Two-Higgs-doublet type-II seesaw model

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Motivated by the new observed scalar boson of 126 GeV at ATLAS and CMS, various phenomena in the two-Higgs-doublet model are investigated broadly in the literature. For considering the model that possesses a solution to the massive neutrinos, we study the simplest extension of conventional type-II seesaw model to two Higgs doublets. We find that the new interactions in the scalar potential cause the sizable mixture of charged Higgses in a triplet and doublets. As a result, we have a completely different decay pattern for doubly charged Higgs ($\delta^{\pm\pm}$); even the vacuum expectation value of a Higgs triplet is at GeV level, which is limited by the precision measurement for the ρ parameter. For illustrating the new characters of the model, we study the influence of new interactions on the new open channels $\delta^{++} \to (H_1^+ W^{+^{(*)}}, H_1^+ H_1^+)$ with H_1^+ being the lightest charged Higgs. Additionally, due to the new mixing effect, the triplet charged Higgs could couple to quarks in the model; therefore, the search for δ^{++} via $\delta^{++} \to tbW^+ \to b\bar{b}W^+W^+$ by mediated H_1^+ becomes significant.

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The recent observation of a new scalar particle at 126 GeV by ATLAS [1] and CMS [2] shows that the Higgs mechanism is a right direction not only for the origin of masses of gauge bosons but also for the masses of quarks and charged leptons in the standard model (SM). By this point of view, the most mysterious observed phenomenon in particle physics is the masses of neutrinos. Besides the undetermined mechanism of neutrino masses, we also know nothing about their mass ordering, which is classified by normal ordering, inverted ordering, and quasidegeneracy in the literature [3].

Before the observations of neutrino oscillations, numerous mechanisms for generating the neutrino masses had been proposed. For instance, the type-I seesaw [4] mechanism introduced the heavy right-handed neutrinos, while the type-II seesaw mechanism [5,6] extended the SM by including a $SU(2)$ Higgs triplet. Additionally, other possibilities were also investigated such as adding the new triplet fermions [7], radiative corrections [8–10], etc. Because of the similarity in the mass generation mechanism between the type-II seesaw and Higgs mechanism, we focus the study on the simplest extension to the type-II seesaw model.

The characters of the type-II seesaw model with one Higgs doublet and one Higgs triplet can be briefly summarized as follows. First, doubly charged Higgs decays to the same sign charged gauge bosons (WW) and leptons ($\ell\ell$), where the former coupling is associated with vacuum expectation value (VEV) of the triplet denoted by v_Δ and the latter is related to the multiplication of Yukawa couplings and v_Δ. The involved parameters are limited to be small by the observed neutrino masses. Second, for achieving the small v_Δ, one needs to require either a small massive coupling for the $H^T i\tau_2 \Delta^\dagger H$ term or a heavy mass scale for the Higgs triplet; we will see this point later. If we adopt the mass scale of the Higgs triplet to be of $O(100)$ GeV, it is then inevitable to have a hierarchy in the massive parameters of the Lagrangian. For instance, if one requires leptonic decays of a doubly charged Higgs to be dominant, because of the requirement of vacuum stability, the coefficient μ of the $H^T i\tau_2 \Delta^\dagger H$ term in the scalar potential has to be $\mu \sim v_\Delta < 10^{-4}$ GeV. Third, the singly charged Higgs of the triplet does not couple to quarks.

From a theoretical viewpoint, the two-Higgs-doublet model (THDM) was proposed for solving the weak and strong CP problems [11,12]. Despite the original motivation, the THDM itself provides rich phenomena in particle physics. By the new discovery of the 126 GeV scalar boson at ATLAS and CMS, the phenomenology of the THDM has been further investigated broadly in the literature, e.g., Refs. [13–15]. Since the THDM does not have the mechanism to generate the masses of neutrinos, according to the discussions on the conventional type-II seesaw model (CTTSM), the massive neutrinos indeed could originate from a Higgs triplet with a nonvanished VEV. Therefore, in this paper, we study the extension of the CTTSM by including one extra Higgs doublet, i.e., the two-Higgs-doublet (THD) and one Higgs triplet model. We find that, unlike the case in the CTTSM, the couplings μ_j of $H_j^T i\tau_2 \Delta^\dagger H_k$ terms in the scalar potential could be as large as the electroweak scale when the small v_Δ is satisfied. Moreover, the μ_j terms cause new large mixing effects in singly charged Higgses and new decay channels for doubly and singly charged Higgses. Consequently, these new effects will change the search of doubly charged Higgs

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at colliders [16–28] and affect the rare decays in low-energy physics, such as $b \to s\gamma$, $B \to \tau\nu$, $B \to D^{(*)}\tau\nu$, etc. [29].

To better understand the new characters of the extended model, in the following, we briefly introduce the model. The involved Higgs doublets and triplet are denoted by $H_{1,2}$ and Δ, respectively. Their representations in the $SU(2)$ group are chosen as

$$H_1 = \begin{pmatrix} H_1^+ \\ (v_1 + \rho_1 + i\eta_1)/\sqrt{2} \end{pmatrix},$$
$$H_2 = \begin{pmatrix} H_2^+ \\ (v_2 + \rho_2 + i\eta_2)/\sqrt{2} \end{pmatrix},$$
$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ (v_\Delta + \delta^0 + i\eta^0)/\sqrt{2} & -\delta^+/\sqrt{2} \end{pmatrix}, \quad (1)$$

where $v_{1,2,\Delta}$ stand for the VEVs of neutral components of H_1, H_2, and Δ respectively. As is known, the general THDM will cause flavor-changing neutral currents (FCNCs) at tree level in the Yukawa sector. For avoiding the FCNC effects, we impose a Z_2 symmetry at the Yukawa interactions. Under the symmetry, the transformations of matter fields are given by

$$H_2 \to -H_2, \qquad U_R \to -U_R, \quad (2)$$

with U_R being the right-handed up-type quarks. The other fields are unchanged in the Z_2 transformation. Accordingly, the Yukawa couplings are written by

$$-\mathcal{L}_Y = \bar{Q}\mathbf{Y^d} D_R H_1 + \bar{Q}\mathbf{Y^u} U_R \tilde{H}_2 + \bar{L}\mathbf{Y}^\ell \ell_R H_1 + \frac{1}{2}[L^T C \mathbf{h} i\sigma_2 \Delta P_L L + \text{H.c.}], \quad (3)$$

where we have suppressed all flavor indices; $Q^T = (u, d)_L$ and $L^T = (\nu, \ell)_L$ are the $SU(2)_L$ doublets of quarks and leptons; (D_R, U_R, ℓ_R) in turn denotes the $SU(2)_L$ singlet for down-type, up-type quarks, and charged leptons; and $\tilde{H} = i\sigma_2 H^*$, with σ_2 being the second Pauli matrix. The detailed discussions for the Yukawa couplings could refer to Ref. [29]. Since the signal of the doubly charged Higgs is clearer and unique in the type-II seesaw model, in this study, we will focus on the decays associated with $\delta^{\pm\pm}$. By Eq. (3), the relevant interactions with leptons are given by

$$\mathcal{L}_{\delta^{\pm\pm}\ell\ell} = \frac{1}{2}\ell^T C \mathbf{h} P_L \ell \delta^{++} + \text{H.c.},$$
$$\mathbf{h} = \frac{\sqrt{2}}{v_\Delta} U^*_{\text{PMNS}} \mathbf{m}_\nu^{\text{dia}} U^\dagger_{\text{PMNS}}. \quad (4)$$

