Grand unified hidden-sector dark matter

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We explore $G \times G$ unified theories with the visible and the hidden or dark sectors paired under a $\mathbb{Z}_2$ symmetry. Developing a system of "asymmetric symmetry breaking" we motivate such models on the basis of their ability to generate dark baryons that are confined with a mass scale just above that of the proton, as motivated by asymmetric dark matter. This difference is achieved from the distinct but related confinement scales that develop in unified theories that have the two factors of G spontaneously breaking in an asymmetric manner. We show how Higgs potentials that admit different gauge group breaking chains in each sector can be constructed, and demonstrate the capacity for generating different fermion mass scales. Lastly we discuss supersymmetric extensions of such schemes.

I. INTRODUCTION

Observations have established that our Universe is composed of ~32% matter and ~68% dark energy. Of the matter, only ~15% is accounted for by the particles that make up the standard model. The makeup of the remaining dark matter (DM) is one of the chief concerns of present day physics. The visible matter (VM) is composed of three generations of quarks and leptons interacting under $SU(3) \times SU(2) \times U(1)$ gauge interactions, plus a Higgs boson. It is common to consider that DM may be a similar set of particles charged under a different gauge group with only limited interactions with ordinary matter. The two sectors, the visible and the dark, provide an explanation for why evidence of DM has only been encountered so far through gravitational effects and the question of how these sectors could form to be so separate is an interesting challenge.

Asymmetric dark matter models, a broad category within hidden-sector scenarios, relate the creation of the mass density in the visible sector to the generation of matter in the dark sector. The fact that the mass densities of DM and VM in the Universe are seen to be of the same order,

$$\Omega_{DM} \simeq 5\Omega_{VM}, \qquad (1)$$

suggests that the mechanism by which VM was created in the early Universe is connected to the production of DM. The established origin of the relic density of VM relies on a baryon asymmetry, in which a small excess of baryons over antibaryons developed, and after the antibaryons had all annihilated with opposing baryons only a baryon density remained. In asymmetric dark matter models the asymmetry in each sector is connected by the conservation of a global quantum number. Once the symmetric parts in each sector have annihilated away then the number densities of the remaining particles in each sector are related to each other [1–3]. However Eq. (1) is a mass-density relation. In order to explain it, a theory of how the DM mass is related to the proton mass is needed in addition to related number densities. Now, grand unified theories (GUTs) unite the fundamental forces of particle physics into a single gauge group at high energy along with their coupling constants. The purpose of this paper is to explore how GUTs can relate a dark-sector confinement scale to the QCD scale.

We approach this by demonstrating an "asymmetric symmetry breaking" mechanism in which isomorphic and $\mathbb{Z}_2$ related gauge groups $G_V \times G_D$ of the visible and dark sectors naturally differ from each other after symmetry breaking. Each sector then features different mass scales for visible and dark baryons. We now briefly review how this mass is generated in our own sector.

The dependence of the running coupling constant of QCD, $\alpha_s(\mu)$, on the scale μ can be expressed in two ways. The first is as a function of a reference scale μ_0 which gives an equation of the form

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + (\beta_0/4\pi)\alpha_s(\mu_0)\ln(\mu^2/\mu_0{}^2)}, \qquad (2)$$

where α_s is known at the reference scale. Alternatively the dependence can be expressed as

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)}, \qquad (3)$$

in which the parameter Λ is the confinement scale, the value at which the strong coupling constant becomes large as the energy scale decreases. This is a distinct feature of asymptotic freedom in which $\beta_0 > 0$. At first order the beta function for SU(3) is

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$$\beta_0 = 11 - \frac{2}{3} n_f, \tag{4}$$

where n_f is the number of quark flavors that appear in the loop corrections at a given energy scale. If one then knows the value of the strong coupling constant at a high energy scale U, for instance at a GUT scale, it is possible to calculate the value of the confinement scale by evolving the coupling constant and taking into account quark mass thresholds. The threshold values are actually at twice the mass of each quark as this is the amount of energy needed to switch on the relevant loop correction. The resulting equation is dependent on this high reference scale, U, α_s at said scale, and the masses of the fermions in the range between the two scales:

$$\Lambda = 2^{2/9} e^{-2\pi/9\alpha_s(U)} U^{\frac{7}{9}} m_c^{\frac{2}{27}} m_b^{\frac{2}{27}} m_t^{\frac{2}{27}}, \tag{5}$$

where $m_{t,b,c}$ are the top-, bottom-, and charm-quark masses. One can obtain the one-loop version of this equation by calculating the value of α_s at the largest quark mass threshold from the reference scale U using Eq. (2) and repeating this process for each quark mass threshold, taking the previous threshold as a starting point until the confinement scale is reached. This calculation can then be rearranged to express the confinement scale in terms of the first reference scale U and the value of each mass threshold in between. For a more general theory the confinement scale is given by

$$\Lambda = 2^{1 - \frac{b_u}{b_c}} e^{-2\pi/\alpha_s(U)b_c} U^{\frac{b_u}{b_c}} m_1^{\frac{b_c - b_b}{b_c}} m_3^{\frac{b_t - b_u}{b_c}} m_2^{\frac{b_b - b_t}{b_c}}. \tag{6}$$

The terms labeled b_x in this form of the equation denote the values of β_0 for different numbers of contributing quark flavors. For instance, b_b is the value above twice the charm mass but below the bottom mass. We use this notation for the sake of the more generalized relationship between energy thresholds and the DM confinement scale where the number of massive quarks and the masses that they have are initially completely free parameters. Only the masses of quarks larger than Λ itself appear explicitly in the equation. It is important to note that this equation is very sensitive to the value of the scale U. This sensitivity is avoided, however, in a non-Abelian dark sector if the confining gauge group is also SU(3), as we explain below. To form the alternate gauge groups we develop a systematic way of generating different dark sectors from unified origins, with both containing an unbroken SU(3) factor.

The idea of connecting DM to unified origins is not new, of course, and a large number of models explore the possibility of DM coming from a dark sector which closely resembles our own. In particular our work is related to the theory of mirror matter [4–21], where the two sectors share the same gauge group and the hidden sector is an exact copy of the standard model. Our work demonstrates the capacity for natural symmetry breaking of mirrored GUT groups to two sectors which are manifestly different at low energies in both gauge symmetry and the masses of their particle content. Where a number of other works, in particular [22–25], posit the existence of a hidden non-Abelian gauge group responsible for generating dark baryon mass, or in the case of [26], glueball mass, we aim to show that such confining dark sector groups and in particular SU(3) can appear spontaneously from a unified original gauge symmetry and that the generation of a confinement scale in the dark sector which is different from (often larger than) our own can be a natural consequence of the way in which these unified theories break "asymmetrically."

Recently [27] explored models of composite fermions from SU(5) × SU(5) with a discrete symmetry that used potentials with different symmetry breaking scales to achieve coupling unification and generate confining particles in the TeV range. The intention of this work is to explore the broader possibilities of generating spontaneous differences in $G_V \times G_D$ theories to answer why DM could have a mass of the same order as VM.

