PHYSICAL REVIEW D **90,** 076008 (2014)

Study of $B\bar{B}^*/D\bar{D}^*$ bound states in a Bethe-Salpeter approach

Jun He[*]

Theoretical Physics Division, Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China;
Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China;
and State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

(Received 12 June 2014; published 20 October 2014)

In this work the $B\bar{B}^*/D\bar{D}^*$ system is studied in the Bethe-Salpeter approach with quasipotential approximation. In our calculation both direct and cross diagrams are included in the one-boson-exchange potential. The numerical results indicate the existence of an isoscalar bound state $D\bar{D}^*$ with $J^{PC}=1^{++}$, which may be related to the $X(3872)$. In the isovector sector, no bound state is produced from the interactions of $D\bar{D}^*$ and $B\bar{B}^*$, which suggests the molecular state explanations for $Z_b(10610)$ and $Z_c(3900)$ are excluded.

DOI: 10.1103/PhysRevD.90.076008 PACS numbers: 11.10.St, 14.40.Rt, 21.30.Fe

I. INTRODUCTION

The deuteron is a loosely bound state of two nucleons. It is natural to expect other bound states composed of two hadrons, that is hadronic molecular state [1]. After the observation by the Belle Collaboration [2] the $X(3872)$ was related to a loosely bound state of $D\bar{D}^*$ immediately [3,4] due to its mass near the $D\bar{D}^*$ threshold. Recently, the Belle Collaboration announced two charged bottomonium-like structures $Z_b(10610)$ and $Z_b(10650)$ near the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds [5]. The analysis of the angular distribution indicated both $Z_b(10610)$ and $Z_b(10650)$ favor $I^G(J^P)=1^+(1^+)$. A structure $Z_c(3900)$ close to the $D\bar{D}^*$ threshold was also observed by the BESIII collaboration in the decay of $Y(4260)$, $Y(4260)\to\pi^+\pi^- J/\psi$ [6].

In Refs. [7,8], the $B\bar{B}^*/D\bar{D}^*$ system was studied with a nonrelativistic one-boson-exchange (OBE) model by solving the Schödinger equation. There exists a bound state solution with quantum number $J^{PC}=1^{++}$ from $B\bar{B}^*$ interaction while there exists no bound state solution from $D\bar{D}^*$ interaction. The results also suggested the importance of π exchange [8]. It is easy to understand because the binding energy is small for a hadronic molecular state so that long range interaction should be more important than short range interaction. In the nonrelatvistic OBE model, potential $V(\boldsymbol{r})$ is obtained with a direct Fourier transformation on $\boldsymbol{q}$ [1,7]. However, for hadronic molecular state, a system composed of two constituents with different masses and/or spins is often involved, which is different from the deuteron where proton and neutron are indistinguishable under isospin $SU(2)$ symmetry. Such difference may lead to invalidness of the potential model in coordinate space, which has been ignored always.

The molecular state is a loosely bound state of two hadrons, so the Bethe-Salpeter equation (BSE) is an appropriate tool to deal with the molecular state. For example, BSE was used to study deuteron and experimental data about deuteron and nucleon-nucleon interaction were well reproduced [9,10]. In Ref. [11] the $B\bar{B}^*$ system has been studied in the BSE approach with a quasipotential approximation. However, in their study only direct diagram was included in the calculation. Hence, the most important π exchange as found in Ref. [8], was not included. In Ref. [12] the $Y(4274)$ and its three body decay were studied in the BS equation approach with nonrelativistic approximation [12]. And this method was successfully applied to the $D_0^*(2400)N$ system. The $\Sigma_c(3250)$ reported by the *BABAR* collaboration recently can be explained as a $D_0^*(2400)N$ molecular state [13]. In this work, we will develop a relativistic theoretical frame in BES approach to study the $B\bar{B}^*/D\bar{D}^*$ system with π exchange and search bound sate solution to understand the structures $Z_b(10610)$ and $Z_c(3900)$.

This work is organized as follows. In next section we present theoretical frame to study the $B\bar{B}^*/D\bar{D}^*$ system through solving the BSE. In Sec. III, the OBE potential is derived with the help of the effective Lagrangian from the heavy quark effective theory. The numerical results are given in Sec. IV. In the last section, a brief summary is given.

II. BSE WITH QUASIPOTENTIAL APPROXIMATION

The BSE of the vertex function $\Gamma\rangle$ in general form is [14]

$$|\Gamma\rangle = \mathcal{V}G|\Gamma\rangle, \tag{1}$$

where $\mathcal{V}$ and G are the potential kernel and the propagator.

