Unification of gauge couplings and the Higgs mass in vectorlike particle theories extended into NMSSM

Yi-Lei Tang*

Institute of Theoretical Physics, Chinese Academy of Sciences, and State Key Laboratory of Theoretical Physics, P.O. Box 2735, Beijing 100190, China

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The minimal supersymmetry standard model (MSSM) extended with vectorlike theories have long been discussed. If we extend the vectorlike MSSM theory into NMSSM and let the vectorlike particles couple with the singlet (S), we find out a natural way to generate the vectorlike particle masses near 1 TeV through the breaking of the Z_3 group. Compared with the MSSM + vectorlike models, vectorlike models extended into NMSSM contain more Yukawa couplings and can help us adjust the renormalization group (RG) trajectories of the gauge couplings in order to unify the intersections. They can also help press down the gauge RG-β functions for a $5+\bar{5}+10+\overline{10}$ model, in order for the RG trajectories of the gauge couplings to unify before the Landau pole. We also discuss the Higgs mass contributed from the vectorlike sectors in this case.

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I. INTRODUCTION

The minimal supersymmetry standard model (MSSM) is a way to extend the standard model (SM) [1]. Within this framework, every particle is paired up with a superpartner with a different spin. One of the features of this model is that it can also automatically unify the gauge-coupling constants in the energy scale $\sim 10^{16}$ GeV [2], as is required by grand unification theory (GUT). Another way to extend the SM is to add extra copies of the $U(1)_Y \times SU(2)_L \times SU(3)_C$ multiplets. In order to construct an anomaly-free theory, the simplest way is just to add extra SM generations [3–6]. Compared with the theories extended with chiral fourth generation, theories extended with vectorlike (VL) generation can survive more easily among the experimental limits due to their particular vectorlike mass parameters.

MSSM extended with VL generations have long been discussed [7–13]. In order not to disturb the gauge-coupling unification scale, only copies of $SU(5)$ $5+\bar{5}$ or $10+\overline{10}$ multiplets are the candidates to be added into the theory. Up to one-loop level, one 10 multiplet modifies the renormalization group (RG) β functions of the gauge-coupling constants to the same extent as three 5 multiplet. If we would like the VL particles to be of the mass 1 TeV, and require the gauge couplings to meet with each other before they knock into the Landau pole, we can only choose N_5 copies of $5+\bar{5}$, where $N_5 \leq 3$, or one $10+\overline{10}$ [14].

The situation of four copies of $5+\bar{5}$ theory, or $5+\bar{5}+10+\overline{10}$ theory, is subtle. One-loop calculation unifies the gauge-coupling constants with a value of 2–5, which is too near to the Landau pole. Two-loop corrections usually contribute a positive value to the gauge RG-β functions and thus directly accelerate the gauge RG trajectories to blow up before they meet.

In MSSM, the effective Higgs mass receives extra loop contributions from the Yukawa couplings of the top quark/squark [1]. A higher order of stop mass raises the Higgs mass while it aggravates the tension of fine-tuning, and VL theories supply another source of the Higgs mass. In this case, VL particles should directly couple with the SM-like higgs, and thus affect the Higgs phenomenology [15].

As is well known, MSSM suffers from the μ problem, and adding VL generations cannot solve this problem at all. The most economic way to solve this problem is to extend MSSM into NMSSM by adding a singlet (S) [16]. The vacuum expectation value (VEV) of the S naturally generates roughly $\mu^2 \sim B\mu$, and the superpotential term $\lambda S H_u H_d$ can also add up to the Higgs quartic couplings, thus raising up the Higgs mass.

If we extend the VL-MSSM with a singlet S, just like the NMSSM, we may take advantage of both these theories. Similar consequences have been discussed in [17–19]. Models with a scalar singlet and VL fermions without supersymmetry is also studied in [20]. However, in our model, the VL particles couple with S, so the VL-mass terms naturally come from the VEV of the singlet Higgs rather than being input "by hand." By setting appropriate values to the Yukawa constants near the GUT scale, the mass spectrum of the VL fermions can be partly predicted. Gauge-coupling trajectories also receive extra contributions from the Yukawa coupling constants; thus the unification can be improved by adjusting the values of the Yukawa couplings in the $5+\bar{5}$ and $10+\overline{10}$ models. In the $5+\bar{5}+10+\overline{10}$ model, extra Yukawa coupling constants also contribute a nonignorable minus value in two-loop gauge RG-β functions; thus the gauge-coupling RG trajectories might meet before the perturbative theories lose effect.

II. GENERAL MODEL

If we extend the ordinary Z_3 NMSSM theory, we should assign the VL superfields Q, $\bar{Q}$, U, $\bar{U}$, D, $\bar{D}$ with Z_3 charges

*tangyilei10@itp.ac.cn

TABLE I. Superfields with their assigned quantum numbers.

	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$	Z_3	R-parity	Description
Q	$\frac{1}{6}$	2	3	$e^{i\frac{4\pi}{3}}$	−	Vectorlike quark doublet
$\bar{Q}$	$-\frac{1}{6}$	2	$\bar{3}$	1	−	Vectorlike antiquark doublet
U	$-\frac{2}{3}$	1	$\bar{3}$	1	−	Vectorlike right-handed up-type quark
$\bar{U}$	$\frac{2}{3}$	1	3	$e^{i\frac{4\pi}{3}}$	−	Vectorlike right-handed up-type antiquark
D	$-\frac{1}{3}$	1	$\bar{3}$	1	−	Vectorlike right-handed down-type quark
$\bar{D}$	$\frac{1}{3}$	1	3	$e^{i\frac{4\pi}{3}}$	−	Vectorlike right-handed down-type antiquark
L	$-\frac{1}{2}$	2	1	$e^{i\frac{4\pi}{3}}$	−	Vectorlike lepton doublet
$\bar{L}$	$\frac{1}{2}$	2	1	1	−	Vectorlike antilepton doublet
E	1	1	1	1	−	Vectorlike right-handed electron
$\bar{E}$	−1	1	1	$e^{i\frac{4\pi}{3}}$	−	Vectorlike right-handed antielectron
H_u	$\frac{1}{2}$	2	1	$e^{i\frac{2\pi}{3}}$	+	Up-type Higgs doublet
H_d	$-\frac{1}{2}$	2	1	$e^{i\frac{2\pi}{3}}$	+	Down-type Higgs doublet
S	1	1	1	$e^{i\frac{2\pi}{3}}$	+	NMSSM singlino Higgs
Q_3	$\frac{1}{6}$	2	3	1	−	SM third generation quark doublet
U_3	$-\frac{2}{3}$	1	$\bar{3}$	$e^{i\frac{4\pi}{3}}$	−	SM right-handed top

in order for them to be coupled with the S. These Z_3 charges will also keep VL fermions massless before Z_3 breaks. Appropriate assignment will also forbid the VL particles to mix with the SM Q_3, U_3 fields, which are highly limited by experiments. The quantum numbers assigned to the VL particles are listed in Table I. However, only the $5+\overline{5}+10+\overline{10}$ model involves all the fields listed in Table I. In our discussion about the $5+\overline{5}$ model and the $10+\overline{10}$ model, only part of the fields is needed. We should also note that all MSSM quarks and leptons are ignored except that the effects of the top sectors are considered in our discussions due to their large Yukawa coupling constant.