The next section gives the results on the dark SU(3) confinement scale as a function of dark-quark mass in nonsupersymmetric theories. Section III then explains the basic idea behind asymmetric symmetry breaking in the simplest possible context. With that as a springboard, Sec. IV shows how the mechanism can be implemented in nonsupersymmetric SU(5), while Sec. V deals with how dark-quark mass generation can be automatically different from ordinary quark mass generation. Section VI briefly discusses the rather different and diverse possibilities afforded in SO(10) constructions, and Sec. VII touches on phenomenological constraints. Attention then turns to supersymmetry, with Sec. VIII showing how asymmetric symmetry breaking can be implemented, and Sec. IX displaying the dark confinement scale results. Final remarks are made in Sec. X. Appendixes A and B give further details of the scalar potential analyses in the non-SUSY and SUSY cases, respectively.

II. DARK SU(3)

The goal of these models is to obtain the standard model in one sector and a naturally occurring but distinctly different dark sector with its own SU(3) gauge group that facilitates an asymptotically free strong binding of dark quarks into heavy dark baryons. These baryons could then account for the relative mass density difference between the total visible and DM in a Universe where the number density generation is governed by asymmetric dark matter dynamics.

In a model with unified coupling constants, and where at a high energy the gauge groups of each sector break to SU(3) at the same scale, the two values of the strong coupling constant α_s and α_{s_D} are the same from the GUT

breaking scale all the way down to the scale at which the number of possible fermions in the loop corrections first deviates between the two sectors or further symmetry breaking occurs. This is highly desirable as it allows the equation of the dark confinement scale to be greatly simplified, as the high reference scale can then be chosen to be at the value of α_{s_D}, the deviation point which has just the value of the standard model α_s at the scale of either the top quark or the heaviest of the dark quarks depending on which of these two has greater mass. If we make the further assumption for the sake of simplicity that all heavy dark quarks have the same mass, then our equation becomes a function of just one continuous and one discrete parameter, namely the dark fermion mass scale m and the number of fermions, n_f, that are at such a scale, $\Lambda(n_f, m)$. Figure 1 shows the relation between the dark confinement scale and the mass of the heavy quarks which couple to the dark strong force. One could devise scenarios in which some of the heavy fermions attain an intermediate mass scale and adjust the confinement scale accordingly. Figure 1 shows that a value of Λ at approximately 1 order of magnitude greater than the standard model is compatible with dark quark masses up to 1000 TeV. The baryons themselves form from the light, or massless, quarks and therefore have mass either almost or totally dominated by the confinement scale. If one can build a model of two such sectors that allows for the dark sector to give masses to colored fermions at a low enough energy scale then accordingly one can provide an explanation for the similarity in the mass densities of visible and dark matter.

The focus of this paper is on how asymmetric symmetry breaking patterns from $G \times G$ GUTs can be induced and how quark and dark-quark mass generation may naturally differ, as these are the most important ingredients for determining the dark QCD confinement scale. Other important features for asymmetric DM models, such as
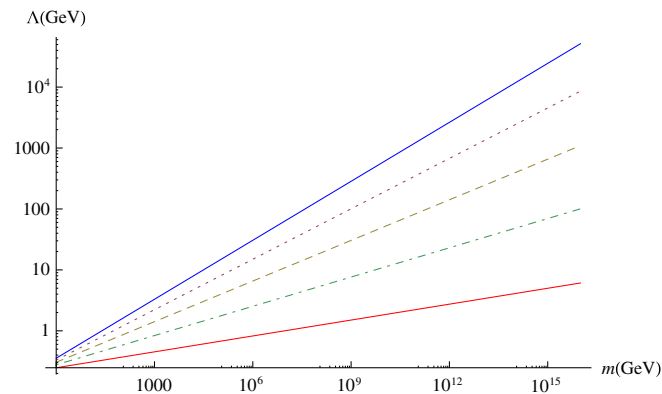

Λ(GeV)

FIG. 1 (color online). Confinement scale dependence on fermion masses with five heavy and one light quark at the top through five light and one heavy quark at the bottom line. All heavy quarks have mass m and all light quarks are taken to have masses below Λ.

the asymmetry generation and transfer mechanism and the annihilation of the symmetric part, plus various issues associated with constructing fully realistic GUTs,[1] are left for future work.

III. ASYMMETRIC SYMMETRY BREAKING

In order to illustrate the range of possible asymmetric symmetry breaking models and explain the basic features that drive asymmetric symmetry breaking we examine in this section a simple toy model that involves all of the most basic terms required and demonstrate what vacuum expectation value (VEV) patterns are possible.

The simple model we use for illustration is based on four real scalars in two $\mathbb{Z}_2$ pairs,

$$\phi_1 \leftrightarrow \phi_2, \qquad \chi_1 \leftrightarrow \chi_2. \tag{7}$$

The general potential can be written without loss of generality as

$$\begin{aligned}
V = {} & \lambda_\phi(\phi_1^2 + \phi_2^2 - v_\phi^2)^2 + \lambda_\chi(\chi_1^2 + \chi_2^2 - v_\chi^2)^2 \\
& + \kappa_\phi(\phi_1^2 \phi_2^2) + \kappa_\chi(\chi_1^2 \chi_2^2) + \sigma(\phi_1^2 \chi_1^2 + \phi_2^2 \chi_2^2) \\
& + \rho(\phi_1^2 + \chi_1^2 + \phi_2^2 + \chi_2^2 - v_\phi^2 - v_\chi^2)^2. \tag{8}
\end{aligned}$$

Terms such as $\phi_1^3 \phi_2 + \phi_1 \phi_2^3$ etc. are taken to be absent because of additional discrete symmetries. If each of the parameters is positive, then each of the six terms in this potential is positive definite. Then each is individually minimized if it is equal to zero. The first four terms are thus minimized by the condition that for each $\mathbb{Z}_2$ pair, one field gains a nonzero VEV while its partner has strictly zero VEV. The fifth term is minimized by the condition that the two nonzero-valued fields do not share a subscript (sector). The last term is then already zero by the previous conditions and the entire potential is minimized by these "asymmetric" configurations:

$$\langle \phi_1 \rangle = v_\phi, \qquad \chi_1 = 0, \langle \phi_2 \rangle = 0, \qquad \chi_2 = v_\chi. \tag{9}$$

Note that it could have been (ϕ_2, χ_1) that gained nonzero VEVs; i.e. we cannot know *a priori* which way the symmetry will break.

