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Structure functions in deep inelastic scattering from gauge/string duality beyond single-hadron final states

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We study deep inelastic scattering at large 't Hooft coupling and finite x from gauge/string duality beyond single-hadron final states, which gives the leading large-N_c contribution. Within the supergravity approximation, we calculate the subleading large-N_c contribution by introducing an extra hadron into the final states. We find that the contribution from these double-hadron final states will dominate in the Bjorken limit $q^2 \to \infty$ compared with the single-hadron states. We discuss the implications of our results.

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I. INTRODUCTION

The gauge/string correspondence, since first conjectured [1–3], has been used widely in studying nonperturbative aspects of QCD. As the first application to deep inelastic scattering (DIS), Polchinski and Strassler [4] employed the correspondence to calculate the structure functions for hadrons in the large-'t Hooft coupling and large-N_c limits by introducing an infrared cutoff in the fifth dimension to mimic the confinement. Since then, there have been many further investigations [5–23] in this direction. Compared to QCD, these studies show that the hadron structures in the strong-coupling limit bear very different features at finite x, while sharing similar features at small x. It turns out that the structure functions in the strong-coupling regime are all power suppressed at finite x, implying that few of the partons gain a finite amount of longitudinal momentum from the target hadron, and that almost all of the partons are squeezed into the small-x region. However, such conclusions can be arrived at only in the large-N_c limit from the contribution of single-hadron final states. It is valuable to investigate the structure functions beyond this limit, which is the major concern of the current work.

It is a nontrivial task to obtain complete contributions to the structure functions at subleading order in large N_c from gauge/string duality. For simplicity, we will restrict ourselves to the supergravity approximation, considering subleading contributions from the processes with only two scalar hadrons involved in final states. Through the specific calculation and power analysis, we find that in the large-N_c expansion (compared with leading contribution) the subleading contribution can be less suppressed in the power expansion of $1/q$. Thus the subleading contribution will dominate in the Bjorken limit $q \to \infty$, which implies that the large-N_c limit and the Bjorken limit do not commute with each other.

The paper is organized as follows. In Sec. II we formulate the DIS on a scalar target in the gauge/string correspondence. In Sec. III we evaluate successively the transition amplitudes, the hadronic tensor, and the structure functions for the DIS process under the supergravity approximation. In Sec. IV we analyze the power dependence on $1/q$ for various channels and phase spaces and extract the leading contribution for the structure functions in the Bjorken limit $q \to \infty$. In Sec. V we discuss our results and give a summary.

II. DIS FROM THE GAUGE/STRING DUALITY

In the one-photon exchange approximation for DIS, the initial lepton interacts with the hadron target by the exchange of a virtual photon, and the hadron absorbs the photon and decays into the final states. The cross section is determined by the hadronic tensor $W^{\mu\nu}$ which is defined as

$$W^{\mu\nu} = \sum_X (2\pi)^4 \delta(p + q - P_X) \langle H|J^\mu(0)|X\rangle\langle X|J^\nu(0)]|H\rangle, \tag{1}$$

where J^μ is the electromagnetic current, q^μ is the momentum of the virtual photon, p^μ denotes the momentum of the initial hadron H, and P_X denotes the total momentum of the final hadron states X. For the spinless or spin-averaged hadrons, the hadronic tensor can be decomposed into

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$$W^{\mu\nu} = F_1(x,q^2)\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \frac{2x}{q^2}F_2(x,q^2)\left(p^\mu + \frac{q^\mu}{2x}\right)\left(p^\nu + \frac{q^\nu}{2x}\right). \quad (2)$$

All the information for the hadron structure is encoded in the structure functions $F_1(x,q^2)$ and $F_2(x,q^2)$.

In the gauge/string duality, scalar hadrons correspond to normalizable supergravity modes of the dilaton and the electromagnetic current corresponds to a non-normalizable mode of a Kaluza-Klein gauge field at the boundary of the anti–de Sitter (AdS) space, AdS_5. The mass gap of hadrons can be generated by breaking the conformal invariance through introducing a sharp cutoff $0 \le z \le z_0 \equiv 1/\Lambda$. The metric in AdS_5 space can be written as

$$ds^2 = \frac{1}{z^2}(\eta_{\mu\nu}dy^\mu dy^\nu + dz^2), \quad (3)$$

where $\eta_{\mu\nu} = (-,+,+,+)$ is the flat-space metric at the boundary. The initial/final dilaton wave function satisfies the Klein-Gorden equation in AdS_5 and the corresponding normalizable solution with the boundary condition $\Phi(y,z_0) = 0$ is given by

$$\Phi(y,z) = c_{\kappa,n}e^{ip\cdot y}z^2 J_\kappa(M_{\kappa,n}z), \quad (4)$$

where $\kappa = \Delta - 2$ with Δ being the conformal dimension of the state, $M_{\kappa,n}z_0$ denotes the nth zero point of the Bessel function J_κ, and $c_{\kappa,n}$ is the normalization factor,

$$c_{\kappa,n} = \frac{\sqrt{2}}{z_0|J_{\kappa+1}(M_{\kappa,n}z_0)|}. \quad (5)$$

In order to calculate the subleading large-N_c contribution from the final multiple-hadron states, we need the bulk-to-bulk propagator of dilatons in AdS_5 space, which is given by

$$G(y,z;y',z') = -\int \frac{d^4k}{(2\pi)^4}e^{-ik\cdot(y-y')} \times \int_0^\infty d\omega \frac{\omega}{\omega^2 + k^2 - i\epsilon} z^2 J_\kappa(\omega z) z'^2 J_\kappa(\omega z'). \quad (6)$$

When considering the boundary condition, the accurate propagator should take the discrete form

$$G(y,z;y',z') = -\int \frac{d^4k}{(2\pi)^4}e^{-ik\cdot(y-y')}\sum_{M_{\kappa,n}} \frac{M_{\kappa,n}c_{\kappa,n}^2}{M_{\kappa,n}^2 + k^2 - i\epsilon} \times z^2 J_\kappa(M_{\kappa,n}z) z'^2 J_\kappa(M_{\kappa,n}z'). \quad (7)$$

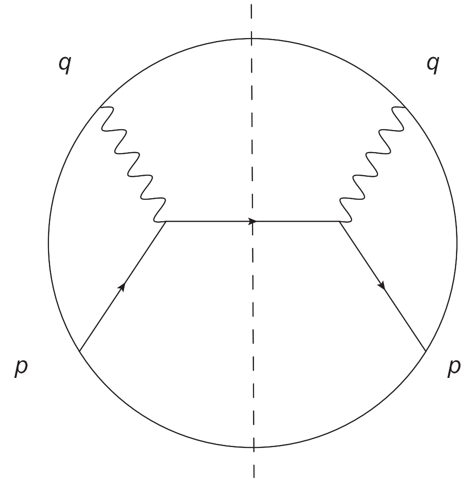

FIG. 1. Leading large-N_c contribution from the single-hadron final states.

The gauge field corresponding to the current satisfies the Maxwell equations in AdS_5 space, and the non-normalizable solution with the boundary condition $A_\mu(y,\infty) = n_\mu e^{iq\cdot y}$ (where n_μ is the virtual photon polarization vector) and the Lorentz-like gauge fixing $\partial_\mu A^\mu + z\partial_z(A_z/z) = 0$ is given by

$$A_\mu = n_\mu e^{iq\cdot y} qzK_1(qz), \qquad A_z = in\cdot q e^{iq\cdot y} zK_0(qz), \quad (8)$$

where K_1 and K_0 are both modified Bessel functions.

When we were working in the leading large-N_c approximation, only a single hadron in the final states was needed. The corresponding Witten diagram for the hadronic tensor is represented in Fig. 1, in which the dashed line denotes the final states. In our present work, we will devote ourselves to calculating the subleading large-N_c contribution and analyzing the power dependence of $1/q$. It should be mentioned that the complete subleading contribution of large N_c can come from different sources; however, in this paper we will restrict ourselves to considering the contribution by introducing an extra hadron into the final states. Doing such a calculation is mainly inspired by the discussion of Polchinski and Strassler in Ref. [4], while it should be emphasized that it is possible that the ignored terms could cancel the leading $1/q$ contribution or that they may have even more important contributions in $1/q$ than those found in the present work. For simplicity, we will ignore these complexities in the following discussion.

For further simplicity, we will only consider the spinless hadron and the final states that include two dilatons, in which the gauge propagator and gravity propagator do not contribute. The relevant supergravity interaction is

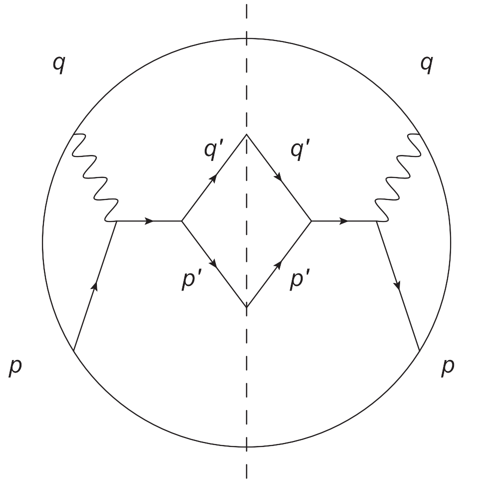

FIG. 2. s-channel contribution from the double-hadron final states.