[*]junhe@impcas.ac.cn

A. BSE with definite quantum numbers

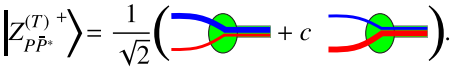
The $P\bar{P}^*$ systems (here and hereafter we mark B or D as P) can be categorized as the isovector (T) and isoscalar (S) states under SU(3) symmetry with corresponding flavor wave functions [7,8],

$$\begin{cases} \left|Z^{(T)\,+}_{P\bar{P}^*}\right\rangle = \frac{1}{\sqrt{2}}(|P^{*+}\bar{P}^0\rangle + cP^+\bar{P}^{*0}), \\ \left|Z^{(T)\,-}_{P\bar{P}^*}\right\rangle = \frac{1}{\sqrt{2}}(|P^{*-}\bar{P}^0\rangle + cP^-\bar{P}^{*0}), \\ \left|Z^{(T)\,0}_{P\bar{P}^*}\right\rangle = \frac{1}{2}[(|P^{*+}P^-\rangle - P^{*0}\bar{P}^0) + c(P^+P^{*-} - P^0\bar{P}^{*0})], \end{cases} \tag{2}$$

$$\left|Z^{(S)\,0}_{P\bar{P}^*}\right\rangle = \frac{1}{2}[(|P^{*+}P^-\rangle + P^{*0}\bar{P}^0) + c(P^+P^{*-} + P^0\bar{P}^{*0})], \tag{3}$$

where $c = \pm$ corresponds to C-parity $C = \mp$, respectively.

Now we introduce the BSE with definite quantum numbers especially isospin I and charge parity C. Here we take the positive charged system $|Z^{(T)\,+}_{P\bar{P}^*}\rangle$ as example to explain how to obtain the BSE for a system with definite quantum numbers. First, the coupled channel BSE for the two-component of the positive charged system,

$\left|Z^{(T)\,+}_{P\bar{P}^*}\right\rangle =$  (4)

is figured in the first line of Fig. 1.

The coupled channel equation can be rewritten as two equations shown in the second line. And we can construct the vertex with definite quantum numbers as Eq. (4). After flavor factors are isolated, two components of $|Z^{(T)\,+}_{P\bar{P}^*}\rangle$ should be the same under isospin $SU(2)$ symmetry. Hence, after rearranging the notations of the momenta in two equations in the second line of Fig. 1 and summing them up, we reach one equation at the last line of Fig. 1. Here the flavor factors are isolated out as I^i_d and I^j_c for direct diagram and cross diagram with i for different exchanged light meson, which is the same as the flavor factors used in the nonrelativistic OBE model [15,16]. In the existing OBE model calculation, the final momenta of cross diagram are not exchanged correctly [3,7,15]. This exchange does not affect the nucleon-nucleon interaction in the study of nuclear force, the u/d quark-quark interaction in the consistent quark model and $P\bar{P}/P^*\bar{P}^*$ system due to the same masses and spins of nucleon, u/d quark and the $P\bar{P}/P^*\bar{P}^*$ under $SU(2)$ symmetry. However, for the $P\bar{P}^*$ system composed of two constituents with different masses and spins, it is essential to make such exchange in the cross diagram.

B. Quasipotential approximation in covariant spectator theory

The BSE of the vertex function Γ for a system composed of a vector meson and a pseudoscalar meson (marked as constituent 1 and 2) is written explicitly as

$$|\Gamma^\mu\rangle = \mathcal{V}^{\mu\nu}G_{\nu\mu'}|\Gamma^{\mu'}\rangle, \tag{5}$$

where the propagator is

$$G^{\mu'\mu} = G_1^{\mu'\mu}G_2 = \frac{-P_1^{\mu'\mu}}{(k_1^2 - m_1^2)(k_2^2 - m_2^2)} \equiv P_1^{\mu'\mu}G_0, \tag{6}$$

where $P_1^{\mu\nu} = -g^{\mu\nu} + \frac{k_1^\mu k_1^\nu}{m_1^2}$ and $k_{1,2}$ and $m_{1,2}$ are the momentum and mass for constituent 1 or 2.

As in the study of nucleon-nucleon interaction, a quasipotential approach should be used to reduce 4-dimension equation to 3-dimension equation. Here we adopt the covariant spectator theory which is suitable to study a system with different constituents [9]. The heavy constituent, here the vector meson marked as constituent 1,