Here, $\mathbf{m}_\nu^{\text{dia}}$ is the diagonalized neutrino mass matrix, and U_{PMNS} is the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [30,31]. From Eq. (4), one can see that the typical coupling of $\delta^{\pm\pm}$ to the lepton pair is proportional to m_ν/v_Δ. Consequently, if we take the masses of neutrinos as the knowns that are determined by experiments, the partial decay rate for $\delta^{\pm\pm} \to \ell^\pm\ell^\pm$ strongly depends on the value of v_Δ.

Besides the leptonic couplings, $\delta^{\pm\pm}$ also couples to a charged gauge boson, and the couplings could be read from the gauge-invariant kinetic terms of Higgs fields. Hence, we write the kinetic terms as

$$\mathcal{L}_{\text{K.T.}} = (D^\mu H_1)^\dagger (D_\mu H_1) + (D^\mu H_2)^\dagger (D_\mu H_2) + \text{Tr}[(D_\mu \Delta)^\dagger D^\mu \Delta]. \quad (5)$$

The covariant derivatives of the associated fields are expressed by

$$D_\mu H_{1(2)} = \left(\partial_\mu - i\frac{g}{\sqrt{2}}(W^+_\mu T^+ + W^-_\mu T^-) - i\frac{g}{C_W} Z_\mu (T^3 - S_W^2 Q) - ieA_\mu Q\right) H_{1(2)},$$
$$D_\mu \Delta = \partial_\mu \Delta - i\frac{g}{\sqrt{2}}(W^+_\mu [T^+, \Delta] + W^-_\mu [T^-, \Delta]), - i\frac{g}{c_W} Z_\mu([T^3, \Delta] - S_W^2 [Q, \Delta]) - ieA_\mu [Q, \Delta], \quad (6)$$

where the $W^\pm_\mu$, Z_μ, and A_μ stand for the gauge bosons in the SM; g is the gauge coupling constant of the $SU(2)$; e is the electromagnetic coupling constant; $S_W(C_W) = \sin\theta_W(\cos\theta_W)$, with θ_W being the Weinberg angle; $T^\pm = (\sigma^1 \pm i\sigma^2)/2$ and $T^3 = \sigma^3/2$ are defined by the Pauli matrices σ^i; and Q is the electric charge operator. After electroweak symmetry breaking (EWSB), the masses of $W^\pm$ and Z bosons are obtained by

$$m_W^2 = \frac{g^2 v^2}{4}\left(1 + \frac{2v_\Delta^2}{v^2}\right),$$
$$m_Z^2 = \frac{g^2 v^2}{4\cos^2\theta_W}\left(1 + \frac{4v_\Delta^2}{v^2}\right), \quad (7)$$

with $v = (v_1^2 + v_2^2)^{1/2}$. As a result, the ρ parameter at tree level could be obtained as

$$\rho = \frac{m_W^2}{m_Z^2 c_W^2} = \frac{1 + 2v_\Delta^2/v^2}{1 + 4v_\Delta^2/v^2}. \quad (8)$$

Taking the current precision measurement for the ρ parameter to be $\rho = 1.0004^{+0.0003}_{-0.0004}$ [3], we get $v_\Delta < 3.4$ GeV when 2σ errors are taken into account. By Eqs. (1), (5), and (6), the interactions of $\delta^{\pm\pm}$ with $W^\mp$ are found by

$$\mathcal{L}_{\delta^{\pm\pm}W^{\mp}} = -ig(\partial_\mu\delta^{++})\delta^- W^{-\mu} + ig\delta^{++}(\partial_\mu\delta^-)W^{-\mu} + \frac{1}{2}(\sqrt{2}g^2 v_\Delta)\delta^{++}W^-_\mu W^{-\mu} + \text{H.c.} \tag{9}$$

We see clearly that the coupling of δ^{++} to W^-W^- is proportional to v_Δ. In the CTTSM, the value of v_Δ determines which decaying channel is the dominant mode, the $\ell\ell$ or WW channel. Since $\delta^{\pm\pm}$ and $\delta^{\pm}$ belong to the same multiplet and get the masses from m_Δ before EWSB, the possible mass difference $m_{\delta^{\pm\pm}} - m_{\delta^{\pm}}$ is at most of $O(m_W)$. Therefore, for $m_{\delta^{++}} > m_{\delta^+}$, the decay $\delta^{++} \to W^+\delta^+$ is suppressed by the phase space. However, by the first two interactions in Eq. (9), where the couplings are independent of v_Δ, the three-body decay $\delta^{++} \to \delta^+ W^{+^*}(\to \ell^+\nu)$ indeed can be significant [24–27]. Nonetheless, when a new charged Higgs is introduced, we will show that the new interactions in the scalar potential will lead to a different decay pattern for a doubly charged Higgs.

In the following, we give detailed discussions on the scalar potential, which is the origin of the crucial effects in our model. The scalar potential of the THD and triplet in $SU(2)_L \times U(1)_Y$ symmetry is expressed as

$$V(H_1, H_2, \Delta) = V_{H_1H_2} + V_\Delta + V_{H_1H_2\Delta}, \tag{10}$$

where $V_{H_1H_2}$ and V_Δ stand for the scalar potential of the THDM and of a pure triplet and $V_{H_1H_2\Delta}$ is the interaction among H_1, H_2, and Δ. Their expressions are given by

$$\begin{aligned} V_{H_1H_2} &= m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 - m_{12}^2(H_1^\dagger H_2 + \text{H.c.}) + \lambda_1(H_1^\dagger H_1)^2 + \lambda_2(H_2^\dagger H_2)^2 + \lambda_3 H_1^\dagger H_1 H_2^\dagger H_2 + \lambda_4 H_1^\dagger H_2 H_2^\dagger H_1 \\ &\quad + \frac{\lambda_5}{2}[(H_1^\dagger H_2)^2 + \text{H.c.}], \\ V_\Delta &= m_\Delta^2 \text{Tr}\Delta^\dagger\Delta + \lambda_9(\text{Tr}\Delta^\dagger\Delta)^2 + \lambda_{10}\text{Tr}(\Delta^\dagger\Delta)^2, \\ V_{H_1H_2\Delta} &= (\mu_1 H_1^T i\tau_2\Delta^\dagger H_1 + \mu_2 H_2^T i\tau_2\Delta^\dagger H_2 + \mu_3 H_1^T i\tau_2\Delta^\dagger H_2 + \text{H.c.}) + (\lambda_6 H_1^\dagger H_1 + \bar{\lambda}_6 H_2^\dagger H_2)\text{Tr}\Delta^\dagger\Delta \\ &\quad + H_1^\dagger(\lambda_7\Delta\Delta^\dagger + \lambda_8\Delta^\dagger\Delta)H_1 + H_2^\dagger(\bar{\lambda}_7\Delta\Delta^\dagger + \bar{\lambda}_8\Delta^\dagger\Delta)H_2. \end{aligned} \tag{11}$$

