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Semileptonic meson decays in point-form relativistic quantum mechanics: Unambiguous extraction of weak form factors

María Gómez-Rocha*

University of Graz, Institute of Physics, NAWI Graz, A-8010 Graz, Austria

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The point-form version of the Bakamjian-Thomas construction is applied to the description of several semileptonic decays of mesons. Weak form factors are extracted without ambiguity for pseudoscalar-to-pseudoscalar as well as for pseudoscalar-to-vector transitions of mesons from the most general covariant decomposition of the weak current. No manifestation of cluster-separability violation appears in the form of nonphysical contributions to the structure of such a current, in contrast to what happens in the electromagnetic case. Moreover, no frame dependence is observed when we extract the form factors from the most general covariant decomposition of the current, which contrasts with analogous front-form calculations that involve vector mesons in the transition. We present our results for heavy-light meson decays, i.e. $B \to D$, as well as for B and D mesons decaying into π, ρ and $K^{(*)}$, and perform a numerical comparison with the analogous front-form approach. Differences between point and front forms that are not seen in the heavy-quark limit of $q\bar{Q}$ systems appear. These differences are attributed to the different role that the nonvalence contributions play in the description of hadronic reactions in each form. It is argued how contributions from missing Z graphs can be estimated.

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I. INTRODUCTION

From the three prominent forms of relativistic Hamiltonian dynamics presented by Dirac [1], the point form (PF) is the least explored. However, it possesses virtues that are worth exploiting for the study of relativistic composite systems. One of the most important features is that of the 10 generators of the Poincaré algebra; those forming the Lorentz subgroup, rotations and boosts, are kinematic (free of interactions). This is to be contrasted to the most familiar instant form (IF), where the boost operators make changes of reference frames challenging, since they carry interaction terms (they are dynamical). This is particularly problematic in quantum field theories, where the number of particles is not conserved.[1] The front form (FF), despite being the form with the larger kinematical group (containing seven generators), has the drawback that rotations are interaction dependent, and thus it makes the addition of angular momentum of relativistic interacting particles troublesome [3,4].

In the last few years, a considerable number of articles have been written with the goal of developing a new formalism able to describe the structure of hadrons—or, more generally, of relativistic bound states—in terms of the properties of their constituents by using the point form of relativistic quantum mechanics (PFRQM) (cf. [5–18]). Relativistic quantum mechanics, unlike quantum field theory, considers a restricted number of degrees of freedom. Poincaré invariance is ensured in this formalism by using the Bakamjian-Thomas construction [3,19,20]. Its point-form version introduces a free velocity operator, that is multiplied by the interacting mass operator and leads to an interaction-dependent four-momentum operator [3,10]. Using a coupled-channel approach for that mass operator, we can describe the physical process from which invariant amplitudes and hadronic currents can be calculated.

An appropriate description of the structure of the current poses several problems, and it is not straightforward to derive electroweak currents with all the required properties. Two basic features are Poincaré covariance and cluster separability [3,21–23]. Our formalism in PFRQM has helped to understand the electroweak structure of hadrons in several ways. It was initially applied to calculate the spectrum and decay widths of vector and axial-vector mesons within the chiral constituent quark model [5,6]. Later, electromagnetic properties of spin-0 and spin-1 two-body bound states with equal-constituent masses were studied [7–9]. More recently, the relativistic multichannel formalism was extended to unequal-mass constituents and to weak decay form factors in the timelike momentum transfer region [11–13]. An additional condition that has to be satisfied by systems of unequal-constituent masses is to respect the heavy-quark symmetry predictions in the extreme case in which one of the masses is infinitely heavier than the other [24–26]. It was shown that our approach respects the required heavy-quark symmetry predictions [11,16]; i.e. relations between electromagnetic and weak form factors appear in the limit $m_Q \to \infty$. This guarantees that the formalism is general enough to be

*maria.gomez-rocha@uni-graz.at

[1]Reference [2] shows how boosting bound states in QCD using instant-form boosts becomes rather intractable.

applied to systems of arbitrary constituent masses, and gives us the freedom to apply it to heavy-to-heavy, heavy-to-light and light-to-light meson transitions. The main goal of this paper is to provide the result of the application of our formalism to all these cases. Reference [11] provides the basis and the main motivation for the present work.

There is a second important issue we would like to address. It is known that the Bakamjian-Thomas construction produces problems related to cluster separability [3], which enter the calculation of form factors and may lead to unphysical contributions in the electromagnetic current [7,9,10]. It was observed in [11] that this is not the case in timelike processes, such as the weak decays we are going to consider in this work. Weak form factors can be extracted unambiguously and there is no need to introduce any additional spurious contribution to ensure the required covariant properties of the hadronic currents. This does not mean, however, that the cluster problem is not present. The cluster problem is intrinsic to the Bakamjian-Thomas construction and we do not know any relativistic quantum mechanical approach that eliminates the cluster problem completely [27].

The need for additional covariants in spacelike processes is similar to the occurrence within the covariant light-front formulation of Carbonell *et al.* [28], in which the orientation of the light front has to be considered explicitly in order to render the front-form approach manifestly covariant. In fact, comparisons between the point- and the front-form electromagnetic form factors show that the number of needed spurious contributions in spin-0 and spin-1 two-body systems coincides in the PF and FF cases [9,29]. In FF, one way to cure this problem is the introduction of pair-creation currents [30–34], such as the so-called Z graphs. This is particularly necessary in FF when one considers timelike processes, where it is not possible to use the very convenient $q^+ = 0$. For $q^+ > 0$, additional covariants associated with zero modes are necessary in order to provide the appropriate Lorentz structure of the weak current and a certain frame dependence of the form factors is encountered in processes that involve spin-1 mesons when the latter are considered as simple valence $q\bar{q}$ bound states [33–39]. These problems are closely related to the violation of rotational invariance in the calculation of one-body-current matrix elements in FF.

As mentioned above, in PF we do not encounter covariance problems of this kind in the current for timelike momentum transfers and there is no need for introducing spurious contributions. Thus, it is now interesting to consider a detailed numerical comparison between the point- and the front-form results for timelike processes. For this comparison, we choose the light-front quark model of Ref. [39], and use the same harmonic-oscillator wave function and adopt the same harmonic-oscillator and mass parameters. Nonvalence contributions are not considered explicitly in this work, nor in the work of Ref. [39]. Since nonvalence contributions enter differently in every form of dynamics, it is expected that considering the meson as a valence quark-antiquark pair only must result in different resulting form factors as well.

The purpose of this paper is therefore twofold: on the one hand we apply the PFRQM approach to several particular cases of semileptonic decays using the harmonic-oscillator wave function that was used in previous works and obtain results that can be compared with experiments and with other approaches. With this we do not intend to make accurate predictions, but rather to explore the applicability of our PFRQM approach to a broader range of reactions. On the other hand, we perform a numerical comparison with an analogous front-form approach in which, as in our case, no additional nonvalence contributions are considered explicitly. Our purpose is to pose the question about the different role that nonvalence contributions such as Z graphs play in each role. The encountered differences reflect the fact that effects coming from vacuum fluctuations have to be treated differently in each form.

This article is organized as follows. Section II condenses the most important steps in the procedure used by the PFRQM approach and applies it to the process of a general weak semileptonic decay. In Sec. III we present and analyze our numerical results obtained in several particular cases for pseudoscalar mesons decaying into a pseudoscalar meson ($P \to P$) as well as to vector mesons ($P \to V$). We compare our results with the analogous front-form approach and discuss the encountered numerical differences. Conclusions and outlook are presented in Sec. IV. Two important concepts of this formalism, *velocity states* and *vertex operators*, are presented in Appendixes A and B, respectively.

