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$B \to K_1\pi(K)$ decays in the perturbative QCD approach

Zhi-Qing Zhang,[1] Zhi-Wei Hou,[1] Yueling Yang,[2] and Junfeng Sun[2]
[1]*Department of Physics, Henan University of Technology, Zhengzhou, Henan 450001, China*
[2]*College of Physics and Information Engineering, Henan Normal University, Xinxiang 453007, China*
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Within the framework of the perturbative QCD approach, we study the two-body charmless decays $B \to K_1(1270)(K_1(1400))\pi(K)$. We find the following results: (i) The decays $\bar{B}^0 \to K_1(1270)^+\pi^-$, $K_1(1400)^+\pi^-$ are incompatible with the present experimental data. There exists a similar situation for the decays $\bar{B}^0 \to a_1(1260)^+K^-$, $b_1(1235)^+K^-$, which are usually considered that the nonperturbative contributions are needed to explain the data. But the difference is that the nonperturbative contributions seem to play opposite roles in these two groups of decays. (ii) The pure annihilation type decays $\bar{B}^0 \to K_1^\pm(1270)K^\mp$, $K_1^\pm(1400)K^\mp$ are good channels to test whether an approach can be used to calculate correctly the strength of the penguin-annihilation amplitudes. Their branching ratios are predicted at 10^{-7} order, which are larger than the QCDF results. (iii) The dependence of the direct CP-violating asymmetries of these decays on the mixing angle θ_{K_1} are also considered.

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I. INTRODUCTION

In general, the mesons are classified in J^{PC} multiplets. There are two types of orbitally excited axial-vector mesons, namely 1^{++} and 1^{+-}. The former includes $a_1(1260)$, $f_1(1285)$, $f_1(1420)$ and K_{1A}, which compose the $^3P_1$-nonet, and the latter includes $b_1(1235)$, $h_1(1170)$, $h_1(1380)$ and K_{1B}, which compose the $^1P_1$-nonet. Except $a_1(1260)$ and $b_1(1235)$, other axial-vector mesons exist mixing problem, which makes their inner structure become more ambiguous, for example, K_{1A} and K_{1B} can mix with each other and form two physical mass eigenstates $K_1(1270)$, $K_1(1400)$. Various values about the mixing angle θ_{K_1} can be found in different literatures, which will be examined in more detail in Sec. III. For the mixings of the SU(3)-singlet and SU(3)-octet mesons, specifically, the $f_1(1285) - f_1(1420)$ mixing angle $\theta_{^3P_1}$ and the $h_1(1170) - h_1(1380)$ mixing angle $\theta_{^1P_1}$, there also exist several values in the phenomenal analysis. Certainly, these two angles can associate with θ_{K_1} through the Gell-Mann-Okubo mass formula. For the lack of sufficient experimental data, none of them can be accurately determined up to now. So the decays involving these mesons become more ambiguous compared with the decays involving $a_1(1260)$ or/and $b_1(1235)$ meson(s), which have been discussed in the previous works [1–6].

In this paper, we would like to discuss the decays $B \to K_1(1270)\pi(K)$, $K_1(1400)\pi(K)$. On the theoretical side, many approaches have been used to study these decays, such as the naive factorization [4], the generalized factorization [5], and the QCD factorization approach [6]. From the predictions of these approaches, One can find that the branching ratios of the decays $B \to K_1(1270)\pi$, $K_1(1400)\pi$ are in the order of 10^{-6}, for example, $\mathrm{Br}(B^0 \to K_1(1270)^+\pi^-) = (3–8) \times 10^{-6}$, $\mathrm{Br}(B^0 \to K_1(1400)^+\pi^-) = (2–5) \times 10^{-6}$, those of almost all the decays $B \to K_1(1270)K$, $K_1(1400)K$ are in the order of 10^{-8}–10^{-7}. While on the experimental side, the large upper limits are given for the decays $B^0 \to K_1(1400)^+\pi^-$ and $B^+ \to K_1(1400)^0\pi^+$ at the 90% level (C.L.) of 1.1×10^{-3} and 2.6×10^{-3}, respectively [7], and the Heavy Flavor Averaging Group (HFAG) gives the following results [8]:

$$\mathrm{Br}(B^+ \to K_1(1270)^0\pi^+) < 40 \times 10^{-6},$$
$$\mathrm{Br}(B^+ \to K_1(1270)^0\pi^+) < 39 \times 10^{-6}, \quad (1)$$

$$\mathrm{Br}(B^0 \to K_1(1270)^+\pi^-) = (17^{+8}_{-11}) \times 10^{-6},$$
$$\mathrm{Br}(B^0 \to K_1(1400)^+\pi^-) = (17^{+7}_{-9}) \times 10^{-6}. \quad (2)$$

The preliminary data are given by *BABAR* [9],

$$\mathrm{BR}(B^0 \to K_1^+(1270)\pi^-) = (12.0 \pm 3.1^{+9.3}_{-4.5}) \times 10^{-6}, \quad (3)$$

$$\mathrm{BR}(B^0 \to K_1^+(1400)\pi^-) = (16.7 \pm 2.6^{+3.5}_{-5.0}) \times 10^{-6}. \quad (4)$$

Furthermore, *BABAR* has also measured the branching ratios $\mathrm{Br}(B^0 \to K_1(1270)^+\pi^- + K_1(1400)^+\pi^-) = 3.1^{+0.8}_{-0.7} \times 10^{-5}$ and $\mathrm{Br}(B^+ \to K_1(1270)^0\pi^+ + K_1(1400)^0\pi^+) = 2.9^{+2.9}_{-1.7} \times 10^{-5}$ with 7.5σ and 3.2σ significance, respectively. In the paper [10], the two sided intervals for some of the decays $B \to K_1(1270)\pi$, $K_1(1400)\pi$ are evaluated at 68% probability ($\times 10^{-5}$):

$$\mathrm{BR}(B^- \to \bar{K}_1(1270)^0\pi^-) \in [0.0, 2.1],$$
$$\mathrm{BR}(B^- \to \bar{K}_1(1400)^0\pi^-) \in [0.0, 2.5], \quad (5)$$

$$\mathrm{BR}(B^0 \to K_1(1270)^+\pi^-) \in [0.6, 2.5],$$
$$\mathrm{BR}(B^0 \to K_1(1400)^+\pi^-) \in [0.8, 2.4]. \quad (6)$$

In view of the differences between the theories and experiments, we are going to use the PQCD approach to explore these decays and analyze whether the nonperturbtive contributions are necessary to explain the experimental data. In the following, $K_1(1270)$ and $K_1(1400)$ are denoted as K_1 in some places for convenience. The layout of this paper is as follows. In Sec. II, the decay constants and the light-cone distribution amplitudes of the relevant mesons are introduced. In Sec. III, we then analyze these decay channels by using the PQCD approach. The numerical results and the discussions are given in Sec. IV. The conclusions are presented in the final part.

II. DECAY CONSTANTS AND DISTRIBUTION AMPLITUDES

For the wave function of the heavy B meson, we take

$$\Phi_B(x,b) = \frac{1}{\sqrt{2N_c}}(\not{P}_B + m_B)\gamma_5\phi_B(x,b). \quad (7)$$

Here only the contribution of Lorentz structure $\phi_B(x,b)$ is taken into account, since the contribution of the second Lorentz structure $\bar{\phi}_B$ is numerically small [11] and has been neglected. For the distribution amplitude $\phi_B(x,b)$ in Eq. (7), we adopt the following model:

$$\phi_B(x,b) = N_B x^2(1-x)^2 \exp\left[-\frac{M_B^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right], \quad (8)$$

where ω_b is a free parameter, we take $\omega_b = 0.4 \pm 0.04$ Gev in numerical calculations, and $N_B = 101.4$ is the normalization factor for $\omega_b = 0.4$.

The distribution amplitudes of the axial-vector K_1 are written as:

$$\begin{aligned}
&\langle K_1(P,\epsilon_L^*)|\bar{q}_{2\beta}(z)q_{1\alpha}(0)|0\rangle \\
&= \frac{i\gamma_5}{\sqrt{2N_c}}\int_0^1 dx e^{ixp\cdot z}[m_{K_1}\not{\epsilon}_L^*\phi_{K_1}(x) + \not{\epsilon}_L^*\not{P}\phi_{K_1}^t(x) \\
&\quad + m_{K_1}\phi_{K_1}^s(x)]_{\alpha\beta}, \\
&\langle K_1(P,\epsilon_T^*)|\bar{q}_{2\beta}(z)q_{1\alpha}(0)|0\rangle \\
&= \frac{i\gamma_5}{\sqrt{2N_c}}\int_0^1 dx e^{ixp\cdot z}[m_{K_1}\not{\epsilon}_T^*\phi_{K_1}^v(x) + \not{\epsilon}_T^*\not{P}\phi_{K_1}(x) \\
&\quad + m_{K_1} i\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu\epsilon_T^{*v}n^\rho v^\sigma\phi_{K_1}^a(x)]_{\alpha\beta},
\end{aligned} \quad (9)$$

where K_1 refers to the two flavor states K_{1A} and K_{1B}, and the corresponding distribution functions can be calculated by using light-cone QCD sum rule and listed as follows:

$$\begin{cases}
\phi_{K_1}(x) = \frac{f_{K_1}}{2\sqrt{2N_c}}\phi_\parallel(x), & \phi_{K_1}^T(x) = \frac{f_{K_1}}{2\sqrt{2N_c}}\phi_\perp(x), \\
\phi_{K_1}^t(x) = \frac{f_{K_1}}{2\sqrt{2N_c}}h_\parallel^{(t)}(x), & \phi_{K_1}^s(x) = \frac{f_{K_1}}{2\sqrt{4N_c}}\frac{d}{dx}h_\parallel^{(s)}(x), \\
\phi_{K_1}^v(x) = \frac{f_{K_1}}{2\sqrt{2N_c}}g_\perp^{(v)}(x), & \phi_{K_1}^a(x) = \frac{f_{K_1}}{8\sqrt{2N_c}}\frac{d}{dx}g_\perp^{(a)}(x).
\end{cases} \quad (10)$$