We note that the imposed Z_2 symmetry is broken spontaneously. To make sure the ultraviolet divergences of higher-order effects are under control, as usual, we keep the Z_2 soft breaking terms $m_{12}^2 H_1^\dagger H_2$ and $\mu_3 H_1^T i\sigma_2\Delta^\dagger H_2$ in the scalar potential, in which the former is mass dimension 2 while the later is mass dimension 3; however, the Z_2 hard breaking terms are suppressed. Since we will not discuss the CP-violating effects, hereafter, we take all couplings in the potential as real values. By Eqs. (10) and (11), the VEVs of neutral scalar fields could be determined by the minimal conditions $\partial\langle V\rangle/\partial v_{1,2,\Delta} = 0$. As a result, we have

$$\begin{aligned} \frac{\partial\langle V\rangle}{\partial v_1} &\approx m_1^2 v_1 - m_{12}^2 v_2 + \lambda_1 v_1^3 + \lambda_L v_2^2 v_1 \approx 0, \\ \frac{\partial\langle V\rangle}{\partial v_2} &\approx m_2^2 v_2 - m_{12}^2 v_1 + \lambda_2 v_2^3 + \lambda_L v_1^2 v_2 \approx 0, \\ \frac{\partial\langle V\rangle}{\partial v_\Delta} &\approx m_\Delta^2 v_\Delta - \frac{1}{\sqrt{2}}(v_1^2 + \mu_1 + v_2^2\mu_2 + v_1 v_2\mu_3) \\ &\quad + \left[\frac{\lambda_6 + \lambda_7}{2}v_1^2 + \frac{\bar{\lambda}_6 + \bar{\lambda}_7}{2}v_2^2\right]v_\Delta \approx 0, \end{aligned} \tag{12}$$

where the terms associated with v_Δ in the first two equations and v_Δ^3 in the third equation have been ignored due to $v_\Delta \ll v_{1,2}$. From the last equation, the VEV of the neutral triplet is obtained by

$$v_\Delta \approx \frac{1}{\sqrt{2}}\frac{\mu_1 v_1^2 + \mu_2 v_2^2 + \mu_3 v_1 v_2}{m_\Delta^2 + (\lambda_6 + \lambda_7)v_1^2/2 + (\bar{\lambda}_6 + \bar{\lambda}_7)v_2^2/2}. \tag{13}$$

By this result, we see that with $\mu_2 = \mu_3 = 0$ the small v_Δ indicates the small μ_1 or large m_Δ in the CTTSM. However, when the μ_2 and μ_3 effects are introduced, the necessity of small v_Δ could be accommodated by the massive parameters $\mu_{1,2,3}$ and m_Δ, which can be in the same order of magnitude. Hence, the magnitude of v_Δ indeed could be adjusted by the free parameters of the new scalar potential without introducing a hierarchy to the massive parameters.

By counting the physical degrees of freedom, we have three CP-even neutral particles, two CP-odd pseudoscalar bosons, two singly charged Higgses, and one doubly charged Higgs in the model. The new interactions such as $\mu_{1,2,3}$ terms in $V_{H_1H_2\Delta}$ could cause interesting effects on the couplings of SM-like Higgs, pseudoscalars, charged Higgses, and doubly charged Higgs; moreover, their producing and decaying channels are also modified. For illustrating the features of this model, we concentrate on the new mixing effects of singly charged Higgses and on the new decaying channels of the doubly charged Higgs. The complete analysis of the model will be given elsewhere.

We have shown the couplings of $\delta^{\pm\pm}$ to leptons and the W-gauge boson in Eqs. (3) and (9). For discussing the singly charged Higgs effects, like the conventional THDM, we combine both doublets H_1 and H_2 to be

$$\bar{h} = \cos\beta H_1 + \sin\beta H_2 = \begin{pmatrix} G^+ \\ (v + h^0 + iG^0)/\sqrt{2} \end{pmatrix},$$

$$\bar{H} = -\sin\beta H_1 + \cos\beta H_2 = \begin{pmatrix} H^+ \\ (H^0 + iA^0)/\sqrt{2} \end{pmatrix}, \quad (14)$$

where only the doublet $\bar{h}$ has the VEV after EWSB and $\sin\beta(\cos\beta) = v_2/v(v_1/v)$. As is known, in the THDM, h^0 and H^0 are the CP-even scalars, and they are not physical states; A^0 is the physical CP-odd scalar boson; and $H^\pm$ is the physical charged Higgs particle. When the $SU(2)$ triplet Δ is included to the model, δ^0, η^0, and $\delta^\pm$ of the $SU(2)$ triplet will mix with (h^0, H^0), A^0, and $H^\pm$, respectively. In this study, we will concentrate on the new mixing effects of charged Higgses and their implications. For simplifying numerical analysis and preserving the requirement of $v_\Delta \ll v_1,\ v_2$, we adopt the relation

$$\mu_3 \sim -\frac{\mu_1 v_1^2 + \mu_2 v_2^2}{v_1 v_2}. \quad (15)$$

For completeness, we also show the mass matrices of CP-odd and CP-even Higgs bosons in the Appendix. Hence, in terms of the triplet representation in Eq. (1), doublet representations in Eq. (14), and scalar potentials in Eq. (11), the mass matrix for G^+, H^+, and δ^+ is written by

$$(G^- H^- \delta^-) \begin{pmatrix} 0 & 0 & m^2_{G^-\delta^+} \\ 0 & m^2_{H^-H^+} & m^2_{H^-\delta^+} \\ m^2_{G^-\delta^+} & m^2_{H^-\delta^+} & m^2_{\delta^-\delta^+} \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \\ \delta^+ \end{pmatrix}, \quad (16)$$

where the elements of the mass matrix are found by

$$\begin{aligned} m^2_{G^-\delta^+} &\approx 0, \\ m^2_{H^-H^+} &\equiv m^2_{H^\pm} = \frac{m^2_\pm}{\sin\beta\cos\beta}, \qquad m^2_\pm = m^2_{12} - \frac{\lambda_4 + \lambda_5}{2} v_1 v_2, \\ m^2_{H^-\delta^+} &= \frac{v}{2\sin\beta\cos\beta}[\mu_1 \cos^4\beta - \mu_2 \sin^4\beta + (\mu_1 - \mu_2)\sin^2\beta\cos^2\beta], \\ m^2_{\delta^-\delta^+} &\equiv m^2_{\delta^\pm} = m^2_\Delta + \frac{v_1^2}{4}(2\lambda_6 + \lambda_7 + \lambda_8) + \frac{v_2^2}{4}(2\bar{\lambda}_6 + \bar{\lambda}_7 + \bar{\lambda}_8). \end{aligned} \quad (17)$$