II. RELATIVISTIC MULTICHANNEL FORMALISM AND HADRON CURRENTS

The starting point of the derivation of currents and form factors in PFRQM is the physical processes in which such form factors are measured. In this work we examine semileptonic decays of mesons. In order to describe the processes in a fully Poincaré-invariant way, a multichannel version of the Bakamjian-Thomas construction [3,19] is employed. In the point-form version of the Bakamjian-Thomas construction the four-momentum operator factorizes into an interacting mass operator and a free four-velocity operator:

$$\hat{P}^\mu = \hat{P}^\mu_{\text{free}} + \hat{P}^\mu_{\text{int}} = \hat{M}\hat{V}^\mu_{\text{free}} = (\hat{M}_{\text{free}} + \hat{M}_{\text{int}})\hat{V}^\mu_{\text{free}}. \quad (1)$$

The four-velocity operator is free of interactions and is defined by $\hat{V}^\mu_{\text{free}} := \hat{P}^\mu_{\text{free}}/\hat{M}_{\text{free}} = \hat{P}^\mu/\hat{M}$. It describes the overall motion of the system. The mass operator $\hat{M}$, which depends on internal variables only, is the quantity of interest, since it contains the information of the internal structure of the system.

The procedure to calculate invariant amplitudes of hadronic reactions, from which currents and form factors can be extracted, has been elaborately explained throughout several works, in which both electromagnetic and weak decays are considered (see [7,8,11] for illustration, and [9,16] for deep details). We summarize here the most important steps that are required for the study of the processes in which we are interested, and will refer to the more extended literature when necessary.

We will consider $P \to P$ meson transitions as well as $P \to V$ meson transitions.

A. Derivation of the optical potential and identification of hadronic currents

Our point-form approach is a coupled-channel formalism for a Bakamjian-Thomas mass operator formulated in the point form of Hamiltonian dynamics. Due to the form of Eq. (1) the problem reduces to solving an eigenvalue equation for the mass operator $\hat{M}$:

$$\hat{M}|\psi\rangle = m|\psi\rangle, \tag{2}$$

where $\hat{M}$ is the coupled channel mass operator for the Bakamjian-Thomas construction in its point-form version. For a weak process of a meson α decaying into another meson α' the mass operator $\hat{M}$ needs—at least—four channels. They are needed in order to account for two possible time-ordered contributions, which are depicted in Fig. 1:

$$\hat{M} = \begin{pmatrix} \hat{M}^{\rm conf}_{q\bar{q}} & 0 & \hat{K}_{q'\bar{q}W\to q\bar{q}} & \hat{K}_{q\bar{q}We\bar{\nu}_e\to q\bar{q}} \\ 0 & \hat{M}^{\rm conf}_{q'\bar{q}e\bar{\nu}_e} & \hat{K}_{q'\bar{q}W\to q'\bar{q}e\bar{\nu}_e} & \hat{K}_{q\bar{q}We\bar{\nu}_e\to q'\bar{q}e\bar{\nu}_e} \\ \hat{K}^\dagger_{q'\bar{q}W\to q\bar{q}} & \hat{K}^\dagger_{q'\bar{q}W\to q'\bar{q}e\bar{\nu}_e} & \hat{M}^{\rm conf}_{q'\bar{q}W} & 0 \\ \hat{K}^\dagger_{q\bar{q}We\bar{\nu}_e\to q\bar{q}} & \hat{K}^\dagger_{q\bar{q}We\bar{\nu}_e\to q'\bar{q}e\bar{\nu}_e} & 0 & \hat{M}^{\rm conf}_{q\bar{q}We\bar{\nu}_e} \end{pmatrix}. \tag{3}$$

The mass eigenstate $|\psi\rangle$ on which $\hat{M}$ acts is a direct sum of $|\psi_{q\bar{q}}\rangle$, $|\psi_{q'\bar{q}e\bar{\nu}_e}\rangle$, $|\psi_{q'\bar{q}W}\rangle$ and $|\psi_{q\bar{q}We\bar{\nu}_e}\rangle$ Hilbert spaces. In point form it is convenient to use a *velocity-states* basis [28,40,41], defined in Eq. (A1). The nondiagonal elements of $\hat{M}$ are *vertex operators*, $\hat{K}^\dagger$ and $\hat{K}$, that describe the emission and absorption of the exchanged W boson, respectively. They are appropriately related to the weak interaction Lagrangian density through Eq. (B1) (see Refs. [10,42] and Appendix B). The instantaneous confining $q\bar{q}$ interaction is included in the diagonal elements of the matrix, which are denoted by "conf" (see [8,11]). For instance,

$$M^{\rm conf}_{q\bar{q}W}|\underline{v};\vec{\underline{k}}_W,\underline{\mu}_W;\vec{\underline{k}}_\alpha,\underline{\mu}_\alpha,\alpha\rangle = (\omega_{\underline{k}_W}+\omega_{\underline{k}_\alpha})|\underline{v};\vec{\underline{k}}_W,\underline{\mu}_W;\vec{\underline{k}}_\alpha,\underline{\mu}_\alpha,\alpha\rangle, \tag{4}$$

where $\underline{\mu}_\alpha$ denotes the spin orientation of the confined $q\bar{q}$ bound state and α represents the remaining discrete quantum numbers necessary to specify it uniquely. The energy of the $q\bar{q}$ bound state with quantum numbers α and mass m_α is represented by $\omega_{\underline{k}_\alpha}$ and expressed below Eq. (A3). Underlined velocities, momenta and spin projections distinguish states with a confined $q\bar{q}$ pair from those with a free $q\bar{q}$ pair, which are not underlined.

The system of equations (2) can be transformed into an equation for $|\psi_{q\bar{q}}\rangle$ by means of a Feshbach reduction, leading to the required expression for the optical potential that describes the entire process of the W-boson exchange, including both time-ordered contributions (cf. Fig. 1):

$$(\hat{M}^{\rm conf}_{q\bar{q}} + \hat{V}^{q\bar{q}\to q'\bar{q}e\bar{\nu}_e}_{\rm opt}(m))|\psi_{q\bar{q}}\rangle = m|\psi_{q\bar{q}}\rangle, \tag{5}$$

where

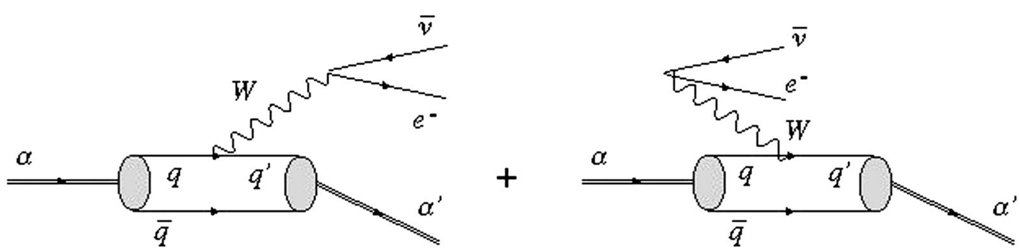

FIG. 1. Time-ordered contributions to the semileptonic decay of a meson α to α'.

$$\hat{V}_{\rm opt}^{q\bar{q}\to q'\bar{q}e\bar{\nu}_e}(m) = \hat{K}_{q'\bar{q}W\to q'\bar{q}e\bar{\nu}_e}(m - M^{\rm conf}_{q'\bar{q}W})^{-1}\hat{K}^\dagger_{q'\bar{q}W\to q\bar{q}} + \hat{K}_{q\bar{q}We\bar{\nu}_e\to q'\bar{q}e\bar{\nu}_e}(m - \hat{M}^{\rm conf}_{q\bar{q}We\bar{\nu}_e})^{-1}\hat{K}^\dagger_{q\bar{q}We\bar{\nu}_e\to q\bar{q}}. \tag{6}$$