Here we use f_{K_1} to present both the longitudinally and transversely polarized states $K_{1A}(K_{1B})$ by assuming $f_{K_{1A}}^T = f_{K_{1A}} = f_{K_1}$ for K_{1A} and $f_{K_{1B}} = f_{K_{1B}}^T = f_{K_1}$ for K_{1B}, respectively. It is similar for the case of $a_1(b_1)$ states, and the difference is that here K_{1A} and K_{1B} are not the mass eigenstates. In Eq. (10), the twist-2 distribution functions are in the first line and can be expanded as:

$$\phi_{\parallel,\perp} = 6x(1-x)\left[a_0^{\parallel,\perp} + 3a_1^{\parallel,\perp}t + a_2^{\parallel,\perp}\frac{3}{2}(5t^2-1)\right], \quad (11)$$

the twist-3 light-cone distribution amplitudes (LCDAs) are used the following forms for K_{1A} and K_{1B} states:

$$\begin{aligned}
h_\parallel^{(t)}(x) &= 3a_0^\perp t^2 + \frac{3}{2}a_1^\perp t(3t^2-1), \\
h_\parallel^{(s)}(x) &= 6x(1-x)(a_0^\perp + a_1^\perp t), \\
g_\perp^{(a)}(x) &= 6x(1-x)(a_0^\parallel + a_1^\parallel t), \\
g_\perp^{(v)}(x) &= \frac{3}{4}a_0^\parallel(1+t^2) + \frac{3}{2}a_1^\parallel t^3,
\end{aligned} \quad (12)$$

where $t = 2x-1$ and the Gegenbauer moments [12] $a_0^\perp(K_{1A}) = 0.26_{-0.22}^{+0.03}$, $a_0^\parallel(K_{1B}) = -0.15 \pm 0.15$, $a_0^\parallel(K_{1A}) = a_0^\perp(K_{1B}) = 1$, $a_1^\perp(K_{1A}) = -1.08 \pm 0.48$, $a_1^\perp(K_{1B}) = 0.30_{-0.31}^{+0.00}$, $a_1^\parallel(K_{1A}) = -0.30_{-0.00}^{+0.26}$, $a_1^\parallel(K_{1B}) = -1.95 \pm 0.45$, $a_2^\parallel(K_{1A}) = -0.05 \pm 0.03$, $a_2^\perp(K_{1B}) = 0.09_{-0.18}^{+0.16}$.

The wave functions for the pseudoscalar (P) mesons K, π are given as:

$$\begin{aligned}
\Phi_{K(\pi)}(P,x,\zeta) \equiv \frac{1}{\sqrt{2N_C}}\gamma_5[&\not{P}\phi_{K(\pi)}^A(x) + m_0\phi_{K(\pi)}^P(x) \\
&+ \zeta m_0(\not{v}\not{n} - v\cdot n)\phi_{K(\pi)}^T(x)],
\end{aligned} \quad (13)$$

where the parameter ζ is either $+1$ or -1 depending on the assignment of the momentum fraction x. The chiral scale parameter m_0 is defined as $m_0 = \frac{m_\pi^2}{m_u+m_d}$ for π meson and $m_0 = \frac{m_K^2}{m_u+m_s}$ for K meson. The distribution amplitudes are expanded as:

$$\begin{aligned}
\phi_{K(\pi)}^A(x) = \frac{3f_{K(\pi)}}{\sqrt{6}}x(1-x)[&1 + a_{1K(\pi)}C_1^{3/2}(t) \\
&+ a_{2K(\pi)}C_2^{3/2}(t) + a_{4K(\pi)}C_4^{3/2}(t)],
\end{aligned} \quad (14)$$

$$\phi^P_{K(\pi)}(x) = \frac{3f_{K(\pi)}}{2\sqrt{6}}\left[1 + \left(30\eta_3 - \frac{5\rho^2_{K(\pi)}}{2}\right)C_2^{1/2}(t) - 3\left(\eta_3\omega_3 + \frac{9\rho^2_{K(\pi)}}{20}(1 + 6a_{2K(\pi)})\right)C_4^{1/2}(t)\right], \tag{15}$$

$$\phi^T_{K(\pi)}(x) = \frac{-f_{K(\pi)}t}{2\sqrt{6}}\left[1 + 6\left(5\eta_3 - \frac{\eta_3\omega_3}{2} - \frac{7\rho^2_{K(\pi)}}{20} - \frac{3\rho^2_{K(\pi)}a_{2K(\pi)}}{5}\right)(1 - 10x + 10x^2)\right], \tag{16}$$

where the decay constants $f_K = 0.16$ GeV, $f_\pi = 0.13$ GeV and the Gegenbauer moments, Gegenbauer polynomials are defined as:

$$a_{1K} = 0.17 \pm 0.17, \qquad a_{1\pi} = 0,$$
$$a_{2K} = a_{2\pi} = 0.115 \pm 0.115, \qquad a_{4K} = a_{4\pi} = -0.015,$$
$$C_1^{3/2}(t) = 3t, \qquad C_2^{3/2}(t) = \frac{3}{2}(5t^2 - 1),$$
$$C_4^{3/2}(t) = \frac{15}{8}(1 - 14t^2 + 21t^4), \qquad C_2^{1/2}(t) = \frac{1}{2}(3t^2 - 1),$$
$$C_4^{1/2}(t) = \frac{1}{8}(3 - 30t^2 + 35t^4), \tag{17}$$

and the constants $\eta_3 = 0.015$, $\omega_3 = -3$, the mass ratio $\rho_{K(\pi)} = m_{K(\pi)}/m_{0K(\pi)}$ with $m_K = 0.49$ GeV, $m_{0K} = 1.7$ GeV, $m_\pi = 0.135$ GeV, $m_{0\pi} = 1.4$ GeV.

III. THE PERTURBATIVE QCD CALCULATION

The PQCD approach is an effective theory to handle hadronic B decays [13–15]. Because it takes into account the transverse momentum of the valence quarks in the hadrons, one will encounter the double logarithm divergences when the soft and the collinear momenta overlap. Fortunately, these large double logarithm can be resummed into the Sudakov factor [16]. There also exist another type of double logarithms which arise from the loop corrections to the weak decay vertex. These double logarithms can also be resummed and resulted in the threshold factor [17]. This factor decreases faster than any other power of the momentum fraction in the threshold region, which removes the endpoint singularity. It is often parametrized into a simple form which is independent on channels, twists and flavors [18]. Certainly, when the higher order diagrams only suffer from soft or collinear infrared divergence, it is ease to cure by using the eikonal approximation [19]. Controlling these kinds of divergences reasonably makes the PQCD approach more self-consistent.

For these two axial vector mesons, their mass eigenstates and flavor eigenstates are not the same with each other, and the former can be obtained by the latter through a mixing angle θ_{K_1}:

$$K_1(1270) = K_{1A}\sin\theta_{K_1} + K_{1B}\cos\theta_{K_1},$$
$$K_1(1400) = K_{1A}\cos\theta_{K_1} - K_{1B}\sin\theta_{K_1}. \tag{18}$$

Unfortunately, there are many uncertainties about this mixing angle. From various phenomenological analysis and experimental data on the masses of these two physical states, it indicates that this mixing angle is around either 33° or 58° [20–29]. Certainly, the author of [30] stresses that the sign of θ_{K_1} depends on the relative sign of flavor states K_{1A} and K_{1B}, which can be determined by fixing the relative sign of the decay constants of K_{1A} and K_{1B}. If the decay constants f_{1A}, f_{1B} are the same in sign (it means that the transitions $B \to K_{1A}$ and $B \to K_{1B}$ have the opposite signs), then the mixing angle θ_{K_1} defined in (18) is positive. It is noticed that the mixing angle for the antiparticle states $\bar{K}_1(1270)$, $\bar{K}_1(1400)$, which is denoted as $\theta_{\bar{K}_1}$, is of opposite sign to that for the particle states $K_1(1270)$, $K_1(1400)$. But even so, we cannot confirm whether θ_{K_1} is larger or less than 45° up to now. Different approaches and models are used and different values of the mixing angle are obtained. In order to pin down it, Cheng [30] advocates to determine the mixing angles $\theta_{^3P_1}$ and $\theta_{^1P_1}$ between $f_1(1285) - f_1(1420)$ and $h_1(1170) - h_1(1380)$, respectively, which in turn depend on the $K_{1A} - K_{1B}$ mixing angle θ_{K_1} through the mass relation. Through analyzing the present data of the h_1, f_1 mesons' strong/radiative decay modes, the author prefers $\theta_{K_1} \sim 33°$ over 58°. In view of the present limited data, we will still include the mixing angle $\theta_{K_1} \sim 58°$ in our calculations.

It is just because of the ambiguous mixing angle that makes the study very difficult. Here we take the decay $\bar{B}^0 \to \bar{K}_1(1270)^0\pi^0$ as an example, which is contributed by the decays $\bar{B}^0 \to \bar{K}^0_{1A}\pi^0$ and $\bar{B}^0 \to \bar{K}^0_{1B}\pi^0$. Figure 1 is for the Feynman diagrams of the decay $\bar{B}^0 \to \bar{K}^0_{1A}\pi^0$ (it is similar to the decay $\bar{B}^0 \to \bar{K}^0_{1B}\pi^0$), through which the amplitudes can be calculated directly, and the total amplitudes of the decay $\bar{B}^0 \to \bar{K}_1(1270)^0\pi^0$ can be obtained by combining the two sets of flavor state amplitudes according to Eq. (18):