The null elements in Eq. (16) arise from the neglect of small v_Δ that has been used in Eq. (12) for minimal conditions. Since $m^2_{G^-\delta^+}$ is also proportional to v_Δ, for self-consistency, the v_Δ terms should be dropped. As a result, we get $m^2_{G^-\delta^+} \approx 0$; i.e., $G^\pm$ are the Goldstone bosons and decouple with $H^\pm$ and $\delta^\pm$. With this approximation, we find that the 3×3 mass square matrix in Eq. (16) could be reduced to be a 2×2 matrix. The physical charged Higgs states could be regarded as the combination of $H^\pm$ and $\delta^\pm$, and their mixture could be parametrized by

$$\begin{pmatrix} H_1^\pm \\ H_2^\pm \end{pmatrix} = \begin{pmatrix} \cos\theta_\pm & \sin\theta_\pm \\ -\sin\theta_\pm & \cos\theta_\pm \end{pmatrix} \begin{pmatrix} H^\pm \\ \delta^\pm \end{pmatrix}. \quad (18)$$

The masses of charged Higgs particles and their mixing angle are derived as

$$\begin{aligned} (m_{H^\pm_{1,2}})^2 &= \frac{1}{2}(m^2_{\delta^\pm} + m^2_{H^\pm}) \mp \frac{1}{2}[(m^2_{\delta^\pm} - m^2_{H^\pm})^2 + 4m^4_{H^-\delta^+}]^{1/2}, \\ \tan 2\theta_\pm &= -\frac{2m^2_{H^-\delta^+}}{m^2_{\delta^\pm} - m^2_{H^\pm}}. \end{aligned} \quad (19)$$

Here, $H_1^\pm$ is identified as the lighter charged Higgs.

Besides the couplings of $\delta^{\pm\pm}$ that exist in the CTTSM, the scalar potentials in Eq. (11) provide new couplings to $H^\pm$. The relevant interactions could be found as

$$\begin{aligned} -\mathcal{L}_{\delta^{\pm\pm}(H^\mp,\delta^\mp)} = &\frac{1}{2}(2\mu_1 + 2\mu_2)\delta^{++}H^-H^- \\ &- \frac{1}{2}(\sqrt{2}\lambda_{10} v_\Delta)\delta^{++}\delta^-\delta^- \\ &+ \frac{v\sin 2\beta}{4}\left[(\lambda_7 - \lambda_8) - (\bar{\lambda}_7 - \bar{\lambda}_8)\right]\delta^{++}H^-\delta^- \\ &+ \text{H.c.}, \end{aligned} \quad (20)$$

where the first and third terms on the rhs do not exist in the CTTSM. If the charged Higgs $H^\pm$ is much lighter than $\delta^{\pm\pm}$, we see that the new decay channel $\delta^{++} \to H^+H^+$ will be opened. Unlike the Feynman rules for the interactions of $\delta^{++}\ell\ell$ and $\delta^{++}W^-W^-$, the new interactions are not suppressed by m_ν/v_Δ or v_Δ. In other words, the decay rate of the H^+H^+ mode is much larger than that of $\ell^+\ell^+$ and W^+W^+; therefore, the current limit on the mass of the doubly charged Higgs may be relaxed. Furthermore, the new decay channel $\delta^{++} \to H^+W^{+(*)}$ now is also allowed through the mixing angle $\theta_\pm$.

Next, we discuss the numerical analysis for δ^{++} decays. According to the earlier discussions, the relevant free

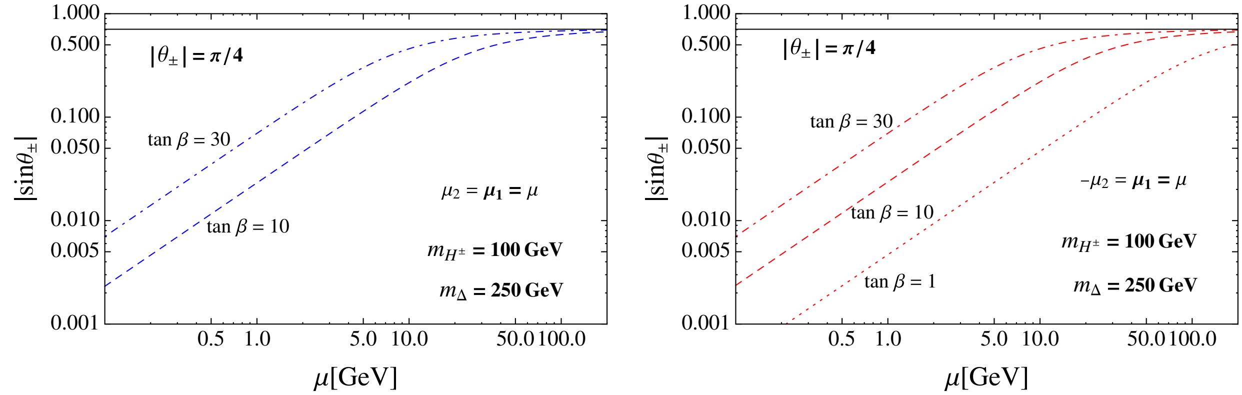

FIG. 1 (color online). The mixing effect $|\sin\theta_{\pm}|$ of $H^{\pm}$ and $\delta^{\pm}$ as a function of μ with $m_{H^{\pm}} = 100$ GeV and $m_{\delta^{\pm}} = 250$ GeV. The left panel is for scheme I, while the right panel is for scheme II. The dotted, dashed, and dotted-dashed lines stand for $\tan\beta = 1$, 10, and 30, respectively.

parameters are angle β, $\lambda_{6,7,8}$, $\bar{\lambda}_{6,7,8}$, $\lambda_{4,5}$, $\mu_{1,2}$, v, v_{Δ}, m^2_{12}, and m_{Δ}. For reducing the free parameters and simplifying the numerical analysis, we take $v \approx 2m_W/g$ as an input and assume $m_{\Delta} \sim m_{\delta^{\pm\pm}} \sim m_{\delta^{\pm}}$. The involved parameters that we use for presentation are set to be angles β, $m_{H^{\pm}}$, m_{Δ}, v_{Δ}, and $\mu_{1,2}$. Since the parameters $\mu_{1,2}$ are the important effects in our model, we adopt two different schemes for numerical discussions: (I) $\mu_1 = \mu_2 = \mu$ and (II) $\mu_1 = -\mu_2 = \mu$. We note that by Eqs. (17) and (19) the mixing angle $\theta_{\pm}$ is not a free parameter but is determined. Because of the tiny neutrino masses, the value of v_{Δ} is much less than 1 GeV.