On-shell matrix elements of such optical potential have the structure of the invariant $\alpha \to \alpha'^{(*)}e\bar{\nu}_e$ decay amplitude resulting from leading-order covariant perturbation theory. The calculation requires the insertion, in the appropriate places, of the spectral decomposition of the unity operators, written in the velocity-states basis (cf. Appendix A). Since the calculation is tedious, it is not presented here in detail. We refer to Ref. [16] for technicalities, where the required matrix elements are given explicitly. From the structure of the invariant decay amplitude it is straightforward to identify the microscopic hadron current:

$$\begin{aligned}
&\langle \underline{v}'; \underline{\vec{k}}'_e, \underline{\mu}'_e; \underline{\vec{k}}'_{\bar{\nu}_e}; \underline{\vec{k}}'_{\alpha'}, \underline{\mu}'_{\alpha'}, \alpha' | \hat{V}_{\rm opt}^{b\bar{d}\to c\bar{d}e\bar{\nu}_e}(m) | \underline{\vec{k}}_\alpha, \underline{\mu}_\alpha, \alpha\rangle_{\rm os} \\
&= \underline{v}_0 \delta^3(\underline{\vec{v}}' - \underline{\vec{v}}) \frac{(2\pi)^3}{\sqrt{(\omega_{\underline{k}'_e} + \omega_{\underline{k}'_{\bar{\nu}_e}} + \omega_{\underline{k}'_{\alpha'}})^3}\sqrt{\omega^3_{\underline{k}_\alpha}}} \\
&\times \frac{e^2}{2\sin^2\vartheta_{\rm w}} V_{cb} \frac{1}{2} \underbrace{\bar{u}_{\underline{\mu}'_e}(\underline{\vec{k}}'_e)\gamma^\mu(1-\gamma^5)v_{\underline{\mu}'_{\bar{\nu}_e}}(\underline{\vec{k}}'_{\bar{\nu}_e})}_{j^\mu_{\nu_e\to e}(\underline{\vec{k}}'_e,\underline{\mu}'_e;\underline{\vec{k}}'_{\bar{\nu}_e},\underline{\mu}'_{\bar{\nu}_e})} \frac{(-g_{\mu\nu})}{(\underline{k}'_e + \underline{k}'_{\bar{\nu}_e})^2 - m_W^2} \frac{1}{2} J^\nu_{\alpha\to\alpha'}(\underline{\vec{k}}'_{\alpha'}, \underline{\mu}'_{\alpha'}; \underline{\vec{k}}_\alpha, \underline{\mu}_\alpha),
\end{aligned} \tag{7}$$

where $\vartheta_{\rm w}$ is the electroweak mixing angle, e the elementary electric charge and V_{cb} the Cabibbo-Kobayashi-Maskawa matrix element occurring at the Wbc vertex. Note that the covariant structure of the W propagator is only achieved when the sum of both time-ordering contributions in Fig. 1 are considered [16].

The semileptonic current extracted from Eq. (7) in the cases of a $P \to P$ transition has the structure

$$\begin{aligned}
J^\nu_{\alpha\to\alpha'}(\underline{\vec{k}}'_{\alpha'}; \underline{\vec{k}}_\alpha = \vec{0}) &= \frac{\sqrt{\omega_{\underline{k}_\alpha}\omega_{\underline{k}'_{\alpha'}}}}{4\pi} \int \frac{d^3\tilde{k}'_{\bar{q}}}{2\omega_{k_q}} \sqrt{\frac{\omega_{\tilde{k}'_{q'}} + \omega_{\tilde{k}'_{\bar{q}}}}{\omega_{k'_{q'}} + \omega_{k'_{\bar{q}}}}} \sqrt{\frac{\omega_{\tilde{k}_q}\omega_{\tilde{k}_{\bar{q}}}}{\omega_{\tilde{k}'_{q'}}\omega_{\tilde{k}'_{\bar{q}}}}} \Bigg\{ \sum_{\mu_q, \mu'_{q'} = \pm\frac{1}{2}} \bar{u}_{\mu'_{q'}}(\vec{k}'_{q'})\gamma^\nu(1-\gamma^5)u_{\mu_q}(\vec{k}_q) \\
&\times D^{1/2}_{\mu_q\mu'_{q'}}\left[R_{\rm W}\left(\frac{\tilde{k}'_{\bar{q}}}{m_{\bar{q}}}, B_c(v'_{q'\bar{q}})\right)R^{-1}_{\rm W}\left(\frac{\tilde{k}'_{q'}}{m_{q'}}, B_c(v'_{q'\bar{q}})\right)\right]\Bigg\} \psi^*_{\alpha'}(|\vec{\tilde{k}}'_{\bar{q}}|)\psi_\alpha(|\vec{\tilde{k}}_{\bar{q}}|),
\end{aligned} \tag{8}$$

and for $P \to V$ transitions[2]

$$\begin{aligned}
J^\nu_{\alpha\to\alpha'^*}(\underline{\vec{k}}'_{\alpha'^*}, \underline{\mu}'_{\alpha'^*}; \underline{\vec{k}}_\alpha = \vec{0}) &= \frac{\sqrt{\omega_{\underline{k}_\alpha}\omega_{\underline{k}'_{\alpha'^*}}}}{4\pi} \int \frac{d^3\tilde{k}'_{\bar{q}}}{2\omega_{k_q}} \sqrt{\frac{\omega_{\tilde{k}'_{q'}} + \omega_{\tilde{k}'_{\bar{q}}}}{\omega_{k'_{q'}} + \omega_{k'_{\bar{q}}}}} \sqrt{\frac{\omega_{\tilde{k}_q}\omega_{\tilde{k}_{\bar{q}}}}{\omega_{\tilde{k}'_{q'}}\omega_{\tilde{k}'_{\bar{q}}}}} \Bigg\{ \sum_{\mu_b, \mu'_{q'}, \tilde{\mu}'_{q'}, \tilde{\mu}'_{\bar{q}} = \pm\frac{1}{2}} \bar{u}_{\mu'_{q'}}(\vec{k}'_{q'})\gamma^\nu(1-\gamma^5)u_{\mu_q}(\vec{k}_q) \\
&\times \sqrt{2}(-1)^{\frac{1}{2}-\mu_q} C^{1\mu'_{\alpha'^*}}_{\frac{1}{2}\tilde{\mu}'_{q'}\frac{1}{2}\tilde{\mu}'_{\bar{q}}} D^{1/2}_{\tilde{\mu}'_{q'}\mu'_{q'}}\left[R^{-1}_{\rm W}\left(\frac{\tilde{k}'_{q'}}{m_{q'}}, B_c(v'_{q'\bar{q}})\right)\right] D^{1/2}_{\tilde{\mu}'_{\bar{q}}-\mu_q} \\
&\times \left[R^{-1}_{\rm W}\left(\frac{\tilde{k}'_{\bar{q}}}{m_{\bar{q}}}, B^{-1}_c(v'_{q'\bar{q}})\right)\right]\Bigg\} \psi^*_{\alpha'^*}(|\vec{\tilde{k}}'_{\bar{q}}|)\psi_\alpha(|\vec{\tilde{k}}_{\bar{q}}|).
\end{aligned} \tag{9}$$

The procedure presented here yields expressions for the hadronic currents that satisfy the required covariant properties; i.e. they transform as four-vectors. The proof requires us to transform the velocity states to the physical momenta via a canonical boost $B_c(v)$ [8].

