PHYSICAL REVIEW D **90,** 077301 (2014)

Prediction of leptonic *CP* phase from perturbatively modified tribimaximal (or bimaximal) mixing

Sin Kyu Kang[1,2] and C. S. Kim[3]

[1]*School of Liberal Arts, Seoul-Tech, Seoul 139-743, Korea*
[2]*PITT PACC, Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA*
[3]*Department of Physics and IPAP, Yonsei University, Seoul 120-749, Korea*

(Received 16 July 2014; published 7 October 2014)

We consider the perturbatively modified tribimaximal (or bimaximal) mixing to estimate the (Dirac-type) CP phase in the neutrino mixing matrix. The expressions for the CP phase are derived from the equivalence between the standard parametrization of the neutrino mixing matrix for the Majorana neutrino and modified tribimaximal or bimaximal mixing matrices with appropriate CP phases. Carrying out numerical analysis based on the current experimental results for neutrino mixing angles, we can predict the values of the CP phase for several possible cases.

DOI: 10.1103/PhysRevD.90.077301 PACS numbers: 14.60.Pq, 14.60.Lm, 11.30.Er

I. INTRODUCTION

Recent measurements of not-so-small values of the reactor neutrino mixing angle have opened up new windows to probe leptonic CP violation (LCPV) [1]. Establishing LCPV is one of the most challenging tasks in future neutrino experiments [2]. The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [3] is presented by 3×3 unitary matrix which contains, in addition to the three angles, a Dirac type CP violating phase in general as it exists in the quark sector, and two extra phases if neutrinos are Majorana particles. Although we do not yet have compelling evidence for LCPV, the current fit to neutrino data indicates nontrivial values of the Dirac-type CP phase [4,5]. Several experiments have been proposed or are being scheduled to establish CP violation in neutrino oscillations [6]. In this situation, it must be worthwhile to investigate possible size of LCPV detectable through neutrino oscillations. From the point of view of *calculability*, it is conceivable that a Dirac type LCPV phase may be calculable with regards to some observables [7]. In this brief report, we propose possible forms of neutrino mixing matrix that lead us to estimate the size of LCPV phase, particularly, in terms of two neutrino mixing angles *only*, in the PDG-type standard parametrization [8]. The estimation of LCPV phase is carried out by the following procedure:

(i) Constructing the neutrino mixing matrix with appropriate CP phases so as to accommodate the current neutrino oscillation data in such a way to perturb conventional (tri)bimaximal matrix.
(ii) Deriving the master formulas linking the Dirac-type CP phase with neutrino mixing angles from the equivalence principle that any forms of neutrino mixing matrix should be equivalent to the standard parametrization of the PMNS mixing matrix.

As will be shown later, the neutrino mixing matrices we adopt at the first step contain a maximal mixing angle which plays a crucial role in deriving the relations among neutrino mixing angles and Dirac-type CP phase in the standard parametrization. Substituting values of neutrino mixing angles into those equations obtained at the second step, we perform numerical analysis on observables for the LCPV and present the results.

II. NEUTRINO MIXING MATRICES

In the leading order approximation, the conventional neutrino mixing matrices in the flavor basis can be given by

$$U_0^{\rm PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (1)$$

Taking $\sin\theta$ to be either $1/\sqrt{3}$ or $1/\sqrt{2}$ leads to the so-called tribimaximal mixing $U_0^{\rm TBM}$ or bimaximal mixing $U_0^{\rm BM}$, respectively [9,10]. Although the tribimaximal and/or bimaximal ones are theoretically well-motivated patterns of the neutrino mixing matrix, they are challenged by the current experimental results for three neutrino mixing angles. While the bimaximal mixing has already been ruled out by the nonmaximal mixing for the solar angle, the current measurements of nonzero θ_{13} definitely disfavor the exact tribimaximal mixing either. Since the measured values of θ_{13} have been turned out to be of the order of required deviation of θ_{12} from maximal, the tribimaximal mixing can be treated on the same footing with the bimaximal mixing as leading order approximation of the neutrino mixing matrix.