For understanding how the mixing angle $\theta_{\pm}$ depends on the free parameters, we plot $|\sin\theta_{\pm}|$ as a function of μ in Fig. 1, where we have used $m_{H^{\pm}} = 100$ GeV and $m_{\Delta} = 250$ GeV; the left (right) panel denotes scheme I (II); the dotted, dashed, and dotted-dashed lines stand for $\tan\beta = 1$, 10, and 30, respectively; and the horizontal line corresponds to $|\theta_{\pm}| = \pi/4$. For scheme I, due to $m^2_{H^-\delta^+} = 0$ at $\tan\beta = 1$, the mixing angle vanishes; therefore, we only have two curves in the left panel. By the plots, we see that when the value of μ is taken toward to $O(100)$ GeV the mixing effect is approaching the maximum. The value of μ cannot be arbitrarily large; otherwise, the mass square of the lighter charged Higgs $H_1^{\pm}$ in Eq. (19) will become a negative.

Now, it is known that the magnitude of the mixing effect of $H^{\pm}$ and $\delta^{\pm}$ strongly depends on the values of $\mu_{1,2}$. We believe that the interactions arisen from $\mu_j H_j^T i\Delta^{\dagger} H_j$ (j = 1, 2) could lead to a new decay pattern for a doubly charged Higgs. For more clarity, we present the couplings of $\delta^{\pm\pm}$ to the physical states $H^{\pm}_{1,2}$ and $W^{\pm}$ in Table I. From the table, we see that the involved free parameter for the vertex $\delta^{\pm\pm}$-$H_1^{\mp}$-$W^{\mp}$ is only the angle $\theta_{\pm}$. Although the coupling for the vertex $\delta^{\pm\pm}$-$H_1^{\mp}$-$H_1^{\mp}$ could be comparable with that for $\delta^{\pm\pm}$-$H_1^{\mp}$-$W^{\mp}$, due to phase space suppression, the decay rate for the $H_1^{\mp}H_1^{\mp}$ mode usually will be smaller than that for $H_1^{\mp}W^{\mp}$ mode, except in the case with $\tan\beta = 1$ and the case constrained by the kinematic requirement. Applying these interactions, the partial decay rates for $\delta^{\pm\pm} \to H^{\pm}_{1(2)}X$ ($X = H^{\pm}_{1(2)}, W^{\pm}$) could be formulated by

$$\Gamma\left(\delta^{\pm\pm} \to H^{\pm}_{1(2)}W^{\pm}\right) = \frac{g^2 m^3_{\delta^{++}}}{16\pi m^2_W}\sin^2\theta_{\pm}(\cos^2\theta_{\pm})\left[\lambda\left(\frac{m^2_W}{m^2_{\delta^{++}}}, \frac{m^2_{H^+_{1(2)}}}{m^2_{\delta^{++}}}\right)\right]^{\frac{3}{2}}, \quad (21)$$

$$\Gamma\left(\delta^{\pm\pm} \to H^{\pm}_{1(2)}W^{\pm *}\right) = \frac{9g^4 m^2_{\delta^{++}}}{128\pi^3}\sin^2\theta_{\pm}(\cos^2\theta_{\pm})G\left(\frac{m^2_W}{m^2_{\delta^{++}}}, \frac{m^2_{H^+_{1(2)}}}{m^2_{\delta^{++}}}\right), \quad (22)$$

$$\Gamma\left(\delta^{\pm\pm} \to H^{\pm}_{1(2)}H^{\pm}_{1(2)}\right) = \frac{(\mu_1+\mu_2)^2}{4\pi m^2_{\delta^{++}}}\cos^4\theta_{\pm}(\sin^4\theta_{\pm})\sqrt{m^2_{\delta^{++}} - 4m^2_{H^+_{1(2)}}}, \quad (23)$$

TABLE I. The couplings of $\delta^{\pm\pm}$ to $H^{\pm}_{1,2}$ and $W^{\pm}$.

Vertex	Coupling	Vertex	Coupling
$\delta^{\pm\pm}H_2^{\mp}W_{\mu}^{\mp}$	$-ig\cos\theta_{\pm}(p_{\delta^{\pm\pm}} - p_{H_2^{\mp}})_{\mu}$	$\delta^{\pm\pm}H_1^{\mp}W_{\mu}^{\mp}$	$-ig\sin\theta_{\pm}(p_{\delta^{\pm\pm}} - p_{H_1^{\mp}})_{\mu}$
$\delta^{\pm\pm}H^{\mp}_{1(2)}H^{\mp}_{1(2)}$	$2(\mu_1+\mu_2)\cos^2\theta_{\pm}(\sin^2\theta_{\pm})$	$\delta^{\pm\pm}H_1^{\mp}H_2^{\mp}$	$2(\mu_1+\mu_2)\cos\theta_{+}\sin\theta_{+}$

TABLE II. Selected benchmark points in scheme I.

	m_Δ	$m_{H^\pm}$	$\tan\beta$	μ	$m_{H_2^\pm}$	$m_{H_1^\pm}$	$\vert\sin\theta_+\vert$
BP1	250 GeV	100 GeV	1	100 GeV	250 GeV	100 GeV	0
BP2	500 GeV	400 GeV	10	100 GeV	579 GeV	274 GeV	0.57
BP3	500 GeV	400 GeV	30	50 GeV	628 GeV	124 GeV	0.62
BP4	120 GeV	80 GeV	10	5 GeV	133 GeV	56 GeV	0.47

TABLE III. Selected benchmark points in scheme II.