In order to proceed to extract the form factors by using the obtained current matrix elements we need to specify the system kinematics. We make the most natural choice, in which a meson α, initially at rest, decays into a meson α' moving in the x direction with momenta $\kappa_{\alpha'}$:

$$\underline{k}_\alpha = \begin{pmatrix} m_\alpha \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \underline{k}_{\alpha'} = \begin{pmatrix} \sqrt{m^2_{\alpha'} + \kappa^2_{\alpha'}} \\ \kappa_{\alpha'} \\ 0 \\ 0 \end{pmatrix} \tag{10}$$

[2]In the sequel, an asterisk is used to label a meson with total spin 1.

with

$$\kappa_{\alpha'}^2 = \frac{1}{4m_\alpha^2}(m_\alpha^2 + m_{\alpha'}^2 - \underline{q}^2)^2 - m_{\alpha'}^2. \tag{11}$$

The modulus of the α' meson center-of-mass momentum $\kappa_{\alpha'} = |\vec{\underline{k}}_{\alpha'}|$ is thus restricted by $0 \le \kappa_{\alpha'}^2 \le (m_\alpha^2 - m_{\alpha'}^2)^2/(4m_\alpha^2)$. The allowed values of the 4-momentum transfer squared are then

$$0 \le \underline{q}^2 \le (m_\alpha - m_{\alpha'})^2. \tag{12}$$

B. Form factors

Form factors are obtained by equating matrix elements of the obtained hadronic currents to their most general decomposition in terms of covariants and Lorentz-invariant functions. An appropriate decomposition of the $P \to P$ current can be written as [43]

$$J^\nu_{\alpha\to\alpha'}(\vec{\underline{p}}'_{\alpha'};\vec{\underline{p}}_\alpha) = \left((\underline{p}_\alpha + \underline{p}'_{\alpha'})^\nu - \frac{m_\alpha^2 - m_{\alpha'}^2}{\underline{q}^2}\underline{q}^\nu\right)F_1(\underline{q}^2) + \frac{m_\alpha^2 - m_{\alpha'}^2}{\underline{q}^2}\underline{q}^\nu F_0(\underline{q}^2), \tag{13}$$

where $\underline{q} = (\underline{p}_\alpha - \underline{p}_{\alpha'})$ is the timelike, 4-momentum transfer. And

$$\begin{aligned} J^\nu_{\alpha\to\alpha'^*}(\vec{\underline{p}}'_{\alpha'^*},\underline{\sigma}'_{\alpha'^*};\vec{\underline{p}}_\alpha) = {}& \frac{2i\epsilon^{\nu\mu\rho\sigma}}{m_\alpha + m_{\alpha'^*}}\epsilon^*_\mu(\vec{\underline{p}}'_{\alpha'^*},\underline{\sigma}'_{\alpha'^*})\underline{p}'_{\alpha'^*\rho}\underline{p}_{\alpha\sigma}V(\underline{q}^2) - (m_\alpha + m_{\alpha'^*})\epsilon^{*\nu}(\vec{\underline{p}}'_{\alpha'^*},\underline{\sigma}'_{\alpha'^*})A_1(\underline{q}^2) \\ & + \frac{\epsilon^*(\vec{\underline{p}}'_{\alpha'^*},\underline{\sigma}'_{\alpha'^*})\cdot \underline{q}}{m_\alpha + m_{\alpha'^*}}(\underline{p}_\alpha + \underline{p}'_{\alpha'^*})^\nu A_2(\underline{q}^2) + 2m_{\alpha'^*}\frac{\epsilon^*(\vec{\underline{p}}'_{\alpha'^*},\underline{\sigma}'_{\alpha'^*})\cdot \underline{q}}{\underline{q}^2}\underline{q}^\nu A_3(\underline{q}^2) \\ & - 2m_{\alpha'^*}\frac{\epsilon^*(\vec{\underline{p}}'_{\alpha'^*},\underline{\sigma}'_{\alpha'^*})\cdot \underline{q}}{\underline{q}^2}\underline{q}^\nu A_0(\underline{q}^2), \end{aligned} \tag{14}$$

in the $P \to V$ case [43]. $\epsilon^*(\vec{\underline{p}}'_{\alpha'^*},\underline{\sigma}'_{\alpha'^*})$ is the polarization 4-vector of the α'^* meson and $A_3(\underline{q}^2)$ the linear combination

$$A_3(\underline{q}^2) = \frac{m_\alpha + m_{\alpha'^*}}{2m_{\alpha'^*}}A_1(\underline{q}^2) - \frac{m_\alpha - m_{\alpha'^*}}{2m_{\alpha'^*}}A_2(\underline{q}^2). \tag{15}$$

With the kinematics adopted in Eq. (10) the polarization vectors read

$$\begin{aligned} \epsilon(\vec{\underline{k}}'_{\alpha'^*},\pm 1) &= \frac{1}{\sqrt{2}}\left(\mp\frac{\kappa_{\alpha'^*}}{m_{\alpha'^*}}, \mp\sqrt{1+\left(\frac{\kappa_{\alpha'^*}}{m_{\alpha'^*}}\right)^2}, -i, 0\right), \\ \epsilon(\vec{\underline{k}}'_{\alpha'^*},0) &= (0,0,0,1). \end{aligned} \tag{16}$$

The calculation of the form factors requires the insertion of the expressions found in Eq. (8) and in Eq. (9) in the left-hand sides of Eqs. (13) and (14), respectively (see Ref. [11] for details). Note that Eq. (14) expresses a system of four equations with four unknowns for every polarization vector $\epsilon(\vec{k},\mu)$, where μ can be 1, −1 or 0. The kinematics used in Eq. (10) leads to ten nonvanishing matrix elements, namely $J^2(0)$, $J^3(0)$, $J^\mu(\pm 1)$, $\mu = 0,1,2,3$, where $J^\nu(\underline{\mu}'_{\alpha'^*}) := J^\nu_{\alpha\to\alpha'^*}(\vec{\underline{k}}'_{\alpha'^*},\underline{\mu}'_{\alpha'^*};\vec{\underline{k}}_\alpha)$. Since $J^\mu(1)$ and $J^\mu(-1)$ are simply related by a space reflection one is left with six matrix elements of the current, from which only four are independent. Consequently, the form factors are determined uniquely.

This is an important achievement, since analogous calculations in the front form of dynamics fail in the attempt to extract the form factors unambiguously in $P \to V$ meson transitions. Difficulties originated by violation of rotational invariance in the front form make the description of the dynamics of spin-1 mesons troublesome. We will refer to this issue later.

It was demonstrated and explained in [11] that in timelike processes there is no manifestation of cluster separability violation that may appear in the form of nonphysical contribution to the decomposition of the current. That occurred, by contrast, in the electromagnetic case [7,8], and the problem was attributed to the cluster-separability violation caused by the Bakamjian-Thomas construction [3].

We are now in the position to present our numerical results for several weak decays and discuss the comparison with analogous results in the front form of dynamics.

III. NUMERICAL STUDIES

The method presented here to derive hadronic currents and to extract form factors has been tested in [11]. The work presented in Ref. [11] extended the application of the PF formalism to the weak interaction, and considered mesons with different constituent-quark masses. Those studies allowed us to see, in a comprehensive way, how the predictions of heavy-quark symmetry arise when one of the quark masses increases asymptotically. As predicted by heavy-quark symmetry, electromagnetic and weak form factors are related in the exact limit $m_q \to \infty$. Such examinations provided analytic evidence for the expected connection between these two types of interactions, and consequently, a sign of reliability for our treatment of relativistic composite systems of different constituent masses. Reference [11] focused on the study of heavy-quark symmetry as well as on cluster-separability properties of this point-form approach. For that purpose the same harmonic-oscillator wave function with parameter $a = 0.55$ GeV was used for all numerical studies.

In the present work, however, we want to test the PFRQM approach in another way. We introduce a flavor dependence in the wave function, by assuming a different harmonic-oscillator parameter for each meson. Even taking into account this flavor dependence, the model remains very simple. Thus, it might not be sophisticated enough to establish quantitative predictions which could be compared with experiments. Nonetheless, it is necessary to carry out such calculations for several decays in order to understand how the point-form approach compares with other approaches and to learn at least qualitatively how the transition form factors depend on the kind of transition considered. Numerical studies within such a simple model will serve as our starting point for future developments in PFRQM.