A key feature of these asymmetric models is the ability of one asymmetry to induce further asymmetry in additional $\mathbb{Z}_2$-related fields. If we take a second set of four fields just as in the above case,

$$\Omega_1 \leftrightarrow \Omega_2, \qquad \eta_1 \leftrightarrow \eta_2, \tag{10}$$

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[1]In particular, the problem of unsuccessful quark-lepton mass relations requires a nonminimal scalar sector. This will not affect our results provided that all colored components of these multiplets receive GUT-scale masses, as is required in any case to solve the doublet-triplet splitting problem.

our new general potential can be written in the form,

$$
\begin{aligned}
V = {} & \lambda_\phi(\phi_1^2 + \phi_2^2 - v_\phi^2)^2 + \lambda_\chi(\chi_1^2 + \chi_2^2 - v_\chi^2)^2 + \kappa_\phi(\phi_1^2\phi_2^2) + \kappa_\chi(\chi_1^2\chi_2^2) \\
& + \sigma(\phi_1^2\chi_1^2 + \phi_2^2\chi_2^2) + \rho(\phi_1^2 + \chi_1^2 + \phi_2^2 + \chi_2^2 - v_\phi^2 - v_\chi^2)^2 \\
& + \lambda_\Omega(\Omega_1^2 + \Omega_2^2 - v_\Omega^2)^2 + \lambda_\eta(\eta_1^2 + \eta_2^2 - v_\eta^2)^2 + \kappa_\Omega(\Omega_1^2\Omega_2^2) + \kappa_\eta(\eta_1^2\eta_2^2) \\
& + \sigma_1(\Omega_1^2\eta_1^2 + \Omega_2^2\eta_2^2) + \rho_1(\Omega_1^2 + \eta_1^2 + \Omega_2^2 + \eta_2^2 - v_\Omega^2 - v_\eta^2)^2 \\
& + \sigma_2(\Omega_1^2\chi_1^2 + \Omega_2^2\chi_2^2) + \rho_2(\Omega_1^2 + \chi_2^2 + \Omega_2^2 + \chi_1^2 - v_\Omega^2 - v_\chi^2)^2 \\
& + \sigma_3(\phi_1^2\eta_1^2 + \phi_2^2\eta_2^2) + \rho_3(\phi_1^2 + \eta_2^2 + \phi_2^2 + \eta_1^2 - v_\eta^2 - v_\phi^2)^2 \\
& + \rho_4(\Omega_1^2 + \phi_1^2 + \Omega_2^2 + \phi_2^2 - v_\Omega^2 - v_\phi^2)^2 + \sigma_4(\Omega_1^2\phi_2^2 + \Omega_2^2\phi_1^2) \\
& + \rho_5(\chi_2^2 + \eta_2^2 + \chi_1^2 + \eta_1^2 - v_\eta^2 - v_\chi^2)^2 + \sigma_5(\chi_2^2\eta_1^2 + \chi_1^2\eta_2^2).
\end{aligned} \tag{11}
$$

As before, with each term positive definite, the potential is minimized for the following pattern of VEVs:

$$
\begin{aligned}
\langle\phi_1\rangle &= v_\phi, & \langle\chi_1\rangle &= 0, \\
\langle\phi_2\rangle &= 0, & \langle\chi_2\rangle &= v_\chi, \\
\langle\Omega_1\rangle &= v_\Omega, & \langle\eta_1\rangle &= 0, \\
\langle\Omega_2\rangle &= 0, & \langle\eta_2\rangle &= v_\eta.
\end{aligned} \tag{12}
$$

As usual this vacuum is degenerate with its $\mathbb{Z}_2$ transform. The potential has been constructed in such a way that the minima are when nonzero ϕ, Ω VEVs share a sector, and the same is true for χ, η. This associated asymmetry allows us to link together particular subgroups from gauge symmetry breaking with appropriate Higgs multiplets for that specific sector to give different masses to fermions. This idea will be explored further in Sec. V. Large systems of many representations of scalar fields can take an initially mirrored GUT group and naturally populate each sector with nonzero VEVs of different scales which are given to different representations, thus making the two sectors highly divergent in their features though identical in their origins. This toy model will serve as a proof of concept for the more involved scenarios that we move on to, that is, replacing these singlet fields with representations of GUT groups.

IV. SU(5) × SU(5) ASYMMETRIC SYMMETRY BREAKING

We now consider how an asymmetric VEV structure allows for separate mechanisms to generate fermion masses in each sector. This section explores an illustrative model of asymmetrical symmetry breaking that uses the SU(5) GUT candidate. Paired with a discrete symmetry our $\mathrm{SU}(5)_v \times \mathrm{SU}(5)_d$ will be broken to different gauge groups in the two sectors but with both featuring unbroken SU(3) subgroups which have quantitative differences. This then allows a numerical difference in the value of the dark sector confinement scale. To accomplish this we build a symmetry breaking potential out of four scalar multiplets making use of two different representations of SU(5), namely the **24** and the **10**, each of which will have one of two multiplets become the sole attainer of a nonzero VEV in just one sector, thus facilitating the different symmetry breaking patterns. In its most basic form this is just an extension of the simple model of Sec. III in which the two sectors are the visible and dark and the fields ϕ_1, ϕ_2 are now **24** dimensional multiplets while χ_1, χ_2 become two copies of the **10** representation of SU(5),

$$
\begin{aligned}
\phi_v &\sim (24, 1), & \chi_v &\sim (10, 1), \\
\phi_d &\sim (1, 24), & \chi_d &\sim (1, 10).
\end{aligned} \tag{13}
$$

Consider firstly the **10** representation of SU(5) which one uses to spontaneously break

$$
\mathrm{SU}(5)_d \to \mathrm{SU}(3) \times \mathrm{SU}(2) \tag{14}
$$

by appropriate choice of the sign of parameters in a general quartic scalar potential. The general renormalizable potential for a scalar multiplet $\chi \sim 10$ is

$$
V_{10} = -\mu_t^2\chi_{ij}\chi^{ji} + \lambda_{t1}(\chi_{ij}\chi^{ji})^2 + \lambda_{t2}\chi_{ij}\chi^{jk}\chi_{kl}\chi^{li}. \tag{15}
$$

Note that $i, j = 1, \ldots, 5$ are SU(5) gauge indices with $\chi_{ij} = -\chi_{ji}$, and the subscript t denotes "ten." Choosing the parameter λ_{t2} to be negative produces a VEV that breaks SU(5) to SU(3) × SU(2) [28].

In the other sector the method of breaking SU(5) to the standard model is to use scalar fields in the adjoint representation. The quartic potential is

$$
V_{24} = -\mu_a^2\phi_j^i\phi_i^j + \lambda_{a1}(\phi_j^i\phi_i^j)^2 + \lambda_{a2}\phi_j^i\phi_k^j\phi_h^k\phi_i^h, \tag{16}
$$

where the subscript a is for "adjoint," and ϕ is Hermitian traceless. Choosing λ_{a2} to be positive gives us the breaking

$$SU(5)_v \rightarrow SU(3) \times SU(2) \times U(1). \qquad (17)$$

In this model we have four representations of scalar fields in the two $\mathbb{Z}_2$ pairs of Eqs. (15) and (16). The complete, general fourth-order, gauge-invariant scalar potential invariant under the discrete symmetry is written in Appendix A. It contains two copies of each of the above two potentials for the multiplets in each sector as well as all possible gauge-invariant contractions between the **24** and **10** in each individual sector, that is, of the style $\chi_v \chi_v \phi_v \phi_v$.