The simplest (i.e., minimally modified) possible forms of the neutrino mixing matrix without CP phases deviated from the (tri)bimaximal mixing patterns are given by [11,12]

$$\begin{cases} U_0^{(\mathrm{T})\mathrm{BM}} \cdot U_{ij}(\theta), \\ U_{ij}^{\dagger}(\theta) \cdot U_0^{(\mathrm{T})\mathrm{BM}}, \end{cases} \tag{2}$$

where $U_{ij}(\theta)$ represents the unitary matrix corresponding to the rotation with the angle θ in (i, j) plane. Please note that $U_0^{(\mathrm{T})\mathrm{BM}}$ can be achieved in a neutrino model with a flavor symmetry by breaking it down to two different residual symmetries preserved in the neutrino and the charged lepton sector, respectively [13]. In such a model, $U_{ij}(\theta)$ in the upper (lower) form of Eq. (2) is arisen from an appropriate breaking of the residual symmetry of the mass matrix in the neutrino (charged lepton) sector by adding a breaking term in (i, j) and (j, i) entries of the mass matrix. Once the mixing angle θ can be perturbatively treated [11], then Eq. (2) possibly gives rise to nonzero value of the reactor angle as well as deviation from the maximal for the solar angle. As will be explained shortly, eight forms among twelve possible ones in Eq. (2) are consistent with present neutrino data within 3σ C.L. In this respect, we call those eight forms of the neutrino mixing matrix *minimally-modified (tri)bimaximal* (M(T)BM) parametrizations. It is worthwhile to notice that those forms of the neutrino mixing matrix keep a column or a row in (tri)bimaximal mixing matrix unchanged, which may be regarded as a remnant of a possible horizontal symmetry leading to (tri)bimaximal mixing. The column vectors orthogonal to the ith and jth ones in $U_0^{(\mathrm{T})\mathrm{BM}}$ are unchanged for $U_0^{(\mathrm{T})\mathrm{BM}}U_{ij}(\theta)$, whereas the row vectors orthogonal to the ith and jth ones are unchanged for $U_{ij}^{\dagger}(\theta)U_0^{(\mathrm{T})\mathrm{BM}}$. The multiplication of $U_{ij}(\theta)$ represents unitary transformation of the symmetry operator which corresponds to the rotation of two column vectors in the mixing matrix. Thus, a symmetry argument[1] can still be applied to the origin of the neutrino mixing matrices in the M(T)BM parametrizations.

Since the Dirac-type CP phase δ_D is accompanied by θ_{13} in the standard parametrization, it is natural to involve CP phases when construct neutrino mixing matrix so as to generate nonzero θ_{13}. Interesting points in this work based on the simplest forms of neutrino mixing matrix aforementioned are that θ_{13} is related with either θ_{12} or θ_{23}, and δ_D can be related in the standard parametrization with two neutrino mixing angles as long as we identify the M(T)BM parameterizations with the standard one. Therefore, it is highly desirable to predict the Dirac-type CP phase with complex perturbations $U_{ij}(\theta, \xi)$ containing a phase ξ. Among the above twelve forms of the mixing matrix given in Eq. (2), the forms $U_0^{(\mathrm{T})\mathrm{BM}}U_{12}(\theta, \xi)$ and $U_{23}^{\dagger}(\theta, \xi)U_0^{(\mathrm{T})\mathrm{BM}}$ still lead to vanishing reactor mixing angle, and thus predict no CP violation. We do not consider these cases any longer. Therefore, all the possible forms of the MT(B)M mixing matrix eligible for our aim are presented as follows;