We are particularly interested in comparisons with front-form results and in the role of nonvalence contributions in the description of currents and form factors. In front form, such nonvalence contributions turn out to become important when one goes from spacelike to timelike momentum transfers, and thus they may play a role in the point-form approach as well. For timelike momentum transfer it is not possible to use the $q^+ = 0$ frame in front form. As a consequence, nonvalence configurations leading to Z-graph contributions (quark-antiquark pairs created from the vacuum) can occur. Such Z-graph contributions have been analyzed in Ref. [34]. Applying analytic continuation ($q_\perp \to iq_\perp$) from the spacelike to the timelike momentum transfer region to the transition from factors calculated in a $q^+ = 0$ frame for spacelike momentum transfers, it is shown that the outcome is the same as the results from a direct calculation of the decay form factors in the timelike region (where $q^+ \neq 0$), provided that the Z-graphs contributions are appropriately taken into account. The importance of the Z-graph contributions decreases with increasing the mass of the heavy quark and it vanishes in the heavy-quark limit, since an infinitely heavy quark-antiquark pair cannot be produced out of the vacuum [33]. The numerical values obtained for the Isgur-Wise function within the point-form approach agree with those obtained within the analogous front-form quark model [11]. As soon as the decay form factors are calculated for finite physical masses of the heavy quarks, differences between the point- and front-form approach must appear.

Another—but related—particular issue we would like to address in the context of these comparisons concerns the frame dependence that appears in the calculation of form factors of $P \to V$ transitions in the front-form approach. In the light-front quark model of Ref. [39], the authors choose a frame in which the momentum transfer is purely longitudinal, i.e. $q_\perp = 0$, $q^2 = q^+ q^-$. Working in this way, form factors of processes that involve vector mesons cannot be extracted unambiguously, and the form factors exhibit a dependence on whether the daughter meson goes in the positive or negative z direction. On the other hand, it was shown in Ref. [11] that in the point form there is no frame dependence of the form factors in timelike processes and they can be determined unambiguously from the different components of the current.

In order to quantify all these differences, let us define first the wave function and the parameters employed in these numerical studies.

A. Meson wave function

The form factors are solely determined by the $q\bar{q}$ bound-state wave function and the constituent quark masses. One is free to use any model wave function obtained from a particular bound-state problem. We choose the harmonic-oscillator wave function defined as

$$\psi(\kappa) = \frac{2}{\pi^{\frac{1}{4}} a^{\frac{3}{2}}} \exp\left(-\frac{\kappa^2}{2a^2}\right), \qquad (17)$$

which allows us for a direct comparison with Ref. [39]. The numerical results presented here have been computed using the model parameters quoted in Table I, which have been taken from Ref. [39] as well.

TABLE I. Harmonic-oscillator parameters and quark masses (in GeV) used for the calculation of transition form factors in this work. They were determined in Ref. [39] by fitting the wave functions to the experimental values for the decay constants. The Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ as well as the physical meson masses are those quoted by the Particle Data Group [44].

a_π	a_ρ	a_K	a_{K^*}	a_D	a_{D^*}	a_B	a_{B^*}	$m_{u,d}$	m_b	m_c	m_s
0.33	0.30	0.38	0.31	0.46	0.47	0.55	0.55	0.25	4.8	1.6	0.40

TABLE II. $F_1(0)$, or equivalently $f_+(0)$, form factor for $P \to P$ transitions, corresponding to Figs. 2–3, obtained in front form (FF) and in point form (PF).

Decay	FF [39]	PF [this work]	Experiment [44]
$B \to D$	0.70	0.68	⋯
$B \to \pi$	0.26	0.26	⋯
$D \to \pi$	0.64	0.57	0.661 ± 0.022
$D \to K$	0.75	0.68	0.727 ± 0.011

B. $P \to P$ transitions

For pseudoscalar-to-pseudoscalar transitions, in order to allow for comparison with other works, besides $F_0(q^2)$ and $F_1(q^2)$, also $f_-(q^2)$ is depicted for all computed decays. $f_-(q^2)$ and $f_+(q^2)$ are defined by

$$J^\mu(p_1, p_2) = f_+(q^2)(p_1 + p_2)^\mu + f_-(q^2)(p_1 - p_2)^\mu, \tag{18}$$

where p_1 and p_2 are the initial and final meson 4-momenta. Their relation with $F_0(q^2)$ and $F_1(q^2)$ is given by

$$\begin{aligned} F_1(q^2) &= f_+(q^2), \\ F_0(q^2) &= f_+(q^2) + \frac{q^2}{m_\alpha{}^2 - m_{\alpha'}{}^2} f_-(q^2). \end{aligned} \tag{19}$$

The values at $q^2 = 0$ for $F_1(0)$, or equivalently for $f_+(0)$, are shown in Table II together with the results obtained within the light-front quark model [39]. For heavy-to-heavy transitions, i.e. $B \to D$, as well as for $B \to \pi$ transitions, both FF and PF results seem to agree quite well, whereas they differ slightly for $D \to \pi(K)$.

We do not have a definitive explanation for this fact, but we suspect that these differences are due to the different way in which Z graphs and other nonvalence contributions enter the form factors in either approach. There is a particular frame, namely the $q^+ = 0$ frame, in the front form, where Z graphs disappear. In point form a particular $q^+ = 0$ frame can be realized for lepton-hadron scattering by taking the limit of infinitely large Mandelstam s, which corresponds to the infinite-momentum frame of the hadron (cf. [14,15]). This explains, e.g., the equality of our point-form results for electromagnetic meson form factors (for $q^2 < 0$) with corresponding front-form results [8,11]. In the $q^+ = 0$ frame however, weak decays cannot take place, since the process is necessarily timelike ($q^2 = q^+q^- - q_\perp > 0$) or lightlike at the point for maximal recoil ($q^2 = 0$). In the light-front quark model of Ref. [39], the calculations are done in a frame where the momentum transfer is purely longitudinal, this is $q_\perp = 0$, $q^2 = q^+q^-$. At $q^2 = 0$ either q^+ or q^- must vanish which corresponds to the daughter meson going either in the $+$ or in the $-z$ direction, respectively. Since the pseudoscalar decay form factors do not depend on whether the daughter meson goes into the $+$ or $-z$ direction, one can assume $q^+ = 0$. This implies, however, that Z contributions vanish at the maximum recoil point. For $q^2 > 0$ there is, however, no argument to exclude Z-graph contributions in the decay form factors. In point form one does not even have an argument at $q^+ = 0$ (apart from the mass of the produced $Q\bar{Q}$ pair) that Z graphs should vanish.

A quantitative estimate of the Z-graph contribution is not within the scope of this work. We have seen, however, in the previous work of Ref. [11] that the point-form results reproduce the front-form ones exactly in the heavy-quark limit. One can therefore expect that for heavy-to-heavy transitions point-form and front-form results show a greater resemblance than for heavy-to-light transitions. For heavy-to-light processes nonvalence contributions are expected to be more important. It is thus not surprising that the results differ in both approaches. In the $D \to K$ and $D \to \pi$ cases, point- and front-form results differ considerably, the front-form results being somewhat closer to the experimental data [44].