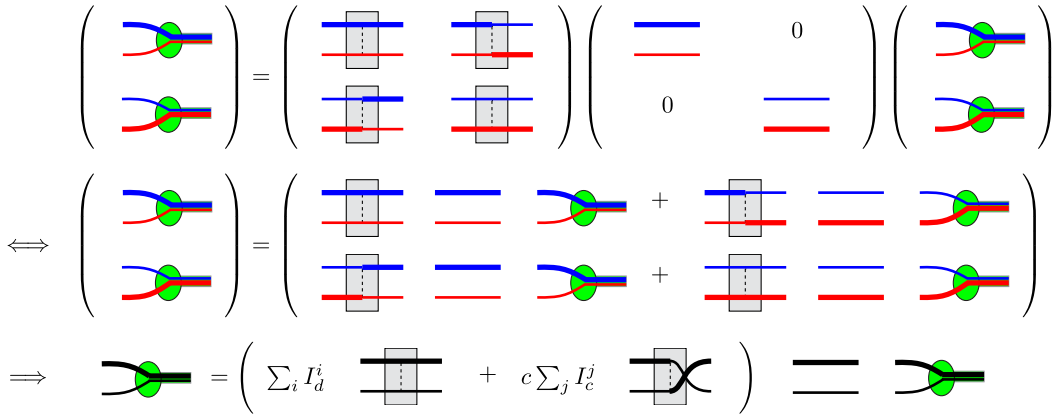


FIG. 1 (color online). The BS equation for $|Z^{(T)\,+}_{P\bar{P}^*}\rangle$. The thick and thin lines are for pseudoscalar and vector mesons, respectively. The red (gray) and blue (black) lines are for charged and neutral mesons, respectively. The black lines are for the diagram after isolating the flavor factors to I^i. In the last line the $SU(2)$ symmetry is applied.

is treated as on-shell, so the numerator of propagator can be written as $P_1^{\mu\nu} = \epsilon_{1\lambda}^{\mu}\epsilon_{1\lambda}^{\nu}$ with λ being the helicity of vector meson. Here and hereafter the sum notation about helicity λ is omitted. After multiplying the polarized vector $\epsilon_{1\lambda}^{\mu}$, we have

$$|\Gamma_\lambda\rangle = \mathcal{V}_{\lambda\lambda'} G_0 |\Gamma_{\lambda'}\rangle, \tag{7}$$

with $|\Gamma_\lambda\rangle = \epsilon_{1\lambda}^{\mu}|\Gamma_\mu\rangle$ and $\mathcal{V}_{\lambda\lambda'} = \epsilon_{1\lambda}^{\mu}\cdot\mathcal{V}_{\mu\nu}\cdot\epsilon_{1\lambda'}^{\nu}$.

The propagator written down in the center of mass frame where $p = (W, \boldsymbol{0})$ is

$$G_0 = 2\pi i \frac{\delta^+(k_1^2 - m_1^2)}{k_2^2 - m_2^2} = 2\pi i \frac{\delta^+(k_1^0 - E_1))}{2E_1[(W-E_1)^2 - E_2^2]}, \tag{8}$$

where $E_{1,2} = \sqrt{m_{1,2}^2 + |\boldsymbol{k}|^2}$. After moving a factor,

$$A = \sqrt{\frac{2E_2}{W - E_1 + E_2}}, \tag{9}$$

to potential kernel and vertex, we have $\bar{\mathcal{V}}_{\lambda\lambda'} = A\mathcal{V}_{\lambda\lambda'}A'$, $|\bar{\Gamma}_\lambda\rangle = A|\Gamma_\lambda\rangle$ and $\bar{G}_0 = G_0/A^2$. The vertex function can be related to the Bethe-Salpeter bound state wave function as $|\psi_\lambda\rangle = \bar{G}_0|\bar{\Gamma}_\lambda\rangle$ and we reach the BSE for the wave function

$$\bar{G}_0^{-1}|\psi_\lambda\rangle = \bar{\mathcal{V}}_{\lambda\lambda'}|\psi_{\lambda'}\rangle. \tag{10}$$

The normalization of the wave function can be obtained by the normalization of the vertex,

$$\begin{aligned} 1 &= i\frac{\langle\Gamma^\mu|G^{\mu\nu} - G^{\mu\mu'}\mathcal{V}^{\mu'\nu'}G^{\nu'\nu}|\Gamma^\nu\rangle}{p^2 - M^2} \\ &= i\langle\psi_\lambda|(-iN^2\delta_{\lambda\lambda'} - \bar{\mathcal{V}}_{\lambda\lambda'})'|\psi_{\lambda'}\rangle. \end{aligned} \tag{11}$$

Here, $\psi \to 0$ when $|\boldsymbol{k}| \to \infty$ and A is stable with small $|\boldsymbol{k}|$. As usual we assume the dependence of $\mathcal{V}$ on W is small. Hence, $(\bar{\mathcal{V}})'$ is negligible. The normalized wave functions can be introduced as $|\phi\rangle = N|\psi\rangle$ with $N = \sqrt{2E_1 2E_2}/\sqrt{(2\pi)^5 2W}$. The integral equation can be written explicitly as

$$(W - E_1(\boldsymbol{k}) - E_2(\boldsymbol{k}))\phi_\lambda(\boldsymbol{k}) = \int \frac{d\boldsymbol{k}'}{(2\pi)^3} V_{\lambda\lambda'}(\boldsymbol{k},\boldsymbol{k}',W)\phi_{\lambda'}(\boldsymbol{k}'), \tag{12}$$

with

$$V_{\lambda\lambda'}(\boldsymbol{k},\boldsymbol{k}',W) = \frac{i\bar{\mathcal{V}}_{\lambda\lambda'}(\boldsymbol{k},\boldsymbol{k}',W)}{\sqrt{2E_1(\boldsymbol{k})2E_2(\boldsymbol{k})2E_1'(\boldsymbol{k}')2E_2'(\boldsymbol{k}')}}. \tag{13}$$