Here we are going to take a short description about the basic NMSSM. The superpotential is [16]

$$W_{NMSSM} = \lambda H_u H_d S + \frac{1}{3} S^3 + y_t Q_3 H_u U_3, \qquad (1)$$

and together we show the supersymmetry soft-breaking terms

$$V^{\rm soft}_{NMSSM} = m^2_{Hu}|\tilde{H}_u|^2 + m^2_{Hd}|\tilde{H}_d|^2 + M^2_S|\tilde{S}|^2 + \left(\lambda A_\lambda \tilde{H}_u \tilde{H}_d \tilde{S} + \frac{1}{3}\kappa A_\kappa \tilde{S}^3 + y_t A_{y_t} \tilde{Q}_3 \tilde{H}_u \tilde{U}_3 + {\rm H.c.}\right). \qquad (2)$$

The convention of the vacuum expectation values of the Higgs fields is

$$H^0_u = v_u + \frac{H_{uR} + iH_{uI}}{\sqrt{2}}$$
$$H^0_d = v_d + \frac{H_{dR} + iH_{dI}}{\sqrt{2}}$$
$$S = v_s + \frac{S_R + iS_I}{\sqrt{2}}, \qquad (3)$$

so the MSSM-like superpotential term $\mu_{eff} H_u H_d = \lambda v_s H_u H_d$ is generated.

Since the VL fermions receive mass terms from v_s, their masses are actually in the same quantity as v_s in most cases. In NMSSM, we are usually concerned about the $\mu_{\rm eff}$ and λ, and then $v_s = \frac{\mu_{\rm eff}}{\lambda}$. If $100\,{\rm GeV} < \mu_{\rm eff} < 1\,{\rm TeV}$, $0.01 < \lambda < 1$, which is usually applied for successful electroweak symmetry breaking, we can derive that $1\ {\rm TeV} < v_s < 100\ {\rm TeV}$. Collider bounds on VL quarks have been reviewed in [21], and the CMS Collaboration recently published their lower bound of the VL toplike quark mass to a value of 687–782 GeV [22], so in our discussions below, we assume all of our VL fermions lie in the mass scale 1 TeV, which is near the bound, although it is very easy to accumulate the VL mass towards 10–100 TeV by lowering λ or raising $\mu_{\rm eff}$. We also set 1 TeV as the turning point in our RG-trajectory calculations.

A. $5+\overline{5}$ model

The $5+\overline{5}$ model is the simplest model. It only contains vectorlike down-type right-handed quarks and antiquarks, D and $\bar{D}$, together with vectorlike leptonic doublets L and $\bar{L}$. The VL particles can only couple with the S, as the superfield shows below,

$$W_{5+\bar{5}} = \lambda_D \bar{D} D S + \lambda_L \bar{L} L S. \qquad (4)$$

In the literature, the right-handed neutrino N might be introduced so that LH_uN vertices are discussed; however, here we ignore them.

The supersymmetry breaking soft terms should be added:

$$V^{\rm soft}_{5+\bar{5}} = m^2_D(\tilde{D}\tilde{D}^\dagger + \tilde{\bar{D}}\tilde{\bar{D}}^\dagger) + m^2_L(\tilde{L}\tilde{L}^\dagger + \tilde{\bar{L}}\tilde{\bar{L}}^\dagger) + (\lambda_D A_{\lambda_D} \tilde{\bar{D}}\,\tilde{D}\,\tilde{S} + \lambda_L A_{\lambda_L} \tilde{\bar{L}}\,\tilde{L}\,S + {\rm H.c.}). \qquad (5)$$

Notice that we have assumed that $\tilde{D}$, $\tilde{\bar{D}}$, or $\tilde{L}$, $\tilde{\bar{L}}$ share the same soft-mass term only for simplicity. We can observe from (4), (5) that only singlet Higgs sectors are involved here. However, as to be discussed below, this still contributes to the SM-like Higgs mass.

B. $10 + \overline{10}$ model

The $10 + \overline{10}$ model contains vectorlike quark doublets Q and $\bar{Q}$, vectorlike up-type quark singlets U and $\bar{U}$, and vectorlike electron singlets E and $\bar{E}$. The Yukawa coupling structure is richer than $5 + \bar{5}$ theory due to the appearance of Higgs doublets:

$$W_{10+\overline{10}} = \lambda_Q \bar{Q} Q S + \lambda_U \bar{U} U S + \lambda_E \bar{E} E S + y_U Q H_u U + y_{\bar{U}} \bar{Q} H_d \bar{U}. \tag{6}$$

The corresponding soft terms are listed below:

$$\begin{aligned} V^{\text{soft}}_{10+\overline{10}} = {} & m_Q^2 (Q Q^\dagger + U U^\dagger) + m_E^2 E E^\dagger \\ & + (A_{\lambda_Q} \lambda_Q \tilde{\bar{Q}}\, \tilde{Q}\, \tilde{S} + A_{\lambda_U} \lambda_U \tilde{\bar{U}}\, \tilde{U}\, \tilde{S} \\ & + A_{\lambda_E} \lambda_E \tilde{\bar{E}}\, \tilde{E}\, \tilde{S} + A_{y_U} y_U \tilde{Q}\, \tilde{H}_u\, \tilde{U} \\ & + A_{y_{\bar{U}}} y_{\bar{U}} \tilde{\bar{Q}}\, \tilde{H}_d\, \tilde{\bar{U}} + \text{H.c.}). \end{aligned} \tag{7}$$

C. $5 + \bar{5} + 10 + \overline{10}$ model

The $5 + \bar{5} + 10 + \overline{10}$ model is not only a combination of $5 + \bar{5}$ and $10 + \overline{10}$; new terms also rise up:

$$\begin{aligned} W_{5+\bar{5}+10+\overline{10}} = {} & W_{5+\bar{5}} + W_{10+\overline{10}} + y_d Q H_d D + y_{\bar{d}} \bar{Q} H_u \bar{D} \\ & + y_L L H_d E + y_{\bar{L}} \bar{L} H_u \bar{E}. \end{aligned} \tag{8}$$

The corresponding soft terms are

$$\begin{aligned} V^{\text{soft}}_{5+\bar{5}+10+\overline{10}} = {} & V_{5+\bar{5}} + V_{10+\overline{10}} + (A_{y_d} y_d \tilde{Q} \tilde{H}_d \tilde{D} + A_{y_{\bar{D}}} y_{\bar{d}} \tilde{\bar{Q}} \tilde{H}_u \tilde{\bar{D}} \\ & + A_{y_L} y_L \tilde{L} \tilde{H}_d \tilde{E} + A_{y_{\bar{L}}} y_{\bar{L}} \tilde{\bar{L}} \tilde{H}_u \tilde{\bar{E}} + \text{H.c.}). \end{aligned} \tag{9}$$

During our discussions of the $5 + \bar{5} + 10 + \overline{10}$ model, we would like to set $m_Q^2 = m_D^2$ and $m_L^2 = m_E^2$ for simplicity.

III. $5 + \bar{5}$ MODEL

For the simplest $5 + \bar{5}$ model, the extra vectorlike particles couple with the S and thus contribute to the Higgs mass. It is much easier to calculate this contribution than the circumstances in $10 + \overline{10}$, or $5 + \bar{5} + 10 + \overline{10}$, because we only need to diagonalize 2×2 mass(-squared) matrices here. There are two down-type squarks, and the corresponding mass-squared matrix is

$$\begin{bmatrix} m_D^2 + \lambda_D^2 v_s^2 & v_s^2 \kappa \lambda_D - v_d v_u \lambda \lambda_D + A_{\lambda_D} \lambda_D v_s \\ v_s^2 \kappa \lambda_D - v_u v_d \lambda \lambda_D + A_{\lambda_D} \lambda_D v_s & m_D^2 + \lambda_D^2 v_s^2 \end{bmatrix}. \tag{10}$$

The mass matrix of the two slepton doublets is

$$\begin{bmatrix} m_L^2 + \lambda_L^2 v_s^2 & v_s^2 \kappa \lambda_L - v_d v_u \lambda \lambda_L + A_{\lambda_L} \lambda_L v_s \\ v_s^2 \kappa \lambda_L - v_u v_d \lambda \lambda_L + A_{\lambda_L} \lambda_L v_s & m_L^2 + \lambda_L^2 v_s^2 \end{bmatrix}. \tag{11}$$