We can take these basic potentials written above and use them to write a simple outline of the full potential. We first duplicate each of the above potentials to accommodate each one's dark counterpart, and add in the cross terms such as $\mathrm{Tr}(\phi_v^2)\mathrm{Tr}(\phi_d^2)$. We term these

$$V_A = V_{24} + V'_{24} + \kappa_a \mathrm{Tr}[\phi_v^2]\mathrm{Tr}[\phi_d^2] \qquad (18)$$

and

$$V_T = V_{10} + V'_{10} + \kappa_t \chi_{vij}\chi_v^{ji}\chi_{dnm}\chi_d^{mn}. \qquad (19)$$

To this there are five remaining contractions that we must add to write the general renormalizable potential. A portion of this potential, displayed in full in Appendix A, can then be written as

$$V = V_A + V_T + C_0 (\chi_{dnm}\chi_d^{mn}\mathrm{Tr}[\phi_v^2]$$
$$+ \chi_{vij}\chi_v^{ji}\mathrm{Tr}[\phi_d^2]) + \dots \qquad (20)$$

Extending the analysis of Sec. III we find that for a particular region of parameter space in this potential, the global minimum is at

$$\langle \phi_v \rangle = v_v \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & -3/2 \end{pmatrix},$$

$$\langle \chi_v \rangle = 0,$$

$$\langle \phi_d \rangle = 0,$$

$$\langle \chi_d \rangle = v_d \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \qquad (21)$$

By using the principles of the simple model and its parameter space from Sec. III, this potential is seen to induce the two SU(5) gauge groups to indeed break differently in each sector. In one sector the **10**

representation attains a VEV breaking SU(5) to SU(3) × SU(2) and the positive definite contraction terms push the **24** in that sector to attain a VEV of zero. In the other sector the **10** representation is driven to have a VEV of zero by contraction terms with its counterpart and this forces the **24** to attain a VEV that breaks this second SU(5) to the standard model gauge group. There is once again no way of knowing which is the visible and which is the dark sector prior to symmetry breaking. Once the symmetry is broken to the lowest state it shall simply be that we label the SU(5) which is broken to the standard model group the gauge symmetry of the visible sector and the alternatively broken symmetry is then the dark sector gauge group.

We now explore fermion mass generation with a view to having the visible and dark color SU(3) gauge coupling constants evolve differently under the renormalization group.

V. FERMION MASSES

In SU(5) theories the fermions of the standard model are assigned to the $\bar{\mathbf{5}}$ and **10** representations. The product of these allows for mass generation through Yukawa couplings to Higgs fields in **5**, **10**, **45** or **50** dimensional representations. As an example, we aim to have two different representations for our mass generation, a **5** to accommodate the standard model Higgs doublet in the visible sector and another representation which attains a nonzero VEV in the dark sector to give a different form of mass generation for the dark sector quarks.[2]

The **10** representation already employed in the symmetry breaking only gives mass to leptons and is thus unsuitable. We therefore choose to examine how a **5** and a **45** in each sector can allow for a difference in the scale of quark and dark-quark masses. The **45** has the interesting property of automatically leaving one dark quark massless [29], which is a very useful feature for our application. The fermion multiplets are the same in each sector, again respecting our initial mirror symmetry:

$$\psi_{v_5} \sim (\bar{5}, 1), \qquad \psi_{d_5} \sim (1, \bar{5}),$$
$$\psi_{v_{10}} \sim (10, 1), \qquad \psi_{d_{10}} \sim (1, 10), \qquad (22)$$

and the Higgs multiplets which take the place of the fields Ω, η from Sec. III are

$$H_{v_5} \sim (5, 1), \qquad H_{d_5} \sim (1, 5),$$
$$H_{v_{45}} \sim (45, 1), \qquad H_{d_{45}} \sim (1, 45). \qquad (23)$$

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[2]The idea of a non-Abelian gauge sector responsible for confining DM has been detailed in a number of different works such as [22] in which the range of SU(N) groups and ultraviolet boundary conditions of the coupling constants that allow for TeV-scale-confined DM were investigated. In [26] the scale of gluinos and glueballs in an SU(N) hidden sector was seen to be adjustable to produce TeV-scale glueball DM that could agree with a number of astrophysical constraints of self-interacting DM.

The Yukawa Lagrangian is

$$\mathcal{L}_F = y_1 \overline{\psi_{v_5}} H^*_{v_5} \psi_{v_{10}} + y_2 \overline{\psi_{v_{10}}} H_{v_{45}} \psi_{v_{10}} + y_1 \overline{\psi_{d_5}} H^*_{d_5} \psi_{d_{10}}$$
$$+ y_2 \overline{\psi_{d_{10}}} H_{d_{45}} \psi_{d_{10}} + \text{H.c.} \qquad (24)$$

The methodology of Sec. IV can be extended to include the $\mathbb{Z}_2$ scalar pairs responsible for fermion mass generation. The asymmetric symmetry breaking described in Sec. III can induce consecutive asymmetries in more sets of fields. The dependence for which way the asymmetry in the second set will fall is entirely dependent on the weighting of the cross terms between the two sets.

It is in this manner that we arrange for the H_{45} in the visible sector to have a zero VEV, while in the dark sector it gives mass to five of the six quarks at an indeterminate scale v_d and reduces the dark sector symmetry from SU(3) $\times$ SU(2) to SU(3). The invariant component of $H_{d_{45}}$ is

$$\langle H_{d_{45}} \rangle^{b5}_a = v_d \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \qquad (25)$$

On the other hand the H_5 has a VEV of zero in the dark sector and a nonzero VEV in the visible sector as per the minimal SU(5) model of giving mass to the fermions:

$$\langle H_v \rangle = v_v (0,0,0,0,1), \qquad \langle H_d \rangle = 0. \qquad (26)$$

The scale v_d can then be compared to the top line in Fig. 1 from Sec. II in which we have five heavy dark quarks and a single massless dark quark. In such a scenario, if the masses of the quarks are less than 1000 TeV then they produce dark confinement scales less than 14 GeV. The remaining massless quark, a dark up-quark, forms a set of neutral $\Delta(uuu)$ baryonlike states, lighter than all other possible dark color singlets and with mass completely dominated by the confinement scale. This forms a dark analogue of the visible sector nucleon but with mass that is an order of magnitude greater. If we consider minimal differences in the magnitude of the mass generating VEVs, which is quite natural to obtain if parameters are of similar order, then at around the electroweak scale, $\sim$246 GeV, a confinement scale of 2.1 GeV is generated in the dark sector. This is around an order of magnitude higher than the standard model QCD scale of 0.217 GeV.

VI. SO(10) × SO(10)

We now briefly touch on the subject of SO(10) $\times$ SO(10) and other GUT models and their scope with regard to asymmetric symmetry breaking. In extending the grand unification from SU(5) to SO(10) we open up a number of possible pathways to break down to the standard model.

