Vorticity and magnetic field production in relativistic ideal fluids

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In the framework of relativistic ideal hydrodynamics, we study the production mechanism for vorticity and magnetic field in relativistic ideal fluids. It is demonstrated that in the uncharged fluids the thermal vorticity will always satisfy the Kelvin's theorem and the circulation must be conserved. However, in the charged fluids, the vorticity and magnetic field can be produced by the interaction between the entropy gradients and the fluid velocity gradients. Especially, in the multiple charged fluids, the vorticity and magnetic field can be produced by the interaction between the inhomogeneous charge density ratio and the fluid velocity gradients even if the entropy distribution is homogeneous, which provides another mechanism for the production of vorticity and magnetic field in relativistic plasmas or in the early universe.

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I. INTRODUCTION

It is well known that the Universe is filled with vorticities and magnetic fields on all scales [1–5]. However the origin of these vorticities and magnetic fields is still one of the most challenging open problems in theoretical physics [5–8]. In the nonrelativistic ideal fluid, Kelvin's circulation theorem in hydrodynamics or generalized version in magnetic hydrodynamics forbids the vorticity or magnetic fields to emerge from a zero initial value when the fluid is barotropic [9–11]. In order to produce seed vorticity or magnetic field, we must resort to the baroclinic effects or go beyond the ideal fluids by including diffusive terms [9–14]. The presence of large-scale magnetic fields in the Universe [15–21], indicates the very possibility that the magnetic fields should have been present in the early universe [22–24], in which the temperature of the Universe is very high and the velocities of the fluid and the particle components are both relativistic. Hence we need relativistic hydrodynamics to deal with these very hot fluid systems. Besides, relativistic hydrodynamics is also a very important theoretical tool in high- energy heavy-ion physics. The ideal and dissipative hydrodynamics has succeeded greatly in describing the collective flow from the data of RHIC and LHC. The study of the vorticity and magnetic field production in relativistic ideal fluids is very relevant to the important chiral effects, called the chiral-magnetic effects [25–29], chiral-vorticity effects [28,30], and local polarization effects [31,32], which can be expected in the noncentral heavy-ion collisions, because all these effects depend on the production of the vorticity and magnetic field in the quark-gluon plasma.

In the relativistic ideal fluids, there exists a similar covariant version of the Kelvin's circulation theorem [33–35]. It turns out that there are some subtleties when we deal with the relativistic case, which have been pointed out in Refs. [36,37] that vorticity and magnetic field can be produced in relativistic purely ideal fluid due to space-time distortion caused by the special relativity. However, all these relativistic investigations up to date on the vorticity and magnetic field, as far as we know, are only limited to the systems with single conserved charge and the particle components in the fluids are also specified with finite mass.

It is well known that there exist the systems without any conserved charges theoretically or realistically, such as neutral ϕ^4 field theory or photon gas. In particular, the hydrodynamic simulation used in relativistic heavy-ion collisions at RHIC or LHC are all based on the ones without charges and all the possible charge imbalance is neglected [38–45]. Hence it is very valuable to investigate both the neutral fluids and charged fluids all together and to see what could be missed only from the neutral hydrodynamic equations and how the novel phenomenology could appear in the single or multiple charged ones.

In this paper, we will extend these investigations to more general cases by direct manipulation of the relativistic hydrodynamic equations. We will not assume in advance that the particle components are massive or not, and the systems we will consider can have multiple conserving charges or no conserving charge at all. We find that the thermal vorticity will always satisfy the Kelvin's circulation theorem and be conserved in the uncharged fluids. However, in the charged fluids, especially in the multiple charged fluids, the vorticity and magnetic field can be produced not only by the interaction between inhomogeneous entropy and inhomogeneous fluid velocity magnitude but also by the interaction between inhomogeneous

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charge density ratio and inhomogeneous fluid velocity magnitude. The latter provides another new mechanism for the production of vorticity and magnetic field in the early universe or in the quark gluon plasma produced in heavy-ion collisions at RHIC or LHC.