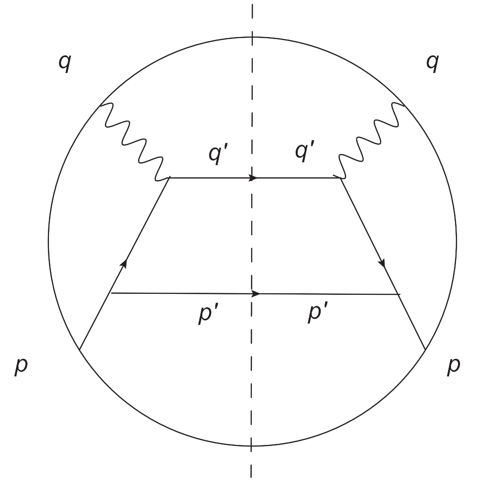

FIG. 3. t-channel contribution from the double-hadron final states.

$$\begin{aligned} S &= -\int d^5x\sqrt{-g}\left[\sum_{i=1}^{3} D^m\Phi_i D_m\Phi_i^* + \sum_{i=1}^{3}\mu_i^2\Phi_i^*\Phi_i + \lambda\Phi_1\Phi_2^*\Phi_3^* + \lambda\Phi_1^*\Phi_2\Phi_3\right] \\ &= -\int d^5x\sqrt{-g}\left[\sum_{i=1}^{3}\partial^m\Phi_i\partial_m\Phi_i^* + \sum_{i=1}^{3}\mu_i^2\Phi_i^*\Phi_i + A^mA_m\sum_{i=1}^{3}\mathcal{Q}_i^2\Phi_i^*\Phi_i \right. \\ &\quad \left. + iA^m\sum_{i=1}^{3}\mathcal{Q}_i(\Phi_i\partial_m\Phi_i^* - \Phi_i^*\partial_m\Phi_i) + \lambda\Phi_1\Phi_2^*\Phi_3^* + \lambda\Phi_1^*\Phi_2\Phi_3\right], \end{aligned} \tag{9}$$

where we have introduced three different dilatons ($i = 1, 2, 3$) which have different charges $\mathcal{Q}_i$ (with $\mathcal{Q}_1 + \mathcal{Q}_2 + \mathcal{Q}_3 = 0$) and five-dimensional mass $\mu_i^2 = \Delta_i(\Delta_i - 4)/R^2$, where Δ_i is the conformal dimension of the states and R is the AdS radius. It should be explained that the above action is mainly based on phenomenological considerations, and we assign the parameters $\mathcal{Q}_i$ and λ as small, free coupling constants.

It follows that the subleading contributions of large N_c[1] with two dilatons in the final states come from the Witten diagrams in Figs. 2, 3, and 4, and from the other six crossed-channel diagrams which have not been displayed here. The Witten diagrams with a cutline actually represent the squared amplitudes: the transition amplitude is to the left of the cut line, while the complex conjugate is to the right. Therefore, in order to calculate the hadronic tensor we first need to calculate the transition amplitudes. From the action given in Eq. (9), it is straightforward to write down all the transition amplitudes corresponding to different channels: the s-channel amplitude,

$$\begin{aligned} \mathcal{M}_s &= i\mathcal{Q}_1\int d^5x d^5x'\sqrt{-g(x)}\sqrt{-g(x')}\Phi_1(x)A^M(x)[\partial_M G(x,x')]\Phi_2^*(x')\Phi_3^*(x') \\ &\quad - i\mathcal{Q}_1\int d^5x d^5x'\sqrt{-g(x)}\sqrt{-g(x')}[\partial_M\Phi_1(x)]A^M(x)G(x,x')\Phi_2^*(x')\Phi_3^*(x'); \end{aligned} \tag{10}$$

the t-channel amplitude,

[1]In supergravity, the loop corrections are always equivalent to $1/N$-suppressed corrections. This is because the action of interest to us [e.g., Eq. (9)] will have an overall coupling-constant factor which is proportional to N^2 (for brevity, we have suppressed this overall factor in this paper). Hence, similar to the argument in large-N_c QCD, we can find that the additional loops or external lines will result in an additional suppression of $1/N$.

$$\mathcal{M}_t = -i\mathcal{Q}_2 \int d^5x d^5x' \sqrt{-g(x)}\sqrt{-g(x')}\Phi_1(x)\Phi_3^*(x)[\partial'_M G(x,x')]A^M(x')\Phi_2^*(x') + i\mathcal{Q}_2 \int d^5x d^5x' \sqrt{-g(x)}\sqrt{-g(x')}\Phi_1(x)\Phi_3^*(x)G(x,x')A^M(x')[\partial'_M \Phi_2^*(x')]; \quad (11)$$

and the u-channel amplitude,

$$\mathcal{M}_u = -i\mathcal{Q}_3 \int d^5x d^5x' \sqrt{-g(x)}\sqrt{-g(x')}\Phi_1(x)\Phi_2^*(x)[\partial'_M G(x,x')]A^M(x')\Phi_3^*(x') + i\mathcal{Q}_3 \int d^5x d^5x' \sqrt{-g(x)}\sqrt{-g(x')}\Phi_1(x)\Phi_2^*(x)G(x,x')A^M(x')[\partial'_M \Phi_3^*(x')], \quad (12)$$

where $x = (y,z), x' = (y',x')$, and

$$A^\mu(x) = n^\mu e^{iq\cdot y} q z^3 K_1(qz), \qquad A^z(x) = in\cdot q e^{iq\cdot y} z^3 K_0(qz), \qquad \Phi_1(x) = c_1 z^2 J_{\kappa_1}(M_1 z) e^{ip\cdot y},$$
$$\Phi_2^*(x) = c_2 z^2 J_{\kappa_2}(M_2 z) e^{-iq'\cdot y}, \qquad \Phi_3^*(x) = c_3 z^2 J_{\kappa_3}(M_3 z) e^{-ip'\cdot y}. \quad (13)$$

For brevity, we have used the shorthand $c_i, M_i (i = 1,2,3)$ for $c_{(i)n,k}$ and $M_{(i)n,k}$, where k labels the conformal weight and n labels the state.

The main task of the remaining parts of this work is to calculate the above transition amplitudes, square them to obtain the hadronic tensor, and finally extract the structure functions.

III. CALCULATION OF THE STRUCTURE FUNCTIONS

Substituting the wave functions of the initial or final states in Eq. (13) into the transition amplitudes and integrating out the boundary coordinates y and y' yields

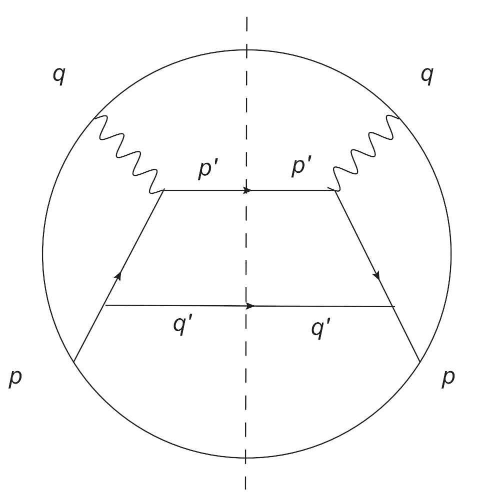

FIG. 4. u-channel contribution from the double-hadron final states.