Gauge D-terms are ignored, this is also done in our remaining sections. Notice that for each slepton doublet, the masses of the charged slepton and the neutral slepton are degenerate in this model. To diagonalize (10) and (11), we acquire

$$\begin{aligned} M_{\tilde{D}_1}^2 &= m_D^2 + \lambda_D^2 v_s^2 + v_s^2 \kappa \lambda_D - v_d v_u \lambda \lambda_D + A_{\lambda_D} \lambda_D v_s, \\ M_{\tilde{D}_2}^2 &= m_D^2 + \lambda_D^2 v_s^2 - v_s^2 \kappa \lambda_D + v_d v_u \lambda \lambda_D - A_{\lambda_D} \lambda_D v_s, \\ M_{\tilde{L}_1^C}^2 = M_{\tilde{L}_1^N}^2 &= m_L^2 + \lambda_L^2 v_s^2 + v_s^2 \kappa \lambda_L - v_d v_u \lambda \lambda_L + A_{\lambda_L} \lambda_L v_s, \\ M_{\tilde{L}_2^C}^2 = M_{\tilde{L}_2^N}^2 &= m_L^2 + \lambda_L^2 v_s^2 - v_s^2 \kappa \lambda_L + v_d v_u \lambda \lambda_L - A_{\lambda_L} \lambda_L v_s, \end{aligned} \tag{12}$$

where $\tilde{D}_{1,2}$ are the two down-type squarks and $\tilde{L}_{1,2}^C$, $\tilde{L}_{1,2}^N$ indicate the two charged sleptons and the two neutral sleptons, respectively.

The masses of the down-type vectorlike quark and the charged (neutral) lepton are

$$\begin{aligned} M_D &= \lambda_D v_s \\ M_L^C = M_L^N &= \lambda_L v_s. \end{aligned} \tag{13}$$

Thus, we can take (12) and (13) into the Coleman-Weinberg potential under the $\overline{MS}$ or $\overline{DR}$ scheme,

$$V_{CW} = \frac{1}{64\pi^2} \left[\sum_{\text{scalars}} m_s^4 N_s \left(\ln \frac{m_s^2}{Q^2} - \frac{3}{2} \right) - \sum_{\text{fermions}} m_f^4 N_f \left(\ln \frac{m_f^2}{Q^2} - \frac{3}{2} \right) \right], \tag{14}$$

where Q is the renormalization scale, and N_s, N_f indicate the degrees of freedom of the particles. N_s and N_f take the value of 6 for colored fermionic or complex scalar particles, and 2 for colorless ones. Notice that the sum over fermions means to sum over all Weyl spinors, so each Dirac particle contributes an extra factor of 2 there.

If we assume that the SM-like Higgs mass eigenstates are in alignment with the VEV, that is to say, $\alpha = \frac{\pi}{2} - \beta$, where α is the mixing angle of the Higgs mass eigenstates, the SM-like Higgs mass should be added with a term

$$\Delta m_h^2 = \frac{1}{2}\sin^2\beta\left(\frac{\partial^2 V_{CW}^{5+\bar{5}}}{\partial v_u^2} - \frac{1}{v_u}\frac{\partial V_{CW}^{5+\bar{5}}}{\partial v_u}\right) + \frac{1}{2}\cos^2\beta\left(\frac{\partial^2 V_{CW}^{5+\bar{5}}}{\partial v_d^2} - \frac{1}{v_d}\frac{\partial V_{CW}^{5+\bar{5}}}{\partial v_d}\right) + \sin\beta\cos\beta\frac{\partial^2 V_{CW}^{5+\bar{5}}}{\partial v_u \partial v_d}$$
$$= \frac{3}{8\pi^2}\sin^2\beta\cos^2\beta\lambda^2\lambda_D^2\left(\ln\frac{M_{\tilde{D}_1}^2}{Q^2} + \ln\frac{M_{\tilde{D}_2}^2}{Q^2}\right) + \frac{1}{4\pi^2}\sin^2\beta\cos^2\beta\lambda^2\lambda_L^2\left(\ln\frac{M_{\tilde{L}_1}^2}{Q^2} + \ln\frac{M_{\tilde{L}_2}^2}{Q^2}\right). \quad (15)$$

It seems strange that the SM-like Higgs mass listed in (15) is Q dependent, which is invisible in MSSM theories. In MSSM, the tree-level quartic coupling among Higgs fields only comes from the gauge D-terms, so loop contributions irrelevant to the gauge terms should not be renormalized in order not to break the gauge invariance. However, in the case of NMSSM, the appearance of $\lambda S H_u H_d$ also contributes to the Higgs quartic coupling and receives the quantum correction from the field-strength renormalization constant Z_S of S. We can then define $\lambda_{\rm eff} = 1 + \frac{3}{64\pi^2}\lambda_D^2(\ln\frac{M_{\tilde{D}_1}^2}{Q^2} + \ln\frac{M_{\tilde{D}_2}^2}{Q^2}) + \frac{1}{32\pi^2}\lambda_L^2(\ln\frac{M_{\tilde{L}_1}^2}{Q^2} + \ln\frac{M_{\tilde{L}_2}^2}{Q^2})$ and replace λ with $\lambda_{\rm eff}$ in the tree-level term $\lambda^2 v^2 \sin^2 2\beta$; then we can also reach (15). Such kind of corrections have appeared in the literature, e.g., [23], although the method the authors used is too complicated to show the Q dependence.

(12), (13), and (14) also contribute to the CP-even singlet Higgs mass. Expanding the consequence up to λ_D^4 and λ_L^4 gives

$$\begin{aligned}\Delta m_S^2 &= \frac{1}{2}\left(\frac{\partial^2 V_{CW}^{5+\bar{5}}}{\partial v_s^2} - \frac{1}{v_s}\frac{\partial V_{CW}^{5+\bar{5}}}{\partial v_s}\right)\\
&= \lambda_L^2\frac{3A_{\lambda_L}v_s^2\kappa + 4v_s^3\kappa^2 + A_{\lambda_L}v^2\sin\beta\cos\beta\lambda}{8\pi^2 v_s}\ln\frac{m_L^2}{Q^2}\\
&+ 3\lambda_D^2\frac{3A_{\lambda_D}v_s^2\kappa + 4v_s^3\kappa^2 + A_{\lambda_D}v^2\sin\beta\cos\beta\lambda}{16\pi^2 v_s}\ln\frac{m_D^2}{Q^2}\\
&+ \frac{\lambda_L^4}{48\pi^2 m_L^4}\Big(-2A_{\lambda_L}^4 v_s^2 + 24A_{\lambda_L}^2 m_L^2 v_s^2 - 15A_{\lambda_L}^3 vs^3\kappa + 90A_{\lambda_L}m_L^2 v_s^3\kappa - 36A_{\lambda_L}^2 v_s^4\kappa^2\\
&+ 72m_L^2 v_s^5\kappa^2 - 35A_{\lambda_L}v_s^5\kappa^3 - 12v_s^6\kappa^4 + 3A_{\lambda_L}^3 v_d v_s v_u\lambda - 18A_{\lambda_L}m_L^2 v_d v_s v_u\lambda\\
&+ 24A_{\lambda_L}^2 v_d v_s^2 v_u\kappa\lambda - 48m_L^2 v_d v_s^2 v_u\kappa\lambda + 45A_{\lambda_L}v_d v_s^3 v_u\kappa^2\lambda + 24v_d v_s^4 v_u\kappa^3\lambda\\
&- 9A_{\lambda_L}v_d^2 v_s v_u^2\kappa\lambda^2 - 12v_d^2 v_s^2 v_u^2\kappa^2\lambda^2 - A_{\lambda_L}v_d^3 v_u^3\lambda^3 + 24m_L^4 v_s^2\ln\frac{m_L^2}{\lambda^2 v_s^2}\Big) +\\
&+ \frac{\lambda_D^4}{32\pi^2 m_D^4}\Big(-2A_{\lambda_D}^4 v_s^2 + 24A_{\lambda_D}^2 m_D^2 v_s^2 - 15A_{\lambda_D}^3 vs^3\kappa + 90A_{\lambda_D}m_D^2 v_s^3\kappa - 36A_{\lambda_D}^2 v_s^4\kappa^2\\
&+ 72m_D^2 v_s^5\kappa^2 - 35A_{\lambda_D}v_s^5\kappa^3 - 12v_s^6\kappa^4 + 3A_{\lambda_D}^3 v_d v_s v_u\lambda - 18A_{\lambda_D}m_D^2 v_d v_s v_u\lambda\\
&+ 24A_{\lambda_D}^2 v_d v_s^2 v_u\kappa\lambda - 48m_D^2 v_d v_s^2 v_u\kappa\lambda + 45A_{\lambda_D}v_d v_s^3 v_u\kappa^2\lambda + 24v_d v_s^4 v_u\kappa^3\lambda\\
&- 9A_{\lambda_D}v_d^2 v_s v_u^2\kappa\lambda^2 - 12v_d^2 v_s^2 v_u^2\kappa^2\lambda^2 - A_{\lambda_D}v_d^3 v_u^3\lambda^3 + 24m_D^4 v_s^2\ln\frac{m_D^2}{\lambda^2 v_s^2}\Big). \quad (16)\end{aligned}$$