	m_Δ	$m_{H^\pm}$	$\tan\beta$	μ	$m_{H_2^\pm}$	$m_{H_1^\pm}$	$\vert\sin\theta_+\vert$
BP5	500 GeV	250 GeV	1	100 GeV	503 GeV	243 GeV	0.13
BP6	150 GeV	100 GeV	1	40 GeV	167 GeV	68 GeV	0.48

$$\Gamma(\delta^{\pm\pm} \to H_1^\pm H_2^\pm) = \frac{(\mu_1+\mu_2)^2}{4\pi m_{\delta^{++}}^2}\sin^2\theta_\pm \cos^2\theta_\pm \sqrt{\lambda\left(\frac{m_{H_1^+}^2}{m_{\delta^{++}}^2}, \frac{m_{H_2^+}^2}{m_{\delta^{++}}^2}\right)}, \quad (24)$$

where $W^{\pm *}$ expresses the off-shell W boson and the functions $\lambda(x,y)$ and $G(x,y)$, which are, respectively, associated with momenta of final particles and three-body phase space integration, are found as [19]

$$\lambda(x,y) = 1 + x^2 + y^2 - 2xy - 2x - 2y$$

$$G(x,y) = \frac{1}{12y}\Bigg[2(x-1)^3 - 9(x^2-x)y + 6(x-1)y^2 + 6(1+x-y)y\sqrt{-\lambda(x,y)} \times \left(\arctan\left[\frac{-1+x-y}{\sqrt{-\lambda(x,y)}} + \frac{-1+x+y}{\sqrt{-\lambda(x,y)}}\right]\right) - 3[1+(x-y)^2-2y]y\log x\Bigg]. \quad (25)$$

Since the doubly charged Higgs boson does not mix with other scalar bosons, the formulas for $\delta^{\pm\pm} \to \ell^\pm\ell^\pm$ and $\delta^{\pm\pm} \to W^\pm W^\pm$ decays are the same as those in the CTTSM. Their explicit expressions could be found from Refs. [19,27].

Since there are still four new free parameters involved in our assumption, in order to illustrate the characters of $\delta^{\pm\pm}$ in this model, we adopt several benchmark points (BPs) for the numerical analysis, and they are given in Table II (III) for scheme I (II). In the tables, we regard the values of $m_{\Delta,H^\pm}$, μ, and $\tan\beta$ as inputs; then, $m_{H_{1,2}^\pm}$ and $\sin\theta_\pm$ are determined accordingly.

In the following, we describe the characteristic of each BP and display the associated results in Fig. 2. In BP1, we consider the case for $m_{\delta^{++}} > 2m_{H_1^+}$ and set $\tan\beta = 1$. Because of $\theta_\pm = 0$, the decay $\delta^{++} \to H_1^+ W^+$ is suppressed. For comparison, we show the branching ratios (BRs) for the decays $\delta^{++} \to (\ell^+\ell^+, H_1^+H_1^+)$ in Fig. 2(a). In this paper, we use the normal ordering for neutrino masses to estimate the decay rate of $\delta^{++} \to \ell^+\ell^+$. By the plot, it is clear that the new open channel always dominates in the displayed region of v_Δ. We note that in any circumstance, comparing to the new decay channel, the WW mode is very small and negligible. Hereafter, we will not mention the results of the WW mode. In BP2 and BP3, we select a heavier m_Δ and $\Delta m = m_\Delta - m_{H^\pm} = 100$ GeV. From Table II, we find that if we use $\mu \sim O(\Delta m)$ the mixing effect is $O(1)$, and the mass splitting between $m_{H_1^\pm}$ and $m_{H_2^\pm}$ is significant. Additionally, with a larger value of $\tan\beta$, we see that the mixing angle and mass splitting are enlarged. We plot the BRs of δ^{++} decays for BP2 and BP3 in Figs. 2(b) and 2(c). Since $m_{\delta^{++}} < 2m_{H_1^+}$ in BP2, only $\ell^+\ell^+$ and $H_1^+W^+$ modes in Fig. 2(b) are allowed. From Figs. 2(b) and 2(c), we confirm the previous inference for $\mathrm{BR}(\delta^{++} \to H_1^+H_1^+) < \mathrm{BR}(\delta^{++} \to H_1^+W^+)$. In BP4, we use a lower mass for $m_{\delta^{++}} = 120\,\mathrm{GeV}$ and $m_{H^\pm} = 80\,\mathrm{GeV}$. In this case, we find that the allowed value of μ cannot be over 7.7 GeV; otherwise, $m_{H_1^+}^2$ will be negative. Because of the kinematic requirement, either W^+ or H_1^+ in the $H_1^+W^+$ mode should be off shell. Since the couplings of $H^\pm$ to quarks and leptons are related to the masses of fermions, for lighter charged Higgs decays, the decay rate for $\delta^{++} \to H_1^{+^*}(\to f_1 f_2)W^+$ is suppressed by the masses of lighter fermions. Therefore, we present the BRs for $\delta^{++} \to (\ell^+\ell^+, H_1^+H_1^+, H_1^+W^{+^*})$ in Fig. 2(d). Because of the phase space, we see $\mathrm{BR}(\delta^{++} \to H_1^+H_1^+) > \mathrm{BR}(\delta^+ + \to H_1^+W^{+^*})$ in this case. Moreover, we also find that the decay $\delta^{++} \to \ell^+\ell^+$ could become dominant when v_Δ is of order of 10^{-9}.

For scheme II, the selected values of parameters are categorized in BP5 and BP6. Since the coupling of $\delta^{++}H_1^-H_1^-$ vanishes in this scheme, the decay $\delta^{++} \to H_1^+H_1^+$ is suppressed. Additionally, the results with $\tan\beta = 10$ and 30 for $\delta^{++} \to H_1^+W^+$ are similar to those in scheme I; therefore, we will not repeatedly discuss the