$$\mathcal{M}_s = \mathcal{Q}_1 c_1 c_2 c_3 (2\pi)^4 \delta^4(p+q-p'-q') n\cdot\left(2p + \frac{1}{x}q\right) \times \int dz dz' \frac{q}{z'} J_{\kappa_1}(M_1 z) K_1(qz) G_s(z,z') J_{\kappa_2}(M_2 z') \times J_{\kappa_3}(M_3 z') - \mathcal{Q}_1 (2\pi)^4 \delta^4(p+q-p'-q') n\cdot q \times \frac{1}{q}\int dz' z'^2 J_{\kappa_1}(M_1 z') K_1(qz') J_{\kappa_2}(M_2 z') J_{\kappa_3}(M_3 z'), \quad (14)$$

$$\mathcal{M}_t = \mathcal{Q}_2 c_1 c_2 c_3 (2\pi)^4 \delta^4(p+q-p'-q') n\cdot\left(2q' + \frac{1}{y'}q\right) \times \int dz dz' \frac{q}{z} J_{\kappa_1}(M_1 z) J_{\kappa_3}(M'_3 z) G_t(z,z') K_1(qz') \times J_{\kappa_2}(M_2 z') + \mathcal{Q}_2 (2\pi)^4 \delta^4(p+q-p'-q') n\cdot q \times \frac{1}{q}\int dz z^2 J_{\kappa_1}(M_1 z) K_1(qz) J_{\kappa_2}(M_2 z) J_{\kappa_3}(M_3 z), \quad (15)$$

$$\mathcal{M}_u = \mathcal{Q}_3 c_1 c_2 c_3 (2\pi)^4 \delta^4(p+q-p'-q') n\cdot\left(2p' + \frac{1}{x'}q\right) \times \int dz dz' \frac{q}{z} J_{\kappa_1}(M_1 z) J_{\kappa_2}(M'_2 z) G_u(z,z') K_1(qz') \times J_{\kappa_3}(M_3 z') + \mathcal{Q}_3 (2\pi)^4 \delta^4(p+q-p'-q') n\cdot q \times \frac{1}{q}\int dz z^2 J_{\kappa_1}(M_1 z) K_1(qz) J_{\kappa_2}(M_2 z) J_{\kappa_3}(M_3 z), \quad (16)$$

where we have defined three scalar variables,

$$x = -\frac{q^2}{2p\cdot q}, x' = -\frac{q^2}{2p'\cdot q}, y' = -\frac{q^2}{2q'\cdot q}, \quad (17)$$

and the reduced bulk-to-bulk propagators in the holographic radial coordinate, which are given by

$$G_s(z,z') = -\int_0^\infty d\omega \frac{\omega c_s^2}{\omega^2 + (p+q)^2 - i\epsilon} \times z^2 J_{\kappa_1}(\omega z) z'^2 J_{\kappa_1}(\omega z'), \quad (18)$$

$$G_t(z,z') = -\int_0^\infty d\omega \frac{\omega c_t^2}{\omega^2 + (p'-p)^2 - i\epsilon} \times z^2 J_{\kappa_2}(\omega z) z'^2 J_{\kappa_2}(\omega z'), \quad (19)$$

$$G_u(z,z') = -\int_0^\infty d\omega \frac{\omega c_u^2}{\omega^2 + (p'-q)^2 - i\epsilon} \times z^2 J_{\kappa_3}(\omega z) z'^2 J_{\kappa_3}(\omega z'), \quad (20)$$

which correspond to the s-channel, t-channel, and u-channel, respectively. It should be noted that for brevity we will use the integral notation instead of the sum notation in the propagator. In order to be consistent with the cutoff in the AdS space, we have introduced the normalization factors c_s, c_t, and c_u which are given (respectively) by

$$c_s = \frac{\sqrt{2}}{z_0(|J_{\kappa_1+1}(\omega z_0)| + |J_{\kappa_1}(\omega z_0)|)}, \quad (21)$$

$$c_t = \frac{\sqrt{2}}{z_0(|J_{\kappa_2+1}(\omega z_0)| + |J_{\kappa_2}(\omega z_0)|)}, \quad (22)$$

$$c_u = \frac{\sqrt{2}}{z_0(|J_{\kappa_3+1}(\omega z_0)| + |J_{\kappa_3}(\omega z_0)|)}. \quad (23)$$

The above normalization is very proper because it is always finite and will reduce to the usual normalization (5) when the propagator is on-shell.

The total transition amplitude is obtained by summing over all the contributions from different channels,

$$\begin{aligned} \mathcal{M} &= \mathcal{M}_s + \mathcal{M}_u + \mathcal{M}_t \\ &= c_1 c_2 c_3 (2\pi)^4 \delta^4(p+q-p'-q') \Big[n\cdot\Big(2p + \frac{1}{x}q\Big) C_s + n\cdot\Big(2q' + \frac{1}{y'}q\Big) C_t + n\cdot\Big(2p' + \frac{1}{x'}q\Big) C_u \Big], \end{aligned} \quad (24)$$

where we have defined

$$C_s = \mathcal{Q}_1 \int dz dz' \frac{q}{z'} J_{\kappa_1}(M_1 z) K_1(qz) G_s(z,z') J_{\kappa_2}(M_2 z') J_{\kappa_3}(M_3 z'), \quad (25)$$

$$C_t = \mathcal{Q}_2 \int dz dz' \frac{q}{z} J_{\kappa_1}(M_1 z) J_{\kappa_3}(M_3 z) G_t(z,z') K_1(qz') J_{\kappa_2}(M_2 z'), \quad (26)$$

$$C_u = \mathcal{Q}_3 \int dz dz' \frac{q}{z} J_{\kappa_1}(M_1 z) J_{\kappa_2}(M_2 z) G_u(z,z') K_1(qz') J_{\kappa_3}(M_3 z'). \quad (27)$$

From the relation between the hadronic tensor and the squared transition amplitude,

$$n^\mu n^\nu W_{\mu\nu} = \mathcal{M}\mathcal{M}^*, \quad (28)$$

and the definitions of the structure functions in Eq. (2), we can extract the structure functions in the Bjorken limit $q \to \infty$ with x fixed,

$$\begin{aligned} F_1(x,q^2) &= c_1^2 \sum_{M_2} \sum_{M_3} c_2^2 c_3^2 \int \frac{d^3\mathbf{p}'}{2E_{p'}(2\pi)^3} \frac{d^3\mathbf{q}'}{2E_{q'}(2\pi)^3} (2\pi)^4 \delta^4(p+q-p'-q') \\ &\times 2q^2 \{ [v_u^2 + 4x^2 (v_s\cdot v_u)^2] C_u C_u^* + [v_t^2 + 4x^2 (v_s\cdot v_t)^2] C_t C_t^* + [v_u\cdot v_t + 4x^2 (v_s\cdot v_u)(v_s\cdot v_t)](C_u C_t^* + C_t C_u^*) \}, \end{aligned} \quad (29)$$

$$\begin{aligned} F_2(x,q^2) &= c_1^2 \sum_{M_2} \sum_{M_3} c_2^2 c_3^2 \int \frac{d^3\mathbf{p}'}{2E_{p'}(2\pi)^3} \frac{d^3\mathbf{q}'}{2E_{q'}(2\pi)^3} (2\pi)^4 \delta^4(p+q-p'-q') \\ &\times 4xq^2 \{ [v_s^2 + 12x^2 v_s^4] C_s C_s^* + [v_u^2 + 12x^2 (v_u\cdot v_s)^2] C_u C_u^* \\ &+ [v_t^2 + 12x^2 (v_t\cdot v_s)^2] C_t C_t^* + [v_s\cdot v_t + 12x^2 (v_t\cdot v_s) v_s^2](C_s C_t^* + C_t C_s^*) \\ &+ [v_s\cdot v_u + 12x^2 (v_u\cdot v_s) v_s^2](C_s C_u^* + C_u C_s^*) + [v_u\cdot v_t + 12x^2 (v_t\cdot v_s)(v_u\cdot v_s)](C_u C_t^* + C_t C_u^*) \}, \end{aligned} \quad (30)$$

where we have defined the three vectors

$$v_s^\mu = \frac{1}{q}\left(p^\mu + \frac{q^\mu}{2x}\right), \qquad v_u^\mu = \frac{1}{q}\left(p'^\mu + \frac{q^\mu}{2x'}\right), \qquad v_t^\mu = \frac{1}{q}\left(q'^\mu + \frac{q^\mu}{2y'}\right). \tag{31}$$

In order to extract the leading contribution in the Bjorken limit $q \to \infty$ with x fixed, it is convenient to define the following scaled variables:

$$\hat{p}'^\mu = p'^\mu/q, \qquad \hat{q}'^\mu = q'^\mu/q, \qquad \hat{\omega} = \omega/q, \qquad \hat{z} = qz, \qquad \hat{z}' = qz'. \tag{32}$$