II. VORTICITY IN RELATIVISTIC IDEAL UNCHARGED FLUIDS

In this section, we consider relativistic fluids without any conserving current, in which the hydrodynamical equations are just the energy-momentum conservation

$$\partial_\nu T^{\mu\nu} = 0, \tag{1}$$

where $T^{\mu\nu}$ is the energy-momentum tensor. In the ideal hydrodynamics, $T^{\mu\nu}$ can be decomposed into the following form,

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu}, \tag{2}$$

where ε is the energy density in the local frame, P is the pressure of the fluid, the metric tensor $g^{\mu\nu}$ is chosen as $(1, -1, -1, -1)$, and the fluid 4-velocity $u^\mu = (\gamma, \gamma \boldsymbol{v})$ with the relativistic kinematic factor $\gamma = 1/\sqrt{1-v^2}$ and the normalization $u^2 = 1$. Substituting Eq. (2) into Eq. (1) and contracting both sides with fluid velocity u^μ, we can have

$$u^\nu \partial_\nu \varepsilon + (\varepsilon + P)\partial_\nu u^\nu = 0. \tag{3}$$

With the general equations from thermodynamics,

$$T ds = d\varepsilon, \tag{4}$$

$$T s = \varepsilon + P, \tag{5}$$

it is easy to verify that Eq. (3) is just the entropy current conservation,

$$\partial_\mu (s u^\mu) = 0. \tag{6}$$

Using Eq. (5), we can rewrite the energy-momentum tensor as

$$T^{\mu\nu} = T s u^\mu u^\nu - P g^{\mu\nu}. \tag{7}$$

With the entropy conservation (6), the energy-momentum conservation can be rewritten by

$$s u^\nu \partial_\nu (T u^\mu) - \partial^\mu P = 0. \tag{8}$$

Using the Gibbs relation $dp = s dT$, we can have the following identity,

$$u^\nu \partial_\nu (T u^\mu) - \partial^\mu T = 0. \tag{9}$$

It is convenient to define the antisymmetric thermal vorticity tensor[1] $\Xi^{\mu\nu}$ by

$$\Xi^{\mu\nu} = \partial^\nu (T u^\mu) - \partial^\mu (T u^\nu), \tag{10}$$

which is in complete analogy to the definition of electromagnetic field tensor and can be regarded as inertia and thermal forces from the fluid. With such a definition, we can rewrite Eq. (9) as

$$\Xi^{\mu\nu} u_\nu = 0. \tag{11}$$

The circulation of the 4-vector temperature current $T u^\mu$ along the covariant loop $L(s)$ where s denotes the proper time is given by

$$\frac{d}{ds} \oint_{L(s)} T u^\mu dx_\mu = \oint_{L(s)} \Xi^{\mu\nu} u_\nu dx_\mu = 0, \tag{12}$$

which is just the relativistic Kelvin circulation theorem. For a specific observer, the vorticity is always defined in a fixed frame; hence, we need to consider the circulation of 3-vector temperature current $T\gamma \boldsymbol{v}$ along the synchronic loop $L(t)$. We specify the components of thermal vorticity tensor $\Xi^{\mu\nu}$ in the three-dimensional space as

$$\Xi^{\mu\nu} = \begin{pmatrix} 0 & -\mathcal{E}^1 & -\mathcal{E}^2 & -\mathcal{E}^3 \\ \mathcal{E}^1 & 0 & -\mathcal{B}^3 & \mathcal{B}^2 \\ \mathcal{E}^2 & \mathcal{B}^3 & 0 & -\mathcal{B}^1 \\ \mathcal{E}^3 & -\mathcal{B}^2 & \mathcal{B}^1 & 0 \end{pmatrix}, \tag{13}$$

with the 3-vector definition,

$$\begin{aligned} \mathcal{E} &= (\mathcal{E}^1, \mathcal{E}^2, \mathcal{E}^3) = [\nabla(T\gamma) - \partial_t (T\gamma \boldsymbol{v})], \\ \mathcal{B} &= (\mathcal{B}^1, \mathcal{B}^2, \mathcal{B}^3) = \nabla \times (T\gamma \boldsymbol{v}) \end{aligned} \tag{14}$$

With the above definition, we can express the space components of Eq. (11) as

$$\mathcal{E} + \boldsymbol{v} \times \mathcal{B} = 0, \tag{15}$$

or

$$\partial_t \left(\frac{T\boldsymbol{v}}{\sqrt{1-\boldsymbol{v}^2}} \right) + \left(\nabla \times \frac{T\boldsymbol{v}}{\sqrt{1-\boldsymbol{v}^2}} \right) \times \boldsymbol{v} = -\nabla \frac{T}{\sqrt{1-\boldsymbol{v}^2}}. \tag{16}$$

It should be noted that all through our paper the spatial hypersurfaces are always defined by the observer in the lab frame instead of the comoving frame with velocity u^μ. Using the above identity, we can immediately obtain the conservation of the thermal current circulation in synchronic space,