Another resemblance with the front-form results is that $f_-(q^2) \sim -f_+(q^2)$ for $B \to \pi$ and to a lesser extent for $D \to \pi$ (cf. Figs. 2 and 3). Near zero recoil

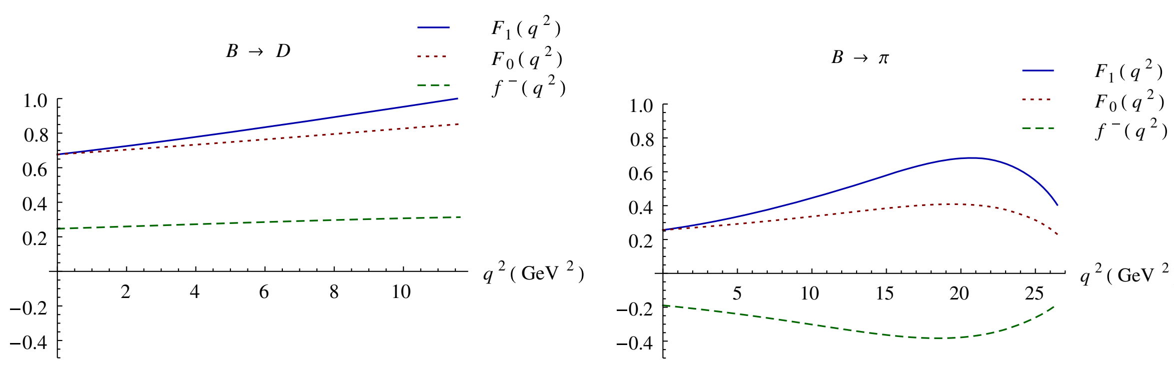

FIG. 2 (color online). $B \to D$ and $B \to \pi$ transition form factors in the whole range $0 \le q^2 \le (M_B - M_{D(\pi)})^2$. Parameters for the quark masses and harmonic-oscillator wave functions are taken from Table I. For the meson masses the current values given by the Particle Data Group have been taken [44].

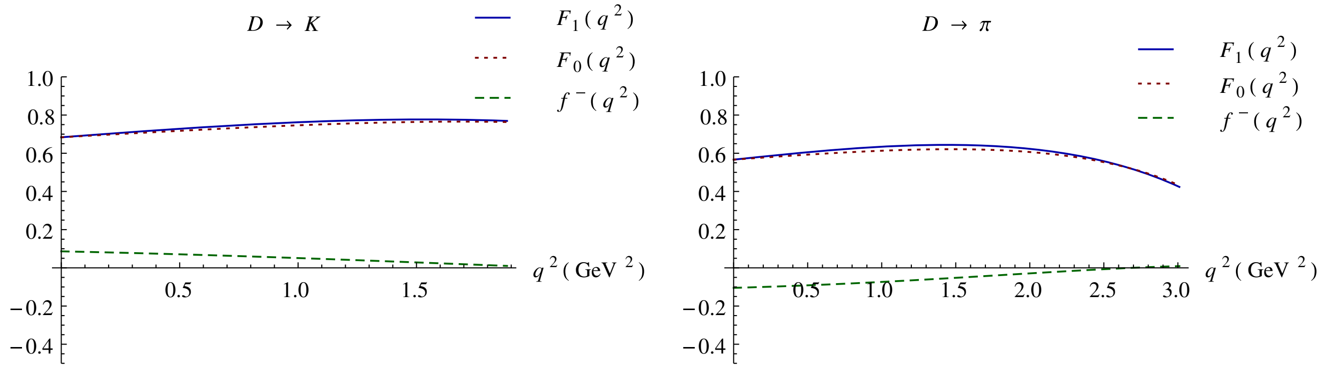

FIG. 3 (color online). Same as in Fig. 2 for $D \to K$ and $D \to \pi$ transition form factors.

(where q^2 is maximal) heavy-quark symmetry predicts $(f_+ + f_-)^{B(D)\pi} \sim \frac{1}{\sqrt{m_{B(D)}}}$. In our case we have

$$(f_+ + f_-)^{B\pi}_{q_{\max}} \sim 0.22, \qquad (f_+ + f_-)^{D\pi}_{q_{\max}} \sim 0.43, \quad (20)$$

whereas $1/\sqrt{m_B} \sim 0.43$ and $1/\sqrt{m_D} \sim 0.73$.

In order to get an idea of the reliability of the results as a function of the quality of the wave function, we have recomputed the above given results using a unique parameter $a = 0.42$ GeV, which is the average of the values considered in this work. The form factors F_1 at $q^2 = 0$ are given in Table III. One can appreciate a considerable difference with respect to those given in Table II, as a consequence of the need for distinguishing the meson considered.

C. $P \to V$ transitions

The comparison for transitions that involve mesons with spin is more interesting. In the light-front quark model [39], the form factors for $P \to V$ meson transitions extracted in the $q_\perp = 0$ frame exhibit a certain frame dependence. For a given q^2, the form factors depend on whether the recoiling daughter moves in the positive "+" or negative "−" z direction relative to the parent meson. In the light-front quark model the results for the form factors are larger in the "+" frame than in the "−" one. The exact vanishing of Z graphs at $q^2 = 0$ in the "+" frame is taken as an argument in Ref. [39] to conclude that Z graphs are less important in the "+" frame than in the "−" frame.

In Table IV results for both frames together with the point-form results obtained in this work are given at $q^2 = 0$. The authors of [39] interpret the difference between the results at $q^2 = 0$ in the "+" and "−" frames as a measure for the Z-graph contribution present in the "−" frame. In the point form all timelike form factors can be extracted without ambiguity and no frame dependence appears in our description of weak decays. Again, the scope of this work does not allow us to give a precise estimate of Z-graph contributions. One could perhaps guess that they are of the same order of magnitude as the difference between "+" and "−" frames in front form.

In Tables IV–VII our form-factor results at $q^2 = 0$ are compared with those of Ref. [39] for several decays. One observes that the results obtained in the point form for $A_0(0)$, $A_1(0)$ and $A_2(0)$ are very similar in all the computed transitions, whereas they differ notably in the front form. There seems to be a good agreement between both approaches for $V(0)$ and $A_0(0)$. For these two form factors one sees that for the heavy-to-heavy transition the point-form result lies between the obtained ones in the front form in the "+" and "−" frames, being closer to the "+" one.

TABLE III. $F_1(0)$, or equivalently $f_+(0)$, form factor obtained for $P \to P$ transitions, using the same oscillator parameter $a = 0.42$ GeV.

$B \to D$	$B \to \pi$	$D \to \pi$	$D \to K$
0.37	0.32	0.64	0.71

TABLE IV. Form factors at $q^2 = 0$ for the $B \to D^*$ transition obtained within the light-front quark model in Ref. [39] (FF) in the frames where the recoiling daughter moves in the positive z direction ("+" frame) and negative z direction ("−" frame) in comparison with the results obtained in the point form (PF).

$B \to D^*$	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$
FF [39] in the "+" frame	0.78	0.73	0.68	0.61
FF [39] in the "−" frame	0.62	0.58	0.59	0.61
PF (this work)	0.76	0.72	0.72	0.72

TABLE V. Form factors at $q^2 = 0$ for the $D \to K^*$ transition obtained within the light-form quark model (FF) in the frame where the recoiling daughter moves in the positive z direction, i.e. "+" frame, and in the point form (PF) of relativistic quantum mechanics.

$D \to K^*$	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$
FF [39]	0.87	0.71	0.62	0.46
PF [this work]	0.87	0.70	0.71	0.73

TABLE VI. Same comparison as in Table V but for the $B \to \rho$ transition.

$B \to \rho$	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$
FF [39]	0.30	0.28	0.20	0.18
PF [this work]	0.31	0.28	0.27	0.26

TABLE VII. Same comparison as in Table V but for the $D \to \rho$ transition.

$D \to \rho$	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$
FF [39]	0.78	0.63	0.51	0.34
PF [this work]	0.80	0.63	0.64	0.64

$A_1(0)$ and $A_2(0)$ turn out to be larger in the point form in all cases.