$$\sqrt{2}A(\bar{K}_1(1270)^0\pi^0)$$
$$= -\xi_t(f_{K_{1A}}\sin\theta_{K_1} + f_{K_{1B}}\cos\theta_{K_1})F_{e\pi}^{LL}\left(a_4 - \frac{1}{2}a_{10}\right) - \xi_t(M_{e\pi}^{LL;K_{1A}}\sin\theta_{K_1} + M_{e\pi}^{LL;K_{1B}}\cos\theta_{K_1})\left(C_3 - \frac{1}{2}C_9\right)$$
$$- \xi_t(M_{e\pi}^{LR;K_{1A}}\sin\theta_{K_1} + M_{e\pi}^{LR;K_{1B}}\cos\theta_{K_1})\left(C_5 - \frac{1}{2}C_7\right) - \xi_t(M_{a\pi}^{LL;K_{1A}}\sin\theta_{K_1} + M_{a\pi}^{LL;K_{1B}}\cos\theta_{K_1})\left(C_3 - \frac{1}{2}C_9\right)$$
$$- \xi_t(M_{a\pi}^{LR;K_{1A}}\sin\theta_{K_1} + M_{a\pi}^{LR;K_{1B}}\cos\theta_{K_1})\left(C_5 - \frac{1}{2}C_7\right) - \xi_t f_B(F_{a\pi}^{LL;K_{1A}}\sin\theta_{K_1} + F_{a\pi}^{LL;K_{1B}}\cos\theta_{K_1})\left(a_4 - \frac{1}{2}a_{10}\right)$$
$$- \xi_t f_B(F_{a\pi}^{SP;K_{1A}}\sin\theta_{K_1} + F_{a\pi}^{SP;K_{1B}}\cos\theta_{K_1})\left(a_6 - \frac{1}{2}a_8\right) + f_\pi(F_{eK_{1A}}^{LL}\sin\theta_{K_1} + F_{eK_{1B}}^{LL}\cos\theta_{K_1})\left[\xi_u a_1 - \xi_t\left(\frac{3C_9}{2} + \frac{C_{10}}{2}\right.\right.$$
$$\left.\left.- \frac{3C_7}{2} - \frac{C_8}{2}\right)\right] + (M_{eK_{1A}}^{LL;\pi}\sin\theta_{K_1} + M_{eK_{1B}}^{LL;\pi}\cos\theta_{K_1})\left[\xi_u C_2 - \xi_t\frac{3C_{10}}{2}\right] - \xi_t(M_{eK_{1A}}^{SP;\pi}\sin\theta_{K_1} + M_{eK_{1B}}^{SP;\pi}\cos\theta_{K_1})\frac{3C_8}{2}, \quad (19)$$

where $\xi_u = V_{ub}V_{us}^*$, $\xi_t = V_{tb}V_{ts}^*$, $F_{e(a)M_1}^{M_2}$ and $M_{e(a)M_1}^{M_2}$ denote the amplitudes of factorizable and nonfactorizable emission (annihilation) diagrams, where the subscript meson M_1 is involved in the $\bar{B}^0$ meson transition, the superscript meson M_2 is the emitted particle. The other superscript in each amplitude denotes different current operators, $(V-A)(V-A)$, $(V-A)(V+A)$ and $(S-P)(S+P)$ corresponding to LL, LR and SP, respectively. If exchanging the positions of K_{1A} and π^0 in Figs. 1(a), 1(b), 1(c) and 1(d), we will get the new Feynman diagrams, which can also contribute to the decay $\bar{B}^0 \to \bar{K}_{1A}^0\pi^0$, and the corresponding amplitudes are given in the last three lines of Eq. (19). The amplitudes for the decay $\bar{B}^0 \to \bar{K}_{1A}^0(\bar{K}_{1B}^0)\pi^0$ can be obtained from those for the decay $B \to K\pi$ which can be found in [31], only changing the variables of K meson with those of $K_{1A}^0(K_{1B}^0)$ meson. So we do not list the analytic expressions for these amplitudes. Certainly, it is noticed that if the axial-vector meson $K_{1A}(K_{1B})$ is on the emitted position in the factorizable emission diagrams, there is no scalar or pseudoscalar current contribution. The total amplitudes for the other three $B \to K_1(1270)\pi$ decay modes can also be written out similarly:

$$A(K_1(1270)^-\pi^+)$$
$$= (f_{K_{1A}}\sin\theta_{K_1} + f_{K_{1B}}\cos\theta_{K_1})F_{e\pi}^{LL}(\xi_u a_1 - \xi_t(a_4 + a_{10})) + (M_{e\pi}^{LL;K_{1A}}\sin\theta_{K_1} + M_{e\pi}^{LL;K_{1B}}\cos\theta_{K_1})(\xi_u C_1 - \xi_t(C_3 + C_9))$$
$$- \xi_t(M_{e\pi}^{LR;K_{1A}}\sin\theta_{K_1} + M_{e\pi}^{LR;K_{1B}}\cos\theta_{K_1})(C_5 + C_7) - \xi_t(M_{a\pi}^{LL;K_{1A}}\sin\theta_{K_1} + M_{a\pi}^{LL;K_{1B}}\cos\theta_{K_1})\left(C_3 - \frac{1}{2}C_9\right)$$
$$- \xi_t(M_{a\pi}^{LR;K_{1A}}\sin\theta_{K_1} + M_{a\pi}^{LR;K_{1A}}\cos\theta_{K_1})\left(C_5 - \frac{1}{2}C_7\right) - \xi_t f_B(F_{a\pi}^{LL;K_{1A}}\sin\theta_{K_1} + F_{a\pi}^{LL;K_{1B}}\cos\theta_{K_1})\left(a_4 - \frac{1}{2}a_{10}\right)$$
$$- \xi_t f_B(F_{a\pi}^{SP;K_{1A}}\sin\theta_{K_1} + F_{a\pi}^{SP;K_{1B}}\cos\theta_{K_1})\left(a_6 - \frac{1}{2}a_8\right), \quad (20)$$

$$\sqrt{2}A(K_1(1270)^-\pi^0)$$
$$= (f_{K_{1A}}\sin\theta_{K_1} + f_{K_{1B}}\cos\theta_{K_1})F_{e\pi}^{LL}[\xi_u a_1 - \xi_t(a_4 + a_{10})] + (M_{e\pi}^{LL;K_{1A}}\sin\theta_{K_1} + M_{e\pi}^{LL;K_{1B}}\cos\theta_{K_1})[\xi_u C_1 - \xi_t(C_3 + C_9)]$$
$$- \xi_t(M_{e\pi}^{LR;K_{1A}}\sin\theta_{K_1} + M_{e\pi}^{LR;K_{1B}}\cos\theta_{K_1})(C_5 + C_7) + (M_{a\pi}^{LL;K_{1A}}\sin\theta_{K_1} + M_{a\pi}^{LL;K_{1B}}\cos\theta_{K_1})[\xi_u C_1 - \xi_t(C_3 + C_9)]$$
$$- \xi_t(M_{a\pi}^{LL;K_{1A}}\sin\theta_{K_1} + M_{a\pi}^{LL;K_{1B}}\cos\theta_{K_1})(C_5 + C_7) + f_B(F_{a\pi}^{LL;K_{1A}}\sin\theta_{K_1} + F_{a\pi}^{LL;K_{1B}}\cos\theta_{K_1})[\xi_u a_2 - \xi_t(a_4 + a_{10})]$$
$$- f_B(F_{a\pi}^{SP;K_{1A}}\sin\theta_{K_1} + F_{a\pi}^{SP;K_{1B}}\cos\theta_{K_1})\xi_t(a_6 + a_8) + f_\pi(F_{eK_{1A}}^{LL}\sin\theta_{K_1} + F_{eK_{1B}}^{LL}\cos\theta_{K_1})\left[\xi_u a_1 - \xi_t\left(\frac{3C_9}{2} + \frac{C_{10}}{2}\right.\right.$$
$$\left.\left.- \frac{3C_7}{2} - \frac{C_8}{2}\right)\right] + (M_{eK_{1A}}^{LL;\pi}\sin\theta_{K_1} + M_{eK_{1B}}^{LL;\pi}\cos\theta_{K_1})\left[\xi_u C_2 - \xi_t\frac{3C_{10}}{2}\right] - \xi_t(M_{eK_{1A}}^{SP;\pi}\sin\theta_{K_1} + M_{eK_{1B}}^{SP;\pi}\cos\theta_{K_1})\frac{3C_8}{2}, \quad (21)$$

$$A(\bar{K}_1(1270)^0\pi^-) \\ = -\xi_t(f_{K_{1A}}\sin\theta_{K_1} + f_{K_{1B}}\cos\theta_{K_1})F_{e\pi}^{LL}\left(a_4 - \frac{1}{2}a_{10}\right) - \xi_t(M_{e\pi}^{LL;K_{1A}}\sin\theta_{K_1} + M_{e\pi}^{LL;K_{1B}}\cos\theta_{K_1})\left(C_3 - \frac{1}{2}C_9\right) \\ - \xi_t(M_{e\pi}^{LR;K_{1A}}\sin\theta_{K_1} + M_{e\pi}^{LR;K_{1B}}\cos\theta_{K_1})\left(C_5 - \frac{1}{2}C_7\right) + (M_{e\pi}^{LL;K_{1A}}\sin\theta_{K_1} + M_{e\pi}^{LL;K_{1B}}\cos\theta_{K_1})[\xi_u C_1 - \xi_t(C_3 + C_9)] \\ - \xi_t(M_{e\pi}^{LR;K_{1A}}\sin\theta_{K_1} + M_{e\pi}^{LR;K_{1B}}\cos\theta_{K_1})(C_5 + C_7) + f_B(F_{a\pi}^{LL;K_{1A}}\sin\theta_{K_1} + F_{a\pi}^{LL;K_{1B}}\cos\theta_{K_1})[\xi_u a_2 - \xi_t(a_4 + a_{10})] \\ - \xi_t f_B(F_{a\pi}^{SP;K_{1A}}\sin\theta_{K_1} + F_{a\pi}^{SP;K_{1B}}\cos\theta_{K_1})(a_6 + a_8). \tag{22}$$

It is easy to get the total amplitudes for the decay modes including $\bar{K}_1(1400)^0/K_1(1400)^-$ by making the replacements with $\sin\theta_{K_1} \to \cos\theta_{K_1}$, $\cos\theta_{K_1} \to -\sin\theta_{K_1}$ in Eqs. (19)–(22), respectively. The total amplitudes for each $B \to K_1(1270)K$, $K_1(1400)K$ decay are given in the Appendix.