The BSE can be related to the nonrelativistic OBE model [3,7] by nonrelativization and the Fourier transformation. The reduced equation is

$$\left[\frac{\nabla^2}{2\mu} - E\right]\phi(\boldsymbol{r}) = \int d\boldsymbol{r} V(\boldsymbol{r},\boldsymbol{r}')\phi(\boldsymbol{r}'), \tag{14}$$

where μ is reduced mass and $E = m_1 + m_2 - W$ is the binding energy. The potential in coordinate space can be defined as

$$V(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{(2\pi)^3}\frac{1}{2}\int d\boldsymbol{q} d\boldsymbol{q}' e^{i[\boldsymbol{q}'\frac{\boldsymbol{r}-\boldsymbol{r}'}{2} - \boldsymbol{q}\frac{\boldsymbol{r}+\boldsymbol{r}'}{2}]} V(\boldsymbol{k},\boldsymbol{k}'), \tag{15}$$

where $\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}$ and $\boldsymbol{q}' = \boldsymbol{k}' + \boldsymbol{k}$. For direct diagram the potential after nonrelativization can be written as $V(\boldsymbol{q})$ which can be transformed to $V(\boldsymbol{r})\delta(\boldsymbol{r} - \boldsymbol{r}')$ in coordinate space. The Schrödinger equation can be obtained. For cross diagram the potential is in the form $V(\boldsymbol{q}')$, which is transformed to $V(\boldsymbol{r})\delta(\boldsymbol{r} + \boldsymbol{r}')$, so the wave functions ϕ in the two sides of Eq. (14) are about $\boldsymbol{r}$ and $-\boldsymbol{r}$. It is no longer feasible to treat this issue with the Shrödinger equation. However, the same treatment for direct diagram and cross diagram was mistakenly applied by many authors [1,3,7]. In the current paper, we do not make such nonrelativization but partial wave expansion of BSE.

C. Partial wave expansion

The wave function has an angular dependence as

$$\phi_\lambda(\boldsymbol{k}) = \sqrt{\frac{2J+1}{4\pi}} D^{J*}_{\lambda_R,\lambda}(\phi,\theta,0)\phi^J_{\lambda,\lambda_R}(|\boldsymbol{k}|), \tag{16}$$

where J partial wave is considered and $D^{J*}_{\lambda_R,\lambda}(\phi,\theta,0)$ is the rotation matrix with λ_R being the helicity of bound state.

The potential after partial wave expansion is [17]

$$V^J_{\lambda\lambda'}(|\boldsymbol{k}|,|\boldsymbol{k}|') = 2\pi\int d\cos\theta_{k,k'} d^J_{\lambda,\lambda'}(\theta_{k,k'})V_{\lambda\lambda'}(\boldsymbol{k},\boldsymbol{k}'), \tag{17}$$

where $\theta_{k,k'}$ is angle between $\boldsymbol{k}$ and $\boldsymbol{k}'$.

Now we reach a one-dimensional integral equation,

$$\begin{aligned} &(W - E_1(|\boldsymbol{k}|) - E_2(|\boldsymbol{k}|))\phi^J_\lambda(|\boldsymbol{k}|) \\ &\quad = \int \frac{|\boldsymbol{k}'|^2 d|\boldsymbol{k}'|}{(2\pi)^3} V^J_{\lambda\lambda'}(|\boldsymbol{k}'|,|\boldsymbol{k}'|)\phi^J_{\lambda'}(|\boldsymbol{k}'|). \end{aligned} \tag{18}$$

For the wave function, we have relations $\phi_\lambda = \phi_{-\lambda}$ for system with quantum number $J^P = 1^+, 0^-$ and $\phi_\lambda = -\phi_{-\lambda}$ for system with 1^-. For potential we have $V_{\lambda\lambda'} = V_{-\lambda-\lambda'}$. Hence, we can only consider the independent wave functions and potentials. In this paper we choose $\phi_1 = \sqrt{2}\phi_\pm$ and ϕ_0 for system with $J^P = 1^+$, ϕ_0 for system with $J^P = 0^+$ and $\phi_1 = \sqrt{2}\phi_\pm$ for system with $J^P = 1^-$ with proper normalization.