The two leading terms are similar to (15), which result from the λ_{eff} defined in the previous text, while the $\lambda_{L,D}^4$ terms, especially the $24m_{L,D}^4 v_s^2 \ln\frac{m_{L,D}^2}{\lambda^2 v_s}$, reflect the fact that the mass of the singlet Higgs also receives the corrections from the mass hierarchy of the corresponding vectorlike fermions and sfermions, which are Q independent.

To see the possible mass spectrum of the vectorlike fermions, we look into the RG trajectories of the coupling constants. We can learn from (A4) that gauge terms contribute negative values to all Yukawa RG-β functions, while the Yukawa terms always contribute positive ones. S is a SM gauge singlet, so the lack of minus terms decides the quasifixing point of κ to be actually 0. H_u and H_d are not SM gauge singlets; however, we coupled many things on S and if λ_D and λ_L are too large, λ also tends to be small. At the GUT point, if we set $\lambda(Q_{\rm GUT}) = \kappa(Q_{\rm GUT}) = 3$, which is near the perturbative limit of $\sqrt{4\pi}$, apply $\lambda_D(Q_{\rm GUT}) = \lambda_L(Q_{\rm GUT})$ with different values, and then run the RG trajectories down, we can see the relationship between these coupling constants near 1 TeV through Fig. 1.

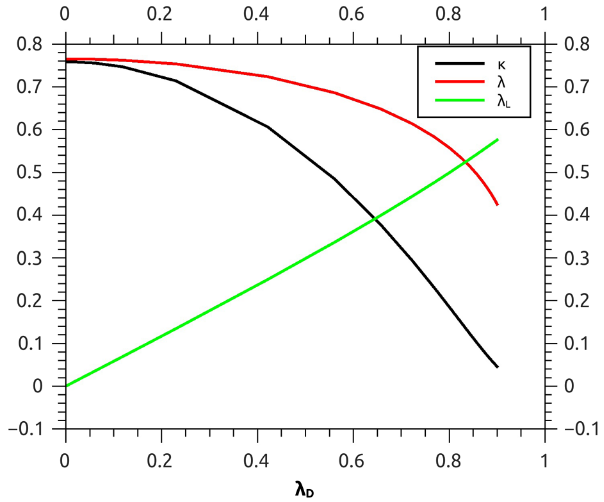

FIG. 1 (color online). The coupling constants near $Q = 1$ TeV in the boundary condition that $\lambda(Q_{\mathrm{GUT}}) = \kappa(Q_{\mathrm{GUT}}) = 3$ and $\lambda_D(Q_{\mathrm{GUT}}) = \lambda_L(Q_{\mathrm{GUT}})$. The GUT scale is defined as $Q_{\mathrm{GUT}} = 1.81 \times 10^{16}$ GeV, and different values of $\lambda_D(Q_{\mathrm{GUT}}) = \lambda_L(Q_{\mathrm{GUT}})$ are taken into the RG input parameters.

From the two-loop β functions of gauge couplings listed in (A4), we can learn that the Yukawa coupling constants play crucial roles if ever they are large enough. It is known that the unification of gauge-coupling constants is not that good even in the circumstance of supersymmetry, although it has been improved greatly when compared with the case of SM. If we want to adjust the Yukawa couplings in order to drive the gauge couplings into unifying in MSSM or NMSSM, there is not much room left in the parameter space because we do not have many notable Yukawa coupling constants to be adjusted, and the top Yukawa $Q_3H_ut_3$ affects on all g_1, g_2, g_3, making it difficult to converge the intersection points. In our case, λ_D strongly influences the trajectory of g_3; however, it slightly modifies g_1 due to D and $\bar{D}$'s relatively small hypercharge $\frac{1}{3}$, and λ_L only has effects on g_1 and g_2, so we can move the intersection point separately by adjusting λ_D and λ_L. After several attempts, we can reach a boundary condition

$$g_1 = g_2 = g_3 = 0.789, \qquad \lambda_D = 1.2, \qquad \lambda_L = 1,$$
$$\lambda = \kappa = 3, \qquad y_t = 0.9,$$
$$\text{at the scale } Q = 1.81 \times 10^{16}\ \text{GeV}, \tag{17}$$

and if we run down into $Q = 1$ TeV, the gauge-coupling constants are accurately in accordance with the low-energy data $g_1(1\ \text{TeV}) = 0.4670$, $g_2(1\ \text{TeV}) = 0.6388$, $g_3(1\ \text{TeV}) = 1.063$. See Fig. 2 for the trajectories.

IV. $10 + \overline{10}$ MODEL

Without the help of extra "vectorlike neutrino" N, the $5 + \bar{5}$ model can only contribute to the SM-like Higgs mass through S. However, the $10 + \overline{10}$ model contains direct vertices QH_uU and $\bar{Q}H_d\bar{U}$. Unlike the $5 + \bar{5}$ case, the $10 + \overline{10}$ model contains four $\frac{2}{3}$ charged squarks and thus a 4×4 matrix needs to be diagonalized, so a simple analytical solution does not exist.

The 4×4 mass-squared matrix of up-type squarks is shown below:

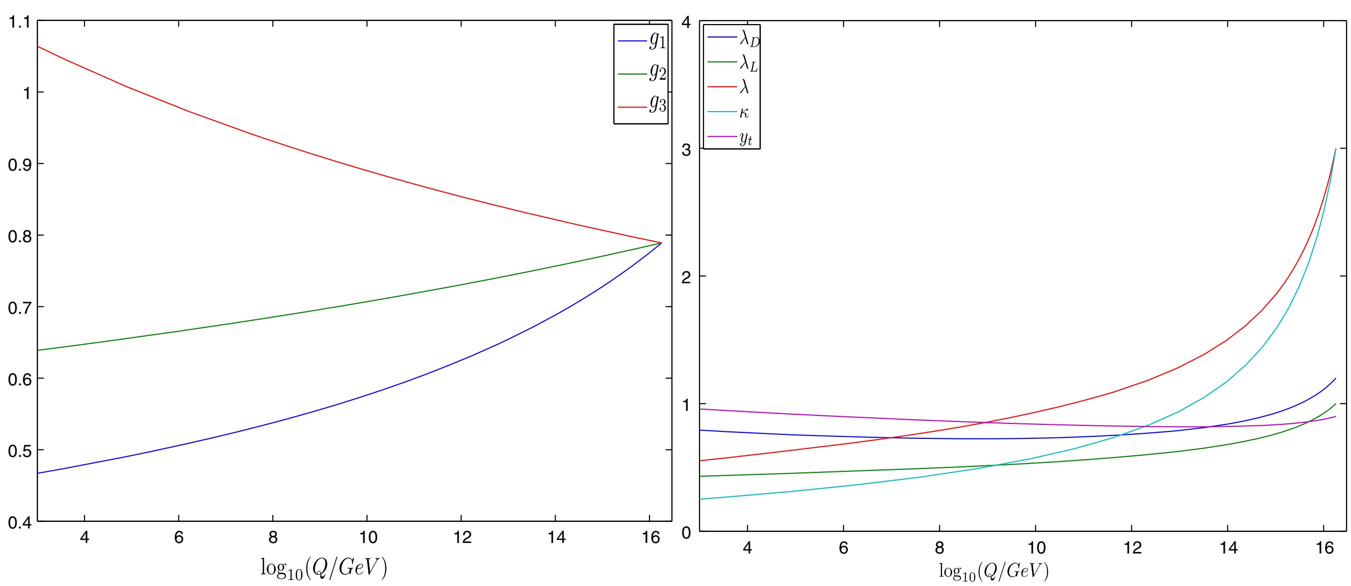

FIG. 2 (color online). Running couplings in $5 + \bar{5}$ theory. The left panel shows the trajectories of the gauge couplings. Notice that they converge into one point accurately. The right panel shows the corresponding Yukawa couplings. It is the Yukawa couplings that contribute into the gauge RG-β functions so that they can converge into one point at the scale $Q = 1.81 \times 10^{16}$ GeV.

$$\begin{bmatrix} \lambda_Q^2 v_s^2 + y_u^2 v_u^2 + m_Q^2 & \kappa\lambda_Q v_s^2 - \lambda\lambda_Q v_u v_d & -y_u \lambda v_d v_s & -y_{\bar{u}}\lambda_Q v_d v_s + \lambda_U y_{\bar{u}} v_u v_s \\ \kappa\lambda_Q v_s^2 - \lambda\lambda_Q v_u v_d & \lambda_Q^2 v_s^2 + y_d^2 v_d^2 + m_Q^2 & -\lambda_U y_{\bar{u}} v_d v_s + \lambda_Q y_u v_u v_s & \lambda y_{\bar{u}} v_u v_s \\ -y_u \lambda v_d v_s & -\lambda_U y_{\bar{u}} v_d v_s + \lambda_Q y_u v_u v_s & \lambda_U^2 v_s^2 + y_u^2 v_u^2 + m_Q^2 & \kappa\lambda_U v_s^2 - \lambda\lambda_U v_u v_d \\ -y_{\bar{u}}\lambda_Q v_d v_s + \lambda_U y_{\bar{u}} v_u v_s & \lambda y_{\bar{u}} v_u v_s & \kappa\lambda_U v_s^2 - \lambda\lambda_U v_u v_d & \lambda_U^2 v_s^2 + v_d^2 y_{\bar{u}}^2 + m_Q^2 \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & A_{\lambda_Q}\lambda_Q v_s & A_{y_u} y_u v_u & 0 \\ A_{\lambda_Q}\lambda_Q v_s & 0 & 0 & -A_{y_{\bar{u}}} y_{\bar{u}} v_d \\ A_{y_u} y_u v_u & 0 & 0 & A_{\lambda_U}\lambda_U v_s \\ 0 & -A_{y_{\bar{u}}} y_{\bar{u}} v_d & A_{\lambda_U}\lambda_U v_s & 0 \end{bmatrix}, \qquad (18)$$

where we can observe that unlike the consequence of the MSSM $+ (10 + \overline{10})$VL model (e.g., in [9]), many off-diagonal terms automatically appear, so the diagonalizing process becomes much more difficult. The mass-squared matrix of the two down-type squarks is

$$\begin{bmatrix} m_Q^2 + \lambda_Q^2 v_s^2 & -v_s^2\kappa\lambda_Q + v_d v_u \lambda\lambda_Q - A_{\lambda_Q}\lambda_Q v_s \\ -v_s^2\kappa\lambda_Q + v_u v_d \lambda\lambda_Q - A_{\lambda_Q}\lambda_Q v_s & m_Q^2 + \lambda_Q^2 v_s^2 \end{bmatrix}. \qquad (19)$$

The mass matrix of two VL fermionic up-type quarks is

$$\begin{bmatrix} \lambda_Q v_s & y_u v_u \\ -y_{\bar{u}} v_d & \lambda_U v_s \end{bmatrix}, \qquad (20)$$

while there is only one VL down-type quark,

$$M_{Q_D} = \lambda_Q v_s. \qquad (21)$$

Direct calculation diagonalizing (18) is lengthy and troublesome, so we expand the result in a series of y_u, $y_{\bar{u}}$, λ and κ, and set $A_{y_u} = A_{y_{\bar{u}}}$, $A_{\lambda_Q} = A_{\lambda_U}$. According to experience, the coupling constants of leptons are usually smaller than quarks because leptons do not have colors; thus their quasifixing points are smaller. The leptons also do not receive N_C enhancements, so, for simplicity, we ignore all leptonic contributions here. If we would like a relatively large $\tan\beta$, say, $\tan\beta > 2$, the SM-like lightest Higgs will mainly be H_u^0 and thus $y_{\bar{u}}$ can also be ignored. Let us define

$$\delta\lambda_Q = \lambda_U - \lambda_Q, \qquad (22)$$

and expand the final result according to y_u, $\delta\lambda_Q$, λ, κ. Similar to the process in (15), we acquire

$$\begin{aligned}\Delta m_h^2 = \frac{1}{16\pi^2 M_{\tilde{Q}}^4} v^2 \sin^2\beta \Big[& 36 M_{FQ}^4 \lambda^2 \lambda_Q^2 \cos^2\beta \ln\left(\frac{M_{\tilde{Q}}^2}{Q^2}\right) \\ & - y_u^2 \sin^2\beta (A_{y_u}^4 + 2A_{y_u}^2 M_{FQ}^2 + M_{FQ}^4 - 12 A_{y_u}^2 M_{\tilde{Q}}^2 \\ & - 6 M_{FQ}^2 M_{\tilde{Q}}^2 + 10 M_{\tilde{Q}}^4) \\ & + 12 y_u^2 \sin^2\beta M_{\tilde{Q}}^2 \ln\left(\frac{M_{\tilde{Q}}^2}{M_{FQ}^2}\right) \\ & - 6 y_u^2 A_{y_u} M_{FQ} M_{\tilde{Q}}^2 \lambda\lambda_Q \sin(2\beta) \Big], \end{aligned} \qquad (23)$$

where $M_{FQ} = \lambda_Q v_s$, $M_{\tilde{Q}} = \sqrt{m_Q^2 + \lambda_Q^2 v_s^2}$ are the estimated masses of fermionic and bosonic up-type quarks. We cut the series up to y_u^4, $\delta\lambda^2$, λ^2 and κ^2. However, $\delta\lambda$ disappears in the final result, telling us that the difference between λ_Q and λ_U does not exert a large effect on the SM-like Higgs mass.

Similar to (16), the singlet Higgs also receives one-loop quantum corrections. However, in spite of the similar λ_Q^2, λ_U^2 terms, the Q-independent terms are so complicated that we do not show them in this paper.