$$V = \begin{cases} U_0^{\mathrm{TBM}}U_{23}(\theta, \xi) & \text{(Case-A)}, \\ U_0^{\mathrm{TBM}}U_{13}(\theta, \xi) & \text{(Case-B)}, \\ U_{12}^{\dagger}(\theta, \xi)U_0^{\mathrm{TBM}} & \text{(Case-C)}, \\ U_{13}^{\dagger}(\theta, \xi)U_0^{\mathrm{TBM}} & \text{(Case-D)}, \\ U_{12}^{\dagger}(\theta, \xi)U_0^{\mathrm{BM}} & \text{(Case-E)}, \\ U_{13}^{\dagger}(\theta, \xi)U_0^{\mathrm{BM}} & \text{(Case-F)}, \\ U_0^{\mathrm{BM}}U_{23}(\theta, \xi) & \text{(Case-G)}, \\ U_0^{\mathrm{BM}}U_{13}(\theta, \xi) & \text{(Case-H)}. \end{cases} \tag{3}$$

While the cases (A), (C), and (E) have been studied in [12], the other cases have not been considered yet. For the completeness of possibility, we here propose a general way to extract Dirac-type CP phase from all possible forms given in Eq. (3), and show that not only the cases (A), (C), (E) but also the other cases (B), (D), (F) are still viable from the recent fit of neutrino mixing angles up to 3σ C.L. [5].

III. CALCULATION OF LEPTONIC CP VIOLATION

Now we demonstrate how to derive δ_D in terms of neutrino mixing angles in the standard parametrization. This can be done from the equivalence between one of the parametrizations in Eq. (3) and the standard parametrization, shown in Eq. (4).

Assuming that neutrinos are Majorana particles, we begin by explicitly presenting the PMNS neutrino mixing matrix in the PDG-type standard parametrization as follows [8],

$$U^{\mathrm{ST}} = U^{\mathrm{PMNS}} \cdot P_{\phi} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_D} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_D} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_D} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_D} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_D} & c_{23}c_{13} \end{pmatrix} P_{\phi}, \tag{4}$$

[1]C. S. Lam has shown [14] that the column vectors of the lepton mixing matrix can be eigenvectors of certain horizontal flavor symmetry. Keeping the symmetry point of view, we can construct the M(T)BM parametrizations of the neutrino mixing matrix by appropriately multiplying either tribimaximal or bimaximal mixing matrix by a unitary matrix while keeping a column or a row vector characterizing the horizontal symmetry unchanged.

where $P_\phi \equiv \mathrm{Diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ is a 3×3 phase matrix. Note that one of three phases in P_ϕ is redundant. Incorporating phase matrices P defined above, the neutrino mixing matrices in Eq. (3) are given by

$$U^{\mathrm{ST}} = P_\alpha \cdot V \cdot P_\beta.$$

Without those two phase matrices P_α and P_β, in general, we cannot equate the M(T)BM parametrizations with the standard parametrization given in Eq. (4). Please note that such biunitary transformation is regarded as a general basis change of leptonic fields [15]. The equivalence between both parametrizations dictates the following relation,

$$V_{ij} e^{i(\alpha_i+\beta_j)} = U^{\mathrm{ST}}_{ij} = U^{\mathrm{PMNS}}_{ij} e^{i\phi_j}. \tag{5}$$

Applying $|V_{13}| = |U^{\mathrm{ST}}_{13}|$ and $|V_{11}/V_{12}| = |U^{\mathrm{ST}}_{11}/U^{\mathrm{ST}}_{12}|$ to Cases A, B, G, and H, we obtain the relations between the solar and reactor mixing angles,

$$s_{12}^2 = \begin{cases} 1 - \frac{2}{3(1-s_{13}^2)} & \text{(Case-A)}, \\ \frac{1}{3(1-s_{13}^2)} & \text{(Case-B)}, \\ 1 - \frac{1}{2(1-s_{13}^2)} & \text{(Case-G)}, \\ \frac{1}{2(1-s_{13}^2)} & \text{(Case-H)}. \end{cases} \tag{6}$$

Those relations indicate that nonzero values of s_{13}^2 lead to $s_{12}^2 < 1/3$ for Case A and $s_{12}^2 > 1/3$ for Case B. While the results for Case A are consistent with the current experimental values of s_{12}^2 at 1σ C.L., those for Case B are so at 2σ C.L. It turns out that the above relations for Cases G and H are not consistent with experimental results up to 3σ C.L., and thus ruled out.