Hence, we have the coupled equation,

$$(W - E_1(\boldsymbol{k}) - E_2(\boldsymbol{k}))\phi_i^J(|\boldsymbol{k}|) = \sum_j \int \frac{|\boldsymbol{k}'|^2 d|\boldsymbol{k}'|}{(2\pi)^3} V_{ij}^J(|\boldsymbol{k}'|, |\boldsymbol{k}'|)\phi_j^J(|\boldsymbol{k}'|), \quad (19)$$

with the normalization as

$$\sum_i \int |\boldsymbol{k}|^2 d|\boldsymbol{k}|\phi_i^{J2}(|\boldsymbol{k}|) = 1. \quad (20)$$

The potential V_{ij} can be written with $V_{\lambda\lambda'}^J$ as

$$V(1^+) = \begin{pmatrix} V_{11}^1 + V_{1-1}^1 & \sqrt{2}V_{10}^1 \\ \frac{1}{\sqrt{2}}(V_{01}^1 + V_{0-1}^1) & V_{00}^1 \end{pmatrix},$$
$$V(1^-) = V_{11}^1 - V_{1-1}^1,$$
$$V(0^+) = V_{00}^0. \quad (21)$$

D. Numerical solution of the BSE

The coupled one-dimensional integral Eq. (22) can be rewritten in the form as

$$\phi_i^J(|\boldsymbol{k}|) = \int d|\boldsymbol{k}'|A_{ij}(|\boldsymbol{k}'|, |\boldsymbol{k}'|)\phi_j^J(|\boldsymbol{k}'|). \quad (22)$$

To solve the integral equation, we discrete the $|\boldsymbol{k}|$ and $|\boldsymbol{k}'|$ to $|\boldsymbol{k}|_k$ and $|\boldsymbol{k}|_{k'}$ by the Gauss quadrature, then the above equation can be transformed to a matrix equation,

$$W\phi_{ik} = \sum_{jk'} A_{ik,jk'}(W)\omega_j\phi_{jk'}. \quad (23)$$

We remark the indices ik and jk' to new indices i and j and absorb ω to A,and have

$$W\phi_i = \sum_j \tilde{A}_{ij}(W)\phi_j, \quad (24)$$

which can be written in a compact matrix form,

$$W\phi = \tilde{A}(W)\phi. \quad (25)$$

Due to the dependence of the total energy W of $\tilde{A}$, the solution of above equation (25) is a nonlinear spectral problem. In this paper the recursion method in Ref [18,19] is adopted. The values of W and ψ are obtained by performing a sequence of approximations by using the recursion relation,

$$W_n^{(l)}\psi = \tilde{A}(W_s^{(l-1)})\psi, \quad (26)$$

where the upper index (l) and the lower index $n = 1, 2, \ldots s, \ldots$ are the iteration number and eigenvalue number, respectively. At the first iteration, we choose a sought eigenvalue and substitute it into the kernel $\tilde{A}$. In the current problem, the binding energy is very small, so the total energy W is chosen as the sum of the masses of two constituents $m_1 + m_2$. The nth eigenvalues can be obtained with the help of the code of DGEEV function in the NAG Fortran Library. If we want to obtain the sth eigenvalues, the corresponding eigenvalue should be substituted in kernel $\tilde{A}$ on each iterative loop. In the current paper, we choose ground state. With new kernel, the linear spectral problem is solved again. On each iteration stopping criterion $|W_s^{(l)} - W_s^{(l-1)}| < \epsilon$, which is related to precision of the final results, is tested. Once stopping criterion fulfilled, the iterative process is terminated. The eigenvalue $W_s^{(l)}$ and eigenfunction $\phi_s^{(l)}$ obtained on the last iteration are adopted as the solution.

III. THE ONE-BOSON-EXCHANGE POTENTIAL

To give the potential, we adopt the effective Lagrangians of the pseudoscalar and vector mesons with heavy flavor mesons from the heavy quark effective theory [20,21],