IV. NUMERICAL RESULTS AND DISCUSSIONS

The input parameters in the numerical calculations [32,33] are listed as follows:

$$f_B = 210 \text{ MeV}, \quad f_{K_{1A}} = 250 \text{ MeV}, \quad f_{K_{1B}}^{\perp} = 190 \text{ MeV} \tag{23}$$

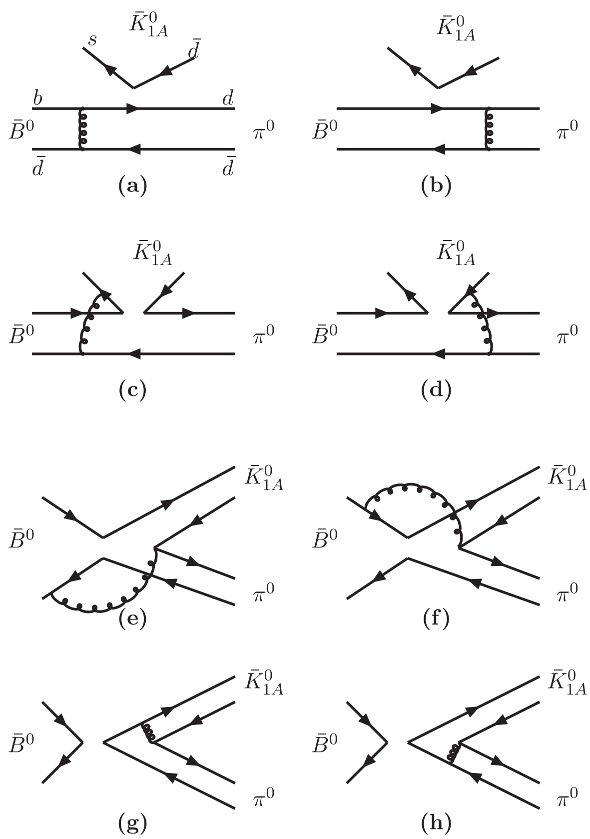

FIG. 1. Diagrams contributing to the decay $\bar{B}^0 \to \bar{K}^0_{1A}\pi^0$.

$$\tau_{B^\pm} = 1.638 \times 10^{-12}\ \text{s}, \qquad \tau_{B^0} = 1.525 \times 10^{-12}\ \text{s}, \tag{24}$$

$$|V_{ud}| = 0.974, \quad |V_{td}| = 8.67 \times 10^{-3}, \quad |V_{ub}| = 3.51 \times 10^{-3}, \tag{25}$$

$$|V_{ts}| = 0.0404, \qquad |V_{us}| = 0.22534, \qquad |V_{tb}| = 0.999. \tag{26}$$

Using the input parameters and the wave functions as specified in this section and Sec. II, it is easy to get the branching ratios for the considered decays which are listed in Table I, where the first error comes from the uncertainty in the B meson shape parameter $\omega_b = 0.40 \pm 0.04$ GeV, the second error is from the hard scale t, which we vary from $0.8t$ to $1.2t$, and the third error is from the combined uncertainties of the Gegenbauer moments $a_1^{\perp}(K_{1A}) = -1.08 \pm 0.48$ and $a_1^{\parallel}(K_{1B}) = -1.95 \pm 0.45$. From Table I we can find that the branching ratios of $B \to K_1(1270)\pi$, $K_1(1400)\pi$ decays fall in 10^{-6} order. The experimental data for the branching ratios of the decays $\bar{B}^0 \to K_1(1270)^-\pi^+$, $K_1(1400)^-\pi^+$, which are given as $(12.0 \pm 3.1^{+9.3}_{-4.5}) \times 10^{-6}$ and $(16.7 \pm 2.6^{+3.5}_{-5.0}) \times 10^{-6}$, respectively, are large and incompatible with all the present theory predictions. Even for the two sided intervals $\text{Br}(\bar{B}^0 \to K_1(1270)^-\pi^+) \in [0.6, 2.5] \times 10^{-5}$ and $\text{Br}(\bar{B}^0 \to K_1(1270)^-\pi^+) \in [0.8, 2.4] \times 10^{-5}$, they almost cannot contain the different theoretical results. While the branching ratios of the charged B decays can be explained by the theories for the large uncertainties of the intervals $\text{Br}(B^- \to \bar{K}_1(1270)^0\pi^-) \in [0.0, 2.1] \times 10^{-5}$, $\text{Br}(B^- \to \bar{K}_1(1400)^0\pi^-) \in [0.0, 2.5] \times 10^{-5}$. The large differences between theories and experiments do not happen to the decays $\bar{B}^0 \to a_1(1260)^{\pm}\pi^{\mp}$, which are tree-dominated. If the decay constants f_{a_1}, f_π and the form factors $V_0^{B\to a_1}$, $F_0^{B\to\pi}$ can be well determined, it is not difficult for us to predict the branching ratios of the decays $\bar{B}^0 \to a_1(1260)^{\pm}\pi^{\mp}$ accurately, because the penguin contributions can be neglected and there are fewer uncertainties. For the considered decays $\bar{B}^0 \to K_1^{\pm}\pi^{\mp}$, the tree operators are suppressed by the CKM matrix elements $V_{ub}V_{us}^*/(V_{cb}V_{cs}^*) \sim 0.02$, and the penguin operators will

TABLE I. Branching ratios (in units of 10^{-6}) for the decays $B \to K_1(1270)\pi$, $K_1(1400)\pi$ and $B \to K_1(1270)K$, $K_1(1400)K$ for mixing angle $\theta_{\bar{K}_1} = -33°$. Other model predictions are also presented here for comparison. It is noticed that the results of [4] and [5] are obtained for mixing angle 32°, while those in [6] are obtained for mixing angle −37°.

	[4]	[5]	[6]	This work
$\bar{B}^0 \to K_1^-(1270)\pi^+$	4.3	7.6	$3.0^{+0.8+1.5+4.2}_{-0.6-0.9-1.4}$	$4.6^{+0.3+0.9+1.5}_{-0.1-0.8-1.2}$
$\bar{B}^0 \to \bar{K}_1^0(1270)\pi^0$	2.3	0.4	$1.0^{+0.0+0.6+1.7}_{-0.0-0.3-0.6}$	$1.4^{+0.1+0.7+0.6}_{-0.1-0.5-0.5}$
$B^- \to \bar{K}_1^0(1270)\pi^-$	4.7	5.8	$3.5^{+0.1+1.8+5.1}_{-0.1-1.1-1.9}$	$3.5^{+0.4+1.9+1.6}_{-0.2-1.1-1.2}$
$B^- \to K_1^-(1270)\pi^0$	2.5	4.9	$2.7^{+0.1+1.1+3.1}_{-0.1-0.7-1.0}$	$3.9^{+0.9+1.0+1.1}_{-0.5-0.7-1.0}$
$\bar{B}^0 \to K_1^-(1400)\pi^+$	2.3	4.0	$5.4^{+1.1+1.7+9.9}_{-1.0-1.3-2.8}$	$3.0^{+0.5+0.1+0.9}_{-0.3-0.1-0.7}$
$\bar{B}^0 \to K_1^0(1400)\pi^0$	1.7	3.0	$2.9^{+0.3+0.7+5.5}_{-0.3-0.6-1.7}$	$3.3^{+0.9+0.1+1.0}_{-0.7-0.0-0.8}$
$B^- \to \bar{K}_1^0(1400)\pi^-$	2.5	3.0	$6.5^{+1.0+2.0+11.6}_{-0.9-1.6-3.6}$	$5.0^{+1.3+1.0+1.4}_{-0.7-0.8-1.1}$
$B^- \to K_1^-(1400)\pi^0$	0.7	1.0	$3.0^{+0.4+1.1+5.2}_{-0.4-0.7-1.3}$	$1.8^{+0.3+0.1+0.4}_{-0.2-0.2-0.3}$
$\bar{B}^0 \to K_1^-(1270)K^+$			$0.01^{+0.01+0.00+0.02}_{-0.00-0.00-0.01}$	$0.13^{+0.01+0.00+0.23}_{-0.01-0.01-0.08}$
$\bar{B}^0 \to K_1^+(1270)K^-$			$0.06^{+0.01+0.00+0.46}_{-0.01-0.00-0.06}$	$0.26^{+0.02+0.05+0.19}_{-0.02-0.04-0.12}$
$B^- \to K_1^0(1270)K^-$	0.22		$0.25^{+0.01+0.15+0.39}_{-0.01-0.08-0.09}$	$1.11^{+0.01+0.19+0.43}_{-0.01-0.03-0.35}$
$B^- \to K_1^-(1270)K^0$	0.02		$0.05^{+0.02+0.07+0.10}_{-0.02-0.03-0.04}$	$1.84^{+0.37+0.29+0.65}_{-0.28-0.25-0.42}$
$\bar{B}^0 \to \bar{K}_1^0(1270)K^0$	0.02		$2.30^{+0.16+1.13+1.43}_{-0.15-0.61-0.61}$	$1.71^{+0.34+0.27+0.51}_{-0.26-0.23-0.43}$
$\bar{B}^0 \to K_1^0(1270)\bar{K}^0$	0.20		$0.24^{+0.01+0.11+0.33}_{-0.01-0.07-0.13}$	$0.26^{+0.03+0.17+0.14}_{-0.06-0.01-0.08}$
$\bar{B}^0 \to K_1^-(1400)K^+$			$0.09^{+0.01+0.00+0.23}_{-0.01-0.00-0.09}$	$0.64^{+0.14+0.00+0.13}_{-0.06-0.01-0.08}$
$\bar{B}^0 \to K_1^+(1400)K^-$			$0.02^{+0.00+0.00+0.04}_{-0.00-0.00-0.00}$	$0.31^{+0.02+0.11+0.12}_{-0.00-0.01-0.09}$
$B^- \to K_1^0(1400)K^-$	0.12		$0.48^{+0.08+0.15+0.81}_{-0.08-0.12-0.26}$	$0.90^{+0.13+0.11+1.21}_{-0.08-0.09-0.16}$
$B^- \to K_1^-(1400)K^0$	4.4		$0.01^{+0.00+0.01+0.14}_{-0.00-0.00-0.01}$	$1.33^{+0.14+0.31+0.33}_{-0.10-0.22-0.22}$
$\bar{B}^0 \to \bar{K}_1^0(1400)K^0$	4.1		$0.08^{+0.01+0.17+0.59}_{-0.01-0.06-0.08}$	$1.46^{+0.16+0.31+0.33}_{-0.13-0.25-0.28}$
$\bar{B}^0 \to K_1^0(1400)\bar{K}^0$	0.11		$0.50^{+0.08+0.13+0.92}_{-0.07-0.11-0.32}$	$0.14^{+0.04+0.04+0.07}_{-0.03-0.03-0.02}$

play a significant role. If the future data are really larger than the present predictions for here considered decays, the authors [6] claimed that there are two possible reasons: one is because the larger corrections from the weak annihilation and the hard spectator contributions, the other is from the charming penguin contributions. In our calculations, the hard spectator contributions which correspond to the nonfactorization emission diagram ones are very small. Although the factorizable annihilation contributions are more important, they cannot promote the branching ratios too much. So we consider that the charming penguins are more likely to explain the large data. Unfortunately, the charming penguins are nonperturbative in nature and remain untouched by many theory approaches. While it is helpful to consider these decays by using the soft-collinear-effective-theory (SECT) [34], where the charming penguin contributions from loop diagrams are included. Certainly, these contributions can also be incorporated in the final-state interactions [35]. There exists the similar situation for the decays $\bar{B}^0 \to a_1(1260)^+K^-$, $b_1(1235)^+K^-$ [1], where the PQCD predictions are larger than the data. The nonperturbative contributions, such as the final state interactions or the charming penguins, are suggested to explain the data. The penguin contributions from the factorization annihilation diagrams in the $K_{1B}\pi$ modes are much larger than those in the $K_{1A}\pi$ modes. So we can find that the branching ratios of $B \to K_{1B}\pi$ decays are always larger than those of $B \to K_{1A}\pi$ decays, which is shown in Table II.