With these scaled variables, we can rewrite the structure functions as

$$F_1(x, q^2) = c_1^2 \sum_{M_2}\sum_{M_3} c_2^2 c_3^2 \int \frac{d^3\hat{\mathbf{p}}'}{2\hat{E}_{p'}(2\pi)^3}\frac{d^3\hat{\mathbf{q}}'}{2\hat{E}_{q'}(2\pi)^3}(2\pi)^4\delta^4(\hat{p}+\hat{q}-\hat{p}'-\hat{q}')$$
$$\times \frac{2}{q^6}\{[v_u^2 + 4x^2(v_s\cdot v_u)^2]\hat{C}_u\hat{C}_u^* + [v_t^2 + 4x^2(v_s\cdot v_t)^2]\hat{C}_t\hat{C}_t^* + [v_u\cdot v_t + 4x^2(v_s\cdot v_u)(v_s\cdot v_t)](\hat{C}_u\hat{C}_t^* + \hat{C}_t\hat{C}_u^*)\}, \tag{33}$$

$$F_2(x, q^2) = c_1^2 \sum_{M_2}\sum_{M_3} c_2^2 c_3^2 \int \frac{d^3\hat{\mathbf{p}}'}{2\hat{E}_{p'}(2\pi)^3}\frac{d^3\hat{\mathbf{q}}'}{2\hat{E}_{q'}(2\pi)^3}(2\pi)^4\delta^4(\hat{p}+\hat{q}-\hat{p}'-\hat{q}')$$
$$\times \frac{4x}{q^6}\{[v_s^2 + 12x^2 v_s^4]\hat{C}_s\hat{C}_s^* + [v_u^2 + 12x^2(v_u\cdot v_s)^2]\hat{C}_u\hat{C}_u^*$$
$$+ [v_t^2 + 12x^2(v_t\cdot v_s)^2]\hat{C}_t\hat{C}_t^* + [v_s\cdot v_t + 12x^2(v_t\cdot v_s)v_s^2](\hat{C}_s\hat{C}_t^* + \hat{C}_t\hat{C}_s^*)$$
$$+ [v_s\cdot v_u + 12x^2(v_u\cdot v_s)v_s^2](\hat{C}_s\hat{C}_u^* + \hat{C}_u\hat{C}_s^*) + [v_u\cdot v_t + 12x^2(v_t\cdot v_s)(v_u\cdot v_s)](\hat{C}_u\hat{C}_t^* + \hat{C}_t\hat{C}_u^*)\}, \tag{34}$$

where

$$\hat{C}_s = \mathcal{Q}_1 \int d\hat{z}d\hat{z}' \frac{1}{\hat{z}'} J_{\kappa_1}(\hat{M}_1\hat{z})K_1(\hat{z})\hat{G}_s(\hat{z},\hat{z}')J_{\kappa_2}(\hat{M}_2\hat{z}')J_{\kappa_3}(\hat{M}_3\hat{z}'), \tag{35}$$

$$\hat{C}_u = \mathcal{Q}_3 \int d\hat{z}d\hat{z}' \frac{1}{\hat{z}} J_{\kappa_1}(\hat{M}_1\hat{z})J_{\kappa_2}(\hat{M}_2\hat{z})\hat{G}_u(\hat{z},\hat{z}')K_1(\hat{z}')J_{\kappa_3}(\hat{M}_3\hat{z}'), \tag{36}$$

$$\hat{C}_t = \mathcal{Q}_2 \int d\hat{z}d\hat{z}' \frac{1}{\hat{z}} J_{\kappa_1}(\hat{M}_1\hat{z})J_{\kappa_3}(\hat{M}_3\hat{z})\hat{G}_t(\hat{z},\hat{z}')K_1(\hat{z}')J_{\kappa_2}(\hat{M}_2\hat{z}'), \tag{37}$$

with

$$\hat{G}_s(\hat{z},\hat{z}') = -\int_0^\infty d\hat{\omega}\frac{\hat{\omega}c_s^2}{\hat{\omega}^2 - \hat{s} - i\epsilon}\hat{z}^2 J_{\kappa_1}(\hat{\omega}\hat{z})\hat{z}'^2 J_{\kappa_1}(\hat{\omega}\hat{z}'), \tag{38}$$

$$\hat{G}_t(\hat{z},\hat{z}') = -\int_0^\infty d\hat{\omega}\frac{\hat{\omega}c_t^2}{\hat{\omega}^2 - \hat{t} - i\epsilon}\hat{z}^2 J_{\kappa_2}(\hat{\omega}\hat{z})\hat{z}'^2 J_{\kappa_2}(\hat{\omega}\hat{z}'), \tag{39}$$

$$\hat{G}_u(\hat{z},\hat{z}') = -\int_0^\infty d\hat{\omega}\frac{\hat{\omega}c_u^2}{\hat{\omega}^2 - \hat{u} - i\epsilon}\hat{z}^2 J_{\kappa_3}(\hat{\omega}\hat{z})\hat{z}'^2 J_{\kappa_3}(\hat{\omega}\hat{z}'). \tag{40}$$

It can be noticed that we need to deal with the integrals over triple Bessel functions, which in general cannot be calculated analytically. However, we can choose some special cases, e.g., we can set $\kappa_1 = 1$, $\kappa_2 = 0$, and $\kappa_3 = 1$ and use the integral formula [24],

$$\int_0^\infty d\hat{z}\hat{z}^2 J_0(a\hat{z})J_0(b\hat{z})K_1(\hat{z}) = \frac{2\sqrt{2}(a^2+b^2+1)}{[(a+b)^2+1]^{\frac{3}{2}}[(a-b)^2+1]^{\frac{3}{2}}}, \tag{41}$$

$$\int_0^\infty d\hat{z}\hat{z}^2 J_1(a\hat{z})J_1(b\hat{z})K_1(\hat{z}) = \frac{8\sqrt{2}ab}{[(a+b)^2+1]^{\frac{3}{2}}[(a-b)^2+1]^{\frac{3}{2}}}, \tag{42}$$

$$\int_0^\infty d\hat{z}\,\hat{z}\,J_1(a\hat{z})J_1(b\hat{z})J_0(c\hat{z}) = \frac{\sqrt{2}(a^2+b^2-c^2)\Theta(a+b-c)\Theta(c-|a-b|)}{\pi^2 ab[(a+b)^2-c^2]^{\frac{1}{2}}[c^2-(a-b)^2]^{\frac{1}{2}}}, \tag{43}$$

We then have

$$\hat{C}_s = -\mathcal{Q}_1\int_{|\hat{M}_3-\hat{M}_2|}^{\hat{M}_3+\hat{M}_2} d\hat{\omega}\frac{\hat{\omega}c_s^2}{\hat{\omega}^2-\hat{s}-i\epsilon}\frac{8\sqrt{2}\hat{M}_1\hat{\omega}}{[(\hat{M}_1+\hat{\omega})^2+1]^{\frac{3}{2}}[(\hat{M}_1-\hat{\omega})^2+1]^{\frac{3}{2}}}\frac{\sqrt{2}[\hat{M}_3^2+\hat{\omega}^2-\hat{M}_2^2]}{\hat{M}_3\hat{\omega}[(\hat{M}_3+\hat{M}_2)^2-\hat{\omega}^2]^{\frac{1}{2}}[\hat{\omega}^2-(\hat{M}_3-\hat{M}_2)^2]^{\frac{1}{2}}}, \tag{44}$$

$$\hat{C}_t = -\mathcal{Q}_2\int_{|\hat{M}_3-\hat{M}_1|}^{\hat{M}_3+\hat{M}_1} d\hat{\omega}\frac{\hat{\omega}c_t^2}{\hat{\omega}^2-\hat{t}-i\epsilon}\frac{\sqrt{2}[\hat{M}_1^2+\hat{M}_3^2-\hat{\omega}^2]}{\hat{M}_1\hat{M}_3[(\hat{M}_1+\hat{M}_3)^2-\hat{\omega}^2]^{\frac{1}{2}}|\hat{\omega}^2-(\hat{M}_1-\hat{M}_3)^2|^{\frac{1}{2}}}\frac{2\sqrt{2}(\hat{M}_2^2+\hat{\omega}^2+1)}{[(\hat{M}_2+\hat{\omega})^2+1]^{\frac{3}{2}}[(\hat{M}_2-\hat{\omega})^2+1]^{\frac{3}{2}}}, \tag{45}$$

$$\hat{C}_u = -\mathcal{Q}_3\int_{|\hat{M}_2-\hat{M}_1|}^{\hat{M}_2+\hat{M}_1} d\hat{\omega}\frac{\hat{\omega}c_u^2}{\hat{\omega}^2-\hat{u}-i\epsilon}\frac{\sqrt{2}[\hat{M}_1^2+\hat{\omega}^2-\hat{M}_2^2]}{\hat{M}_1\hat{\omega}[(\hat{M}_2+\hat{M}_1)^2-\hat{\omega}^2]^{\frac{1}{2}}[\hat{\omega}^2-(\hat{M}_2-\hat{M}_1)^2]^{\frac{1}{2}}}\frac{8\sqrt{2}\hat{M}_3\hat{\omega}}{[(\hat{M}_3+\hat{\omega})^2+1]^{\frac{3}{2}}[(\hat{M}_3-\hat{\omega})^2+1]^{\frac{3}{2}}}. \tag{46}$$

Now let us choose the center-of-mass frame of the initial dilaton and virtual photon, where

$$p^\mu = \left(\frac{q}{2\sqrt{x(1-x)}}, \frac{q}{2\sqrt{x(1-x)}}, 0, 0\right), \qquad q^\mu = \left(\frac{(1-2x)q}{2\sqrt{x(1-x)}}, -\frac{q}{2\sqrt{x(1-x)}}, 0, 0\right). \tag{47}$$