Now we are going to unify the gauge couplings. It is much more difficult to converge the intersection points in this circumstance than in the $5 + \overline{5}$ model, because the gauge couplings run into a larger value, ~1.2, which is much less sensitive to the adjusting of the large Yukawa couplings. If we set

$$\begin{aligned} & g_1 = g_2 = g_3 = 1.155, \quad \lambda_Q = 0.27657, \quad \lambda_U = 0.3, \quad \lambda_E = 0.3, \\ & y_u = y_{\bar{u}} = 3, \quad \lambda = 3, \quad \kappa = 2.19446, \quad y_t = 2.3, \\ & \text{at scale } Q = 3.62 \times 10^{16}\ \text{GeV}, \end{aligned} \qquad (24)$$

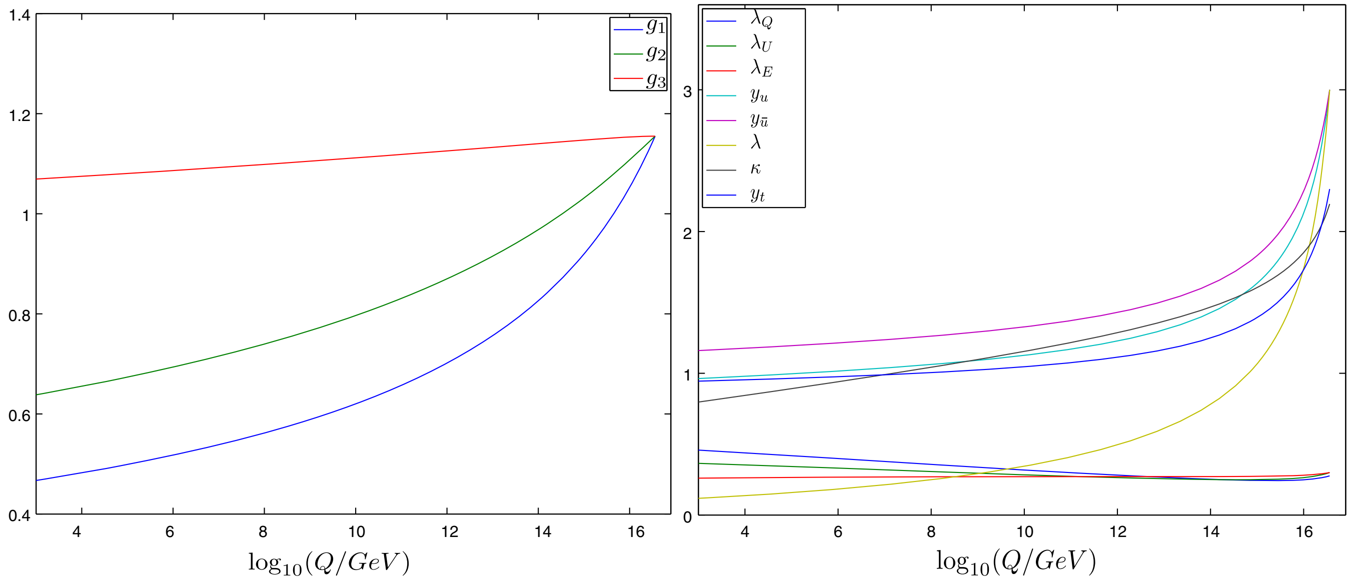

FIG. 3 (color online). The coupling constant trajectories of $10+\overline{10}$ theory. The left panel shows the gauge couplings, and the right panel shows the Yukawa couplings.

after running down to $Q = 1000$ GeV, we get $g_1 = 0.4670$, $g_2 = 0.6382$, $g_3 = 1.069$. See Fig. 3 for trajectories. There is a little deviation from (A3) in the Appendix.

V. $5+\bar{5}+10+\overline{10}$ MODEL

If we ignore all the Yukawa terms, put all the extra particles beyond SM at 1 TeV, and calculate the gauge RG-β functions up to the two-loop level, the g_2 trajectory actually blows up before g_1 can meet g_3. See Fig. 4.

Then we can add up Yukawa couplings to modify the coupling constants' trajectories. Now that g_2 runs the fastest, we add up λ_Q, λ_L, $y_{u,\bar{u},d,\bar{d},E,\bar{E}}$ to press the g_2-β function. However, the RG equations are not stable enough if we run from $Q = 1$ TeV upwards to GUT scale. If we apply the relatively large Yukawa coupling constants to "press" the gauge RG-β functions, it is easy for the Yukawa coupling constants to blow up before the GUT is reached. However, it is much better to run from GUT scale downwards to 1 TeV, and by adjusting the Yukawa coupling constants, we can acquire the correct values near $Q = 1$ TeV.

There is another severe problem that SH_uH_d receives so many corrections through the self-energy diagrams on S. These corrections depress λ very much, forcing the v_s to be extremely large. If we want to discuss the VL particles of roughly 1 TeV, this is not good news, because this requires $\lambda_{Q,U,D,L,E}$ to be rather small.

However, finally, we are still able to get a group of parameters, with the $\lambda(Q = 1\ \text{TeV}) \sim 0.03$, and $\lambda_Q(Q = 1\ \text{TeV}) \sim 0.2$. If we set $\mu_{\text{eff}} \sim 200$ GeV, $M_Q \sim 1.5$ TeV can still be reached. We are also able to converge the intersection points, and all of the GUT-scale g_1, g_2, and g_3 are smaller than $\sqrt{4\pi} \sim 3$. See Fig. 5; the boundary conditions are

$$\begin{aligned}&g_1=g_2=g_3=2.239,\quad \lambda_Q=0.1,\quad \lambda_U=0.13,\quad \lambda_D=0.1,\\ &\lambda_L=1.7,\ \lambda_E=0.3,\quad y_u=3,\ y_{\bar{u}}=1.5,\quad y_d=1.5,\ y_{\bar{d}}=0.8,\\ &y_e=0.5,\quad y_{\bar{e}}=1,\quad \lambda=3,\quad \kappa=3,\quad y_t=1.5,\\ &\text{at the scale } Q=7.15\times10^{16}\ \text{GeV}. \end{aligned} \tag{25}$$

The mass-squared matrices of the up-type squarks and the down-type squarks are both 4×4 shaped, so the complete formulas of Δm_h^2 and Δm_S^2 are too complicated

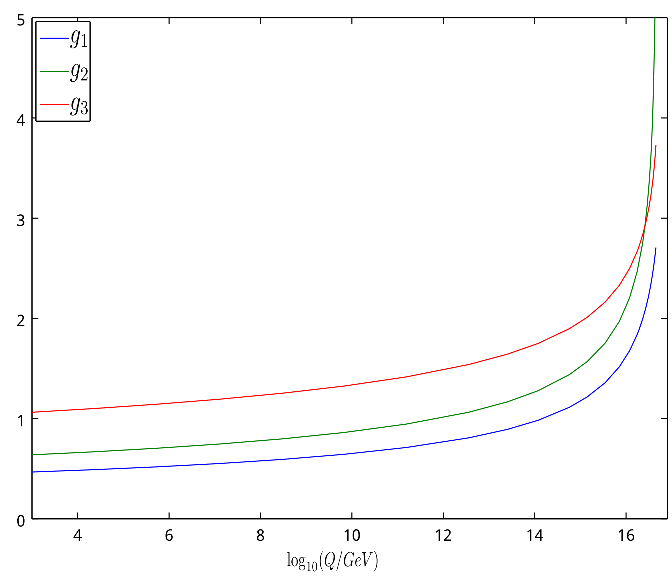

FIG. 4 (color online). The two-loop trajectories of three gauge-coupling constants under $5+\bar{5}+10+\overline{10}+$NMSSM theory. All Yukawa couplings are closed and the trajectories are all run from $Q = 1$ TeV.