$$\mathcal{L}_{P^*P\mathbb{P}} = -\frac{2g\sqrt{m_P m_{P^*}}}{f_\pi}(P_b P_{a\lambda}^{*\dagger} + P_{b\lambda}^* P_a^\dagger)\partial^\lambda \mathbb{P}_{ba} + \frac{2g\sqrt{m_P m_{P^*}}}{f_\pi}(\tilde{P}_{a\lambda}^{*\dagger}\tilde{P}_b + \tilde{P}_a^\dagger \tilde{P}_{b\lambda}^*)\partial^\lambda \mathbb{P}_{ab},$$
$$\mathcal{L}_{P^*P\mathbb{V}} = -i\sqrt{2}\lambda g_V \varepsilon_{\lambda\alpha\beta\mu}(P_a^{*\mu\dagger}\overleftrightarrow{\partial}^\lambda P_b + P_a^\dagger \overleftrightarrow{\partial}^\lambda P_b^{*\mu})(\partial^\alpha \mathbb{V}^\beta)_{ba} - i\sqrt{2}\lambda g_V \varepsilon_{\lambda\alpha\beta\mu}(\tilde{P}_a^{*\mu\dagger}\overleftrightarrow{\partial}^\lambda \tilde{P}_b + \tilde{P}_a^\dagger \overleftrightarrow{\partial}^\lambda \tilde{P}_b^{*\mu})(\partial^\alpha \mathbb{V}^\beta)_{ab},$$
$$\mathcal{L}_{PP\mathbb{V}} = -i\frac{\beta g_V}{\sqrt{2}} P_a^\dagger \overleftrightarrow{\partial}_\mu P_b \mathbb{V}_{ba}^\mu + i\frac{\beta g_V}{\sqrt{2}}\tilde{P}_a^\dagger \overleftrightarrow{\partial}_\mu \tilde{P}_b \mathbb{V}_{ab}^\mu,$$
$$\mathcal{L}_{P^*P^*\mathbb{V}} = i\frac{\beta g_V}{\sqrt{2}} P_a^{*\dagger}\overleftrightarrow{\partial}_\mu P_b^* \mathbb{V}_{ba}^\mu - i2\sqrt{2}\lambda g_V m_{P^*} P_b^{*\mu} P_a^{*\nu\dagger}(\partial_\mu \mathbb{V}_\nu - \partial_\nu \mathbb{V}_\mu)_{ba} - i\frac{\beta g_V}{\sqrt{2}}\tilde{P}_a^{*\dagger}\overleftrightarrow{\partial}_\mu \tilde{P}_b^* \mathbb{V}_{ab}^\mu - i2\sqrt{2}\lambda g_V m_{P^*}\tilde{P}_a^{*\mu\dagger}\tilde{P}_b^{*\nu}(\partial_\mu \mathbb{V}_\nu - \partial_\nu \mathbb{V}_\mu)_{ab},$$
$$\mathcal{L}_{PP\sigma} = -2g_\sigma m_P P_a^\dagger P_a \sigma - 2g_\sigma m_P \tilde{P}_a^\dagger \tilde{P}_a \sigma,$$
$$\mathcal{L}_{P^*P^*\sigma} = 2g_\sigma m_{P^*} P_a^{*\dagger} P_a^* \sigma + 2g_\sigma m_{P^*}\tilde{P}_a^{*\dagger}\tilde{P}_a^* \sigma,$$

where the octet pseudoscalar and nonet vector meson matrices read as

$$\mathbb{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix},$$
$$\mathbb{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \quad (27)$$

TABLE I. The flavor factors I_d^i and I_c^i for direct and cross diagrams and different exchange mesons.

	Direct diagram			Cross diagram			
	$\mathbb{V}$		$\mathbb{S}$	$\mathbb{P}$		$\mathbb{V}$	
Exchanged meson	ρ	ω	σ	π	η	ρ	ω
$Z_{P\bar{P}^*}^{(T)}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{2}$
$Z_{P\bar{P}^*}^{(S)}$	$\frac{3}{2}$	$\frac{1}{2}$	1	$-\frac{3}{2}$	$\frac{1}{6}$	$\frac{3}{2}$	$\frac{1}{2}$

Here we choose parameters $g = 0.59$, $\beta = 0.9$, $\lambda = 0.56\ \mathrm{GeV}^{-1}$, $g_V = 5.8$ and $g_\sigma = g_\pi/(2\sqrt{6})$ with $g_\pi = 3.73$ [22,23].

With above Lagrangians, the potential kernel $\mathcal{V}$ can be written as

$$\mathcal{V}_{\mathbb{V}}^{\mathrm{Direct}} = i\frac{\beta^2 g_V^2}{2}\left[\frac{(k_1+k_1')\cdot(k_2+k_2')}{q^2-m_{\mathbb{V}}^2}\right]\epsilon_1\cdot\epsilon_1'$$

$$\mathcal{V}_{\sigma}^{\mathrm{Direct}} = i4g_\sigma^2 m_P m_{P^*}\frac{\epsilon_1\cdot\epsilon_1'}{q^2-m_\sigma^2}$$

$$\mathcal{V}_{\mathbb{P}}^{\mathrm{Cross}} n = i\frac{4g^2 m_P m_{P^*}}{f_\pi^2}\frac{q\cdot\epsilon_1 q\cdot\epsilon_1'}{q^2-m_{\mathbb{P}}^2}$$

$$\mathcal{V}_{\mathbb{V}}^{\mathrm{Cross}} = i8\lambda^2 g_V^2\frac{1}{q^2-m_{\mathbb{V}}^2}(q\cdot\epsilon_1 q\cdot\epsilon_1' k_2\cdot k_2' + \epsilon_1\cdot\epsilon_1'(k_2\cdot q k_2'\cdot q - k_2\cdot k_2' q^2)), \quad (28)$$

where $q = k_1' - k_1$ for direct diagram and $q = k_2' - k_1 = k_2 - k_1'$ for cross diagram. A form factor $h(k^2) = \frac{\Lambda^4}{(m^2-k^2)^2+\Lambda^4}$ is introduced to compensate for the off-shell effect of heavy meson. In the propagator of exchanged meson we make a replacement $q^2 \to -|q^2|$ to remove singularities as Ref. [17]. The form factor for light meson is chosen as $f(q^2) = \frac{\Lambda^2-m^2}{\Lambda^2+|q^2|}$. The flavor factors I_d^i and I_c^i for direct and cross diagrams are presented in the following Table I.

