WMAP large-angle polarization constrain the scalar spectral index to $n_s = 0.9603 \pm 0.0073$. Our goal is to check the possibility to get a value of n_s in the models investigated here. We describe the inflationary dynamics for two considered models that have unstable de Sitter solutions with $U_f > 0$. Observe that the existence of an unstable de Sitter solution may not be a necessary condition for inflation. Stable de Sitter solutions may be the basis for eternal inflation. From another side, stable de Sitter solutions may appear to be true inflationary solutions subject to the condition that the exit from inflation occurs due to some

TABLE II. Parameter values for the $SU(5)$ inflationary scenario.

ξ	g	N_e	$\check{\Theta}_{\text{end}} (\epsilon = 1)$	$\check{\Theta}_N$	n_s	r	α_s
0.04	0.15	50	1.000868906	1.0121	0.963	0.070	0.00731
0.04	0.15	55	1.000868906	1.0126	0.965	0.063	0.00643
0.04	0.15	60	1.000868906	1.0132	0.968	0.058	0.00660
0.04	0.15	65	1.000868906	1.0137	0.969	0.0535	0.00540
0.045	0.2	50	1.001564816	1.02152	0.958	0.066	0.00699
0.045	0.2	55	1.001564816	1.02252	0.960	0.0595	0.00638
0.045	0.2	60	1.001564816	1.023475	0.963	0.054	0.00579
0.045	0.2	65	1.001564816	1.024388	0.965	0.0495	0.00548

other scenario. We do not consider such inflationary scenarios that can be suitable for the $SU(2)$ model, in this paper.

Note that for the $SU(5)$ model, the de Sitter solutions obtained exist for $U_f < 0$ as well. In this case one cannot use the conformal transformation to formulate the model in the Einstein frame. Also, stability conditions can be violated [48]. The possibility to develop the inflationary

scenario without pathologies in this case demands a more detailed analysis, which will be carried out in future works.

B. Scalar electrodynamics

For scalar electrodynamics, the functions V and U are given by (20); therefore,

$$V_E = \frac{16\pi^2(8\pi^2\lambda + 9e^4 \ln(\sigma^2))}{3\kappa^4(32\xi\pi^2 + e^2 \ln(\sigma^2))^2}, \quad (57)$$

$$Q = \frac{6e^4 \ln(\sigma^2)^2 + 4[3e^2 + 8\pi^2(1 + 12\xi)]e^2 \ln(\sigma^2) + 6e^4 + 384\pi^2 e^2 \xi + 1024\pi^4 \xi(1 + 6\xi)}{\kappa^2 \mu^2 \sigma^2 [32\pi^2 \xi + e^2 \ln(\sigma^2)]^2}, \quad (58)$$

where we use the dimensionless quantity $\sigma \equiv \phi/\mu$. Note that the slow-roll parameters ϵ and η do not depend on the dimensionless combination $\kappa\mu$. We choose the parameters so that $U_f > 0$, which means $\lambda < 36e^2\xi$. Note that the de Sitter solutions correspond to the condition $V'_E(\phi_f) = 0$. Solutions are unstable; so, in this point the potential V_E has a maximum. In Fig. 2 we see that the potential is very flat near the maximum and decreases more rapidly than σ , when it tends to zero.

In our calculations, we put $\lambda = 18e^2\xi$, which gives $\phi_f = \pm\mu$ ($\sigma_f = \pm 1$). At the de Sitter point ϕ_f the value of the RG-improved potential coincides with the value of the tree-level one. We consider positive values for σ and for the parameters in the action. The corresponding inflationary parameters are listed in Table I.

One can see that, for the values of the parameters e and ξ presented in Table I, the corresponding values of n_s and r are in good agreement with the observational data [2,4]. The potential $V_E(\sigma)$ with these values of parameters is presented in Fig. 2.

