PHYSICAL REVIEW D **90,** 074017 (2014)

Effective confining potentials for QCD

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(Received 6 April 2014; published 15 October 2014)

We observe that the linear potential used as a leading approximation for describing color confinement in the instant form of dynamics corresponds to a quadratic confining potential in the front form of dynamics. In particular, the instant-form potentials obtained from lattice gauge theory and string models of hadrons agree with the potentials determined from models using front-form dynamics and light-front holography, not only in their shape, but also in their numerical strength.

DOI: 10.1103/PhysRevD.90.074017 PACS numbers: 12.38.Aw, 11.15.Kc, 12.39.-x, 12.39.Pn

I. INTRODUCTION

A key question in QCD is to understand the nonperturbative dynamics underlying the confinement of quarks and gluons [1,2] from first principles. Various approaches to nonperturbative QCD such as lattice gauge theory, AdS/QCD and string theory appear to describe color confinement in very different ways. In this paper we will show that despite their apparent analytic differences, these approaches have essential elements in common if one takes into account the fact that the shape of the confinement potential depends on the form of dynamics; e.g., the instant form (IF) versus the front form (FF) [3], the latter called also the light-front (LF) dynamics in the literature.

Nonrelativistic analyses such as heavy quark effective theory are based on the commonly used IF dynamics where the Hamiltonian is the usual time evolution operator. Relativistic bound-state problems such as confinement of light quarks are usually formulated in the FF Hamiltonian dynamic framework, since it provides a rigorous frame-independent formalism. In this case, the LF Hamiltonian is the time-evolution operator $H_{\rm LF} = i\frac{\partial}{\partial\tau}$ where $\tau = (ct + z)/c$.

It is important to note that the form of the effective potential in each formalism depends on the form of the dynamics which is utilized. In this paper we will compare the physical descriptions, their effective potentials, and the mass scales controlling quark confinement obtained from lattice, string theory, and the FF approach based on LF holography. A crucial observation is that a linear confining potential in the IF of dynamics agrees with a quadratic confining potential in the FF of dynamics at leading approximation. One thus obtains a common element of quantum-mechanical effective theories which incorporates color confinement, relativity, and essential spectroscopic and dynamical features of hadron physics.

An important tool will be the Wentzel-Kramers-Brillouin (WKB) [4] formalism which allows one to relate the maximum distance of separation between quarks within a meson as predicted by each model. We find that this parameter appears to be universal even among different forms of dynamics. It thus provides a universal point of focus for describing the same phenomenon of color confinement in different approaches.

We begin by recalling that the IF of the nonrelativistic Schrödinger operator for a system made of two strongly interacting particles of identical mass m and with momenta $\vec{p}_q = \vec{p}$ and $\vec{p}_{\bar{q}} = -\vec{p}$, such as J/ψ, Υ or other mesons, is

$$\mathcal{M} = 2m + \frac{\vec{p}^2}{m} + V_{\rm eff}, \tag{1}$$

where $2m + \frac{\vec{p}^2}{m}$ originates from $2\sqrt{m^2 + \vec{p}^2}$. The eigenvalue of $\mathcal{M}$ is the mass of the system. In contrast, the FF formulation of the theory of interacting particles is applicable to nonrelativistic as well as relativistic constituents. It leads to an effective eigenvalue equation for the mass squared operator

$$\mathcal{M}^2 = \frac{k_\perp^2 + m^2}{x(1-x)} + U_{\rm eff}, \tag{2}$$

instead of $\mathcal{M}$ in the IF of dynamics, Eq. (1). The boost-invariant FF variables x and $1-x$ are ratios of longitudinal FF momenta $p_q^+ = p_q^t + p_q^z$ and $p_{\bar{q}}^+ = p_{\bar{q}}^t + p_{\bar{q}}^z$ of the constituents to the longitudinal FF momentum of the meson, $P^+ = P^t + P^z$. The term $\frac{k_\perp^2 + m^2}{x(1-x)}$ is the LF kinetic energy as well as the invariant mass squared $s = (p_q + p_{\bar{q}})^2$ of the $q\bar{q}$ pair.