IV. NUMERICAL RESULTS

In this paper, the states with $J = 0, 1$ are considered and the results of the binding energy $E = m_1 + m_2 - W$ are listed in Table II with cutoffs in the range $0.8 < \Lambda < 5$ GeV and compared with the nonrelativistic OBE model [7,8]. In this paper, only bound states with small binding energy $E < 10$ MeV are considered because hadronic molecular state is defined as a loosely bound state.

In the isoscalar vector, there exist the hidden bottomed bound state solutions with quantum number $I^G(J^{PC}) = 0^-(0^{--})$, $0^-(1^{--})$, $0^-(1^{+-})$ and $0^-(1^{++})$ and the hidden charmed bound state solution with $0^-(1^{+-})$ and $0^-(1^{++})$.

TABLE II. The binding energies E for $D\bar{D}^*$ and $B\bar{B}^*$ systems with different cutoff Λ obtained in this work (BS) and in the nonrelativistic OBE model (OBE) [7]. "$\cdots$" means that no bound state is found. The cutoff Λ and binding energy are in the units of GeV and MeV, respectively.

	$D\bar{D}^*$				$B\bar{B}^*$			
	BS		OBE		BS		OBE	
$I^G(J^{PC})$	Λ	E	Λ	E	Λ	E	Λ	E
$0^-(0^{--})$	$\cdots$	$\cdots$			1.5	1.6		
	$\cdots$	$\cdots$			1.7	4.1		
	$\cdots$	$\cdots$			1.9	6.7		
$0^+(0^{-+})$	$\cdots$	$\cdots$			$\cdots$	$\cdots$		
$0^-(1^{--})$	$\cdots$	$\cdots$			1.6	1.4		
	$\cdots$	$\cdots$			1.7	3.7		
	$\cdots$	$\cdots$			1.8	6.4		
$0^+(1^{-+})$	$\cdots$	$\cdots$			$\cdots$	$\cdots$		
$0^-(1^{+-})$	1.3	0.2	1.4	3.44	1.1	0.6	1.4	1.56
	1.4	6.0	1.5	16.57	1.2	7.8	1.5	12.95
$0^+(1^{++})$	2.0	0.2	1.1	0.61	1.3	0.2	1.1	0.61
	2.2	1.4	1.2	4.42	1.5	3.0	1.2	4.42
	2.4	4.1	1.3	11.78	1.7	7.4	1.3	11.78
$1^+(0^-)$	$\cdots$	$\cdots$			$\cdots$	$\cdots$		
$1^-(0^-)$	$\cdots$	$\cdots$			$\cdots$	$\cdots$		
$1^+(1^-)$	$\cdots$	$\cdots$			$\cdots$	$\cdots$		
$1^-(1^-)$	$\cdots$	$\cdots$			$\cdots$	$\cdots$		
$1^+(1^+)$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	2.1	0.22
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	2.2	1.64
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	2.5	4.74
$1^-(1^+)$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	4.9	0.14
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	5.0	0.41

The $D\bar{D}^*$ bound state with $I^G(J^{PC}) = 0^+(1^{++})$ can be related to the $X(3872)$. This bound state was also found in the nonrelativistic OBE model [7] but with different cutoffs. It is well known that there exists a $c\bar{c}$ component in X(3872) [24]. In the current paper only the hadronic molecular states are considered. The discussion about the $c\bar{c}$ component is beyond the scope of this work and not considered here.

In the isovector sector, the nonrelativistic OBE model predicted a molecular state $B\bar{B}^*$ with $I^G(J^P) = 1^+(1^+)$ with cutoffs about 2 GeV, which is assigned to the observed $Z_b(10610)$ state [7]. The observed $Z_c(3900)$ is also explained as $D\bar{D}^*$ state in literatures [25,26]. In our calculation, there does not exist isovector bound state solution with all cutoffs $\Lambda < 5$ GeV, which suggests the structures $Z_b(10610)$ and $Z_c(3900)$ should be originated from other mechanisms, such as four-quark states or cusp effect [27]. In fact, if we compared the experimental masses for $Z_b(10610)$ and $Z_c(3900)$ and the thresholds for $B\bar{B}^*$ and $D\bar{D}^*$, we can find that these two states are above the thresholds, which conflicts with the molecular state assumption. The lattice results disfavored the possibility of a shallow bound state for $D\bar{D}^*$ interaction also [28,29].

V. SUMMARY

In this paper the $B\bar{B}^*$ and $D\bar{D}^*$ systems are studied in a BSE approach with quasipotential potential approximation by adopting the covariant spectator theory which is suitable to study a system with different constituents. In our calculation, both direct and cross diagrams are considered in the one-boson-exchange potential so that the π exchange which was found more important in the $B\bar{B}^*$ and $D\bar{D}^*$ interactions [7,8] is included. Partial wave expansion is used to reduce the BSE to a one-dimensional equation, which is solved by a recursion method. The numerical results indicate the existence of an isoscalar bound state $D\bar{D}^*$ with $J^{PC} = 1^{++}$, which may be related to the $X(3872)$. In the isovector sector, no bound state is produced from both $D\bar{D}^*$ and $B\bar{B}^*$ interactions, which disfavors the molecular state explanations for $Z_b(10610)$ and $Z_c(3900)$.