Similarly, we get the relations between the atmospheric and reactor mixing angles from $|V_{13}| = |U^{\mathrm{ST}}_{13}|$ and $|V_{23}/V_{33}| = |U^{\mathrm{ST}}_{23}/U^{\mathrm{ST}}_{33}|$,

$$s_{23}^2 = \begin{cases} 1 - \frac{1}{2(1-s_{13}^2)} & \text{(Cases C and E)}, \\ \frac{1}{2(1-s_{13}^2)} & \text{(Cases D and F)}. \end{cases} \tag{7}$$

We see that nonzero values of s_{13}^2 lead to the values of $s_{23}^2 < 1/2$ for Cases C and E and $s_{23}^2 > 1/2$ for Cases D and F. They turned out to be consistent with experimental values of s_{23}^2 at 2σ C.L.

Now, let us derive the relations among δ_D and neutrino mixing angles in the standard parametrization. Since the same method can be applied to all the cases, we only present how to derive the relation only for Case A. From the components of the neutrino mixing matrix for Case A, we see that

$$\frac{V_{23}+V_{33}}{V_{22}+V_{32}} = \frac{V_{13}}{V_{12}}. \tag{8}$$

Applying the relation (5) and $V_{21} = V_{31}$ to Eq. (8), we can get the relation

$$\frac{U^{\mathrm{ST}}_{13}}{U^{\mathrm{ST}}_{12}} = \frac{U^{\mathrm{ST}}_{23}U^{\mathrm{ST}}_{31} + U^{\mathrm{ST}}_{33}U^{\mathrm{ST}}_{21}}{U^{\mathrm{ST}}_{22}U^{\mathrm{ST}}_{31} + U^{\mathrm{ST}}_{32}U^{\mathrm{ST}}_{21}}. \tag{9}$$

Presenting U^{ST}_{ij} in terms of the neutrino mixing angles as well as δ_D, and taking the real part in Eq. (9), we get the equation for δ_D as

$$\cos\delta_D = \frac{-1}{2\tan 2\theta_{23}} \cdot \frac{1-5s_{13}^2}{s_{13}\sqrt{2-6s_{13}^2}}. \tag{10}$$

Notice that the imaginary part in Eq. (9) is automatically canceled. Using the above formulas, we can easily derive the leptonic Jarlskog invariant as follows;

$$J_{\mathrm{CP}}^2 = (\mathrm{Im}[U^{\mathrm{ST}}_{\mu 2}U^{\mathrm{ST}}_{e3}U^{\mathrm{ST}*}_{e2}U^{\mathrm{ST}*}_{\mu 3}])^2$$
$$= \frac{1}{8}\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})\sin\delta_D \tag{11}$$

$$= \frac{1}{12^2}[8s_{13}^2(1-3s_{13}^2) - \cos^2 2\theta_{23} c_{13}^4], \tag{12}$$

where Eq. (11) is obtained [2] by just inserting the entries of $U^{\mathrm{ST}}_{\alpha i}$ given in Eq. (4).

TABLE I. Formulas for $\cos\delta_D$ and J_{CP}^2 for Cases B–F. The second column corresponds to the relation (8) for Case A. $\eta_{ij} = \frac{1}{2\tan 2\theta_{ij}}$, $\kappa_{ij} = \cos^2 2\theta_{ij}\cdot c_{13}^4$, $\xi = \sin 2\theta_{12}$, and $\omega = (s_{13}^2(9s_{12}^2-4) - 3s_{12}^2 + 1)^2$