It will be convenient to define a relative three-vector momentum operator $\vec{p}$ (in the constituent rest frame [5–7]), so that

$$\mathcal{M}^2 = \frac{k_\perp^2 + m^2}{x(1-x)} + U_{\mathrm{eff}} \equiv 4m^2 + 4\vec{p}^2 + U_{\mathrm{eff}}. \qquad (3)$$

We identify $p_\perp^2 = \frac{k_\perp^2}{4x(1-x)}$ and $4m^2 + 4p_3^2 = \frac{m^2}{x(1-x)}$, so $p_3 = \frac{m}{\sqrt{x(1-x)}}(x - \frac{1}{2})$ has an infinite range and is proportional to m. The conjugate variables are $r_\perp = i\frac{\partial}{\partial p_\perp}$ and $r_3 = i\frac{\partial}{\partial p_3}$. Early discussions of models of mesons as two-body systems in the FF dynamics, as alternative to the IF, especially in the infinite momentum frame, can be found in Refs. [8–13].

The central problem then becomes the derivation of the effective interactions V_{eff} and U_{eff}. We observe that nearly all considerations in the IF of the Hamiltonian dynamics lead to the conclusion that the potential between a quark and antiquark at large distances should be linear. This article points out that the linear IF potential V_{eff} implies a quadratic FF potential U_{eff} at large $q\bar{q}$ separation, as implied by Eq. (1), which is the lowest-order approximation, and Eq. (3),

$$U_{\mathrm{eff}} = V_{\mathrm{eff}}^2 + 2\sqrt{\vec{p}^2 + m^2}V_{\mathrm{eff}} + 2V_{\mathrm{eff}}\sqrt{\vec{p}^2 + m^2}. \qquad (4)$$

At large distances, near turning points where kinetic energy is minimal, the potential term V_{eff}^2 dominates the right-hand side. Thus, for a linear IF potential V_{eff}, the FF potential U_{eff} is quadratic. Such FF harmonic oscillator potential predicts linear Regge trajectories [14,15] in the hadron mass square for small quark masses.

In the next sections, various models defined in different forms of dynamics will be discussed. In the last section we will compare the models in terms of their WKB parameter.

II. INSTANT-FORM APPROACHES

The main contemporary tool for studying mesons in the IF of dynamics is lattice QCD, originally formulated in Ref. [16]. One obtains numerical results for hadron properties from calculating their Euclidean propagators [17]. The underlying dynamics can be studied in terms of a potential by calculating the Wilson loops, where quarks are represented by static color sources [18,19]. We focus first on methods which allow one to compute the shape and mass scale of the nonrelativistic potential which confines pairs of infinitely heavy quarks [20–22]. The lattice approach is closely related to the string picture for hadrons (see below).

A. Specific lattice-potential results

The static potential obtained in the quenched approximation of the lattice QCD can be parametrized in the form of the Cornell potential [23]; i.e. (up to a constant term)

$$V_{\mathrm{eff}}^{(\mathrm{lattice})}(r) = -\frac{A}{r} + \sigma r, \qquad (5)$$

where r denotes the distance between infinitely heavy (static) quark and antiquark and σ is called string tension. The string tension due to the gluonic fields connecting static color sources does not include the pair creation mechanism that breaks the string; there is thus no direct relation of the two-body effective potential for QCD to this aspect of the string tension.

The progress made in simulations of QCD on the lattice allows one to calculate coefficients A and σ also for quarks with finite masses. For instance, one of the most recent analyses of charmonium [24,25] found the square root of string tension of magnitude $\sqrt{\sigma} = 394(7)$ MeV, associated with the quark mass 1.74(3) GeV. Despite the fact that our discussion concerns quarks with the phenomenological values of masses which may be different from the charmonium result of 1.74 GeV, the value of $\sqrt{\sigma} = 394$ MeV appears appropriate for our purpose of estimating the behavior of the quark-antiquark effective potential in the configuration where the potential dominates the meson energy. However, it should be mentioned that in the case of static sources the value of $\sqrt{\sigma} \sim 460$ MeV is obtained [26–28]. Lattice estimates for the universal quark-antiquark potentials should be based on calculations for quarks with finite effective mass parameters.

B. Classical string model

An effective description of quark confinement in mesons is the string model for hadrons, where color-electric fields between two static color sources are squeezed into a thin, effectively one-dimensional, flux tube or vortex [29–32]. The string picture of confinement can be considered [18] as the strong coupling limit of the IF Hamiltonian formulation of lattice QCD.