[1]It should be clarified that there exists in Refs. [46,47] another definition of the thermal vorticity from u^μ/T instead of Tu^μ here.

$$\frac{d}{dt}\oint_{L(t)} T\gamma \boldsymbol{v}\cdot d\boldsymbol{x} = \oint_{L(t)}\left[\partial_t\left(\frac{T\boldsymbol{v}}{\sqrt{1-\boldsymbol{v}^2}}\right) + \left(\nabla\times\frac{T\boldsymbol{v}}{\sqrt{1-\boldsymbol{v}^2}}\right)\times\boldsymbol{v}\right]\cdot d\boldsymbol{x}$$
$$= -\oint_{L(t)}\left[\nabla\frac{T}{\sqrt{1-\boldsymbol{v}^2}}\right]\cdot d\boldsymbol{x} = 0, \qquad (17)$$

which implies that the thermal vorticity cannot emerge from a zero initial value. It should be noted that although the circulation of the thermal current $T\gamma\boldsymbol{v}$ is conserved, that of the kinetic current $\gamma\boldsymbol{v}$ can be not conserved generally when the temperature is inhomogeneous. By applying the Stokes theorem, the conservation of circulation can be transformed into the conservation of the flux $\mathcal{B}$ through the surface which moves along with the fluid,

$$\frac{d}{dt}\int_{S(t)}\mathcal{B}\cdot d\boldsymbol{S} = \frac{d}{dt}\oint_{L(t)} T\gamma\boldsymbol{v}\cdot d\boldsymbol{x} = 0, \qquad (18)$$

which means that the vorticity field flux is conserved or the vorticity lines are frozen in.

It should be noted that our result here cannot be naively regarded as a particular case of Ref. [36,37] because all the derivations in [36,37] are based on nonzero charge density. Once we set the charge density to vanish, we need another derivation from the beginning. This is actually what we are devoted to do in this section.

III. VORTICITY AND MAGNETIC FIELDS IN RELATIVISTIC IDEAL MAGNETOHYDRODYNAMICS WITH MULTIPLE CURRENTS

In this section, we are devoted to discussing the relativistic fluids with multiple conserved currents. There are good reasons to investigate the hydrodynamics with multiple currents. For example, in the hot and dense QCD matter produced in heavy-ion collisions at high energy, one should be able to introduce electric charge, baryon number, and strangeness into the system. Therefore multicharge hydrodynamics is important in developing hydrodynamic models in heavy-ion collisions. As we already mentioned in the Introduction, the hydrodynamic simulations used in heavy-ion collisions at RHIC or LHC are all based on the ones without charges [38–45], and so it is very valuable to go beyond the neutral fluids and investigate how the novel phenomenology could appear in the single or multiple charged ones. Besides, in the early universe, the quantum or thermal fluctuations between different charges such as leptonic charge, electric charge, baryonic charge, and so on cannot coincide with each other; hence, it will be very important to investigate if such incoincidence could contribute to the production of vorticity or magnetic fields in the early universe.

Now let us assign one of the currents to the electric current J^μ from the local gauge symmetry, which can interact with the fluids by the magnetohydrodynamic equations. The other m currents $J_i^\mu (i = 1, 2, \ldots, m)$ are from the global symmetry, such as the baryonic current, leptonic current, and so on. Then the magnetohydrodynamic equations for such a system are given by

$$\partial_\nu T^{\mu\nu} = F^{\mu\nu}J_\nu, \qquad (19)$$

$$\partial_\mu J^\mu = 0, \qquad (20)$$

$$\partial_\mu J_i^\mu = 0, \quad (i = 1, 2, \ldots, m), \qquad (21)$$

where $F^{\mu\nu}$ is the electromagnetic stress tensor and can be written in terms of the electromagnetic 4-potential A^μ as $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$.