Cases		$\cos\delta_D$	J_{CP}^2
B	$\frac{V_{21}+V_{31}}{V_{23}+V_{33}} = \frac{V_{11}}{V_{13}}$	$\frac{2-4s_{13}^2}{s_{13}\sqrt{2-3s_{13}^2}}\eta_{23}$	$\frac{1}{6^2}[s_{13}^2(2-3s_{13}^2) - \kappa_{23}]$
C	$\frac{V_{11}+\sqrt{2}V_{12}}{V_{21}+\sqrt{2}V_{22}} = \frac{V_{13}}{V_{23}}$	$\frac{s_{13}^2-(1-3s_{12}^2)(1-3s_{13}^2)}{3s_{13}\sqrt{1-2s_{13}^2}\xi}$	$\frac{1}{12^2}[9c_{12}^2s_{12}^2s_{13}^2(1-2s_{13}^2) - \omega]$
D	$\frac{V_{11}+\sqrt{2}V_{12}}{V_{31}+\sqrt{2}V_{32}} = \frac{V_{13}}{V_{23}}$	$\frac{(1-3s_{13}^2)(1-3s_{12}^2)-s_{13}^2}{3s_{13}\sqrt{1-2s_{13}^2}\xi}$	$\frac{1}{12^2}[9c_{12}^2s_{12}^2s_{13}^2(1-2s_{13}^2) - \omega]$
E	$\frac{V_{12}+V_{11}}{V_{21}+V_{22}} = \frac{V_{13}}{V_{23}}$	$-\frac{1-3s_{13}^2}{s_{13}\sqrt{1-2s_{13}^2}}\eta_{12}$	$\frac{1}{8^2}[4s_{13}^2(1-2s_{13}^2) - \kappa_{12}]$
F	$\frac{V_{11}+V_{12}}{V_{32}+V_{31}} = \frac{V_{13}}{V_{33}}$	$\frac{1-3s_{13}^2}{s_{13}\sqrt{1-2s_{13}^2}}\eta_{12}$	$\frac{1}{8^2}[4s_{13}^2(1-2s_{13}^2) - \kappa_{12}]$

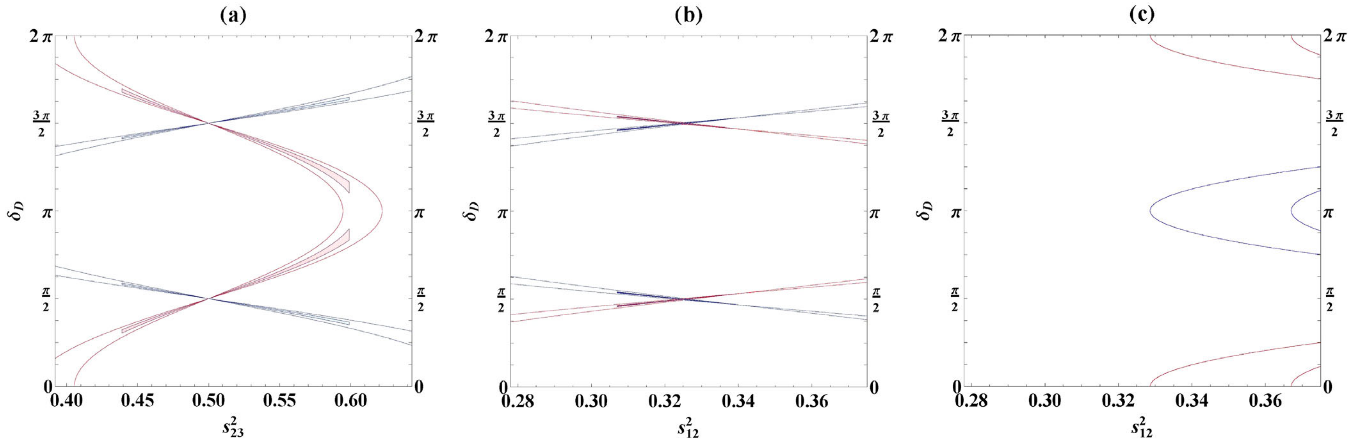

FIG. 1 (color online). Predictions of δ_D in terms of s^2_{23} [(a): Cases A and B] and s^2_{12} [(b): Cases C and D] [(c):Cases E and F] based on the experimental data at 3σ and 1σ (for Cases A-D) C.L. Regions surrounded by blue (red) lines correspond to Cases A, C, and E (B, D, and F).