It follows that

$$F_1(x,q^2) = \sum_{M_2}\sum_{M_3}\frac{c_1^2c_2^2c_3^2|\hat{\mathbf{p}}'|}{4\pi q^6}\sqrt{\frac{x}{1-x}}\int d\theta\sin\theta\{[v_u^2+4x^2(v_s\cdot v_u)^2]\hat{C}_u\hat{C}_u^*+[v_t^2+4x^2(v_s\cdot v_t)^2]\hat{C}_t\hat{C}_t^*$$
$$+[v_u\cdot v_t+4x^2(v_s\cdot v_u)(v_s\cdot v_t)](\hat{C}_u\hat{C}_t^*+\hat{C}_t\hat{C}_u^*)\}, \tag{48}$$

$$F_2(x,q^2) = \sum_{M_2}\sum_{M_3}\frac{c_1^2c_2^2c_3^2|\hat{\mathbf{p}}'|x}{2\pi q^6}\sqrt{\frac{x}{1-x}}\int d\theta\sin\theta$$
$$\times\{[v_s^2+12x^2v_s^4]\hat{C}_s\hat{C}_s^*+[v_u^2+12x^2(v_u\cdot v_s)^2]\hat{C}_u\hat{C}_u^*+[v_t^2+12x^2(v_t\cdot v_s)^2]\hat{C}_t\hat{C}_t^*$$
$$+[v_u\cdot v_t+12x^2(v_t\cdot v_s)(v_u\cdot v_s)](\hat{C}_u\hat{C}_t^*+\hat{C}_t\hat{C}_u^*)$$
$$+[v_s\cdot v_t+12x^2(v_t\cdot v_s)v_s^2](\hat{C}_s\hat{C}_t^*+\hat{C}_t\hat{C}_s^*)+[v_s\cdot v_u+12x^2(v_u\cdot v_s)v_s^2](\hat{C}_s\hat{C}_u^*+\hat{C}_u\hat{C}_s^*)\}, \tag{49}$$

where $|\hat{\mathbf{p}}'|$ is determined by

$$\sqrt{\frac{1-x}{x}} = \sqrt{\hat{\mathbf{p}}'^2+\hat{M}_3^2}+\sqrt{\hat{\mathbf{p}}'^2+\hat{M}_2^2}. \tag{50}$$

IV. POWER ANALYSIS

In order to extract the leading contribution in the Bjorken limit $q \to \infty$, we need to analyze the power dependence of the structure functions on $1/q$ in different kinetic ranges. In this work, we will always assume $\hat{M}_1 \ll 1$ for the initial hadron. Hence, we can classify the kinetic ranges into four different parts according to the masses of the final hadrons: $\hat{M}_2 \sim 1$ & $\hat{M}_3 \sim 1$, $\hat{M}_2 \ll 1$ & $\hat{M}_3 \sim 1$, $\hat{M}_2 \sim 1$ & $\hat{M}_3 \ll 1$, and $\hat{M}_2 \ll 1$ & $\hat{M}_3 \ll 1$. Now let us deal with them one by one.

A. $\hat{M}_2 \sim 1$ & $\hat{M}_3 \sim 1$

In this region, we can reduce the integrals in Eqs. (44)–(46) to

$$\hat{C}_s \approx -Q_1 c_s^2 \frac{16\hat{M}_1}{\hat{M}_3} \int_{|\hat{M}_3-\hat{M}_2|}^{\hat{M}_3+\hat{M}_2} d\hat{\omega} \frac{\hat{\omega}}{(\hat{\omega}^2 - \hat{s} - i\epsilon)(\hat{\omega}^2+1)^3} \times \frac{[\hat{M}_3^2 + \hat{\omega}^2 - \hat{M}_2^2]}{[(\hat{M}_3+\hat{M}_2)^2 - \hat{\omega}^2]^{\frac{1}{2}}[\hat{\omega}^2 - (\hat{M}_3 - \hat{M}_2)^2]^{\frac{1}{2}}}, \tag{51}$$

$$\hat{C}_t \approx -Q_2 c_t^2 \frac{4\pi \hat{M}_1 \hat{M}_3 (\hat{M}_2^2 + \hat{M}_3^2 + 1)}{[(\hat{M}_2+\hat{M}_3)^2+1]^{\frac{3}{2}}[(\hat{M}_2-\hat{M}_3)^2+1]^{\frac{3}{2}}(\hat{M}_3^2 - \hat{t})^2}, \tag{52}$$

$$\hat{C}_u \approx Q_3 c_u^2 \frac{16\pi \hat{M}_1 \hat{M}_3 \hat{u}}{[(\hat{M}_3+\hat{M}_2)^2+1]^{\frac{3}{2}}[(\hat{M}_3-\hat{M}_2)^2+1]^{\frac{3}{2}}(\hat{M}_2^2 - \hat{u})^2}, \tag{53}$$

where we have simply extracted the normalization factors c_s^2, c_t^2, and c_u^2 and assigned them the values that will give a dominant contribution,

$$c_s = \frac{\sqrt{2}}{z_0(|J_{\kappa_1+1}(\sqrt{s}z_0)| + |J_{\kappa_1}(\sqrt{s}z_0)|)}, \tag{54}$$

$$c_t = \frac{\sqrt{2}}{z_0(|J_{\kappa_2+1}(\sqrt{|t|}z_0)| + |J_{\kappa_2}(\sqrt{|t|}z_0)|)}, \tag{55}$$

$$c_u = \frac{\sqrt{2}}{z_0(|J_{\kappa_3+1}(\sqrt{|u|}z_0)| + |J_{\kappa_3}(\sqrt{|u|}z_0)|)}. \tag{56}$$

The normalization coefficient c_1 for the initial hadron in Eqs. (48) and (49) is always of order unity when $\hat{M}_1 \ll 1$, while the normalization coefficients $c_2 \sim q^{\frac{1}{2}}$, $c_3 \sim q^{\frac{1}{2}}$ for the final hadron states by using the asymptotic behavior of the Bessel function. Besides, the sum over M_2 and M_3 contribute $\sum_{M_2}\sum_{M_3} \sim 1$. In order to obtain the final power behavior, we need to divide this kinetic region further according to the momenta of the internal propagators, which are given in Table I. A detailed analysis can be found in Appendix B. It should be pointed out that the subscripts in the structure functions denote the contribution from different channels, e.g., F_{ss} means the contribution to the structure functions from the term $\hat{C}_s\hat{C}_s^*$, F_{ut} means the contribution from $\hat{C}_u\hat{C}_t^* + \hat{C}_t\hat{C}_u^*$, and so on. Since the power dependencies for F_1 and F_2 are the same, we have suppressed the subscript 1 or 2. When there are no corresponding terms, e.g., F_{ss}, F_{st}, and F_{su} in F_1, we will only denote the contribution in F_2. It is easy to show that $\hat{t} + \hat{u} \sim 1$, which implies that we do not need to consider the case where $t \ll 1$ and $u \ll 1$ at the same time.

B. $\hat{M}_2 \ll 1$ & $\hat{M}_3 \sim 1$

In this region, we can have

$$\hat{C}_s \approx Q_1 c_s^2 \frac{16\pi \hat{M}_1 \hat{M}_3 \hat{s}}{(\hat{M}_3^2+1)^3(\hat{s} - \hat{M}_3^2)^2}, \tag{57}$$

TABLE I. $\hat{M}_2 \sim 1$ & $\hat{M}_3 \sim 1$.

Kinetic region	c_s, c_u, c_t	$\hat{C}_s$, $\hat{C}_u$, $\hat{C}_t$	Phase space	Structure functions
$\lvert\hat{t}\rvert \sim 1$	$c_s \sim q^{\frac{1}{2}}$	$\hat{C}_s \sim 1$		$F_{ss} \sim \frac{1}{q^4}$, $F_{uu} \sim \frac{1}{q^4}$
$\lvert\hat{u}\rvert \sim 1$	$c_u \sim q^{\frac{1}{2}}$	$\hat{C}_u \sim 1$	$\int \sin\theta d\theta \sim 1$	$F_{tt} \sim \frac{1}{q^4}$, $F_{su} \sim \frac{1}{q^4}$
	$c_t \sim q^{\frac{1}{2}}$	$\hat{C}_t \sim 1$		$F_{st} \sim \frac{1}{q^4}$, $F_{ut} \sim \frac{1}{q^4}$
$\lvert\hat{t}\rvert \ll 1$	$c_s \sim q^{\frac{1}{2}}$	$\hat{C}_s \sim 1$		$F_{ss} \sim \frac{1}{q^5}$, $F_{uu} \sim \frac{1}{q^5}$
$\lvert\hat{u}\rvert \sim 1$	$c_u \sim q^{\frac{1}{2}}$	$\hat{C}_u \sim 1$	$\int \sin\theta d\theta \sim \frac{1}{q}$	$F_{tt} \sim \frac{1}{q^7}$, $F_{su} \sim \frac{1}{q^5}$
	$c_t \sim 1$	$\hat{C}_t \sim \frac{1}{q}$		$F_{st} \sim \frac{1}{q^6}$, $F_{ut} \sim \frac{1}{q^6}$
$\lvert\hat{t}\rvert \sim 1$	$c_s \sim q^{\frac{1}{2}}$	$\hat{C}_s \sim 1$		$F_{ss} \sim \frac{1}{q^5}$, $F_{uu} \sim \frac{1}{q^9}$
$\lvert\hat{u}\rvert \ll 1$	$c_u \sim 1$	$\hat{C}_u \sim \frac{1}{q^2}$	$\int \sin\theta d\theta \sim \frac{1}{q}$	$F_{tt} \sim \frac{1}{q^5}$, $F_{su} \sim \frac{1}{q^7}$
	$c_t \sim q^{\frac{1}{2}}$	$\hat{C}_t \sim 1$		$F_{st} \sim \frac{1}{q^6}$, $F_{ut} \sim \frac{1}{q^7}$

TABLE II. $\hat{M}_2 \ll 1$ & $\hat{M}_3 \sim 1$.