The constitutive equations for the ideal magnetohydrodynamics read

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - Pg^{\mu\nu} \qquad (22)$$

$$J^\mu = nu^\mu \qquad (23)$$

$$J_i^\mu = n_i u^\mu, \qquad (24)$$

where n is the electric charge density and n_i is charge density corresponding to other global symmetry. The energy-momentum conservation (19) and current conservation (20) can yield

$$nu^\nu\partial_\nu\left(\frac{\varepsilon+P}{n}u^\mu\right) - \partial^\mu P = nF^{\mu\nu}u_\nu. \qquad (25)$$

We can define the generalized thermal vorticity tensor $\Xi^{\mu\nu}$ by

$$\Xi^{\mu\nu} = F^{\mu\nu} + \partial^\nu(fu^\mu) - \partial^\mu(fu^\nu), \qquad (26)$$

with $f = (\varepsilon + P)/n$. We can rewrite Eq. (25) as

$$n\partial^\mu f - \partial^\mu P = n\Xi^{\mu\nu}u_\nu. \qquad (27)$$

Using the thermal equation

$$Tds = d\varepsilon - \mu dn - \sum_i \mu_i dn_i, \qquad (28)$$

$$Ts = \varepsilon + P - \mu n - \sum_i \mu_i n_i, \qquad (29)$$

we can have the Gibbs relation corresponding to the multiple charge components,

$$Td\left(\frac{s}{n}\right) = d\left(\frac{\varepsilon}{n}\right) + Pd\left(\frac{1}{n}\right) - \sum_i \mu_i d\left(\frac{n_i}{n}\right), \quad (30)$$

where μ and μ_i denote the chemical potentials with respect to different conserving charges. Now we can rewrite Eq. (27) as

$$\Xi^{\mu\nu} u_\nu = T\partial^\mu \left(\frac{s}{n}\right) + \sum_i \mu_i \partial^\mu \left(\frac{n_i}{n}\right). \quad (31)$$

It follows that the circulation of the 4-vector current $fu^\mu + A^\mu$ along the covariant loop $L(s)$ is given by

$$\begin{aligned} &\frac{d}{ds}\oint_{L(s)} (fu^\mu + A^\mu) dx_\mu \\ &= \oint_{L(s)} \Xi^{\mu\nu} u_\nu dx_\mu \\ &= \oint_{L(s)} \left[T\partial^\mu \left(\frac{s}{n}\right) + \sum_i \mu_i \partial^\mu \left(\frac{n_i}{n}\right)\right] dx_\mu, \end{aligned} \quad (32)$$

where $fu^\mu + A^\mu$ can be regarded as the canonical momentum or minimal coupling prescription (for details see [48]). It is obvious that the circulation of this 4-vector current is conserved when T and μ_i are constant. Just like we did in the last section, we need to consider the vorticity circulation of the synchronic loop $L(t)$. Let us define the 3-vector,

$$\begin{aligned} \mathcal{E} &= \boldsymbol{E} + [\nabla(f\gamma) - \partial_t(f\gamma\boldsymbol{v})], \\ \mathcal{B} &= \boldsymbol{B} + \nabla\times(f\gamma\boldsymbol{v}). \end{aligned} \quad (33)$$

Then the space components of Eq. (32) can be written as

$$\gamma(\mathcal{E} + \boldsymbol{v}\times\mathcal{B}) = T\nabla\left(\frac{s}{n}\right) + \sum_i \mu_i \nabla\left(\frac{n_i}{n}\right). \quad (34)$$

It follows that

$$\begin{aligned} \frac{d}{dt}\oint_L (f\gamma\boldsymbol{v} + \boldsymbol{A})\cdot d\boldsymbol{x} &= \frac{d}{dt}\int_S \mathcal{B}\cdot d\boldsymbol{S} \\ &= -\int_S \left[\nabla\left(\frac{T}{\gamma}\right)\times\nabla\left(\frac{s}{n}\right)\right]\cdot d\boldsymbol{S} \\ &\quad -\sum_i \int_S \left[\nabla\left(\frac{\mu_i}{\gamma}\right)\times\nabla\left(\frac{n_i}{n}\right)\right]\cdot d\boldsymbol{S}, \end{aligned} \quad (35)$$

where $\boldsymbol{A}$ is the spatial part of A^μ. The second line of the above equation is the source term which can lead to the vorticity or magnetic fields from the zero initial value. If we set $\mu_i = n_i = 0$, we will recover the results obtained in Ref. [36],

$$\frac{d}{dt}\oint_L (f\gamma\boldsymbol{v} + \boldsymbol{A})\cdot d\boldsymbol{x} = -\int_S \left[\nabla\left(\frac{T}{\gamma}\right)\times\nabla\left(\frac{s}{n}\right)\right]\cdot d\boldsymbol{S}. \quad (36)$$