By taking the same procedure described above, we can obtain the formulas for δ_D and J^2_{CP} for Cases B–F as presented in in Table I. Note that the Cases G and H are experimentally ruled out as previously mentioned.

A. Numerical results

For our numerical analysis, we take the current experimental data for three neutrino mixing angles as inputs, which are given at $1\sigma - 3\sigma$ C.L., as presented in Ref. [5]. Here, we perform numerical analysis and present results only for normal hierarchical neutrino mass spectrum. It is straightforward to get numerical results for the inverted hierarchical case. Using experimental results for three neutrino mixing angles, we estimate the values of δ_D and $J_{\rm CP}$ in terms of neutrino mixing angles through Eqs. (10) and (12), respectively, and the formulas presented in Table I.

Figure 1 shows the predictions of δ_D in terms of s^2_{23} [(a): Cases A and B] and s^2_{12} [(b): Cases C and D] [(c): Cases E and F] based on the corresponding experimental data given at 3σ C.L. Regions surrounded by blue (red) lines correspond to Cases A, C, and E (B, D, and F). In particular, the small dark regions in Fig. 1-(a) and (b) correspond to the results obtained by using the experimental data at 1σ C.L. for Cases A-D which apparently indicate CP violation. The width of each bands implies the variation of the other mixing angles, s^2_{12} (Cases A and B) and s^2_{23} (Cases C-F). We see that almost maximal $\delta_D \sim \pi/2, 3\pi/2$ can be achieved by $s^2_{23} \sim 0.5$ for Cases A, B and by $s^2_{12} \sim 0.325$ for Cases C,D. It turns out that the magnitude of CP violation is not large for Cases E and F.

In Figs. 2, 3, and 4, we display contour plots for each value of $|J_{\rm CP}|$ in the planes of (s^2_{23}, s^2_{13}) (a-d) and (s^2_{12}, s^2_{13}) (e,f). The panels (a) [(c)] and (b) [(d)] correspond to the results for Case A(B) obtained by using the experimental data at 1σ and 3σ C.L., respectively. The results for Cases C (D) and E (F) based on the experimental data at 3σ C.L. are displayed in the panels (e) and (f), respectively. We note that the sizes of $|J_{\rm CP}|$ in the lepton sector for Cases A and B can be as large as 0.03–0.04 which are much larger than the values of the quark sector as order of 10^{-5}, and expected to be measurable in foreseeable future. Such a large value of $|J_{\rm CP}|$ can be anticipated from Eq. (11) by imposing the experimental values of neutrino mixing angles for large CP phase $\delta_D \sim \pi/2$, since we are led from Eq. (11) to $J_{\rm CP} \sim 0.035 \sin\delta_D$ for the central values of experimental data for the neutrino mixing angles. For Cases C, D, and F, most parameter space predicts the values $|J_{\rm CP}|$ less than 0.03, as

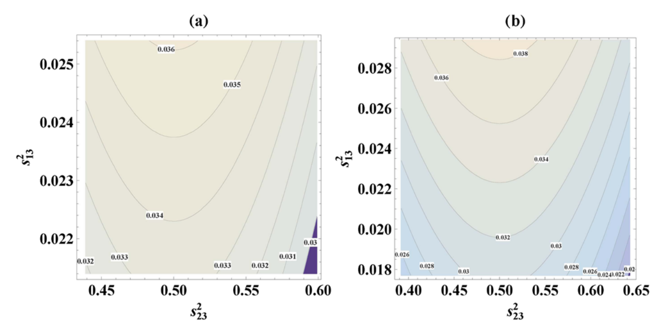

FIG. 2 (color online). Contour plots for each values of $J_{\rm CP}$ in the plane (s^2_{23}, s^2_{13}) for (a) Case A (1σ), (b) A (3σ).