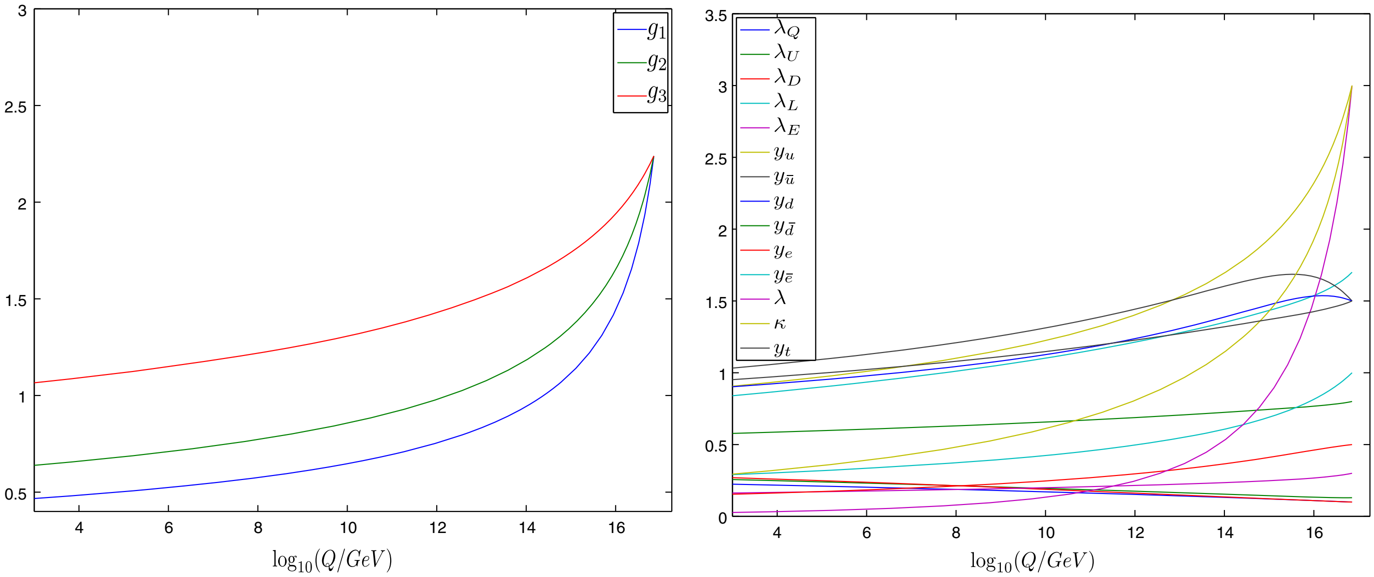

FIG. 5 (color online). The coupling constant trajectories of $5+\bar{5}+10+\overline{10}$ theory. The left panel show the gauge couplings, and the right panel shows the Yukawa couplings.

to be shown in this paper. For SM-like Higgs, naively, we believe that there are two vertices involving H_u, the QH_uU and $\bar{Q}H_u\bar{D}$, so the contribution similar to the (23) can be doubled. However, if we enlarge both y_u and $y_{\bar{d}}$, the λ would be so small that we cannot get TeV-scale VL quarks. In order to accumulate the Higgs mass, we could only choose one of them to be large while giving up another.

VI. CONCLUSION

By combining supersymmetric vectorlike theory and NMSSM and coupling S with the vectorlike particles, we can give a natural source of the vectorlike mass after Z_3 breaks. The mass of the VL fermions is of a similar quantity as the VEV of $\tilde{S}$ in the range of 1–100 TeV. The new Yukawa couplings invented in this theory can help unifying the gauge-coupling constants due to their contribution to the two-loop gauge RG-β functions. It is usually difficult to reach GUT before the Landau pole in a TeV-scale MSSM+$5+\bar{5}+10+\overline{10}$ model. However, with the help of the Yukawa couplings appearing in our models, we succeeded in converging the trajectories of the gauge-coupling constants before they blow up. The coupling between VL particles and Higgs can also contribute into the Higgs mass. Unlike MSSM+VL models, in our model, even $5+\bar{5}$ influences the effective Higgs mass through the couplings between VL particles and S. We have calculated the contributions to Higgs masses analytically in the NMSSM$+5+\bar{5}$ model, and only to the SM-like Higgs mass in the NMSSM+$10+\overline{10}$ model, and we have discussed it briefly in the NMSSM$+5+\bar{5}+10+\overline{10}$ model.

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APPENDIX: THE RG-β FUNCTIONS

Under 1 TeV, we run the gauge-coupling constants through one-loop SM functions [2]:

$$\alpha_1^{-1}(Q) = \alpha_1^{-1}(m_Z) - \frac{41}{20\pi}\ln\frac{Q}{m_Z}$$
$$\alpha_2^{-1}(Q) = \alpha_2^{-1}(m_Z) + \frac{19}{12\pi}\ln\frac{Q}{m_Z}$$
$$\alpha_3^{-1}(Q) = \alpha_3^{-1}(m_Z) + \frac{7}{2\pi}\ln\frac{Q}{m_Z}. \quad \text{(A1)}$$

We start from

$$\alpha_1(m_Z) = 0.0169$$
$$\alpha_2(m_Z) = 0.0338$$
$$\alpha_3(m_Z) = 0.1184, \quad \text{(A2)}$$

where $m_Z = 91.2$ GeV and all the data are calculated according to the values in [24], and then we get

$$g_1(Q = 1\ \text{TeV}) = 0.4670$$
$$g_2(Q = 1\ \text{TeV}) = 0.6388$$
$$g_3(Q = 1\ \text{TeV}) = 1.0633. \tag{A3}$$

Upon 1 TeV, we calculate the β functions according to the steps listed in [25,26]. The gauge RG-β functions are calculated up to two-loop accuracy and the Yukawa RG-β functions are calculated up to one loop.

The gauge RG-β functions are listed below:

$$\begin{aligned}
\beta_{g_1} &= \frac{g_1^3}{16\pi^2}\left(\frac{33}{5}+\frac{N_E}{2}\right)+\frac{g_1^3}{(16\pi^2)^2}\Big[\sum_j(BG_{1j}g_j^2) - C_{\lambda_Q 1}\lambda_Q - C_{\lambda_U 1}\lambda_U - C_{\lambda_D 1}\lambda_D \\
&\quad - C_{\lambda_L 1}\lambda_L - C_{\lambda_E 1}\lambda_E - C_{y_u 1}y_u - C_{y_{\bar u} 1}y_{\bar u} - C_{y_d 1}y_d - C_{y_{\bar d} 1}y_{\bar d} - C_{y_e 1}y_e - C_{y_{\bar e} 1}y_{\bar e} \\
&\quad - C_{\lambda 1}\lambda - C_{y_t 1}y_t\Big] \\
\beta_{g_2} &= \frac{g_2^3}{16\pi^2}\left(1+\frac{N_E}{2}\right)+\frac{g_2^3}{(16\pi^2)^2}\Big[\sum_j(BG_{2j}g_j^2) - C_{\lambda_Q 2}\lambda_Q - C_{\lambda_U 2}\lambda_U - C_{\lambda_D 2}\lambda_D \\
&\quad - C_{\lambda_L 2}\lambda_L - C_{\lambda_E 2}\lambda_E - C_{y_u 2}y_u - C_{y_{\bar u} 2}y_{\bar u} - C_{y_d 2}y_d - C_{y_{\bar d} 2}y_{\bar d} - C_{y_e 2}y_e - C_{y_{\bar e} 2}y_{\bar e} \\
&\quad - C_{\lambda 2}\lambda - C_{y_t 2}y_t\Big] \\
\beta_{g_3} &= \frac{g_3^3}{16\pi^2}\left(-3+\frac{N_E}{2}\right)+\frac{g_3^3}{(16\pi^2)^2}\Big[\sum_j(BG_{3j}g_j^2) - C_{\lambda_Q 3}\lambda_Q - C_{\lambda_U 3}\lambda_U - C_{\lambda_D 3}\lambda_D \\
&\quad - C_{\lambda_L 3}\lambda_L - C_{\lambda_E 3}\lambda_E - C_{y_u 3}y_u - C_{y_{\bar u} 3}y_{\bar u} - C_{y_d 3}y_d - C_{y_{\bar d} 3}y_{\bar d} - C_{y_e 3}y_e - C_{y_{\bar e} 3}y_{\bar e} \\
&\quad - C_{\lambda 3}\lambda - C_{y_t 3}y_t\Big]
\end{aligned} \tag{A4}$$