One can study the spectra of multidimension string models [33–36] such as strings described by the Nambu-Goto action [37,38]. This approach yields a string with a constant energy density per unit length and a static potential which rises linearly as a function of the string length r. In the 4-dimensional space-time, the quark-antiquark potential is thus given (up to a constant term) by [35,39]

$$V_{\mathrm{eff}}^{(\mathrm{string})}(r) = \sigma r\sqrt{1 - \frac{\pi}{6\sigma r^2}}. \qquad (6)$$

From this, one can calculate the dependence of the meson spectrum on the internal angular momentum. By comparing with the empirical Regge trajectories, one finds a slope parameter 470 MeV $< \sqrt{\sigma} <$ 480 MeV for pseudoscalars (π and K), while for other mesons the value of $\sqrt{\sigma}$ varies between 424 and 437 MeV [40]. The string description applies for distances $r \gg r_c = \sqrt{\pi/(6\sigma)}$, and $r_c \simeq$ 0.33 fm for $\sqrt{\sigma} \simeq 430$ MeV. Most of the above results point toward the value about 430 MeV with an ambiguity on the order of 7 MeV. A review of the lattice and the string theories can be found in Refs. [40,41].

C. Stochastic vacuum model

In the stochastic vacuum model (SVM), string formation is a property of the gauge-invariant gluon field-strength correlator, which can be obtained by lattice simulations. It thus connects the lattice with the hadronic string picture [42,43].

The SVM [42] starts with the assumption that the nonperturbative (long-distance) part of the functional integral over the gluon field can be approximated by a Gaussian integration. Wilson loops can be evaluated easily and are determined by the gauge-invariant correlator of the gluon fields; for large loops one derives an area law signifying linear confinement. The resulting nonrelativistic potential begins quadratically and becomes linear at distances comparable to the correlation length of the gluon field. The confinement mechanism is due to the formation of a color-electric string between the quark and antiquark [43]. The string tension is given by [42,43]

$$\sigma = \frac{\pi}{48N_c}\int_0^\infty dz^2 D(z^2), \tag{7}$$

where $D(z^2)$ is the scalar part of the gauge invariant color field correlator $\langle G_{\mu\nu}(z)\Phi(z)G_{\rho\sigma}(0)\rangle$ and $\Phi(z)$ is the color transporter from point 0 to z. $D(z^2)$ can be calculated on the lattice using the cooling method [44]. Using the numerical results of this lattice simulation [44] one obtains for the string tension $\sqrt{\sigma} = 410(11)$ MeV.

III. FRONT-FORM APPROACHES

The lattice gauge theories are not effectively formulated using the FF of dynamics because of difficulties with understanding what to do in the Minkowski space. High-energy experiments require an efficient IF description in the infinite momentum frame or, in a frame-independent way, description using the FF. Since there is no efficient lattice description that could be used, one turns to the FF Hamiltonian methods.

The derivation of the FF QCD Hamiltonian eigenvalue equation that accounts for dynamical effects of all virtual quarks and gluons present in the Fock-space expansion of a hadron state, requires a suitable renormalization group procedure. We focus on the procedure called the similarity renormalization group procedure [45–47], and to its successors, especially the renormalization group procedure for effective particles (RGPEP, see below).

The potential $U_{\rm eff}$ in the FF effective Hamiltonian, Eq. (2), can be found by applying the Ehrenfest principle [48] to quantum field theory [49], in the sense of calculating expectation values which average quantities of interest over all Fock sectors and all effective constituents in them, except for the constituent that is struck by an external probe, called the active one. In every Fock sector, the active constituent moves in an effective potential generated by the remaining constituents, called spectators. Following this line of reasoning, the resulting potential describes the motion of an active constituent around the minimum of its potential energy. Such a potential is expected to be quadratic, $U_{\rm eff}(r) \sim r^2$, as every regular function around its minimum is. Both the Ehrenfest equation and the quadratic potential agree with the requirement of rotational symmetry because all Fock sectors in the bound-state dynamics are included, cf., [50–52]; i.e. multiplets of the spectrum have the mass degeneracy required by the rotational symmetry in 3-dimensions. The quadratic form of the FF Ehrenfest potential around its minimum agrees well with the large-r result that $U_{\rm eff}(r) \sim r^2$ when $V_{\rm eff}(r) \sim r$, and with results of the LF holography. This will be explained after we discuss the LF holography.