Kinetic region	c_s, c_u, c_t	$\hat{C}_s$, $\hat{C}_u$, $\hat{C}_t$	Phase space	Structure functions
$\lvert\hat{t}\rvert \sim 1$	$c_s \sim q^{\frac{1}{2}}$	$\hat{C}_s \sim 1$		$F_{ss} \sim \frac{1}{q^4}$, $F_{uu} \sim \frac{1}{q^4}$
$\lvert\hat{u}\rvert \sim 1$	$c_u \sim q^{\frac{1}{2}}$	$\hat{C}_u \sim 1$	$\int \sin\theta d\theta \sim 1$	$F_{tt} \sim \frac{1}{q^4}$, $F_{su} \sim \frac{1}{q^4}$
	$c_t \sim q^{\frac{1}{2}}$	$\hat{C}_t \sim 1$		$F_{st} \sim \frac{1}{q^4}$, $F_{ut} \sim \frac{1}{q^4}$
$\lvert\hat{t}\rvert \ll 1$	$c_s \sim q^{\frac{1}{2}}$	$\hat{C}_s \sim 1$		$F_{ss} \sim \frac{1}{q^5}$, $F_{uu} \sim \frac{1}{q^5}$
$\lvert\hat{u}\rvert \sim 1$	$c_u \sim q^{\frac{1}{2}}$	$\hat{C}_u \sim 1$	$\int \sin\theta d\theta \sim \frac{1}{q}$	$F_{tt} \sim \frac{1}{q^7}$, $F_{su} \sim \frac{1}{q^5}$
	$c_t \sim 1$	$\hat{C}_t \sim \frac{1}{q}$		$F_{st} \sim \frac{1}{q^6}$, $F_{ut} \sim \frac{1}{q^6}$
$\lvert\hat{t}\rvert \sim 1$	$c_s \sim q^{\frac{1}{2}}$	$\hat{C}_s \sim 1$		$F_{ss} \sim \frac{1}{q^5}$, $F_{uu} \sim \frac{1}{q^9}$
$\lvert\hat{u}\rvert \ll 1$	$c_u \sim 1$	$\hat{C}_u \sim \frac{1}{q^2}$	$\int \sin\theta d\theta \sim \frac{1}{q}$	$F_{tt} \sim \frac{1}{q^5}$, $F_{su} \sim \frac{1}{q^7}$
	$c_t \sim q^{\frac{1}{2}}$	$\hat{C}_t \sim 1$		$F_{st} \sim \frac{1}{q^5}$, $F_{ut} \sim \frac{1}{q^7}$

$$\hat{C}_u \approx -Q_3 c_u^2 \frac{8\pi\hat{M}_3}{\hat{M}_1(\hat{M}_3^2+1)^3} \times \frac{\hat{M}_1^2 - \hat{M}_2^2 + u + \sqrt{(\hat{M}_1^2-\hat{M}_2^2+u)^2 - 4u\hat{M}_1^2}}{\sqrt{(\hat{M}_1^2-\hat{M}_2^2+u)^2 - 4u\hat{M}_1^2}}, \tag{58}$$

$$\hat{C}_t \approx -Q_2 c_t^2 \frac{4\pi\hat{M}_1\hat{M}_3}{(\hat{M}_3^2+1)^2(\hat{M}_3^2-t)^2}. \tag{59}$$

The normalization coefficients $c_2 \sim 1$, $c_3 \sim q^{\frac{1}{2}}$ and the sum over M_2 and M_3 contribute $\sum_{M_2}\sum_{M_3} \sim q$. A detailed power analysis in different kinetic intervals is given in Table II.

C. $\hat{M}_2 \sim 1$ & $\hat{M}_3 \ll 1$

In this region, we can have

$$\hat{C}_s \approx Q_1 c_s^2 \frac{16\pi\hat{M}_1\hat{M}_3\hat{s}}{(\hat{M}_2^2+1)^3(\hat{s}-\hat{M}_2^2)^2}, \tag{60}$$

$$\hat{C}_u \approx Q_3 c_u^2 \frac{16\pi\hat{M}_3\hat{M}_1 u}{(\hat{M}_2^2+1)^3(\hat{M}_2^2-u)^2}, \tag{61}$$

$$\hat{C}_t = -Q_2 c_t^2 \frac{2\pi}{\hat{M}_1\hat{M}_3(\hat{M}_2^2+1)^2} \times \frac{\hat{M}_1^2 + \hat{M}_3^2 - t - \sqrt{(\hat{M}_1^2+\hat{M}_3^2-t)^2 - 4\hat{M}_1^2\hat{M}_3^2}}{\sqrt{(\hat{M}_1^2+\hat{M}_3^2-t)^2 - 4\hat{M}_1^2\hat{M}_3^2}}. \tag{62}$$

The normalization coefficients $c_2 \sim q^{\frac{1}{2}}$, $c_3 \sim 1$ and the sum over M_2 and M_3 contribute $\sum_{M_2}\sum_{M_3} \sim q$. A detailed power analysis in different kinetic intervals is given in Table III.

D. $\hat{M}_2 \ll 1$ & $\hat{M}_3 \ll 1$

In this region, we can have

$$\hat{C}_s \approx Q_1 c_s^2 \frac{16\pi\hat{M}_1\hat{M}_3}{\hat{s}}, \tag{63}$$

TABLE III. $\hat{M}_2 \sim 1$ & $\hat{M}_3 \ll 1$.

Kinetic region	c_s, c_u, c_t	$\hat{C}_s$, $\hat{C}_u$, $\hat{C}_t$	Phase space	Structure functions
$\lvert\hat{t}\rvert \sim 1$	$c_s \sim q^{\frac{1}{2}}$	$\hat{C}_s \sim \frac{1}{q}$		$F_{ss} \sim \frac{1}{q^6}$, $F_{uu} \sim \frac{1}{q^6}$
$\lvert\hat{u}\rvert \sim 1$	$c_u \sim q^{\frac{1}{2}}$	$\hat{C}_u \sim \frac{1}{q}$	$\int \sin\theta d\theta \sim 1$	$F_{tt} \sim \frac{1}{q^6}$, $F_{su} \sim \frac{1}{q^6}$
	$c_t \sim q^{\frac{1}{2}}$	$\hat{C}_t \sim \frac{1}{q}$		$F_{st} \sim \frac{1}{q^6}$, $F_{ut} \sim \frac{1}{q^6}$
$\lvert\hat{t}\rvert \ll 1$	$c_s \sim q^{\frac{1}{2}}$	$\hat{C}_s \sim \frac{1}{q}$		$F_{ss} \sim \frac{1}{q^8}$, $F_{uu} \sim \frac{1}{q^8}$
$\lvert\hat{u}\rvert \sim 1$	$c_u \sim q^{\frac{1}{2}}$	$\hat{C}_u \sim \frac{1}{q}$	$\int \sin\theta d\theta \sim \frac{1}{q}$	$F_{tt} \sim \frac{1}{q^2}$, $F_{su} \sim \frac{1}{q^8}$
	$c_t \sim 1$	$\hat{C}_t \sim q^2$		$F_{st} \sim \frac{1}{q^5}$, $F_{ut} \sim \frac{1}{q^5}$
$\lvert t\rvert \sim 1$	$c_s \sim q^{\frac{1}{2}}$	$\hat{C}_s \sim \frac{1}{q}$		$F_{ss} \sim \frac{1}{q^7}$, $F_{uu} \sim \frac{1}{q^{11}}$
$\lvert u\rvert \ll 1$	$c_u \sim 1$	$\hat{C}_u \sim \frac{1}{q^3}$	$\int \sin\theta d\theta \sim \frac{1}{q}$	$F_{tt} \sim \frac{1}{q^7}$, $F_{su} \sim \frac{1}{q^9}$
	$c_t \sim q^{\frac{1}{2}}$	$\hat{C}_t \sim \frac{1}{q}$		$F_{st} \sim \frac{1}{q^7}$, $F_{ut} \sim \frac{1}{q^9}$

TABLE IV. $\hat{M}_2 \ll 1$ & $\hat{M}_3 \ll 1$.