For the decays $B \to K_1(1270)K$, $K_1(1400)K$, there are no experimental data or upper limits up to now. Although the decays $\bar{B}^0 \to K_1^{\pm}K^{\mp}$ can occur only via annihilation type diagrams, their branching ratios might not be so small as those predicted by the QCDF approach. If our predictions can be confirmed by the future LHCb or the super B experiments, one can say that the PQCD approach is one of the few methods, which can be used to quantitatively calculate the annihilation type contributions. In the previous years both the experimenters and the theorists considered that the branching ratio of $B^0 \to K^+K^-$ was at 10^{-8} order, but two years ago the CDF and LHCb collaborations gave their first measurements of this decay by $(2.3 \pm 1.0 \pm 1.0) \times 10^{-7}$ [36] and $(1.3^{+0.6}_{-0.5} \pm 0.7) \times 10^{-7}$ [37], respectively. Later, these results are confirmed by the PQCD recalculated result 1.56×10^{-7} [38] without introducing too much uncertainties. It shows that the PQCD approach can determine correctly the strength of penguin-annihilation amplitudes. Whether the PQCD approach can give reasonable predictions for the pure annihilation decays $\bar{B}^0 \to K_1(1270)^{\pm}K^{\mp}$, $K_1(1400)^{\pm}K^{\mp}$ also deserves our attention and research. For the decay $\bar{B}^0 \to K_{1B}^0\bar{K}^0$ cannot receive a large emission factorization amplitude, because of the small decay constant $f_{K_{1B}}$ compared with $f_{K_{1A}}$, while it

TABLE II. Branching ratios (in units of 10^{-6}) for the decays $B \to K_{1A}\pi$, $K_{1B}\pi$ and $B \to K_{1A}K$, $K_{1B}K$. The errors for these entries correspond to the uncertainties from $\omega_B = 0.4 \pm 0.04$ GeV, the hard scale t varying from $0.8t$ to $1.2t$, and the Gegenbauer moments $a_1^{\perp}(K_{1A}) = -1.08 \pm 0.48$ for K_{1A} meson, $a_1^{\parallel}(K_{1B}) = -1.95 \pm 0.45$ for K_{1B} meson, respectively.

$\bar{B}^0 \to K_{1A}^- \pi^+$	$2.1^{+1.0+0.1+0.0}_{-0.6-0.1-0.3}$	$\bar{B}^0 \to K_{1B}^- \pi^+$	$5.6^{+0.1+0.8+2.1}_{-0.2-0.9-1.9}$
$\bar{B}^0 \to \bar{K}_{1A}^0 \pi^0$	$1.3^{+0.7+0.2+0.9}_{-0.5-0.2-0.6}$	$\bar{B}^0 \to \bar{K}_{1B}^0 \pi^0$	$3.4^{+0.1+1.0+1.1}_{-0.1-0.7-0.9}$
$B^- \to \bar{K}_{1A}^0 \pi^-$	$3.9^{+1.9+0.6+1.7}_{-1.3-0.5-1.5}$	$B^- \to \bar{K}_{1B}^0 \pi^-$	$4.7^{+0.2+2.2+1.8}_{-0.3-1.5-1.6}$
$B^- \to K_{1A}^- \pi^0$	$2.1^{+0.9+0.2+0.6}_{-0.7-0.2-0.8}$	$B^- \to K_{1B}^- \pi^0$	$3.7^{+0.1+0.7+1.2}_{-0.2-0.8-1.1}$
$\bar{B}^0 \to K_{1A}^- K^+$	$0.47^{+0.03+0.00+0.28}_{-0.04-0.00-0.04}$	$\bar{B}^0 \to K_{1B}^- K^+$	$0.34^{+0.04+0.01+0.14}_{-0.03-0.01-0.07}$
$\bar{B}^0 \to K_{1A}^+ K^-$	$0.14^{+0.01+0.01+0.11}_{-0.00-0.01-0.13}$	$\bar{B}^0 \to K_{1B}^+ K^-$	$0.38^{+0.03+0.03+0.26}_{-0.03-0.02-0.19}$
$B^- \to K_{1A}^0 K^-$	$1.24^{+0.13+0.08+1.74}_{-0.12-0.07-0.65}$	$B^- \to K_{1B}^0 K^-$	$0.60^{+0.04+0.19+0.13}_{-0.04-0.12-0.08}$
$B^- \to K_{1A}^- K^0$	$0.29^{+0.02+0.05+1.26}_{-0.01-0.03-0.03}$	$B^- \to K_{1B}^- K^0$	$2.65^{+0.53+0.48+0.67}_{-0.34-0.41-0.57}$
$\bar{B}^0 \to \bar{K}_{1A}^0 K^0$	$0.10^{+0.00+0.05+0.10}_{-0.00-0.03-0.04}$	$\bar{B}^0 \to \bar{K}_{1B}^0 K^0$	$2.71^{+0.30+0.52+0.66}_{-0.30-0.43-0.58}$
$\bar{B}^0 \to K_{1A}^0 \bar{K}^0$	$0.16^{+0.12+0.06+0.18}_{-0.06-0.03-0.10}$	$\bar{B}^0 \to K_{1B}^0 \bar{K}^0$	$0.17^{+0.01+0.08+0.09}_{-0.01-0.05-0.06}$

has a large annihilation factorization amplitude, which makes its branching ratio slightly larger than that of $\bar{B}^0 \to K_{1A}^0 \bar{K}^0$. The branching ratios of these two decays are at the order of 10^{-7}. But it is very different to the decay $\bar{B}^0 \to \bar{K}_{1B}^0 K^0$: Except having a large annihilation factorization amplitude, it can also obtain a large emission factorization amplitude at the same time, because here the emission meson is K^0 with a larger decay constant $f_K = 0.16$. So this decay gets a large branching ratio, which amounts to 2.71×10^{-6}. Even though the decay $\bar{B}^0 \to \bar{K}_{1A}^0 K^0$ has a small branching ratio, the physical final states $\bar{K}_1(1200)^0 K^0$, $\bar{K}_1(1400)^0 K^0$, which are mixes of the former two group flavor states, still might get a large branching ratio. It has been verified by the different theories, which are shown in Table I. But the branching ratio of the decay $\bar{B}^0 \to \bar{K}_1(1400)^0 K^0$ predicted by the QCDF approach seems too small compared with the results given by the PQCD and the naive factorization approaches, which can be clarified by the future experiments. There exists the similar situation for the decay $B^- \to K_1(1400)^- K^0$. Another decay channel, where exists large divergence between the predictions, is $B^- \to K_1(1200)^- K^0$. The Feynman diagrams of this decay can be obtained from those of the decay $\bar{B}^0 \to \bar{K}_1(1200)^0 K^0$ by replacing the spectator quark d with u, so the difference of the branching ratios of these two decays should not be so large. In a word, the branching ratios of the

TABLE III. Same as Table I except for the mixing angle $\theta_{\bar{K}_1} = -58°$.