A. LF holography

One can write the FF equation of motion for mesons in the form of a single-variable relativisitic eigenvalue equation analogous to the nonrelativistic quark-antiquark radial Schrödinger equation [53]. The same equation for massless quarks arises from the LF holographic mapping [54,55] of the soft-wall model modification of AdS_5 space [56] with any dilaton profile which breaks the maximal symmetry of AdS_5 space. Thus one arrives at a meson equation of motion for zero quark mass, where the fifth-dimension variable z in AdS_5 becomes identified with the boost-invariant transverse $q\bar{q}$ separation variable ζ. One has $\zeta^2 = \frac{1}{4}r_\perp^2 = x(1-x)b_\perp^2$, where $b_\perp = i\frac{\partial}{\partial k_\perp}$ is the transverse distance between the two constituents [57]. The resulting single-variable relativistic equation of motion includes a harmonic oscillator potential

$$U_{\rm eff}^{\rm (LF)}(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1), \tag{8}$$

where J is the total angular momentum of the $q\bar{q}$ meson. The LF-holography is inspired by Maldacena conjecture [58]; it does not require that the number of colors is large.

It has been shown that the harmonic oscillator form of the FF potential arises uniquely when one extends the formalism of de Alfaro, Fubini, and Furlan [59] to the FF Hamiltonian theory [60]. The action of the effective one-dimensional quantum field theory remains conformally invariant, which reflects the underlying conformal invariance of the classical QCD chiral Lagrangian. The constant term $2\kappa^2(J-1)$ is derived from spin-J representations of dynamics in AdS space [55].

The mass parameter κ is determined outside of QCD from a single observable, such as the pion decay constant. One finds consistency with hadron spectroscopy for κ between 540 and 590 MeV.

It is natural to replace $\kappa^4\zeta^2 = \frac{1}{4}\kappa^4 r_\perp^2$ in Eq. (8) by $\frac{1}{4}\kappa^4(r_\perp^2 + r_3^2)$ in the case of massive quarks [61], where the quark masses are input parameters. Then $U_{\rm eff}^{(\rm LF)}$ becomes a 3-dimensional oscillator potential. The corresponding wave function matches phenomenology, e.g. see Ref. [62]. Thus, excitations in the transverse plane are paired with excitations in the 3-direction, and 3-dimensional rotation symmetry is restored in the massive case. This change also establishes connection with the 3-dimensional Eq. (3), and it does not require any change of the value of κ. The same universal value of κ is also obtained when short-range spin-dependent interactions are included [63,64]. It would be interesting to extend these results retaining 3-dimensional rotation symmetry.

B. Gluon condensate model embedded in the RGPEP

A framework based on the RGPEP [61] allows one to develop a relativistic quark model inspired by [65], where the effective particle masses are different from zero and can be set as the input parameters. The FF potential is quadratic as a function of a 3-dimensional quark-antiquark distance r,

$$U_{\rm eff}^{(\rm RGPEP)}(r) = \left(\frac{\pi}{3}\varphi_{\rm glue}\right)^2 r^2, \qquad (9)$$

where $\varphi_{\rm glue}^2$ represents the gluon condensate inside hadrons. In the operator product expansion [66] the expectation value corresponding to gluon condensate can also refer to matrix elements inside hadrons rather than the vacuum [67–73].

The original value of $\varphi_{\rm glue}^2 = 0.012\ {\rm GeV}^4$ obtained by Shifman, Vainshtein, and Zakharov [74] has been updated by Narison [75,76], which in the case of in-hadron condensate implies (see Eq. (20) in Ref. [76])

$$\varphi_{\rm glue}^2 = \frac{1}{\pi}\frac{\langle G|\alpha_s G^{\mu\nu c}G_{\mu\nu}^c|G\rangle}{\langle G|G\rangle} \simeq 0.022(4)\ {\rm GeV}^4, \qquad (10)$$

where α_s is the QCD coupling constant, and $|G\rangle$ represents the gluons condensed inside a meson.