Kinetic region	c_s, c_u, c_t	$\hat{C}_s, \hat{C}_u, \hat{C}_t$	Phase space	Structure functions
$\lvert\hat{t}\rvert \sim 1$	$c_s \sim q^{\frac{1}{2}}$	$\hat{C}_s \sim \frac{1}{q}$		$F_{ss} \sim \frac{1}{q^8}$, $F_{uu} \sim \frac{1}{q^8}$
$\lvert\hat{u}\rvert \sim 1$	$c_u \sim q^{\frac{1}{2}}$	$\hat{C}_u \sim \frac{1}{q}$	$\int \sin\theta d\theta \sim 1$	$F_{tt} \sim \frac{1}{q^8}$, $F_{su} \sim \frac{1}{q^8}$
	$c_t \sim q^{\frac{1}{2}}$	$\hat{C}_t \sim \frac{1}{q}$		$F_{st} \sim \frac{1}{q^8}$, $F_{ut} \sim \frac{1}{q^8}$
$\lvert\hat{t}\rvert \ll 1$	$c_s \sim q^{\frac{1}{2}}$	$\hat{C}_s \sim \frac{1}{q}$		$F_{ss} \sim \frac{1}{q^9}$, $F_{uu} \sim \frac{1}{q^9}$
$\lvert\hat{u}\rvert \sim 1$	$c_u \sim q^{\frac{1}{2}}$	$\hat{C}_u \sim \frac{1}{q}$	$\int \sin\theta d\theta \sim \frac{1}{q}$	$F_{tt} \sim \frac{1}{q^3}$, $F_{su} \sim \frac{1}{q^9}$
	$c_t \sim 1$	$\hat{C}_t \sim q^2$		$F_{st} \sim \frac{1}{q^6}$, $F_{ut} \sim \frac{1}{q^6}$
$\lvert t\rvert \sim 1$	$c_s \sim q^{\frac{1}{2}}$	$\hat{C}_s \sim \frac{1}{q}$		$F_{ss} \sim \frac{1}{q^9}$, $F_{uu} \sim \frac{1}{q^9}$
$\lvert u\rvert \ll 1$	$c_u \sim 1$	$\hat{C}_u \sim \frac{1}{q}$	$\int \sin\theta d\theta \sim \frac{1}{q}$	$F_{tt} \sim \frac{1}{q^9}$, $F_{su} \sim \frac{1}{q^9}$
	$c_t \sim q^{\frac{1}{2}}$	$\hat{C}_t \sim \frac{1}{q}$		$F_{st} \sim \frac{1}{q^9}$, $F_{ut} \sim \frac{1}{q^9}$

$$\hat{C}_u \approx -\mathcal{Q}_3 c_u^2 \frac{8\pi\hat{M}_3}{\hat{M}_1} \frac{\hat{M}_1^2 - \hat{M}_2^2 + u + \sqrt{(\hat{M}_1^2 - \hat{M}_2^2 + u)^2 - 4u\hat{M}_1^2}}{\sqrt{(\hat{M}_1^2 - \hat{M}_2^2 + u)^2 - 4u\hat{M}_1^2}}, \tag{64}$$

$$\hat{C}_t = -\mathcal{Q}_2 c_t^2 \frac{2\pi}{\hat{M}_1\hat{M}_3} \frac{\hat{M}_1^2 + \hat{M}_3^2 - t - \sqrt{(\hat{M}_1^2 + \hat{M}_3^2 - t)^2 - 4\hat{M}_1^2\hat{M}_3^2}}{\sqrt{(\hat{M}_1^2 + \hat{M}_3^2 - t)^2 - 4\hat{M}_1^2\hat{M}_3^2}}. \tag{65}$$

The normalization coefficients $c_2 \sim 1$, $c_3 \sim 1$ and the sum over M_2 and M_3 contribute $\sum_{M_2}\sum_{M_3} \sim 1$. A detailed power analysis in different kinetic intervals is given in Table IV.

E. The final dominant contribution

From the above analysis, we find that the dominant contribution is from the t-channel, where $\hat{M}_1 \ll 1$ $\hat{M}_3 \ll 1$, $\hat{M}_2 \sim 1$ with $|\hat{t}| \ll 1$ and $|\hat{u}| \sim 1$. Hence, the leading contribution is given by

$$F_1(x, q^2) \approx \left(\frac{\Lambda}{q}\right)^2 f_1(x), \quad F_2(x, q^2) \approx \left(\frac{\Lambda}{q}\right)^2 f_2(x), \tag{66}$$

where we have extracted the power dependence and lumped all the others into the functions $f_1(x)$ and $f_2(x)$, which are independent of q or at most dependent on q by $\ln q$. Since we are most interested in the power dependence in this work, we will not present the specific forms of $f_1(x)$ and $f_2(x)$.

V. DISCUSSION AND CONCLUSION

Now let us compare the results in Eq. (66) from the subleading large-N_c contribution with those obtained before from the leading large-N_c contribution, which is given by

$$F_1(x, q^2) = 0, \qquad F_2(x, q^2) \approx \left(\frac{\Lambda}{q}\right)^{2\kappa_1+2} f(x). \tag{67}$$

By setting $\kappa_1 = 1$ (to be consistent with our present specific case), we have

$$F_1(x, q^2) = 0, \qquad F_2(x, q^2) \approx \left(\frac{\Lambda}{q}\right)^4 f(x). \tag{68}$$

We notice two significant differences between them. First, for the leading large-N_c contribution, the structure function $F_1(x, q^2)$ always vanishes, but it will obtain a nonzero contribution at the subleading large-N_c order. As we all know, $F_1(x, q^2)$ is proportional to the Casimir of the scattered hadron under the Lorentz transformation, so it is natural that $F_1(x, q^2)$ vanishes when the virtual photon hits the original scalar target hadron directly at the leading large-N_c order. However, at the subleading large-N_c order, the scalar target hadron can split into two scalar hadrons, and each hadron can have orbital angular momentum and can lead to a nonvanishing $F_1(x, q^2)$ when they are hit by the virtual photon. Such arguments can be verified by Eq. (29), in which only the t-channel and u-channel contribute to $F_1(x, q^2)$, and the s-channel in which the target hadron interacts directly with the virtual photon does not contribute at all. Second, the subleading large-N_c contribution from the double-hadron final states is less power-suppressed than the leading large-N_c one. The power dependence of the structure function is the same as that of the hadron 2 from the leading large-N_c contribution. This conclusion makes sense, because in the dominant contribution (as discussed above) the incoming hadron 1 splits into two hadrons 2 and 3; hadron 2 has the minimum twist $\kappa_2 = 0$, which propagates to the boundary of the AdS space $z = 0$ and interacts with the current. When $\hat{t} \ll 1$, we can regard hadron 2 as an almost on-shell hadron, and hence the final power dependence should be controlled by the twist of hadron 2. Our calculation and analysis verify this argument, which was originally proposed in Ref. [4]. The result that the subleading contribution in N_c will dominate in the Bjorken limit $q^2 \to \infty$ implies that the large-N_c limit and the Bjorken limit do not commute with each other. Such a conclusion can lead to very important consequences in DIS from gauge/gravity duality. When we are calculating within supergravity, we *a priori* take the large-N_c limit first, followed by the Bjorken limit; however, when we are analyzing the process using operator product expansion, we actually *a priori* take the Bjorken limit first, followed by the large-N_c limit. If the large-N_c limit and the Bjorken limit do not commute with each other any more, this mutual comparison and analysis would lose valuable meaning. There is no doubt that we need further investigation in this direction. We postpone such an investigation for a future work.

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APPENDIX A: DERIVATION OF THE TRANSITION AMPLITUDE

In this appendix, we will derive the transition amplitudes in Eqs. (10)–(12) from the action given in Eq. (9). As opposed to the usual calculation of correlators for operators in the conformal field theory that lives at the boundary of the AdS space, here we are interested in the scattering process of physical states. We will follow the ansatz made by Polchinski and Strassler in Ref. [25] for the scattering of gauge-invariant states. The equations of motion corresponding to the action (9) read

$$\frac{1}{\sqrt{-g}}\partial_m(g^{mn}\sqrt{-g}\partial_n\Phi_1) - \mu_1^2\Phi_1 = \frac{-i\mathcal{Q}_1}{\sqrt{-g}}\partial_m(\sqrt{-g}A^m\Phi_1) - i\mathcal{Q}_1 A^m\partial_m\Phi_1 + \lambda\Phi_2^*\Phi_3^*, \tag{A1}$$

$$\frac{1}{\sqrt{-g}}\partial_m(g^{mn}\sqrt{-g}\partial_n\Phi_2) - \mu_2^2\Phi_2 = \frac{-i\mathcal{Q}_2}{\sqrt{-g}}\partial_m(\sqrt{-g}A^m\Phi_2) - i\mathcal{Q}_2 A^m\partial_m\Phi_2 + \lambda\Phi_1^*\Phi_3, \tag{A2}$$

$$\frac{1}{\sqrt{-g}}\partial_m(g^{mn}\sqrt{-g}\partial_n\Phi_3) - \mu_3^2\Phi_3 = \frac{-i\mathcal{Q}_3}{\sqrt{-g}}\partial_m(\sqrt{-g}A^m\Phi_3) - i\mathcal{Q}_3 A^m\partial_m\Phi_3 + \lambda\Phi_1^*\Phi_2, \tag{A3}$$

where we have suppressed the terms $iQ_i^2 A^m A_m \Phi_i^*$ which are not relevant to the process we are considering. The solution up to first order in the coupling of $\mathcal{Q}_i$ or λ is

$$\begin{aligned}\Phi_1(x) = &-i\mathcal{Q}_1\int d^5x' G(x,x')\partial_m[\sqrt{-g(x')}A^m(x')\Phi_1(x')] \\ &+ \int d^5x'\sqrt{-g(x')}G(x,x')[-i\mathcal{Q}_1 A^m(x')\partial_m\Phi_1(x') + \lambda\Phi_2^*(x')\Phi_3^*(x')].\end{aligned} \tag{A4}$$