In particular we could consider breaking to the familiar SU(5) $\times$ SU(5) that we showed previously or instead use asymmetric symmetry breaking to take other paths in both sectors towards a final standard model gauge group and non-Abelian dark sector group. For example the use of the **45** and **54** representations allows one sector to take the Pati-Salam symmetry breaking path [30], and the other that of Georgi-Glashow SU(5) [31]. Alternatively one could bypass SU(5) in one or both sectors altogether. The large number of possibilities raises the prospect of many different ways to adjust the confinement scale for dark QCD. In addition, we could consider the possibilities of how we can adjust the scale of breaking to subgroups in each sector. As a simple example of what we mean we briefly examine a theory which at a high GUT scale M_G breaks as per

$$SU(5)_v \rightarrow SU(3) \times SU(2) \times U(1), \qquad (27)$$

and

$$SU(5)_d \rightarrow SU(4)_d, \qquad (28)$$

and then at an intermediate scale M_i features

$$SU(4)_d \rightarrow SU(3)_d. \qquad (29)$$

Then we can consider how the confinement scale of SU(3)$_d$ changes with the value of M_i. If, by way of a simple example, one assumes that the visible and dark quarks of each sector have the same masses, then the higher values of M_i will yield lower confinement scales as the theory will run as SU(3) for a greater energy span.

Figure 2 shows that higher values of M_i take us closer to the previous analysis of the standard model. This is what one would expect, as in the limit $M_i \rightarrow M_G$ it is as though we have broken directly to SU(3). If we, however, allow for more of the running to be governed by SU(4) then the theory will blow up at a higher scale once we transition to SU(3). In SO(10) $\times$ SO(10) models, the many possibilities warrant a dedicated analysis in future work.
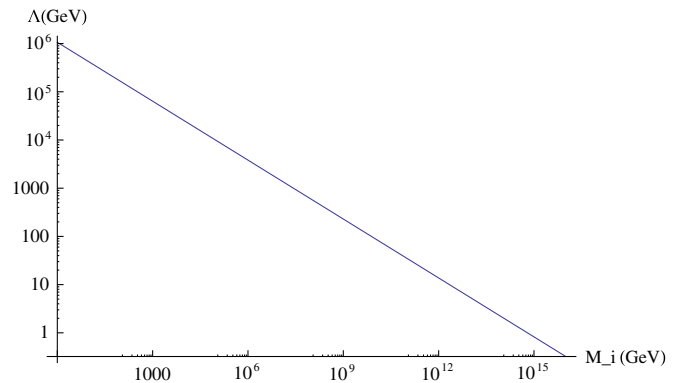

FIG. 2 (color online). The value of the confinement scale for different intermediate breaking scales M_i for the SO(10) $\times$ SO(10) scenario of Eqs. (27)–(29).

VII. PHENOMENOLOGICAL ISSUES

It is important to note that we merely assumed in the previous analysis that the gauge coupling constants of the sectors unify at a high GUT scale. While the scenario of a nonsupersymmetric asymmetric model that we have described does not automatically have gauge coupling unification, it is possible to bring the three coupling constants of the standard model together at the GUT scale by the addition of extra Higgs doublets. One must also consider the constraints from the experimental lower bounds of proton decay. Decay modes from minimal SU(5) models have quite high bounds, $\tau(p \to \pi^0 e^+) \gtrsim 10^{34}$ years [32] and the order of magnitude estimation for the width

$$\Gamma \approx \alpha^2 \frac{m_p^5}{M_X^4}, \tag{30}$$

demands that we must have at least $M_X \approx 4 \times 10^{15}$. In [33] it was shown that consistent proton decay limits and unification could be obtained with the addition of Higgs multiplets in a nonsupersymmetric SU(5).

Bounds on the dark baryons as DM from the bullet cluster observation are similar to that in [34] where the self-interaction cross section of these nucleons $\sigma \sim 10^{-26}$ cm^2 is compared to the upper bound of the DM self-interaction cross section $\leq 10^{-23}$ cm^2 [34–36]. The scale that v_d can take is something that we have not followed in full detail, opting to simply take as a guide the range of scale differences that we can accommodate in the simple model in Sec. III. These lead us to see that the scale of v_d for a factor of 5 difference between ordinary and dark baryons would need to be between ~ 30 GeV to 10^4 TeV depending on how many of the heavy quarks are given mass. The **45** representation of SU(5) would observe the lower bound of ~ 30 GeV as the mass scale would give this exact ratio. If, on the other hand, one only gave mass to a single quark in the dark sector then a very high mass would be compatible with a confinement scale of order the standard model.

It would be interesting to see what additional breaking chains discussed in Sec. VI could allow for the confinement scales to approach this ratio without even considering differences in the fermion masses between the two sectors.

Since the achievement of gauge coupling constant unification in non-SUSY GUT models is somewhat *ad hoc* and, more importantly, suffers from the gauge hierarchy problem, we now turn to SUSY models where these problems are absent.

VIII. SUPERSYMMETRIC ASYMMETRIC SYMMETRY BREAKING

We now develop a supersymmetric analogue of the model in Sec. IV, that is an SU(5) theory with scalar fields in the **10** and **24**. In building the supersymmetric potential we will have to introduce another chiral supermultiplet in the **$\overline{10}$** representation, Y, to make it possible to include

gauge-invariant terms containing $X \sim 10$ in the superpotential. We must of course also introduce a counterpart field Y_d for the sake of the discrete symmetry.

This allows for the construction of a potential including all of the fields from the non-SUSY case. However, in order to facilitate asymmetric symmetry breaking it is key that we have both terms that mix the fields under different representations in each sector and cross terms between the two sectors. This is not possible with the set of fields as they are. To achieve this we add a singlet scalar superfield S which transforms into itself under the discrete symmetry. Doing so allows for the superpotential to generate all of the necessary cross terms for asymmetric symmetry breaking through the F-terms of the scalar potential. The chiral supermultiplets involved are then

$$\Phi_v \sim (24, 1), \qquad X_v \sim (10, 1), \qquad Y_v \sim (\overline{10}, 1),$$
$$\Phi_d \sim (1, 24), \qquad X_d \sim (1, 10), \qquad Y_d \sim (1, \overline{10}), \tag{31}$$

and

$$S \sim (1, 1). \tag{32}$$

The general superpotential

$$\begin{aligned} W &= s_1(X_v Y_v + X_d Y_d) + s_2(\Phi_v \Phi_v + \Phi_d \Phi_d) \\ &\quad + s_3(\Phi_v \Phi_v \Phi_v + \Phi_d \Phi_d \Phi_d) + s_4(X_d \Phi_d Y_d + X_v \Phi_v Y_v) \\ &\quad + s_5(\Phi_v \Phi_v S + \Phi_d \Phi_d S) + s_6(X_v Y_v S + X_d Y_d S) \\ &\quad + s_7 S + s_8 SS + s_9 SSS \end{aligned} \tag{33}$$

satisfies $SU(5)_v \times SU(5)_d$ gauge invariance and the $\mathbb{Z}_2$ discrete symmetry. The symmetry breaking possibilities with this potential are discussed in more detail in Appendix B.