where

$$\begin{aligned}
C_{\lambda_Q} &= [2/5, 6, 4] \\
C_{\lambda_U} &= [16/5, 0, 2] \\
C_{\lambda_D} &= [4/5, 0, 2] \\
C_{\lambda_L} &= [6/5, 2, 0] \\
C_{\lambda_E} &= [12/5, 0, 0] \\
C_{y_u} &= [26/5, 6, 4] \\
C_{y_{\bar u}} &= [26/5, 6, 4] \\
C_{y_d} &= [14/5, 6, 4] \\
C_{y_{\bar d}} &= [14/5, 6, 4] \\
C_{y_e} &= [18/5, 2, 0] \\
C_{y_{\bar e}} &= [18/5, 2, 0] \\
C_{\lambda} &= [6/5, 2, 0] \\
C_{y_t} &= [26/5, 6, 4]
\end{aligned}$$

and

$$[BG_{ij}^{MSSM}] = \begin{bmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{bmatrix}$$
$$[BG_{ij}^{5}] = \begin{bmatrix} 7/30 & 9/10 & 16/15 \\ 3/10 & 7/2 & 0 \\ 2/15 & 0 & 17/3 \end{bmatrix}$$
$$[BG_{ij}^{10}] = \begin{bmatrix} 23/10 & 3/10 & 24/5 \\ 1/10 & 21/2 & 8 \\ 3/5 & 3 & 17 \end{bmatrix},$$

so

$$\begin{aligned}
BG_{ij} &= BG_{ij}^{MSSM} + 2BG_{ij}^{5}, \quad \text{for } 5+\overline{5} \text{ theory}, \\
BG_{ij} &= BG_{ij}^{MSSM} + 2BG_{ij}^{10}, \quad \text{for } 10+\overline{10} \text{ theory}, \\
BG_{ij} &= BG_{ij}^{MSSM} + 2BG_{ij}^{5} + 2BG_{ij}^{10}, \\
&\qquad \text{for } 5+\overline{5}+10+\overline{10} \text{ theory}.
\end{aligned}$$

The RG-β functions of the Yukawa couplings are

$$\beta_{\lambda_Q} = \frac{1}{16\pi^2}\lambda_Q\left[8\lambda_Q^2 + 3\lambda_U^2 + 3\lambda_D^2 + 2\lambda_L^2 + \lambda_E^2 + y_{\bar{u}}^2 + y_{\bar{d}}^2 + y_u^2 + y_d^2 + 2\lambda^2 - 2\left(\frac{1}{60}g_1^2 + \frac{3}{4}g_2^2 + \frac{4}{3}g_3^2\right)\right]$$

$$\beta_{\lambda_U} = \frac{1}{16\pi^2}\lambda_U\left[5\lambda_U^2 + 6\lambda_Q^2 + 3\lambda_D^2 + 2\lambda_L^2 + \lambda_E^2 + y_{\bar{u}}^2 + y_u^2 + 2\lambda^2 - 2\left(\frac{4}{15}g_1^2 + \frac{4}{3}g_3^2\right)\right]$$

$$\beta_{\lambda_D} = \frac{1}{16\pi^2}\lambda_D\left[5\lambda_D^2 + 6\lambda_Q^2 + 3\lambda_U^2 + 2\lambda_L^2 + \lambda_E^2 + y_{\bar{d}}^2 + y_d^2 + 2\lambda^2 - 2\left(\frac{1}{15}g_1^2 + \frac{4}{3}g_3^2\right)\right]$$

$$\beta_{\lambda_L} = \frac{1}{16\pi^2}\lambda_L\left[4\lambda_L^2 + 6\lambda_Q^2 + 3\lambda_D^2 + \lambda_E^2 + y_{\bar{e}}^2 + y_e^2 + 2\lambda^2 - 2\left(\frac{3}{20}g_1^2 + \frac{3}{4}g_2^2\right)\right]$$

$$\beta_{\lambda_E} = \frac{1}{16\pi^2}\lambda_E\left[3\lambda_E^2 + 3\lambda_Q^2 + 3\lambda_U^2 + 3\lambda_D^2 + 2\lambda_L^2 + 2y_e^2 + 2y_{\bar{e}}^2 + 2\lambda^2 - 2\left(\frac{6}{5}g_1^2\right)\right]$$

$$\beta_{y_u} = \frac{1}{16\pi^2}y_u\left[6y_u^2 + 3y_{\bar{d}}^2 + y_d^2 + \lambda_U^2 + \lambda_Q^2 + y_{\bar{e}}^2 + \lambda^2 + 3y_t^2 - 2\left(\frac{13}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2\right)\right]$$

$$\beta_{y_{\bar{u}}} = \frac{1}{16\pi^2}y_{\bar{u}}\left[6y_{\bar{u}}^2 + y_{\bar{d}}^2 + 3y_d^2 + \lambda_U^2 + \lambda_Q^2 + y_e^2 + \lambda^2 - 2\left(\frac{13}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2\right)\right]$$

$$\beta_{y_d} = \frac{1}{16\pi^2}y_d\left[6y_d^2 + y_u^2 + 3y_{\bar{u}}^2 + y_e^2 + \lambda_D^2 + \lambda_Q^2 + \lambda^2 - 2\left(\frac{7}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2\right)\right]$$

$$\beta_{y_{\bar{d}}} = \frac{1}{16\pi^2}y_{\bar{d}}\left[6y_{\bar{d}}^2 + 3y_u^2 + y_{\bar{u}}^2 + y_{\bar{e}^2} + \lambda_D^2 + \lambda_Q^2 + \lambda^2 + 3y_t^2 - 2\left(\frac{7}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2\right)\right]$$

$$\beta_{y_e} = \frac{1}{16\pi^2}y_e\left[4y_e^2 + 3y_{\bar{u}}^2 + 3y_d^2 + \lambda_L^2 + \lambda_E^2 + \lambda^2 + 3y_t^2 - 2\left(\frac{9}{10}g_1^2 + \frac{3}{2}g_2^2\right)\right]$$

$$\beta_{y_{\bar{e}}} = \frac{1}{16\pi^2}y_{\bar{e}}\left[4y_{\bar{e}}^2 + 3y_{\bar{u}}^2 + 3y_{\bar{d}}^2 + \lambda_L^2 + \lambda_E^2 + \lambda^2 - 2\left(\frac{9}{10}g_1^2 + \frac{3}{2}g_2^2\right)\right]$$

$$\beta_{\lambda} = \frac{1}{16\pi^2}\lambda\left[4\lambda^2 + 6\lambda_Q^2 + 3\lambda_U^2 + 3\lambda_D^2 + 2\lambda_L^2 + \lambda_E^2 + 3y_u^2 + 3y_{\bar{u}}^2 + 3y_d^2 + 3y_{\bar{d}}^2 + y_e^2 + y_{\bar{e}}^2 + 3y_t^2 - 2\left(\frac{3}{20}g_1^2 + \frac{3}{4}g_2^2\right)\right]$$

$$\beta_{\kappa} = \frac{1}{16\pi^2}3\kappa[\kappa^2/3 + 3\lambda_Q^2 + 3\lambda_U^2 + 3\lambda_D^2 + 2\lambda_L^2 + \lambda_E^2 + 3\lambda^2]$$

$$\beta_{y_t} = \frac{1}{16\pi^2}y_t\left[6y_t^2 + 3y_{\bar{d}}^2 + 3y_u^2 + 3y_{\bar{e}} + \lambda^2 - 2\left(\frac{13}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2\right)\right].$$

If we do not calculate complete $5 + \overline{5} + 10 + \overline{10}$ RG flows, we can just set the irrelevant Yukawa coupling constants to 0.

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