Integrating the first term by parts gives

$$\begin{aligned}\Phi_1(x) = &\, i\mathcal{Q}_1\int d^5x'\sqrt{-g(x')}\partial'_m G(x,x')A^m(x')\Phi_1(x') \\ &+ \int d^5x'\sqrt{-g(x')}G(x,x')[-i\mathcal{Q}_1 A^m(x')\partial_m\Phi_1(x') + \lambda\Phi_2^*(x')\Phi_3^*(x')].\end{aligned} \tag{A5}$$

The solution up to second order in the coupling of $\mathcal{Q}_i$ or λ is

$$\begin{aligned}\Phi_1(x) = & iQ_1\lambda \int d^5x'd^5x''\sqrt{-g(x')}\sqrt{-g(x'')}\partial'_m G(x,x')A^m(x')G(x',x'')\Phi_2^*(x'')\Phi_3^*(x'') \\ & - iQ_1\lambda \int d^5x'd^5x''\sqrt{-g(x')}\sqrt{-g(x'')}G(x,x')A^m(x')\partial'_m G(x',x'')\Phi_2^*(x'')\Phi_3^*(x'') \\ & - iQ_2\lambda \int d^5x'd^5x''\sqrt{-g(x')}\sqrt{-g(x'')}G(x,x')\Phi_3^*(x')\partial'_m G(x',x'')A^m(x'')\Phi_2^*(x'') \\ & + iQ_2\lambda \int d^5x'd^5x''\sqrt{-g(x')}\sqrt{-g(x'')}G(x,x')\Phi_3^*(x')G(x',x'')A^m(x'')\partial''_m\Phi_2^*(x'') \\ & - iQ_3\lambda \int d^5x'd^5x''\sqrt{-g(x')}\sqrt{-g(x'')}G(x,x')\Phi_2^*(x')\partial'_m G(x',x'')A^m(x'')\Phi_3^*(x'') \\ & + iQ_3\lambda \int d^5x'd^5x''\sqrt{-g(x')}\sqrt{-g(x'')}G(x,x')\Phi_2^*(x')G(x',x'')A^m(x'')\partial''_m\Phi_3^*(x''). \end{aligned} \tag{A6}$$

It follows that the transition amplitudes (10)–(12) can be obtained by contracting the above expression with the initial wave function in Eq. (13). In order to obtain the final result, it should be noted that when $\Phi_1(x)$ is given in Eq. (13) the following identity holds:

$$\int d^5x'\sqrt{-g(x')}G(x,x')\Phi_i(x') = \Phi_i(x). \tag{A7}$$

The first and second terms correspond to the s-channel, the third and fourth terms correspond to the t-channel, and the last two terms are the u-channel. By using the formalism proposed by Polchinski and Strssler in Ref. [25] for the scattering process of gauge-invariant states, we do not need to deal with the boundary value of the fields. Hence we do not meet any UV issues in our calculation. Besides, since what we are more interested in is the power dependence rather than the overall magnitude, UV issues are not very relevant in the present work.

APPENDIX B: DETAILED ANALYSIS OF THE POWER DEPENDENCE

In this appendix, we will take the first case where $\hat{M}_2 \sim 1$ and $\hat{M}_3 \sim 1$ as an example and give a detailed analysis of the power dependence. As we mentioned above, we will always assign $\hat{M}_1 \ll 1$ for the initial hadron. Due to the kinematic constraint, the Mandelstam variable $\hat{s}$ is always of order unity. First, let us consider the power dependence from various of normalization factors: c_1, c_2, c_3, c_s, c_t, and c_u. Recalling the definitions of these factors in Eqs. (5) and (54), it is easy to see that $\hat{M}_1 \ll 1$, $|\hat{t}| \ll 1$, and $|\hat{u}| \ll 1$, i.e., $M_1 \ll q$, $|t| \ll q^2$, and $|u| \ll q^2$ yield $c_1 \sim 1$, $c_t \sim 1$, and $c_u \sim 1$, respectively. When we want to consider $\hat{M}_2 \sim 1$, $\hat{M}_3 \sim 1$, $\hat{s} \sim 1$, $|\hat{t}| \sim 1$, or $|\hat{u}| \sim 1$, i.e., $M_2 \sim q \to \infty$, $M_3 \sim q \to \infty$, $s \sim q^2 \to \infty$, $|t| \sim q^2 \to \infty$, or $|u| \sim q^2 \to \infty$, we need the asymptotic behavior of the Bessel function $J_\nu(z)$ in the limit $|z| \to \infty$,

$$J_\nu(z) \sim \sqrt{\frac{2}{\pi z}}\cos\left(z - \frac{\nu\pi}{2} - \frac{\pi}{4}\right). \tag{B1}$$

When $z \to \infty$, the nth zero point $z_n \sim (2n+1)\pi + \frac{\nu\pi}{2} + \frac{\pi}{4} \pm \frac{\pi}{2}$. With this asymptotic expression, $\hat{M}_2 \sim 1$, $\hat{M}_3 \sim 1$, $\hat{s} \sim 1$, $|\hat{t}| \sim 1$. and $|\hat{u}| \sim 1$ yield $c_2 \sim q^{\frac{1}{2}}$, $c_3 \sim q^{\frac{1}{2}}$, $c_s \sim q^{\frac{1}{2}}$, $c_t \sim q^{\frac{1}{2}}$. and $c_u \sim q^{\frac{1}{2}}$. respectively. From the expressions for $\hat{C}_s$ in Eq. (51), $\hat{s} \sim 1$ yields $\hat{C}_s \sim c_s^2\hat{M}_1 \sim 1$. From the expressions for $\hat{C}_t$ in Eq. (52), we have $\hat{C}_t \sim c_t^2\hat{M}_1$. Then $|\hat{t}| \sim 1$ and $|\hat{t}| \ll 1$ yield $\hat{C}_t \sim 1$ and $\hat{C}_t \sim \frac{1}{q}$, respectively, and $|\hat{t}| \sim 1$ and $|\hat{u}| \sim 1$ yield $\hat{C}_u \sim c_u^2\hat{M}_1 \sim 1$ and $\hat{C}_t \sim c_t^2\hat{M}_1 \sim 1$, respectively. In a similar way, from the expressions for $\hat{C}_u$ in Eq. (53) we have $\hat{C}_u \sim c_u^2\hat{M}_1$. Then $|\hat{u}| \sim 1$ and $|\hat{u}| \ll 1$ yield $\hat{C}_u \sim 1$ and $\hat{C}_u \sim \frac{1}{q^2}$, respectively. It is very straightforward (but tedious) to verify that the factors involving v_s^2, v_t^2, v_u^2, $v_s \cdot v_u$, $v_u \cdot v_t$, and $v_s \cdot v_t$ in Eqs. (48) and (49) can be of order unity. It is very easy to show that $|\hat{\mathbf{p}}'|$ can always be of order unity. Hence, every term in $F_1(x, q^2)$ or $F_2(x, q^2)$ from different channels behaves as

$$F_{\alpha\beta} \sim \frac{1}{q^6}c_1^2c_2^2c_3^2\sum_{M_2}\sum_{M_3}\int d\theta\sin\theta(\hat{C}_\alpha\hat{C}_\beta^* + \hat{C}_\beta\hat{C}_\alpha^*), \tag{B2}$$

where both α and β denote the different types of channels (s, t, and u). Since the hadrons with $\hat{M}_2 \sim 1$ and $\hat{M}_3 \sim 1$ are very limited, the sum over M_2 and M_3 contributes $\sum_{M_2}\sum_{M_3} \sim 1$. The integral over the phase space $\int d\theta \sin\theta$ depends on the interval of $\hat{t}$ or $\hat{u}$. $|\hat{t}| \sim 1$ and $|\hat{u}| \sim 1$ yield $\int d\theta\sin\theta \sim 1$, and $|\hat{t}| \sim 1$ and $|\hat{u}| \ll 1$ or $|\hat{u}| \sim 1$ and $|\hat{t}| \ll 1$ yield $\int d\theta\sin\theta \sim \frac{1}{q}$. Putting all of these together, we can finally obtain the results in Table I.

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