The complete potential has contributions from the F-terms of the superpotential, the D-terms from those fields which are charged under one of the SU(5) symmetries and soft mass and trilinear terms. Since we have a complete singlet S, the nonholomorphic trilinear terms are taken to be absent [37]. The equation is

$$\begin{aligned} V &= W^{i*} W_i + \frac{1}{2} \sum_a (g \Phi_i T_a \Phi^i)^2 - m_X (X_{vij} X_v^{ji} + X_{dij} X_d^{ji}) \\ &\quad - m_Y (Y_{vij} Y_v^{ji} + Y_{dij} Y_d^{ji}) - m_\Phi (\Phi_v \Phi_v + \Phi_d \Phi_d) - m_S S^2 \\ &\quad - a_1 (\Phi_d \Phi_d \Phi_d + \Phi_v \Phi_v \Phi_v) - a_2 (X_d Y_d \Phi_d + X_v Y_v \Phi_v), \end{aligned} \tag{34}$$

where

$$W^i = \frac{\partial W}{\partial \phi_i} \tag{35}$$

and each ϕ_i is one our fields. There are nine parameters from the superpotential $(s_1, \ldots, s_9)$ and six parameters from the soft terms m_Φ, m_X, m_Y, m_S, a_1, a_2 as well as the SU(5) coupling constant present in the D-terms. With this field content we find that the scalar potential then has the

capacity to display asymmetric symmetry breaking by appropriate choice of the parameters. The singlet field S is important here. Without it we could not arrive at a scalar potential that has terms such as $\Phi_v\Phi_v\Phi_d\Phi_d$, that is, terms which mix the two sectors. Without these it is not possible to create the necessary dependence between sectors for VEV development to be opposing. There are nonminimal choices one could make for the additional fields that would allow for these terms but for now we choose to simply focus on the simplest case.

Consider a parameter choice with s_4 and s_5 large compared to the other superpotential parameters, and with nonzero values of m_X, m_Y and m_Φ. F-terms of the style $(\Phi_v^2\Phi_d^2)$ or $(X_v X_v \Phi_v \Phi_v)$ can then serve as the cross terms that create the asymmetric acquisition of VEVs. With largely positive quartic terms coming from the D-terms and negative quadratic terms in the form of the soft masses, these cross terms can drive one variety of each multiplet of a given dimensionality to zero in the same manner as the non-SUSY case. It is however the case that many other W-terms can spoil this pattern and so many of the other superpotential parameters must be kept relatively small, at least an order of magnitude. The parameter s_9 we can allow to be large, as it will serve to bring the value of S to zero. In one scenario one can generate a nonzero VEV for Φ_v in the visible sector, again breaking

$$SU(5)_v \rightarrow SU(3) \times SU(2) \times U(1), \quad (36)$$

and in the dark sector we have Φ_d developing a VEV of zero. Then the multiplets X_d and Y_d together acquire nonzero VEVs which break

$$SU(5)_d \rightarrow SU(3) \times SU(2). \quad (37)$$

Being a pair of conjugate representations, they will induce breaking to the maximal stability group of SU(5) according to Michel's conjecture [38,39] which states that this is the case for a potential containing only a real representation or a pair of conjugate representations. This does not strictly apply in this scenario, of course, because we have other fields involved in the potential. However, we invoke it as numerical analysis shows that symmetry breaking of this type occurs within the parameter space that gives asymmetric VEV patterns. Appendix B contains further details of this parameter space. For the **10** dimensional representation, the maximal stability group, or maximal little group, is $SU(3) \times SU(2)$ as it is the only maximal group which observes a singlet within the **10** of SU(5).

The supersymmetric case is more constrained in its ability to display asymmetric configurations, though with suitable additions in particle content we have found that it is a feature that a unified supersymmetric theory can have. Many of the parameters in the superpotential must be kept quite small so as to not overpower the terms essential for guaranteeing asymmetric VEV arrays. It would be interesting to explore this issue further in developing a complete theory and

examining more of the possibilities for asymmetric SUSY sectors; however that is beyond the scope of this work.

We now discuss the dependence of the confinement scale with various parameters in a general supersymmetric theory.

IX. SUPERSYMMETRIC CONFINEMENT

In the case of supersymmetric theories the running coupling is modified by the additional particle content. For SU(3) we are however only interested in those particles with color charge. Note that this analysis is not dependent on any particular choice of GUT group, relying only on an $SU(3)_v \times SU(3)_d$ structure after GUT breaking.

In the MSSM the one loop beta function for SU(3) is altered by the addition of the gluinos and sfermions as per

$$\beta_0 = 11 - \frac{2}{3}n_f - C_g - \frac{2}{6}n_s, \quad (38)$$

where n_f (n_s) is the number of quarks (squarks) and $C_g = 2$ is due to the gluinos. The calculation of the dependence of confinement scale is more model dependent here as one must first of all take into account the mass that visible sector gluinos and squarks take to consider what value the coupling will take at the GUT or high reference scale μ_0. This will alter the precise calculation of the value of α_{3d} at the scale at which the visible and dark sector couplings unify. One can also consider in the dark sector how we might separate the scales of the quarks and squarks. If we take the assumption that the SUSY breaking scale is no higher than the mass scale of the dark quarks in the dark sector then this provides a rough upper bound on the scale at which we place the supersymmetric partners in that sector. This assumption is favorable also as it allows for a similar analysis as before in that, if the two sectors have SU(3) gauge symmetry with the same number of particles of each kind all the way down in energy to the mass of the heaviest dark quark, then we can choose this as our high reference scale and take the value of the coupling at this scale to be the same in both supersymmetric sectors. Then we can establish a range of possible confinement scales that supersymmetric dark QCD could have. We will examine the relationship between the confinement scale and these mass scales as we did in the non-SUSY case. In this case we take the squarks and gluinos of the dark sector to be quite light (under a TeV) and in such a scenario the dependence is similar to the non-SUSY case but with a larger confinement scale, shown in Fig. 3.

We now examine the dependence of the dark confinement scale on the dark SUSY breaking scale for a range of different dark-quark masses. The scale of dark-quark masses is taken to be higher than the SUSY breaking scale in each case. Figures 4–6 show this dependence for different numbers of heavy dark quarks. The value of the confinement scale is in general higher than the non-SUSY case though we do have additional parameters to contend with in the form of the mass scales of the squarks and gluinos.
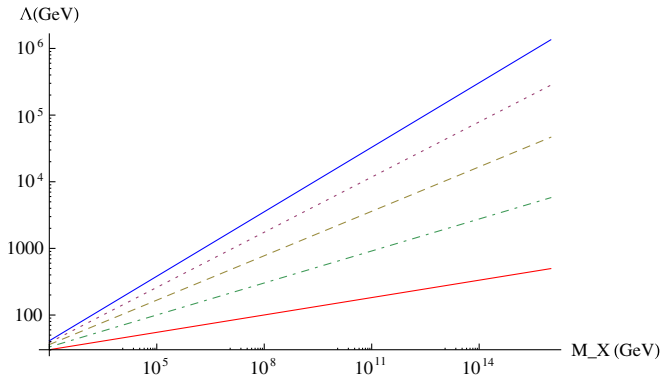
FIG. 3 (color online). Confinement scale dependence on fermion masses, in the simple SUSY case, almost identical to non-SUSY, but with the confinement scale axis multiplied by ~10.
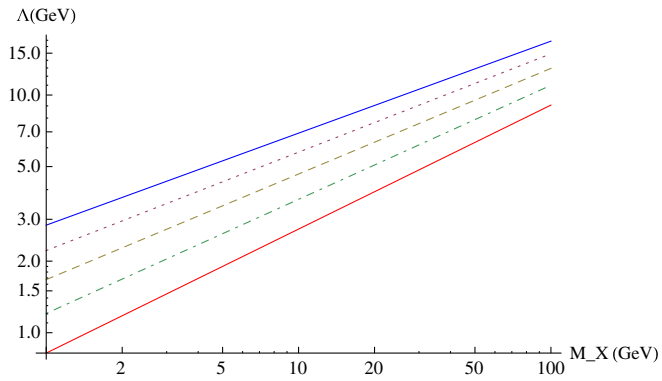