As pointed out in Ref. [36], the source term can be decomposed into the usual baroclinic term,

$$S_b \equiv -\int_S \left[\frac{1}{\gamma}\nabla T\times\nabla\left(\frac{s}{n}\right)\right]\cdot d\boldsymbol{S}, \quad (37)$$

and the pure relativistic term,

$$S_r \equiv -\int_S \left[T\nabla\left(\frac{1}{\gamma}\right)\times\nabla\left(\frac{s}{n}\right)\right]\cdot d\boldsymbol{S}, \quad (38)$$

which is absent in the nonrelativistic limit. When the velocity and entropy gradients are comparable, the baroclinic term can be neglected in the highly relativistic region due to the estimate [36]

$$\frac{|S_r|}{|S_b|} \approx \frac{v^2}{1-v^2} \quad (39)$$

Now when the multiple currents are involved, we notice that an extra new term,

$$S_n \equiv -\sum_i \int_S \left[\nabla\left(\frac{\mu_i}{\gamma}\right)\times\nabla\left(\frac{n_i}{n}\right)\right]\cdot d\boldsymbol{S}, \quad (40)$$

arises. This is the principal result of this paper. It is very interesting that this term will generate the vorticity or magnetic field even when the entropy is homogeneous where the first term in the second line of Eq. (35) will vanish. This new term can be broken into two terms too; one is

$$S_{n\mu} \equiv -\sum_i \int_S \left[\frac{1}{\gamma}\nabla\mu_i\times\nabla\left(\frac{n_i}{n}\right)\right]\cdot d\boldsymbol{S}, \quad (41)$$

and the other is

$$S_{nr} \equiv -\sum_i \int_S \left[\mu_i\nabla\left(\frac{1}{\gamma}\right)\times\nabla\left(\frac{n_i}{n}\right)\right]\cdot d\boldsymbol{S}. \quad (42)$$

Following the similar argument for S_r and S_b above, when the velocity and chemical potential gradients are comparable, $S_{n\mu}$ term can be neglected in the highly relativistic region due to

$$\frac{|S_{nr}|}{|S_{n\mu}|} \approx \frac{v^2}{1-v^2}. \quad (43)$$

Therefore, the dominant contribution will be from the S_{nr} term.

Compared with the results obtained in Refs. [36,37], in which only the single conserved current is included, we have considered multiple currents in our work and obtained new contributions when the ratio of the different charge densities is inhomogeneous. This provides another possible mechanism for the production of vorticity and magnetic field in relativistic plasmas or in the early universe.

IV. DISCUSSION AND CONCLUSION

First, we emphasize that our result Eq. (17) for the ideal fluid without conserving current cannot be derived from the result Eq. (35) with conserving currents by naively taking the limit of $n \to 0$ and $n_i \to 0$ because there exists the $1/n$ term. Take the neutral ϕ^4 field or photon gas as examples. In these systems there is no conserved charge at all, and we cannot introduce the charge density from the beginning. That is why we must consider the ideal fluid without conserving currents separately. Although the result Eq. (36) with single current can be found in the literature everywhere, we failed to find the result Eq. (17) in the literature. Hence we have given the derivation of the result Eq. (17) in our paper in Sec. II. The result reveals that the thermal vorticity always satisfies the Kelvin's circulation theorem and cannot emerge from a zero initial value.

With respect to the result for the multiple currents in Eq. (35), the contribution from the terms of S_n or $S_{n\mu}$ and S_{nr} is new. These terms, especially the S_{nr} term, are very relevant to the production of vorticity or magnetic fields in the early universe, where the particles can carry different charges, such as leptonic charge, electric charge, baryonic charge, and so on. Besides, it is very relevant to the quark gluon plasma produced in heavy-ion collisions. If there is any inhomogeneous local distribution for some different charges, the vorticity or magnetic fields will be induced through the mechanism in Eq. (42). Then the chiral-magnetic effects, chiral-vorticity effects, and local polarization effects [26–32] will follow in the noncentral heavy-ion collisions.

The above result is very relevant to the recent investigation on the chiral vortical effect at RHIC. The baryon-number separation observed by STAR [49] can be explained by the chiral vortical effect; however, such interpretation to a great extent depends on the existence of large vorticity. In the Bjorken scaling scenario [50], which is widely used as the initial condition for the hydrodynamic equations in the relativistic heavy-ion collisions, the initial vorticity must be zero. The results in our present paper show that the vorticity will always be zero when we insist on using the ideal hydrodynamic equations without any conserved currents and the chiral vortical effect does not appear at all. If we still want to be in the regime of ideal hydrodynamics, in order to estimate the possible chiral vortical effect, we must resort to the hydrodynamic equations with single or multiple conserved currents.

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