It is found in our calculation that for cross diagram the BSE can not be transformed to the Schrödinger equation with potential in coordinate space $V(\boldsymbol{r})$. This problem appears in all systems composed of two constituents with different masses and/or spins which can convert to each other, such as the K exchange potential between s quark and u/d quark in the constituent quark model [30] and NN^* interaction [31] where a potential in coordinate space are used. The results obtained in this work show there does not exist isovector $B\bar{B}^*/D\bar{D}^*$ bound state, which is more consistent with the experiment and the lattice QCD [28,29]. Hence, one should be cautious in the direct application of potential in coordinate space obtained by a simple Fourier transformation, which has been widely used in the studies of the hadron spectrum, hadronic molecular states and other fields [1,7,30].

ACKNOWLEDGMENTS

This project is partially supported by the Major State Basic Research Development Program in China (No. 2014CB845405), the National Natural Science Foundation of China (Grants No. 11275235, No. 11035006) and the Chinese Academy of Sciences (the Knowledge Innovation Project under Grant No. KJCX2-EW-N01).

[1] N. A. Tornqvist, Phys. Rev. Lett. **67**, 556 (1991).
[2] S. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003).
[3] N. A. Tornqvist, Phys. Lett. B **590**, 209 (2004).
[4] F. E. Close and P. R. Page, Phys. Lett. B **578**, 119 (2004).
[5] A. Bondar *et al.* (Belle Collaboration), Phys. Rev. Lett. **108**, 122001 (2012).
[6] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **110**, 252001 (2013).
[7] Z.-F. Sun, J. He, X. Liu, Z.-G. Luo, and S.-L. Zhu, Phys. Rev. D **84**, 054002 (2011).
[8] Z.-F. Sun, Z.-G. Luo, J. He, X. Liu, and S.-L. Zhu, Chin. Phys. **C36**, 194 (2012).
[9] F. Gross and A. Stadler, Phys. Rev. C **82**, 034004 (2010).
[10] E. Hummel and J. A. Tjon, Phys. Rev. Lett. **63**, 1788 (1989).
[11] H.-W. Ke, X.-Q. Li, Y.-L. Shi, G.-L. Wang, and X.-H. Yuan, J. High Energy Phys. 04 (2012) 056.
[12] J. He and X. Liu, Eur. Phys. J. C **72**, 1986 (2012).
[13] J. He, D.-Y. Chen, and X. Liu, Eur. Phys. J. C **72**, 2121 (2012).
[14] J. Adam, Jr., J. W. Van Orden, and F. Gross, Nucl. Phys. **A640**, 391 (1998).
[15] L.-L. Shen, X.-L. Chen, Z.-G. Luo, P.-Z. Huang, S.-L. Zhu, P.-F. Yu, and X. Liu, Eur. Phys. J. C **70**, 183 (2010).
[16] B. Hu, X.-L. Chen, Z.-G. Luo, P.-Z. Huang, S.-L. Zhu, P.-F. Yu, and X. Liu, Chin. Phys. **C35**, 113 (2011).
[17] F. Gross and A. Stadler, Phys. Rev. C **78**, 014005 (2008).
[18] T. M. Soloveva, Comput. Phys. Commun. **136**, 208 (2001).
[19] J. He and P.-L. Lü, Nucl. Phys. **A919**, 1 (2013).
[20] P. Colangelo, F. De Fazio, and T. N. Pham, Phys. Rev. D **69**, 054023 (2004).
[21] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. **281**, 145 (1997).
[22] C. Isola, M. Ladisa, G. Nardulli, and P. Santorelli, Phys. Rev. D **68**, 114001 (2003).
[23] A. F. Falk and M. E. Luke, Phys. Lett. B **292**, 119 (1992).
[24] S. Coito, G. Rupp, and E. van Beveren, Eur. Phys. J. C **73**, 2351 (2013).
[25] Q. Wang, C. Hanhart, and Q. Zhao, Phys. Rev. Lett. **111**, 132003 (2013).
[26] F.-K. Guo, C. Hidalgo-Duque, J. Nieves, and M. Pavon-Valderrama, Phys. Rev. D **88**, 054007 (2013).
[27] D.-Y. Chen and X. Liu, Phys. Rev. D **84**, 094003 (2011).
[28] S. Prelovsek and L. Leskovec, Phys. Lett. B **727**, 172 (2013).
[29] D.-Y. Chen, X. Liu, and T. Matsuki, Phys. Rev. D **88**, 036008 (2013).
[30] L. Y. Glozman and D. Riska, Phys. Rep. **268**, 263 (1996).
[31] L. Zhao, P.-N. Shen, Y.-J. Zhang, and B.-S. Zou, Eur. Phys. J. A **49**, 59 (2013).