FIG. 4 (color online). Confinement scale dependence on SUSY breaking scale for a fixed dark-quark mass scale of 100 GeV. The number of heavy quarks at the dark-quark mass scale ranges from five at the top to one at the bottom.
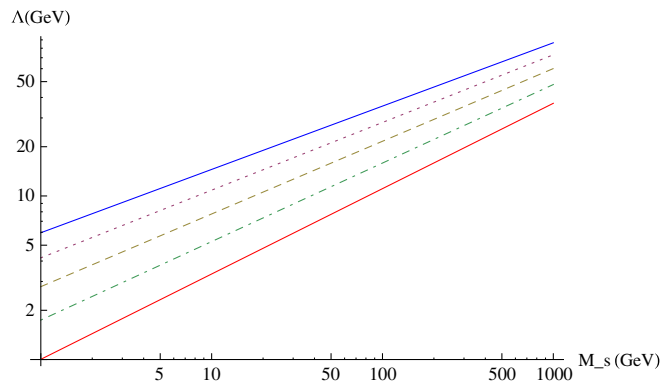

FIG. 5 (color online). As for Fig. 4 but with a dark-quark mass scale of 1000 GeV.

FIG. 6 (color online). As for Fig. 4 but with a dark-quark mass scale of 10^4 GeV.

X. CONCLUSIONS

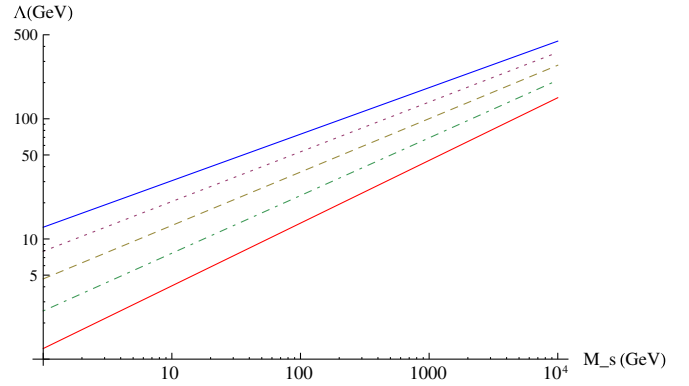Dark matter may be the manifestation of a perhaps quite complicated dark sector that is described by a gauge theory similar to the standard model. The cosmological "coincidence" $\Omega_{\mathrm{DM}} \simeq 5\Omega_{\mathrm{VM}}$ encourages the thought that DM has a similar origin to VM, and serves as the main motivation for asymmetric DM. These models typically succeed in relating the baryon and DM *number* densities, but have nothing very profound to say about the dark matter mass scale. But the latter is as important as the former in considerations of the mass-density coincidence. In most asymmetric DM models, the similar number densities imply that the DM mass should be similar to the proton mass, usually a factor of a few higher. What could be the origin of such a DM mass scale?

We have explored the idea that grand unification may provide an explanation. Beginning with a mirror-matter style $G \times G$ gauge group augmented by $\mathbb{Z}_2$ interchange symmetry, we invented a process termed "asymmetric symmetry breaking" which sees the two factors of G break in different ways. For the asymmetric DM application, we required that both sectors feature unbroken SU(3) subgroups, with different but related confinement scales. The ordinary QCD confinement scale sets the proton mass, while its dark-sector analogue sets the mass scale of the dark baryon that serves as the DM. We demonstrated that a significant region of parameter space furnishes a dark confinement scale within an order of magnitude or so above the QCD scale, as favored by most asymmetric DM models. Much higher scales are also possible, of course. We investigated both nonsupersymmetric and, more compellingly, supersymmetric GUTs of this type, and in the process explained how dark-quark mass generation can naturally differ from quark mass generation. Our analysis serves as a starting point for building fully realistic asymmetric DM models from a grand unification base.

The possibilities inherent in asymmetric symmetry breaking are rather large when one considers $G = \mathrm{SO}(10)$ and other higher-rank options. This seems to offer fruitful avenues for future investigations, and may ultimately serve to provide a truly unified understanding of the microphysics and macrophysics of ordinary and dark matter.

APPENDIX A: SCALAR POTENTIAL FOR NONSUPERSYMMETRIC SU(5) × SU(5) MODEL

In Sec. IV we outlined the construction of an SU(5) × SU(5) potential with asymmetric minima. Here we discuss its features in more detail and explore some of the possibilities in regard to breaking to various subgroups. The full SU(5) × SU(5) potential can be written as

$$
\begin{aligned}
V =\ & \lambda_{a1}(\phi_{vj}^{i}\phi_{vi}^{j} + \phi_{dj}^{i}\phi_{di}^{j} - \mu_a^2)^2 + \kappa_a(\phi_{vj}^{i}\phi_{vi}^{j}\phi_{dk}^{h}\phi_{dh}^{k}) + \lambda_{a2}(\phi_{vj}^{i}\phi_{vk}^{j}\phi_{vh}^{k}\phi_{vi}^{h} + \phi_{dj}^{i}\phi_{dk}^{j}\phi_{dh}^{k}\phi_{di}^{h}) \\
& + \lambda_{t2}(\chi_{vij}\chi_v^{ji} + \chi_{dij}\chi_d^{ji} - \mu_t^2)^2 + \kappa_t(\chi_{vij}\chi_v^{ji}\chi_{dij}\chi_d^{ji}) \\
& + \lambda_{t2}(\chi_{vij}\chi_v^{ij}\chi_{vij}\chi_v^{ij} + \chi_{dij}\chi_d^{ij}\chi_{dij}\chi_d^{ij}) + C_0(\chi_{vij}\chi_v^{ij}\phi_{vj}^{i}\phi_{vi}^{j} + \chi_{dij}\chi_d^{ji}\phi_{dj}^{i}\phi_{di}^{j}) \\
& + C_1(\chi_{vij}\chi_v^{jk}\phi_{vk}^{l}\phi_{vl}^{j} + \chi_{dij}\chi_d^{jk}\phi_{dk}^{l}\phi_{dl}^{j}) + C_2(\chi_{vlq}\chi_v^{ij}\phi_{vj}^{l}\phi_{vi}^{q} + \chi_{dlq}\chi_d^{ij}\phi_{dj}^{l}\phi_{di}^{q}) \\
& + C_3(\chi_{vsu}\chi_v^{pq}\phi_{vm}^{n}\phi_{vj}^{i}\epsilon^{smujt}\epsilon_{pnqit} + \chi_{dsu}\chi_d^{pq}\phi_{dm}^{n}\phi_{dj}^{i}\epsilon^{smujt}\epsilon_{pnqit}) \\
& + C_4(\phi_{vj}^{i}\phi_{vi}^{j}\chi_{dij}\chi_d^{ji} + \phi_{dj}^{i}\phi_{di}^{j}\chi_{vij}\chi_v^{ji}).
\end{aligned}
\tag{A1}
$$