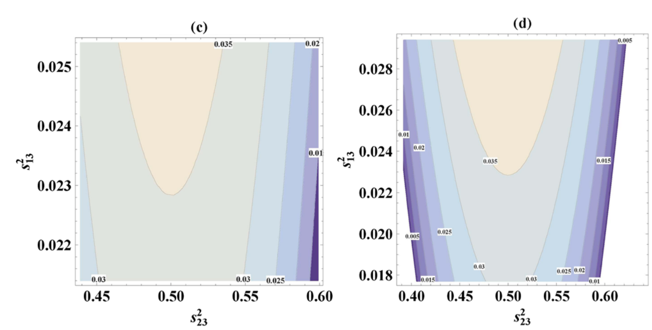

FIG. 3 (color online). Contour plots for each values of $J_{\rm CP}$ in the plane (s^2_{23}, s^2_{13}) for (c) Case B (1σ) and (d) B (3σ).

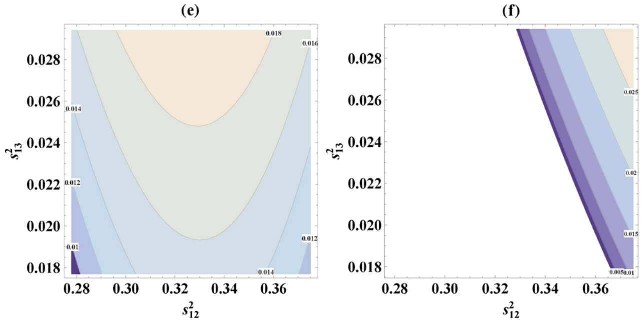

FIG. 4 (color online). Contour plots for each values of J_{CP} in the plane (s^2_{12}, s^2_{13}) for Cases (e) C, D and (f) E, F.

shown in Fig. 4 (e) and (f). We see from Fig. 4 (f) that the region of $s^2_{12} < 0.32$ for Cases E and F is excluded because it leads to $|\cos\delta_D| > 1$ for the experimentally allowed region of s^2_{13}.

ACKNOWLEDGMENTS

The work of C. S. K. was supported by the NRF grant funded by Korea government of the MEST (No. 2011-0017430) and (No. 2011-0020333). The work of S. K. K. was supported by the NRF grant funded by Korea government of the MEST (No. 2011-0029758).

[1] F. P. An *et al.* (Daya Bay Collaboration), Phys. Rev. Lett. **108**, 171803 (2012); Chin. Phys. C **37**, 011001 (2013); J. K. Ahn *et al.* (RENO Collaboration), Phys. Rev. Lett. **108**, 191802 (2012); Y. Abe *et al.* (Double Chooz Collaboration), Phys. Rev. Lett. **108**, 131801 (2012).

[2] G. C. Branco, R. G. Felipe, and F. R. Joaquim, Rev. Mod. Phys. **84**, 515 (2012).

[3] B. Pontecorvo, Sov. Phys. JETP **6**, 429 (1957); **26**, 984 (1968); Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[4] F. Capozzi, G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, and A. Palazzo, Phys. Rev. D **89**, 093018 (2014); see also G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, and A. M. Rotunno, Phys. Rev. D **86**, 013012 (2012).

[5] D. V. Forero, M. Tortola, and J. W. F. Valle, arXiv:1405.7540.

[6] See for examples, S. Choubey *et al.* (IDS-NF Collaboration), arXiv:1112.2853; A. Bandyopadhyay *et al.* (ISS Physics Working Group Collaboration), Rep. Prog. Phys. **72**, 106201 (2009).