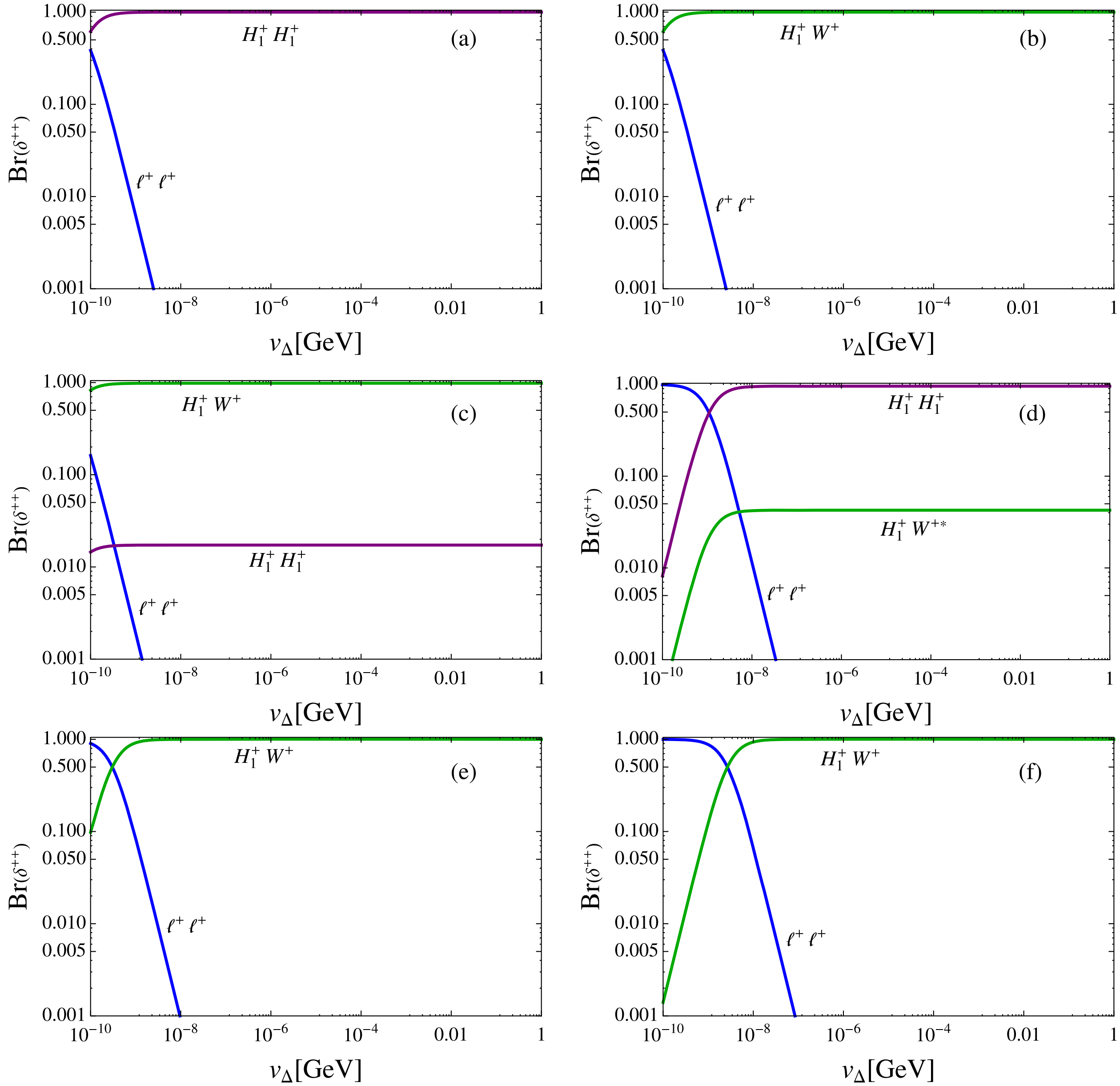

FIG. 2 (color online). BRs for δ^{++} decays. The (a)–(f) plots, respectively, stand for BP1–6, defined in Tables II and III.

cases but focus on the case with $\tan\beta = 1$. Hence, we present the BRs for $\delta^{++} \to (\ell^+\ell^+, H_1^+ W^+)$ in Figs. 2(e) and 2(f), where both BP5 and BP6 have similar behavior but the turning point of the leading decay mode occurs at a different value of v_Δ.

It is known that the neutrinos get their masses at tree level in the type-II seesaw model. However, the neutrino masses could be also generated by loop corrections, e.g., the two-loop effects that are similar to the Zee model [32]. The loop corrections in the CTTSM are actually negligible due to $\mu_i \sim v_\Delta \ll v$. Since we claim that the μ_i could be as large as the VEV v, here it is worth discussing the loop effects in our model. By an order of magnitude estimate, the two-loop effects are roughly expressed by

$$m^\nu_{\ell'\ell} \sim \frac{1}{(4\pi^2)^2} \frac{m^2_{\ell'} m^2_\ell}{v^2 m^2_\Delta} (\mathbf{h})_{\ell'\ell} \mu_1 I_2, \tag{26}$$

where we only consider the contribution of the μ_1 term, $1/(4\pi^2)^2$ denotes the two-loop effect, $m^2_{\ell'} m^2_\ell / v^2$ is from the vertex of $\bar{L} H_1 \ell_R$ in Eq. (3) and the mass insertion in charged lepton propagators, $(\mathbf{h})_{\ell'\ell}$ is the Yukawa coupling of the Higgs triplet in Eq. (3), and I_2 stands for the loop integration. Using $(\mathbf{h})_{\ell'\ell} \sim m_\nu / v_\Delta$, numerically we have $m^\nu_{\ell'\ell} \sim 10^{-12} (\mu_1 / v_\Delta)(m^2_{\ell'} m^2_\ell / m^4_\tau) m_\nu I_2$. Thus, by choosing proper value of v_Δ, the radiative corrections to neutrino masses with $\mu_1 \sim O(v)$ could be still much smaller than the contributions from the tree level.

Although our analysis focuses on the situation for which the Higgs triplet is heavier than Higgs doublets, the reverse case should be also interesting and worth further studying. In the case of reversed mass ordering, the heavier doublet Higgs bosons can decay into a doubly charged Higgs through charged Higgs decay $H^+ \to W^- \delta^{++}$ or through the cascade decay of neutral Higgs $H^0 \to H^+ W^- \to \delta^{++} W^- W^-$. Like SM Higgs, the heavier neutral Higgs would be produced by gluon fusion and have a sizeable production cross section at the LHC. Thus, it is interesting to search for the signal of the process $pp \to H^0 \to H^+ W^- \to \delta^{++} W^- W^-$ that represents the specific signature of the model. Further studies of the collider signals are left as our future work.

Finally, we give a remark on the couplings of triplet particles to quarks. As it is known, $\delta^\pm$ belongs to the $SU(2)$ triplet and cannot couple to quarks directly. However, the interactions of $\delta^\pm$ with quarks are built in our model through the mixing of $\delta^\pm$ and $H^\pm$, which arises from the μ_i terms of the scalar potential. Consequently, we open not only a new channel for the search of $\delta^{\pm\pm}$ but also a new way to look for it. For instance, if $m_{\delta^{++}} \sim 250$ GeV and $m_{H_1^+} \sim 180$ GeV, the signal for the existence of δ^{++} could be read via the decay $\delta^{++} \to H_1^{+(*)} W^+ \to t\bar{b} W^+ \to b\bar{b} W^+ W^+$, i.e., $2b$-jet $+ W^+ W^+$ in the final state, where the signal of δ^{++} becomes completely different from the CTTSM.