	[4]	[5]	[6]	This work
$\bar{B}^0 \to K_1^-(1270)\pi^+$	4.3	7.6	$2.7^{+0.6+1.3+4.4}_{-0.5-0.8-1.5}$	$3.2^{+0.7+0.5+0.8}_{-0.5-0.5-0.8}$
$\bar{B}^0 \to \bar{K}_1^0(1270)\pi^0$	2.1	0.4	$0.8^{+0.1+0.5+1.7}_{-0.1-0.3-0.6}$	$0.5^{+0.2+0.0+0.4}_{-0.0-0.2-0.2}$
$B^- \to \bar{K}_1^0(1270)\pi^-$	4.7	5.8	$3.0^{+0.2+0.1+2.7}_{-0.2-0.2-2.2}$	$3.2^{+1.3+1.2+1.3}_{-0.9-0.8-1.2}$
$B^- \to K_1^-(1270)\pi^0$	1.6	4.9	$2.5^{+0.1+1.0+3.2}_{-0.1-0.7-1.0}$	$3.3^{+1.1+0.7+0.8}_{-0.8-0.6-1.1}$
$\bar{B}^0 \to K_1^-(1400)\pi^+$	2.3	4.0	$2.2^{+1.1+0.7+2.6}_{-0.8-0.6-1.3}$	$4.5^{+0.0+0.3+1.5}_{-0.0-0.5-1.3}$
$\bar{B}^0 \to K_1^0(1400)\pi^0$	1.6	1.7	$1.5^{+0.4+0.3+1.7}_{-0.3-0.3-0.9}$	$4.1^{+0.8+0.7+1.2}_{-0.4-0.4-0.8}$
$B^- \to \bar{K}_1^0(1400)\pi^-$	2.5	3.0	$2.8^{+1.0+0.9+3.0}_{-0.8-0.9-1.7}$	$5.4^{+0.3+1.6+1.5}_{-0.2-1.2-1.4}$
$B^- \to K_1^-(1400)\pi^0$	0.6	1.4	$1.0^{+0.4+0.4+1.2}_{-0.3-0.4-0.5}$	$2.5^{+0.0+0.3+0.8}_{-0.0-0.4-0.7}$
$\bar{B}^0 \to K_1^-(1270)K^+$			$0.01^{+0.00+0.00+0.02}_{-0.00-0.00-0.01}$	$0.19^{+0.01+0.00+0.37}_{-0.01-0.00-0.09}$
$\bar{B}^0 \to K_1^+(1270)K^-$			$0.04^{+0.01+0.00+0.27}_{-0.01-0.00-0.04}$	$0.16^{+0.00+0.02+0.12}_{-0.02-0.03-0.06}$
$B^- \to K_1^0(1270)K^-$	0.22		$0.22^{+0.01+0.12+0.39}_{-0.01-0.07-0.12}$	$1.47^{+0.10+0.16+1.59}_{-0.06-0.10-0.58}$
$B^- \to K_1^-(1270)K^0$	0.75		$0.05^{+0.02+0.09+0.10}_{-0.01-0.03-0.04}$	$0.78^{+0.17+0.09+0.97}_{-0.13-0.08-0.19}$
$\bar{B}^0 \to \bar{K}_1^0(1270)K^0$	0.70		$2.10^{+0.13+1.23+1.31}_{-0.13-0.65-0.57}$	$0.46^{+0.13+0.07+0.17}_{-0.09-0.05-0.13}$
$\bar{B}^0 \to K_1^0(1270)\bar{K}^0$	0.20		$0.26^{+0.10+0.12+0.47}_{-0.01-0.08-0.17}$	$0.23^{+0.09+0.13+0.18}_{-0.06-0.08-0.16}$
$\bar{B}^0 \to K_1^-(1400)K^+$			$0.07^{+0.02+0.00+0.16}_{-0.02-0.00-0.06}$	$0.58^{+0.06+0.01+0.15}_{-0.06-0.01-0.13}$
$\bar{B}^0 \to K_1^+(1400)K^-$			$0.01^{+0.00+0.00+0.16}_{-0.02-0.00-0.06}$	$0.42^{+0.03+0.01+0.22}_{-0.02-0.00-0.16}$
$B^- \to K_1^0(1400)K^-$	0.12		$0.22^{+0.07+0.07+0.24}_{-0.07-0.07-0.13}$	$0.54^{+0.04+0.14+0.76}_{-0.02-0.11-0.13}$
$B^- \to K_1^-(1400)K^0$	3.9		$0.01^{+0+0.02+0.04}_{-0-0.00-0.00}$	$2.39^{+0.34+0.50+0.48}_{-0.25-0.39-0.48}$
$\bar{B}^0 \to \bar{K}_1^0(1400)K^0$	3.6		$0.10^{+0.02+0.21+0.15}_{-0.02-0.08-0.10}$	$2.24^{+0.36+0.40+0.59}_{-0.28-0.34-0.51}$
$\bar{B}^0 \to K_1^0(1400)\bar{K}^0$	0.11		$0.25^{+0.07+0.08+0.31}_{-0.07-0.07-0.15}$	$0.21^{+0.02+0.13+0.09}_{-0.01-0.07-0.07}$

TABLE IV. Direct CP violation (in units of %) for the decays $B \to K_{1A}\pi$, $K_{1B}\pi$ and $B \to K_{1A}K$, $K_{1B}K$. The errors for these entries correspond to the uncertainties from $\omega_B = 0.4 \pm 0.04$ GeV, the hard scale t varying from $0.8t$ to $1.2t$, and the Gegenbauer moment $a_1^{\perp}(K_{1A}) = -1.08 \pm 0.48$ for K_{1A} meson, $a_1^{\parallel}(K_{1B}) = -1.95 \pm 0.45$ for K_{1B} meson, respectively.

$\bar{B}^0 \to K_{1A}^- \pi^+$	$9.1^{+2.4+0.8+3.0}_{-2.0-0.8-3.4}$	$\bar{B}^0 \to K_{1B}^- \pi^+$	$-14.7^{+1.2+0.0+1.1}_{-1.4-0.2-1.6}$
$\bar{B}^0 \to \bar{K}_{1A}^0 \pi^0$	$-6.6^{+1.3+0.9+2.8}_{-1.4-1.0-8.4}$	$\bar{B}^0 \to \bar{K}_{1B}^0 \pi^0$	$-9.2^{+1.0+3.3+1.6}_{-0.7-3.5-1.9}$
$B^- \to \bar{K}_{1A}^0 \pi^-$	$-2.3^{+0.8+0.8+1.5}_{-1.2-0.6-6.8}$	$B^- \to \bar{K}_{1B}^0 \pi^-$	$3.3^{+0.1+0.6+1.9}_{-0.1-0.6-1.3}$
$B^- \to K_{1A}^- \pi^0$	$17.7^{+4.1+3.0+17.1}_{-3.5-3.1-7.4}$	$B^- \to K_{1B}^- \pi^0$	$3.4^{+1.2+0.0+0.0}_{-1.4-4.6-6.8}$
$\bar{B}^0 \to K_{1A}^- K^+$	$43.9^{+1.7+0.5+0.0}_{-1.3-3.1-35.6}$	$\bar{B}^0 \to K_{1B}^- K^+$	$-13.9^{+2.5+1.8+0.4}_{-2.6-2.0-0.4}$
$\bar{B}^0 \to K_{1A}^+ K^-$	$46.5^{+0.5+4.4+40.3}_{-1.3-3.3-29.5}$	$\bar{B}^0 \to K_{1B}^+ K^-$	$-3.3^{+1.1+6.8+1.6}_{-0.7-4.1-1.7}$
$B^- \to K_{1A}^0 K^-$	$6.6^{+1.6+3.1+4.9}_{-1.7-3.8-1.8}$	$B^- \to K_{1B}^0 K^-$	$-80.7^{+1.3+4.4+11.1}_{-1.7-3.5-2.9}$
$B^- \to K_{1A}^- K^0$	$-29.4^{+7.6+2.6+86.7}_{-6.3-1.8-0.0}$	$B^- \to K_{1B}^- K^0$	$0.8^{+2.7+0.4+4.0}_{-3.6-0.5-2.9}$

charged B decays are at or near the order of 10^{-6}, those of the pure annihilation decays are at the order of 10^{-7} by taking the mixing angle $\theta_{K_1} = 33°$.

In order to compare with other theoretical predictions, we also list the branching ratios with the mixing angle $\theta_{\bar{K}_1} = -58°$ shown in Table III. One can find that the branching ratios of the decays $B^- \to K_1^-(1270)K^0$, $\bar{B}^0 \to \bar{K}_1^0(1270)K^0$ have a remarkable decrease from the mixing angles $-33°$ to $-58°$, while those of the decays $B^- \to K_1^-(1400)K^0$, $\bar{B}^0 \to \bar{K}_1^0(1400)K^0$ have a remarkable increase.

Now we turn to the evaluations of the CP-violating asymmetries in the PQCD approach. For the neutral $\bar{B}^0$ (the charged B^-) decays the direct CP-violating asymmetries can be defined as

$$\mathcal{A}_{CP}^{\rm dir} = \frac{\Gamma(\bar{B}^0(B^-) \to f) - \Gamma(B^0(B^+) \to \bar{f})}{\Gamma(\bar{B}^0(B^-) \to f) + \Gamma(B^0(B^+) \to \bar{f})} = \frac{2z \sin\theta \sin\delta}{(1 + 2z\cos\theta\cos\delta + z^2)}, \quad (27)$$

where δ is the relative strong phase between the tree and penguin amplitudes, and θ the CKM weak phase $\theta = \alpha$ for $b \to d$ transition, $\theta = \gamma$ for $b \to s$ transition. Certainly, if the final states are the same for B^0 and $\bar{B}^0$, that is $f = \bar{f}$, the CP-asymmetries may be time-dependent, including not only the direct CP violation but also the mixing-induced CP violation. Using the input parameters and the wave functions as specified in this section and Sec. II, it is easy to get the PQCD predictions (in units of 10^{-2}) for the direct CP-violating asymmetries of B decaying to each flavor final state, which are listed in Table IV. For the real physical final states, which are mixes of the corresponding flavor states, their direct CP-violating asymmetries will be dependent on the mixing angle $\theta_{\bar{K}_1}$. As has been emphasised before, $\theta_{\bar{K}_1}$ for the antiparticle states $\bar{K}_1(1270)$, $\bar{K}_1(1400)$ is of opposite sign to that for the particle states $K_1(1270)$, $K_1(1400)$. For taking the convention of decay constant $f_{K_{1B}}$ in this work, so θ_{K_1} is positive and $\theta_{\bar{K}_1}$ is negative. In Figs. 2–4, we give the dependence of the direct CP-violating asymmetries on the mixing angle $\theta_{\bar{K}_1}$ for each decay. Here taking $\theta_{\bar{K}_1} = -33°$ or $\theta_{\bar{K}_1} = -58°$, we can read each direct CP-violating asymmetry from these figures.