The parameters are $\lambda_{t1}, \mu_t, \lambda_{t2}$ as well as $\lambda_{a1}, \mu_a, \lambda_{a2}, \kappa_a$ and κ_t. In addition to these there are five cross terms arising from nontrivial contractions between our representations, with parameters $(C_0, C_1, C_2, C_3, C_4)$. In general the asymmetry required can be attained by making these additional cross term parameters smaller than C_0 and the other parameters of the model. In minimizing this potential we can reduce the total number of parameters by placing all of our fields in a simplified VEV form. The adjoint can be represented by the traceless matrix

$$
\langle \phi_v \rangle = v_v
\begin{pmatrix}
\alpha_1 & 0 & 0 & 0 & 0 \\
0 & \alpha_2 & 0 & 0 & 0 \\
0 & 0 & \alpha_3 & 0 & 0 \\
0 & 0 & 0 & \alpha_4 & 0 \\
0 & 0 & 0 & 0 & \alpha_5
\end{pmatrix},
\tag{A2}
$$

with $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 0$. For the **10** we have

$$
\langle \chi_d \rangle = v_d
\begin{pmatrix}
0 & \rho_1 & 0 & 0 & 0 \\
-\rho_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \rho_2 & 0 \\
0 & 0 & -\rho_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix},
\tag{A3}
$$

with $\rho_{1,2}$ complex. The **24** and **10** are both reduced to just four total different degrees of freedom each in this form. Working numerically we can however quickly compare the results of using just these 16 degrees of freedom or the full 68; they were found to agree in all cases. The parameter space is directly comparable to that of the simple model of Sec. III. The positive definite terms act exactly like collections of additional fields that one could add to that previous model with the same-sector and cross-sector couplings needed to generate asymmetric VEVs that differentiate entire sets of fields within these multiplets. That is, if κ_a is large enough then if all (ϕ_{vj}^{i}) fields gain a nonzero VEV, all of the fields (ϕ_{dj}^{i}) are encouraged to become zero. Together with (C_1, C_2, C_3, C_4) there is a greater variability for the signs of quartic terms of the potential. Scaling any of these additional quartics too high may alter the VEV pattern from the desired asymmetric pattern. A larger value of C_0 will however ensure the breaking is the extension of that in Sec. III. To be concrete, we display an example of some parameters set along these guidelines and the VEVs that are produced. The parameters

$$
\begin{aligned}
&\lambda_{a1} \simeq 0.4, \quad \kappa_a \simeq 0.4, \quad \kappa_t \simeq 0.4, \quad \lambda_{t1} \simeq 0.8, \\
&\mu_t \simeq 0.2, \quad \mu_a \simeq 0.1, \quad \lambda_{a2} \simeq 0.1, \quad \lambda_{t2} \simeq -0.1, \\
&C_0 \simeq 0.5, \quad C_1 \simeq -0.1, \quad C_2 \simeq -0.1, \quad C_3 \simeq -0.1, \\
&C_4 \simeq -0.1
\end{aligned}
\tag{A4}
$$

give rise to the VEVs

$$
\langle \phi_v \rangle \simeq 0.24
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -3/2 & 0 \\
0 & 0 & 0 & 0 & -3/2
\end{pmatrix},
$$

$$
\langle \chi_v \rangle \simeq 0, \quad \langle \phi_d \rangle \simeq 0,
$$

$$
\langle \chi_d \rangle \simeq 0.1
\begin{pmatrix}
0 & 1+i & 0 & 0 & 0 \\
-1-i & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}.
\tag{A5}
$$

APPENDIX B: SCALAR POTENTIAL FOR SUPERSYMMETRIC SU(5) × SU(5) MODEL

In this section we will discuss further the results of the supersymmetric version of asymmetric symmetry breaking. The analysis here only serves to demonstrate that such asymmetric patterns are possible within the constraints inherent in supersymmetric theories.

Positive definite couplings between fields of different sectors are required to create the anticorrelation between sectors. This is what necessitates a field which transforms into itself under the discrete symmetry. An alternative to this could be to arm the theory with a pair of complete singlets under the discrete symmetry, i.e. S_v, S_d. Without such additions we are unable to create gauge-invariant terms in the superpotential which can allow for cross-sector couplings to appear in the F-terms. The other addition we made of the multiplet Y was based on our choice of complex representations.[3] We wish, however, to demonstrate that the theory which we used previously can be adopted into a supersymmetric form with the same gauge group breaking chains. The terms that we wish to highlight that are derived from the superpotential are the contractions of the form

$$s_4^2(\Phi_v\Phi_v X_v X_v + \Phi_v\Phi_v Y_v Y_v + \Phi_d\Phi_d X_d X_d + \Phi_d\Phi_d Y_d Y_d). \tag{B1}$$

It is clear that the parameter s_4 being larger can help lead to asymmetric VEVs. The other important parameter is s_5 which affects the term

$$s_5^2(\Phi_d\Phi_d\Phi_v\Phi_v). \tag{B2}$$

With just these terms and the additional soft masses one can generate an asymmetric VEV pattern. For the parameter example

$$s_4 = s_5 \simeq 0.02, \qquad g_5 \simeq 0.037, \qquad s_9 \simeq 0.001,$$
$$m_X = m_Y \simeq 0.001, \qquad m_\Phi \simeq 0.1, \qquad m_S = 0, \tag{B3}$$

and all trilinear terms and other parameters set at or close to zero, we obtain nonzero VEVs for the adjoint in one sector and for the fields X_v and Y_d in the other sector which serve to break $SU(5)_v$ to the standard model gauge group and $SU(5)_d$ to the dark sector gauge group with VEVs

$$\langle\Phi_v\rangle \simeq 2.1 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & -3/2 \end{pmatrix},$$

$$\langle X_v\rangle \simeq 0, \qquad \langle Y_v\rangle \simeq 0, \qquad \langle\Phi_d\rangle \simeq 0, \qquad \langle S\rangle \simeq 0,$$

$$\langle X_d\rangle \simeq \begin{pmatrix} 0 & 1.2+2.9i & 0 & 0 & 0 \\ -1.2-2.9i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.53-2.1i & 0 \\ 0 & 0 & 1.5+2.1i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\langle Y_d\rangle \simeq \begin{pmatrix} 0 & 1.2-2.9i & 0 & 0 & 0 \\ -1.2+2.9i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.53+2.1i & 0 \\ 0 & 0 & 1.5i-2.1i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{B4}$$

This demonstrates the capacity for supersymmetric models to display the same asymmetric symmetry breaking as non-SUSY models. There are other terms which can contribute to the asymmetric pattern, i.e. contractions of the style $(X_d X_d X_v X_v)$, but scaling these up to be larger also scales upwards terms that we would need to contend with to maintain the asymmetry.

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[3]This may of course not be necessary, if one was working with two different real representations to facilitate different symmetry breaking in each sector. In that case the procedure would be more straightforward.

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