[7] Below is a list of references showing connections between some of neutrino mixing angles and CP phases in different ways, but we believe that their predictions are not as general as ours, and depend on further parameters or assumptions: S. K. Kang, C. S. Kim, and J. D. Kim, Phys. Rev. D **62**, 073011 (2000); M. Fukugita and M. Tanimoto, Phys. Lett. B **515**, 30 (2001); C. Giunti and M. Tanimoto, Phys. Rev. D **66**, 113006 (2002); Z.-z. Xing, Phys. Lett. B **533**, 85 (2002); W.-l. Guo and Z.-z. Xing, Phys. Lett. B **583**, 163 (2004); S. T. Petcov and W. Rodejohann, Phys. Rev. D **71**, 073002 (2005); R. N. Mohapatra and W. Rodejohann, Phys. Rev. D **72**, 053001 (2005); S. Antusch and S. F. King, Phys. Lett. B **631**, 42 (2005); Z.-z. Xing, H. Zhang, and S. Zhou, Phys. Lett. B **641**, 189 (2006); J. Harada, Europhys. Lett. **75**, 248 (2006); R. N. Mohapatra and H.-B. Yu, Phys. Lett. B **644**, 346 (2007); Z.-z. Xing, H. Zhang, and S. Zhou, Int. J. Mod. Phys. A **23**, 3384 (2008); S.-F. Ge, D. A. Dicus, and W. W. Repko, Phys. Lett. B **702**, 220 (2011); D. Marzocca, S. T. Petcov, A. Romanino, and M. Spinrath, J. High Energy Phys. 11 (2011) 009; H.-J. He and F.-R. Yin, Phys. Rev. D **84**, 033009 (2011); C. Duarah, A. Das, and N. N. Singh, arXiv:1210.8265; N. Razzaghi and S. S. Gousheh, Phys. Rev. D **86**, 053006 (2012); Y. Shimizu and M. Tanimoto, arXiv:1405.1521.

[8] Particle Data Group, http://pdg.lbl.gov; J. Beringer *et al.*, Phys. Rev. D **86**, 010001 (2012).

[9] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B **530**, 167 (2002); G. Altarelli, F. Feruglio, L. Merlo, and E. Stamou, J. High Energy Phys. 08 (2012) 021.

[10] F. Vissani, arXiv:hep-ph/9708483; V. D. Barger, S. Pakvasa, T. J. Weiler, and K. Whisnant, Phys. Lett. B **437**, 107 (1998); A. J. Baltz, A. S. Goldhaber, and M. Goldhaber, Phys. Rev. Lett. **81**, 5730 (1998); H. Fritzsch and Z.-z. Xing, Phys. Lett. B **440**, 313 (1998); R. N. Mohapatra and S. Nussinov, Phys. Lett. B **441**, 299 (1998); S. K. Kang and C. S. Kim, Phys. Rev. D **59**, 091302 (1999); G. Altarelli, F. Feruglio, and L. Merlo, J. High Energy Phys. 05 (2009) 020.

[11] X.-G. He and A. Zee, Phys. Lett. B **645**, 427 (2007); Phys. Rev. D **84**, 053004 (2011); C. H. Albright and W. Rodejohann, Eur. Phys. J. C **62**, 599 (2009); C. H. Albright, A. Dueck, and W. Rodejohann, Eur. Phys. J. C **70**, 1099 (2010); Z.-Z. Xing, Chin. Phys. C **36**, 101 (2012); Chin. Phys. C **36**, 281 (2012); W. Chao and Y.-j. Zheng, J. High Energy Phys. 02 (2013) 044; S. K. Garg and S. Gupta, J. High Energy Phys. 10 (2013) 128.

[12] W. Rodejohann and H. Zhang, Phys. Rev. D **86**, 093008 (2012); D. Marzocca, S. T. Petcov, A. Romanino, and M. C. Sevilla, J. High Energy Phys. 05 (2013) 073; Z.-z. Xing and S. Zhou, Phys. Lett. B **737**, 196 (2014).

[13] D. Hernandez and A. Y. Smirnov, Phys. Rev. D **86**, 053014 (2012); **87**, 053005 (2013).

[14] C. S. Lam, Phys. Rev. D **74**, 113004 (2006);Phys. Lett. B **656**, 193 (2007);Phys. Rev. D **87**, 013001 (2013).

[15] S. Antusch, J. Kersten, M. Lindner, and M. Ratz, Nucl. Phys. **B674**, 401 (2003).