In summary, we have studied the new interactions in a two-Higgs-doublet type-II seesaw model. We find that the small VEV of the Higgs triplet could be satisfied by accommodating the free parameters in the new scalar potential, i.e., $\mu_{1,2,3}$, m_Δ, $v_{1,2}$, etc., where these massive parameters could be the same order of magnitude. By neglecting the contributions of v_Δ, the charged Higgs mixing could be described by one mixing angle $\theta_\pm$. The mixing angle is dictated by the parameters $\mu_{1,2}$ and $\tan\beta$. We have demonstrated that by taking proper values of $\mu_{1,2}$ and $\tan\beta$ the new decay channels $\delta^{++} \to (H_1^+ W^+, H_1^+ H_1^+)$ are dominant in δ^{++} decays, except at very tiny v_Δ. Since the decay pattern of $\delta^{\pm\pm}$ is different from that in the CTTSM, the search for $\delta^{\pm\pm}$ and the limit on its mass should be further studied at the colliders. It will be interesting to see the new phenomena in the model at the LHC.

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APPENDIX: MASS MATRICES FOR *CP*-ODD AND *CP*-EVEN HIGGS BOSONS

Using the potentials in Eq. (11) and the basis of the Higgs doublets in Eq. (14), the mass matrix for the CP-odd components G^0, A^0, and η^0 is written by

$$\frac{1}{2}\begin{pmatrix} G^0 \\ A^0 \\ \eta^0 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & m^2_{G^0\eta^0} \\ 0 & m^2_{A^0A^0} & m^2_{A^0\eta^0} \\ m^2_{G^0\eta^0} & m^2_{A^0\eta^0} & m^2_{\eta^0\eta^0} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \\ \eta^0 \end{pmatrix}, \quad \text{(A1)}$$

where the elements of the mass matrix are obtained as

$$\begin{aligned} m^2_{G^0\eta^0} &\simeq 0 \\ m^2_{A^0A^0} &\equiv m^2_{A^0} = \frac{m^2_{12} - \lambda_5 v_1 v_2}{\cos\beta\sin\beta} \\ m^2_{A^0\eta^0} &= \frac{v}{\sqrt{2}\cos\beta\sin\beta} \\ &\quad \times [\mu_1 \cos^4\beta - \mu_2 \sin^4\beta + (\mu_1 - \mu_2)\cos^2\beta\sin^2\beta] \\ m^2_{\eta^0\eta^0} &\equiv m^2_{\eta^0} = m^2_\Delta + \frac{v_1^2}{2}(\lambda_6 + \lambda_7) + \frac{v_2^2}{2}(\bar{\lambda}_6 + \bar{\lambda}_7). \end{aligned} \quad \text{(A2)}$$

The null elements in Eq. (A2) arise from the neglect of small v_Δ as in the charged Higgs case. By using Eq. (15), we get $m^2_{G^0\eta^0} \propto v_\Delta$. Like the discussion on $m^2_{G^-\delta^+}$, for self-consistency, we should drop the v_Δ effect and take $m^2_{G^0\eta^0} \approx 0$. Thus, the mass matrix could be reduced to a 2×2 matrix. Consequently, the physical states of CP-odd Higgses could be parametrized by one mixing angle, defined by

$$\begin{pmatrix} A_1^0 \\ A_2^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_A & \sin\theta_A \\ -\sin\theta_A & \cos\theta_A \end{pmatrix} \begin{pmatrix} A^0 \\ \eta^0 \end{pmatrix}. \quad \text{(A3)}$$

The masses of CP-odd Higgs particles and the mixing angle are derived as

$$\begin{aligned} (m_{A^0_{1,2}})^2 &= \frac{1}{2}(m^2_{A^0A^0} + m^2_{\eta^0\eta^0}) \\ &\quad \mp \frac{1}{2}\sqrt{(m^2_{A^0A^0} - m^2_{\eta^0\eta^0})^2 + 4(m_{A^0\eta^0})^4}, \\ \tan 2\theta_A &= \frac{2m^2_{A^0\eta^0}}{m^2_{A^0A^0} - m^2_{\eta^0\eta^0}}, \end{aligned} \quad \text{(A4)}$$

where A_1^0 is identified as the lighter CP-odd Higgs.

For CP-even Higgs bosons, first we transform the h^0 and H^0 states to h and H states by

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}, \quad \text{(A5)}$$

where h and H usually are the physical mass eigenstates in the THDM and α is the mixing angle. With Eq. (14), we write $\rho_{1,2}$ in terms of h and H as

$$\begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} = \begin{pmatrix} \cos(\alpha-\beta) & \sin(\alpha-\beta) \\ -\sin(\alpha-\beta) & \cos(\alpha-\beta) \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}. \quad \text{(A6)}$$

In this basis, the mass matrix becomes

$$\frac{1}{2}\begin{pmatrix} h \\ H \\ \delta^0 \end{pmatrix}^T \begin{pmatrix} m_{hh}^2 & 0 & m_{h\delta^0}^2 \\ 0 & m_{HH}^2 & m_{H\delta^0}^2 \\ m_{h\delta^0}^2 & m_{H\delta^0}^2 & m_{\delta^0\delta^0}^2 \end{pmatrix}\begin{pmatrix} h \\ H \\ \delta^0 \end{pmatrix}. \tag{A7}$$

The elements of the mass matrix and $\tan 2\alpha$ are given by

$$\begin{aligned} m_{HH,hh}^2 &= \frac{1}{2}[m_{12}^2(\tan\beta + \cot\beta) + 2(\lambda_1\cos^2\beta + \lambda_2\sin^2\beta)v^2] \\ &\pm \frac{1}{2}\sqrt{[m_{12}^2(\tan\beta - \cot\beta) + 2(\lambda_1\cos^2\beta - \lambda_2\sin^2\beta)v^2]^2 + 4(m_{12}^2 - \lambda_{345}v^2\sin\beta\cos\beta)^2}, \\ m_{H\delta^0}^2 &= \frac{v}{\sqrt{2}}(\mu_1\cot\beta - \mu_2\tan\beta)\sin(\alpha-\beta), \\ m_{h\delta^0}^2 &= \frac{v}{\sqrt{2}}(\mu_1\cot\beta - \mu_2\tan\beta)\cos(\alpha-\beta), \\ \tan 2\alpha &= \frac{2(-m_{12}^2 + \lambda_{345}v^2\sin\beta\cos\beta)}{m_{12}^2(\tan\beta - \cot\beta) + 2(\lambda_1\cos^2\beta - \lambda_2\sin^2\beta)v^2}, \end{aligned} \tag{A8}$$

where we have used Eq. (15) and $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$. Since $m_{h\delta^0}^2$ is not suppressed by v_Δ, the 3×3 mass matrix in general cannot be further reduced. However, for the case with $\sin(\alpha-\beta) \sim -1$ where h is the SM-like Higgs particle, due to $m_{h\delta^0}^2 \sim 0$, the mass matrix then could be reduced to a 2×2 mass matrix. In summary, because of the μ_i terms, large mixing effects between triplet and doublet particles occur in our model.

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