For the whole q^2 range, i.e. $0 \le q^2 \le (m_\alpha - m_{\alpha'})^2$, the form factors $V(q^2), A_0(q^2), A_1(q^2)$ and $A_2(q^2)$ are depicted in Figs. 4–7. If one compares with the corresponding plots in Ref. [39] the observations made already for $q^2 = 0$ are confirmed. For the B decays our form factors resemble very much those of Ref. [39] (in the "+" frame) with $A_2(q^2)$ showing the biggest deviations. For D decays larger differences can be observed, in particular for $A_1(q^2)$ and $A_2(q^2)$, but the qualitative features of the form factors are still quite similar. This discrepancy is, of course, foreseeable since the point- and front-form approaches are not equivalent as long as one does not include nonvalence contributions. The equivalence is only reached in the heavy-quark limit, where the same Isgur-Wise function is obtained [11].

IV. CONCLUSIONS AND OUTLOOK

We have applied the PFRQM approach to several weak decays. Numerical results have been given that can be compared with experiments and with other approaches, e.g. the analogous calculation in front form considered herein. While the harmonic-oscillator wave function, Eq. (17), still might be too simple to make quantitative predictions that can be compared with experiments, it has served as a first step in the understanding of our point-form approach by means of numerical studies that allow for direct comparisons.

While in the front form the obtained results for $P \to V$ transitions exhibit a certain dependence on the reference frame, i.e. on whether the recoiling daughter moves in the positive or negative z direction relative to the parent meson, in the point form all form factors are determined unambiguously.

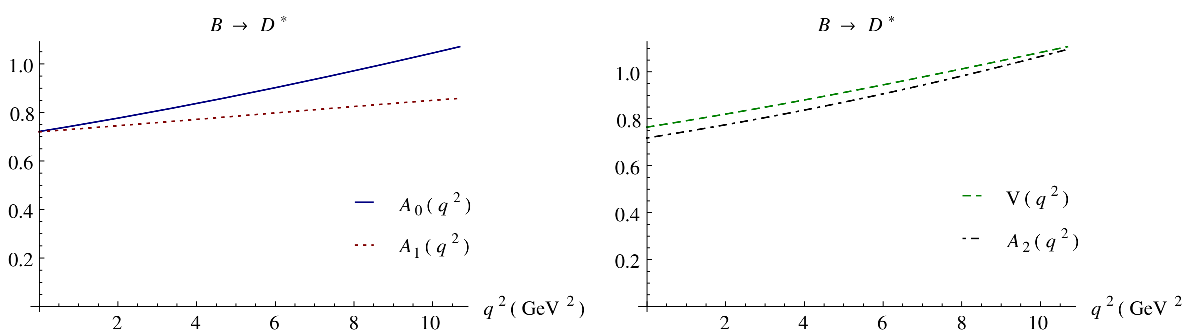

FIG. 4 (color online). $B \to D^*$ transition form factors in the whole range $0 \le q^2 \le (M_B - M_{D^*})^2$. Parameters for the quark masses and harmonic-oscillator wave functions are taken from Table I. For the meson masses the current values given by the Particle Data Group are taken [44].

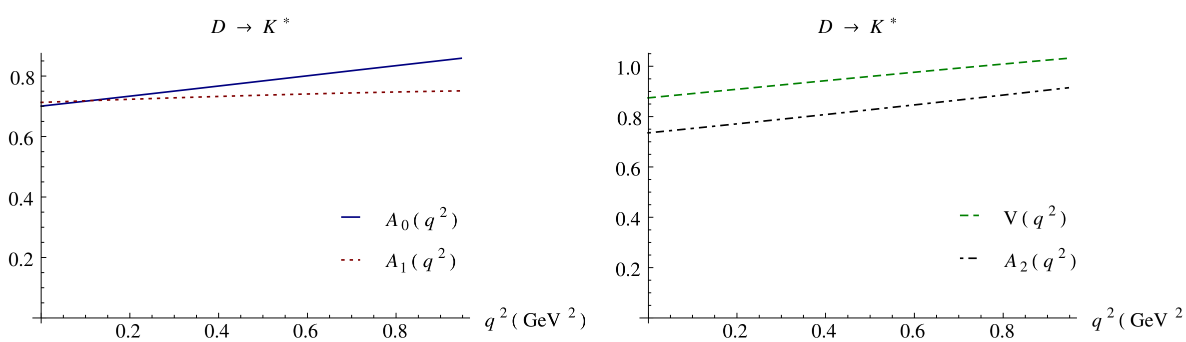

FIG. 5 (color online). Same as in Fig. 4 for $D \to K^*$ transition form factors.

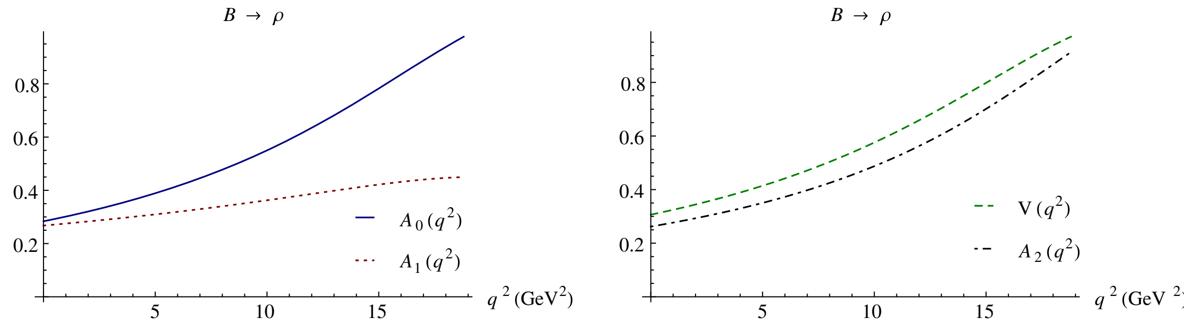

FIG. 6 (color online). Same as in Fig. 4 for $B \to \rho$ transition form factors.

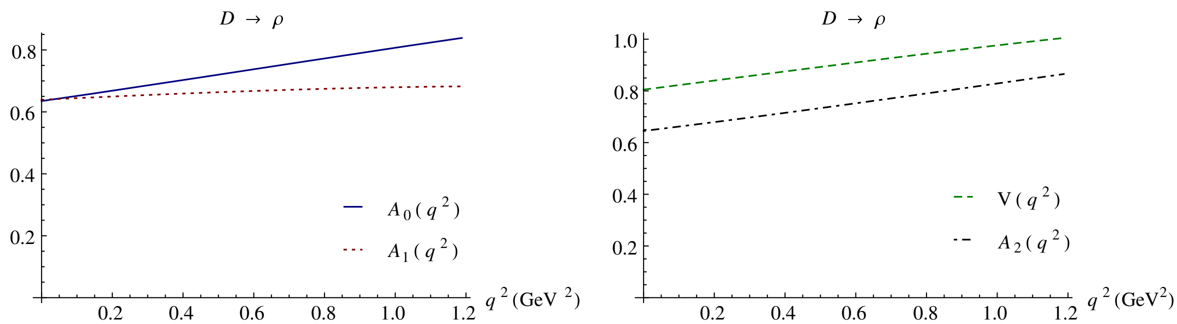

FIG. 7 (color online). Same as in Fig. 4 for $D \to \rho$ transition form factors.

Furthermore, in contrast to what happened using the approach in the electromagnetic case [7,9], we are able to extract all form factors of mesons with spin 0 decaying into mesons of spin 0 and to spin 1 without the need for introducing any nonphysical contribution—spurious form factor—to correct covariant deficiencies of the current.