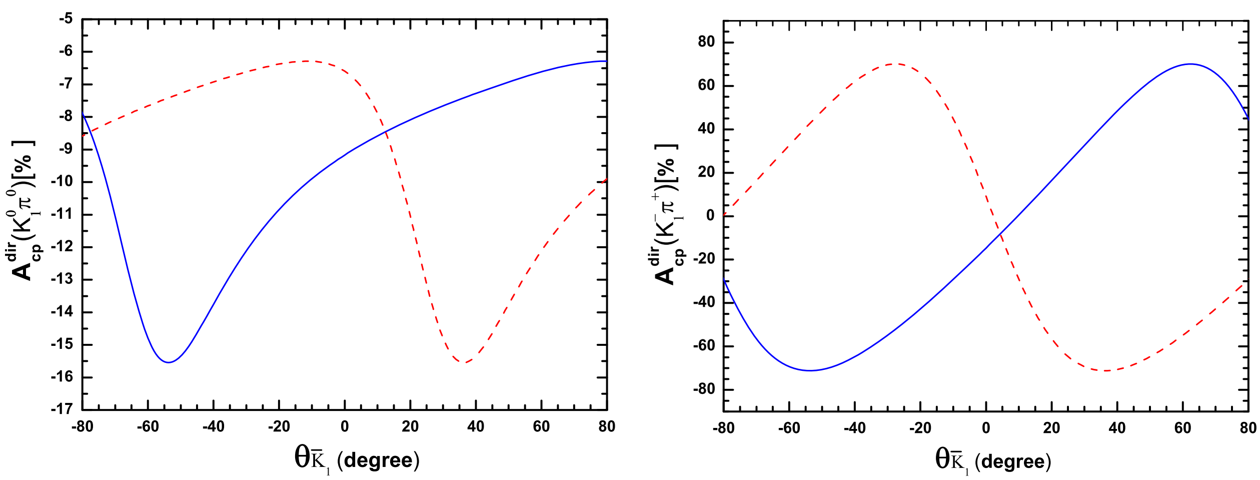

FIG. 2 (color online). The dependence of the direct CP-violating asymmetries on the mixing angle $\theta_{\bar{K}_1}$: the solid lines represent the decays $\bar{B}^0 \to K_1(1270)^0\pi^0$ (left), $\bar{B}^0 \to K_1(1270)^-\pi^+$ (right), and the dashed lines are for the decays $\bar{B}^0 \to K_1(1400)^0\pi^0$ (left), $\bar{B}^0 \to K_1(1400)^-\pi^+$ (right), respectively.

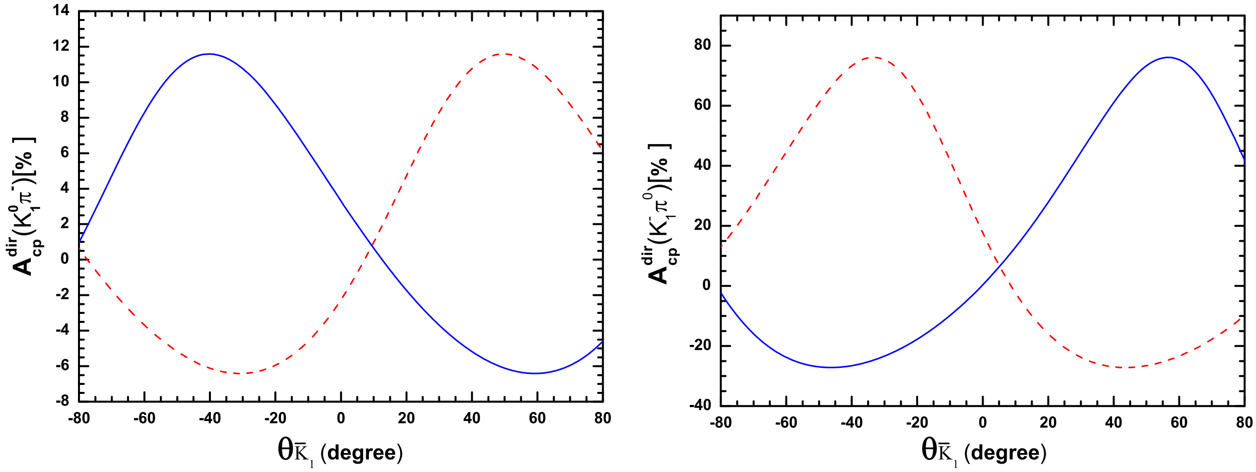

FIG. 3 (color online). The dependence of the direct CP-violating asymmetries on the mixing angle $\theta_{\bar{K}_1}$: the solid lines represent the decays $B^- \to K_1(1270)^0\pi^-$ (left), $B^- \to K_1(1270)^-\pi^0$ (right), and the dashed lines are for the decays $B^- \to K_1(1400)^0\pi^-$ (left), $B^- \to K_1(1400)^-\pi^0$ (right), respectively.

It is noticed that for the decays $\bar{B}^0 \to K_1(1270)^+K^-$, $K_1(1400)^+K^-$, $B^- \to K_1(1270)^0K^-$, $K_1(1400)^0K^-$, which include the particle states, their direct CP-violating asymmetry values are still read at $-33°$ or $-58°$ for $\theta_{K_1} = -\theta_{\bar{K}_1}$ and so the corresponding mixing angle is positive. The signs of the direct CP-violating asymmetries of $B \to K_1(1270)K(\pi)$ and $B \to K_1(1400)K(\pi)$ are opposite at the mixing angle $\theta_{\bar{K}_1} = -33°$ for most of these decays except only two groups, whose direct CP-violating asymmetries are predicted as $\mathcal{A}_{CP}^{\rm dir}(\bar{B}^0 \to \bar{K}_1(1270)^0\pi^0) = -12.6\%$, $\mathcal{A}_{CP}^{\rm dir}(\bar{B}^0 \to \bar{K}_1(1400)^0\pi^0) = -6.7\%$ and $\mathcal{A}_{CP}^{\rm dir}(\bar{B}^0 \to K_1(1270)^+K^-) = 12.2\%$, $\mathcal{A}_{CP}^{\rm dir}(\bar{B}^0 \to K_1(1400)^+K^-) = 9.6\%$, respectively. From Table IV, one can find that the direct CP-violating asymmetries of each decay $B \to K_{1A}\pi$, $K_{1B}\pi$ are not large, while those for some real physical final states become very large. For example, the direct CP-violating asymmetries of the decays $\bar{B}^0 \to K_1(1270)^-\pi^+$, $K_1(1400)^-\pi^+$ are about -58.1% and 68.4% at the mixing angle $-33°$, respectively. Certainly, we only learn phenomenally about the mixing angle θ_{K_1} at present and have no accurate calculations or measurements. Furthermore, the direct CP-violating asymmetries are sensitive to the mixing angle. It is much more complex for some considered decays where the nonperturbative contributions, such as charming penguins, give large corrections, and the corresponding direct CP-violating asymmetries may also change. So we cannot confirm that these decays must have so large direct CP-violating asymmetries. As for the decays $\bar{B}^0 \to \bar{K}_1(1270)^0K^0$, $\bar{K}_1(1400)^0K^0$, there is no tree contribution at the leading order, so the direct CP-violating asymmetry is naturally zero.

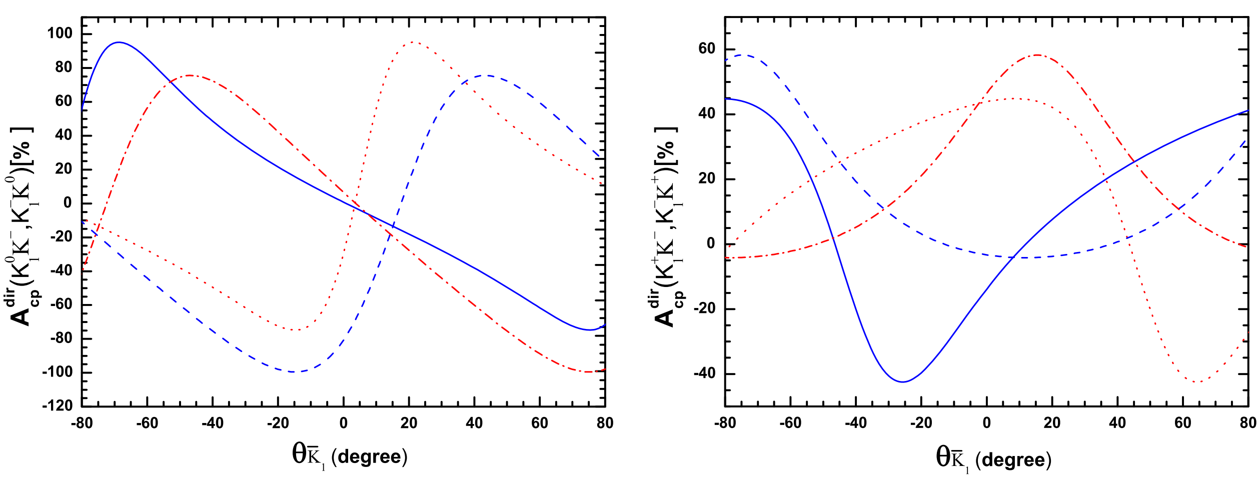

FIG. 4 (color online). The dependence of the direct CP-violating asymmetries on the mixing angle $\theta_{\bar{K}_1}$: the solid lines represent the decays $B^- \to K_1(1270)^-K^0$ (left), $\bar{B}^0 \to K_1(1270)^-K^+$ (right), the dashed lines are for the decays $B^- \to K_1(1270)^0K^-$ (left), $\bar{B}^0 \to K_1(1270)^+K^-$ (right), the dot lines are for the decays $B^- \to K_1(1400)^-K^0$ (left), $B^- \to K_1(1400)^-K^+$ (right), and the dash-dot lines represent the decays $B^- \to K_1(1400)^0K^-$ (left), $\bar{B}^0 \to K_1(1400)^+K^-$ (right), respectively.

V. CONCLUSION

In this paper, by using the decay constants and the light-cone distribution amplitudes derived from the QCD sum-rule method, we research the decays $B \to K_1(1270)\pi(K)$, $K_1(1400)\pi(K)$ in the PQCD approach and find that

(i) All the theoretical predictions for the branching ratios of the decays $\bar{B}^0 \to K_1(1270)^+\pi^-$, $K_1(1400)^+\pi^-$ are incompatible with the present experimental data. There exists the similar situation for the decays $\bar{B}^0 \to a_1(1260)^+K^-$, $b_1(1235)^+K^-$, where the nonperturbative contributions, such as the final state interactions or the charming penguins, are needed to explain the data. But the difference is that the nonperturbative contributions seem to play opposite roles in these two groups of decays. If the future data are really larger than the present predictions for some considered decays, it might indicate that the nonperturbative contributions have pronounced corrections for some decay channels which include the higher resonances in the final states.

(ii) The pure annihilation type decays $\bar{B}^0 \to K_1^{\pm}(1270)K^{\mp}$, $K_1^{\pm}(1400)K^{\mp}$ are good channels to test whether an approach can be used to calculate correctly the strength of the penguin-annihilation amplitudes. Their branching ratios are predicted at 10^{-7} order.