C. The $SU(5)$ model

In the case of a $SU(5)$ RG-improved potential,

$$V_E = \frac{135g^2(\check{\Theta} - 1)\check{\Theta}^{5/4}}{32\kappa^4(\check{\Theta}^{9/8} + 6\xi - 1)^2}, \quad (59)$$

$$Q = \frac{4(\check{\Theta}^{9/8} + 6\xi - 1) + \frac{15}{128\pi^2}(15g^2\check{\Theta}^{1/8} + 16\pi^2(\check{\Theta}^{9/8} + 6\xi - 1))^2}{5(\check{\Theta}^{9/8} + 6\xi - 1)^2\kappa^2\phi^2}. \quad (60)$$

For the $SU(5)$ RG-improved potential (38), the function $\varphi(\phi)$ cannot be written in closed form. Because of this reason, we write the slow-roll parameters as functions of the Jordan-frame scalar field ϕ . The slow-roll parameter ϵ and the number of e -foldings read as follows:

$$\epsilon = \frac{125g^4(4\check{\Theta}^{9/8} - 5(6\xi - 1) + 9\check{\Theta}(6\xi - 1))^2}{288\pi^4(\check{\Theta} - 1)^2\check{\Theta}^2[4(\check{\Theta}^{9/8} + 6\xi - 1) + \frac{15}{128\pi^2}(15g^2\check{\Theta}^{9/8} + 16\pi^2(\check{\Theta}^{9/8} + 6\xi - 1))^2]},$$

and

$$N_e = \frac{36}{125} \int_{\check{\Theta}_{end}}^{\check{\Theta}_N} \frac{\pi^4(\check{\Theta} - 1)\check{\Theta}(4(\check{\Theta}^{9/8} + 6\xi - 1) + \frac{15}{128}(\frac{15\check{\Theta}^{1/8}g^2}{\pi^2} + 16(\check{\Theta}^{9/8} + 6\xi - 1))^2)}{g^4(\check{\Theta}^{9/8} + 6\xi - 1)((9\check{\Theta} - 5)(6\xi - 1) + 4\check{\Theta}^9)} d\check{\Theta}. \quad (61)$$

Note that the slow-roll parameters and N_e depend on the dimensionless function $\check{\Theta}$.

The potential V_E for $\xi = 0.04$ and $\xi = 0.045$ is plotted in Fig. 3. The corresponding inflationary parameters are listed in Table II.

The resulting joint BICEP2 + Planck 2013 analysis yields that the upper limit of the tensor-to-scalar ratio is $r < 0.11$, a slight improvement relative to the Planck analysis alone, which gives $r < 0.13$ (95% C.L.) [4]. We do see that the inflationary parameters of the model considered are in very good agreement with the observational data.

X. CONCLUSIONS

In this paper we have considered the possibility to construct inflationary models for the renormalization-group improved potentials corresponding to scalar electrodynamics and to the $SU(2)$ and $SU(5)$ models. In all cases, the tree-level potential is $\lambda\phi^4 - \xi\phi^2 R$, which corresponds to the cosmological constant in the Einstein frame, and is in no case suitable for inflation. The standard way to get an inflationary model is to add the Hilbert-Einstein term to the action [17,18]. Actually we did not add this term here, but included instead the quantum corrections to the potential coming from to the RG equation. We then analyzed the corresponding inflationary scenario with unstable de Sitter solutions only. This means that the corresponding potential in the Einstein frame should have a maximum. We have found that, for some reasonable values of the parameters, this is indeed the case, both for scalar electrodynamics and for the $SU(5)$ model, and that the corresponding values of the coupling function are indeed positive, $U_f > 0$. For the finite $SU(2)$ model de Sitter solutions exist, if it corresponds to the model with a minimally coupled scalar field and a cosmological constant; thus, this case is not suitable for inflation. In the $SU(2)$ gauge model there exist stable de Sitter solutions for $U_f > 0$ only. Note also that a stable de Sitter solution may appear to be a true inflationary solution, but only under the condition that the exit from inflation

occurs according to some other particular scenarios; but we did not consider such inflationary scenarios in this paper.

In the inflationary models, for both scalar electrodynamics and the $SU(5)$ RG-improved potentials, we have found that these models are in good agreement with the most recent observational data [2,4] provided some reasonable values are taken for the parameters.

Our study indicates that inflation could well be caused by quantum effects of the scalar sector of some convenient GUT theory. We believe this is quite a remarkable result. In this respect, it would be of interest to investigate the possibility of inflation in GUTs with other gauge groups, as the exceptional E_8 group, or GUTs that proceed from the string framework. From another side, adding a RG-improved effective potential to the classical GR action may lead to a qualitative change of the inflationary dynamics that occur in such models. This issue will be discussed elsewhere.

ACKNOWLEDGMENTS

E. E. and S. D. O. are supported in part by MINECO (Spain), Project No. FIS2010-15640, and by the CPAN Consolider Ingenio Project. E. P. and S. V. are supported in part by RFBR Grant No. 14-01-00707 and by the Russian Ministry of Education and Science under Grant No. NSH-3042.2014.2.

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