In the heavy-quark limit, as was shown in Ref. [11], point-form and front-form calculations yield the same numerical result for the Isgur-Wise function. This equivalence is possible because in the heavy-quark limit nonvalence contributions, such as Z graphs, vanish. Numerical comparisons of our outcome with analogous front-form calculations show that the results obtained from both approaches do not coincide exactly outside the limit. This is not surprising. Nonvalence contributions such as Z graphs cannot be present in the $m_Q \to \infty$ limit. On the other hand, such Z-graph contributions as well as other vacuum-induced currents do not exist in the front form, even for finite masses if one choose the $q^+ = 0$ frame, since momentum conservation imposes the "+" sum of momenta at every vertex to be positive. Such kind of nonvalence contributions cannot be excluded, however, in the point form. As long as nonvalence contributions are not calculated explicitly, the point- and the front-form approaches cannot be equivalent.

From the coincidence of both approaches in the heavy-quark limit we conclude that the apparent discrepancy between the point and the front forms outside of the heavy-quark limit must be due to the different way in which vacuum-induced currents enter the description in every form of dynamics.

All this is relevant in order to explore the effect of introducing additional degrees of freedom in the approach. It is the subject of future work to introduce Z-graph contributions explicitly in the coupled channel approach and to investigate how they affect the form factors (for recent advances in this direction, see [14]). Similar studies on this subject were carried out in the front form [34]. Like in Ref. [34] an estimate on Z-graph contributions within our approach could be obtained by calculating the transition form factors in the spacelike region, where one can go into the infinite-momentum frame and continue those results analytically to the timelike momentum-transfer region. The work presented here poses the starting point for this goal. Studies concerning weak decays in the spacelike region and its analytic continuation have been initiated recently in [15,16].

ACKNOWLEDGMENTS

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APPENDIX A: VELOCITY STATES

An n-particle velocity state $|v;\vec{k}_1,\mu_1;\vec{k}_2,\mu_2;\ldots;\vec{k}_n,\mu_n\rangle$ is defined through an overall velocity v and n individual momenta and spin projections $\{\vec{k}_i,\mu_i\}$, such that $\sum_{i=1}^n \vec{k}_i = 0$. A velocity state represents an n-particle system in the rest frame that is boosted to a frame with a total 4-velocity v ($v^\mu v_\mu = 1$) by means of a canonical boost $B_c(v)$ [3]:

$$|v;\vec{k}_1,\mu_1;\vec{k}_2,\mu_2;\ldots;\vec{k}_n,\mu_n\rangle := \hat{U}_{B_c(v)}|\vec{k}_1,\mu_1;\vec{k}_2,\mu_2;\ldots;\vec{k}_n,\mu_n\rangle. \tag{A1}$$

They satisfy the orthogonality and completeness relations [5]:

$$\begin{aligned}\langle v';\vec{k}'_1,\mu'_1;\vec{k}'_2,\mu'_2;\ldots;\vec{k}'_n,\mu'_n|v;\vec{k}_1,\mu_1;\vec{k}_2,\mu_2;\ldots;\vec{k}_n,\mu_n\rangle \\ = v_0\delta^3(\vec{v}'-\vec{v})\frac{(2\pi)^3 2\omega_{k_n}}{(\sum_{i=1}^n \omega_{k_i})^3}\left(\prod_{i=1}^{n-1}(2\pi)^3 2\omega_{k_i}\delta^3(\vec{k}'_i-\vec{k}_i)\right) \\ \times\left(\prod_{i=1}^{n}\delta_{\mu'_i\mu_i}\right)\end{aligned} \tag{A2}$$

and

$$\begin{aligned}\mathbf{1}_{1,\ldots,n} = \sum_{\mu_1=-j_1}^{j_1}\cdots\sum_{\mu_n=-j_n}^{j_n}\int\frac{d^3v}{(2\pi)^3 v_0}\left[\prod_{i=1}^{n-1}\frac{d^3k_i}{(2\pi)^3 2\omega_{k_i}}\right] \\ \times\frac{(\sum_{i=1}^n\omega_{k_i})^3}{2\omega_{k_n}}|v;\vec{k}_1,\mu_1;\vec{k}_2,\mu_2;\ldots;\vec{k}_n,\mu_n\rangle\langle v;\vec{k}_1,\mu_1;\vec{k}_2,\mu_2;\ldots;\vec{k}_n,\mu_n|,\end{aligned} \tag{A3}$$

with m_i, $\omega_{k_i} := \sqrt{m_i^2+\vec{k}_i^2}$, and j_i, being the mass, the energy, and the spin of the ith particle, respectively.

Velocity states transform under Lorentz transformations Λ as

$$\begin{aligned}\hat{U}_\Lambda|v;\vec{k}_1,\mu_1;\vec{k}_2,\mu_2;\ldots;\vec{k}_n,\mu_n\rangle \\ = \sum_{\mu'_1,\mu'_2,\ldots,\mu'_n}\left\{\prod_{i=1}^n D^{j_i}_{\mu'_i\mu_i}[R_{\mathrm{W}}(v,\Lambda)]\right\}|\Lambda v;\overrightarrow{R_{\mathrm{W}}(v,\Lambda)k_1},\mu'_1;\overrightarrow{R_{\mathrm{W}}(v,\Lambda)k_2},\mu'_2;\ldots;\overrightarrow{R_{\mathrm{W}}(v,\Lambda)k_n},\mu'_n\rangle,\end{aligned} \tag{A4}$$

with the Wigner-rotation matrix

$$R_{\mathrm{W}}(v,\Lambda) = B_c^{-1}(\Lambda v)\Lambda B_c(v), \tag{A5}$$

where

$$B_c(v) = \begin{pmatrix} v^0 & \mathbf{v}^T \\ \mathbf{v} & \mathbf{1}+\frac{v^0-1}{\mathbf{v}^2}\mathbf{v}\mathbf{v}^T\end{pmatrix}. \tag{A6}$$

APPENDIX B: VERTEX OPERATORS

The creation and annihilation of particles is introduced in this framework by means of *vertex operators* $\hat{K}$ that are specified by the velocity-state representation and an appropriate relation to the pertinent field-theoretical interaction-Lagrangian density $\hat{\mathcal{L}}_{\mathrm{int}}$. In this work, $\hat{\mathcal{L}}_{\mathrm{int}}$ corresponds to the Lagrangian density of the weak interaction. Due to velocity conservation that follows from the point-form version of the Bakamjian-Thomas construction, one is led to define matrix elements of $\hat{K}$ by [10,42]

$$\begin{aligned}\langle v,\vec{k}_1,\mu_1;\ldots;\vec{k}_{n+1},\mu_{n+1}|\hat{K}^\dagger|v,\vec{k}_1,\mu_1;\vec{k}_2,\mu_2;\ldots;\vec{k}_n,\mu_n\rangle = \langle v,\vec{k}_1,\mu_1;\vec{k}_2,\mu_2;\ldots;\vec{k}_n,\mu_n|\hat{K}|v,\vec{k}_1,\mu_1;\ldots;\vec{k}_{n+1},\mu_{n+1}\rangle^* \\ = \mathcal{N}_{n+1,n}v^0\delta^3(\vec{v}-\vec{v}')\langle v,\vec{k}_1,\mu_1;\ldots;\vec{k}_{n+1},\mu_{n+1}|\hat{\mathcal{L}}_{\mathrm{int}}(0)f(\Delta m)|v,\vec{k}_1,\mu_1;\vec{k}_2,\mu_2;\ldots;\vec{k}_n,\mu_n\rangle,\end{aligned} \tag{B1}$$

where $\mathcal{N}_{n+1,n} = (2\pi)^3/\sqrt{\mathcal{M}'_{n+1}\mathcal{M}'_n}$, $\mathcal{M}'_n = \sum_{i=1}^k \omega_i$ and $f(\Delta m = \mathcal{M}'_{n+1}-\mathcal{M}'_n)$ denotes a vertex form factor that can be introduced in order to account for (part of) the neglected off-diagonal velocity contributions and to regulate integrals.

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