(iii) In the four final neutral flavor states $K^0_{1A}\bar{K}^0$, $K^0_{1B}\bar{K}^0$, $\bar{K}^0_{1A}K^0$, $\bar{K}^0_{1B}K^0$, the decay $\bar{B}^0 \to \bar{K}^0_{1B}K^0$ have the largest branching ratio which is of 10^{-6} order, while the other decays with the branching ratios at 10^{-7} order. So the decays $\bar{B}^0 \to \bar{K}_1(1200)^0K^0$, $\bar{K}_1(1400)K^0$ which include the real physical states can have large branching ratios at the mixing angle $\theta_{\bar{K}_1} = -33°$ compare with the decays $\bar{B}^0 \to K_1(1200)^0\bar{K}^0$, $K_1(1400)\bar{K}^0$.

(iv) The signs of the direct CP-violating asymmetries are opposite between almost of the decays $B \to K_1(1270)K(\pi)$ and $B \to K_1(1400)K(\pi)$ at mixing angle $\theta_{K_1} = -33°$ except only two groups, whose direct CP-violating asymmetries are predicted as $\mathcal{A}^{\rm dir}_{CP}(\bar{B}^0 \to \bar{K}_1(1270)^0\pi^0) = -12.6\%$, $\mathcal{A}^{\rm dir}_{CP}(\bar{B}^0 \to \bar{K}_1(1400)^0\pi^0) = -6.7\%$ and $\mathcal{A}^{\rm dir}_{CP}(\bar{B}^0 \to K_1(1270)^+K^-) = 12.2\%$, $\mathcal{A}^{\rm dir}_{CP}(\bar{B}^0 \to K_1(1400)^+K^-) = 9.6\%$, respectively.

(v) The strong phase introduced by the nonperturbative contributions might produce dramatic effects on some of the considered decays, such as $\bar{B}^0 \to K_1(1270)^-\pi^+$, $K_1(1400)^-\pi^+$, $K_1(1270)^-\pi^0$, $K_1(1270)^-\pi^0$, and these effects could exceed those from the parametric uncertainties in the case of the CP asymmetries.

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APPENDIX: ANALYTIC FORMULAS FOR THE DECAY AMPLITUDES

$$
\begin{aligned}
A(K_1(1270)^0\bar{K}^0)
&= -\xi_t(f_{K_{1A}}\sin\theta_{K_1} + f_{K_{1B}}\cos\theta_{K_1})F^{LL}_{eK}\left(a_4 - \frac{1}{2}a_{10}\right) - \xi_t(M^{LL;K_{1A}}_{eK}\sin\theta_{K_1} + M^{LL;K_{1B}}_{eK}\cos\theta_{K_1})\left(C_3 - \frac{1}{2}C_9\right)\\
&\quad - \xi_t(M^{LR;K_{1A}}_{eK}\sin\theta_{K_1} + M^{LR;K_{1B}}_{eK}\cos\theta_{K_1})\left(C_5 - \frac{1}{2}C_7\right) - \xi_t(M^{LL;K_{1A}}_{aK}\sin\theta_{K_1} + M^{LL;K_{1B}}_{aK}\cos\theta_{K_1})\left(C_3 - \frac{1}{2}C_9\right)\\
&\quad - \xi_t(M^{LL;K_{1A}}_{aK}\sin\theta_{K_1} + M^{LL;K_{1B}}_{aK}\cos\theta_{K_1})\left(C_4 - \frac{1}{2}C_{10}\right) - \xi_t(M^{LR;K_{1A}}_{aK}\sin\theta_{K_1} + M^{LR;K_{1B}}_{aK}\cos\theta_{K_1})\left(C_5 - \frac{1}{2}C_7\right)\\
&\quad - \xi_t(M^{SP;K_{1A}}_{aK}\sin\theta_{K_1} + M^{SP;K_{1B}}_{aK}\cos\theta_{K_1})\left(C_6 - \frac{1}{2}C_8\right) - \xi_t f_B(F^{LL;K_{1A}}_{aK}\sin\theta_{K_1} + F^{LL;K_{1B}}_{aK}\cos\theta_{K_1})\left(a_3 - \frac{1}{2}a_9\right)\\
&\quad - \xi_t f_B(F^{LL;K_{1A}}_{aK}\sin\theta_{K_1} + F^{LL;K_{1B}}_{aK}\cos\theta_{K_1})\left(a_4 - \frac{1}{2}a_{10}\right) - \xi_t f_B(F^{LL;K_{1A}}_{aK}\sin\theta_{K_1} + F^{LL;K_{1B}}_{aK}\cos\theta_{K_1})\left(a_5 - \frac{1}{2}a_7\right)\\
&\quad - \xi_t f_B(F^{SP;K_{1A}}_{aK}\sin\theta_{K_1} + F^{SP;K_{1B}}_{aK}\cos\theta_{K_1})\left(a_6 - \frac{1}{2}a_8\right) - \xi_t(M^{LL;K}_{aK_{1A}}\sin\theta_{K_1} + M^{LL;K}_{aK_{1B}}\cos\theta_{K_1})\left(C_4 - \frac{1}{2}C_{10}\right) \qquad \text{(A1)}
\end{aligned}
$$

$$- \xi_t (M^{SP;K}_{aK_{1A}} \sin\theta_{K_1} + M^{SP;K}_{aK_{1B}} \cos\theta_{K_1})\left(C_6 - \frac{1}{2}C_8\right) - \xi_t f_B (F^{LL;K}_{aK_{1A}} \sin\theta_{K_1} + F^{LL;K}_{aK_{1B}} \cos\theta_{K_1})\left(a_3 - \frac{1}{2}a_9\right)$$
$$- \xi_t f_B (F^{LL;K}_{aK_{1A}} \sin\theta_{K_1} + F^{LL;K}_{aK_{1B}} \cos\theta_{K_1})\left(a_5 - \frac{1}{2}a_7\right), \quad \text{(A1)}$$

$$A(K_1(1270)^0K^-)$$
$$= -\xi_t (f_{K_{1A}} \sin\theta_{K_1} + f_{K_{1B}} \cos\theta_{K_1}) F^{LL}_{eK}\left(a_4 - \frac{1}{2}a_{10}\right) - \xi_t (M^{LL;K_{1A}}_{eK} \sin\theta_{K_1} + M^{LL;K_{1B}}_{eK} \cos\theta_{K_1})\left(C_3 - \frac{1}{2}C_9\right)$$
$$- \xi_t (M^{LR;K_{1A}}_{eK} \sin\theta_{K_1} + M^{LR;K_{1B}}_{eK} \cos\theta_{K_1})\left(C_5 - \frac{1}{2}C_7\right) + (M^{LL;K_{1A}}_{aK} \sin\theta_{K_1} + M^{LL;K_{1B}}_{aK} \cos\theta_{K_1})(\xi_u C_1 - \xi_t(C_3 + C_9))$$
$$- \xi_t (M^{LR;K_{1A}}_{aK} \sin\theta_{K_1} + M^{LR;K_{1B}}_{aK} \cos\theta_{K_1})(C_5 + C_7) + f_B (F^{LL;K_{1A}}_{aK} \sin\theta_{K_1} + F^{LL;K_{1B}}_{aK} \cos\theta_{K_1})(\xi_u a_2 - \xi_t(a_4 + a_{10})$$
$$- \xi_t f_B (F^{SP;K_{1A}}_{aK} \sin\theta_{K_1} + F^{SP;K_{1B}}_{aK} \cos\theta_{K_1})(a_6 + a_8). \quad \text{(A2)}$$

In the upper two formulas, if changing the first term as $-\xi_t f_K (F^{LL}_{eK_{1A}} \sin\theta_{K_1} + F^{LL}_{eK_{1B}} \cos\theta_{K_1})(a_4 - \frac{1}{2}a_{10})) - \xi_t f_K (F^{SP}_{eK_{1A}} \sin\theta_{K_1} + F^{SP}_{eK_{1B}} \cos\theta_{K_1})(a_6 - \frac{1}{2}a_8)$, and at the same time exchanging the positions of $K_{1A}(K_{1B})$ and K in other terms, we will get the decay amplitudes of $\bar{B}^0 \to \bar{K}_1(1270)^0 K^0$ and $B^- \to K_1(1270)^- K^0$, respectively.

$$A(K_1(1270)^+K^-) = (M^{LL;K_{1A}}_{aK} \sin\theta_{K_1} + M^{LL;K_{1B}}_{aK} \cos\theta_{K_1})(\xi_u C_2 - \xi_t(C_4 + C_{10}))$$
$$- \xi_t (M^{SP;K_{1A}}_{aK} \sin\theta_{K_1} + M^{SP;K_{1B}}_{aK} \cos\theta_{K_1})(C_6 + C_8)$$
$$+ f_B (F^{LL;K_{1A}}_{aK} \sin\theta_{K_1} + F^{LL;K_{1B}}_{aK} \cos\theta_{K_1})(\xi_u a_1 - \xi_t(a_3 + a_5 + a_7 + a_9))$$
$$- \xi_t f_B (F^{LL;K_{1A}}_{aK} \sin\theta_{K_1} + F^{LL;K_{1B}}_{aK} \cos\theta_{K_1})\left(a_3 + a_5 - \frac{1}{2}a_7 - \frac{1}{2}a_9\right)$$
$$- \xi_t (M^{LL;K}_{aK_{1A}} \sin\theta_{K_1} + M^{LL;K}_{aK_{1B}} \cos\theta_{K_1})\left(C_4 - \frac{1}{2}C_{10}\right)$$
$$- \xi_t (M^{SP;K}_{aK_{1A}} \sin\theta_{K_1} + M^{SP;K}_{aK_{1B}} \cos\theta_{K_1})\left(C_6 - \frac{1}{2}C_8\right). \quad \text{(A3)}$$

In Eq. (A3), if exchanging the positions of $K_{1A}(K_{1B})$ and K, we will get the total amplitude of the decay $\bar{B}^0 \to K_1(1270)^- K^+$. The total amplitudes of the decays $B \to K_1(1400)K$ can be obtained by making the replacements with $\sin\theta_{K_1} \to \cos\theta_{K_1}$, $\cos\theta_{K_1} \to -\sin\theta_{K_1}$ in Eqs. (A1)–(A3), respectively.

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