IV. DISCUSSION

In the IF models, the confinement potential increases linearly at large distances between static quarks as exemplified in Eqs. (5)–(7). Other terms contribute at small distances. The eigenvalues of the IF Hamiltonian are the energies of the hadrons. In contrast, the eigenvalues of the Hamiltonian in the frame-independent FF of dynamics is quadratic in the hadron mass M: Eqs. (8) and (9). Note that $M^2 = (2m+\epsilon)^2 = 4m^2 + 4m\epsilon + \epsilon^2$ where ϵ is the binding energy. It is essential to retain the ϵ^2-term in the FF eigenvalue equation, Eq. (2), since the ϵ^2-term contributes to the FF potential $U_{\rm eff}$ even if m is large, see Eq. (3). Thus, if the IF potential is linear, then the FF potential in at a large distance between quarks should be quadratic [49,77]. This can be seen straightforwardly in the cases where the mass of constituents m tends to zero.

In order to compare different descriptions of confinement we can adopt the WKB method. It defines the turning point $r_{\rm max}$ where the kinetic energy is completely turned into potential energy. One obtains $M = 2m + \sigma r_{\rm max}$ in the lattice, string and SVM approach, $M^2 = 4m^2 + (\frac{\pi}{3}\varphi_{\rm glue})^2 r_{\rm max}^2$ in the RGPEP approach, and $M^2 = 4m^2 + \frac{1}{4}\kappa^4 r_{\rm max}^2$ in the LF-holography approach. The last factor $\frac{1}{4}$ comes from the fact that $x = \frac{1}{2}$ at the WKB turning point, where $p_\perp$ and p_3 both vanish.

Figure 1 compares the phenomenological results for the coefficient of $r_{\rm max}$. The values for the effective confinement scales derived from the WKB analysis in each model discussed above are sufficiently close to each other that one can argue that the various confinement models describe the same effective two-body system in the IF and in the FF of dynamics. There are different scales of energy in QCD, determined by quantities such as masses of quarks, $\Lambda_{\rm QCD}$, both possibly multiplied by some powers of $\alpha_{\rm QCD}$. Nevertheless, the values of parameters quoted here are of the same order. Finally, we wish to stress that the linear confining potential of the IF of dynamics is consistent with the quadratic confining potential in the FF of dynamics.

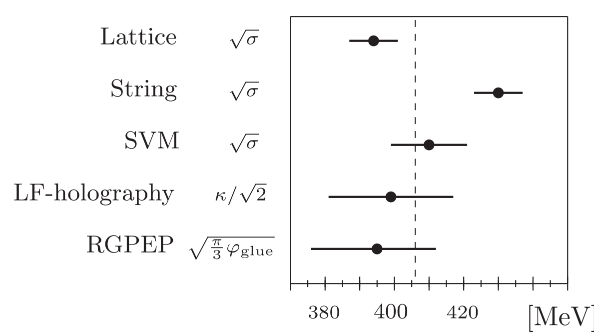

FIG. 1. Phenomenological results for the coefficient of $r_{\rm max}$ obtained using the WKB method (see the text). We compare the coefficients obtained from the lattice approach $\sqrt{\sigma} = 394(7)$ MeV, string theory $\sqrt{\sigma} = 430(7)$ MeV, the stochastic vacuum model $\sqrt{\sigma} = 410(11)$ MeV, the LF holography approach $\kappa/\sqrt{2} = 381 \div 417$ MeV, and the in-hadron gluon condensate in the RGPEP approach $\sqrt{\pi\varphi_{\rm glue}/3} = 395(18)$ MeV. The dashed line is the average of these values.

ACKNOWLEDGMENTS

A. P. T. wants to express his gratitude for the hospitality extended to him at SLAC where this article was written. This work was supported by the Polish-U.S. Fulbright Commission and the Foundation for Polish Science International Ph.D. Projects Programme, co-financed by the EU European Regional Development Fund.

[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).
[2] M. Gell-Mann, Phys. Lett. **8**, 214 (1964).
[3] P. A. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
[4] For the original work, see J. Lioville, J. Math. Pures Appl. **2**, 418 (1837); Lord Rayleigh, Proc. R. Soc. A **86**, 207 (1912); H. Jeffreys, Proc. London Math. Soc. **s2-23**, 428 (1925); G. Wentzel, Z. Phys. **38**, 518 (1926); H. A. Kramers, Z. Phys. **39**, 828 (1926); L. Brillouin, Comptes Rendus **183**, 24 (1926).
[5] P. Danielewicz and J. M. Namysłowski, Phys. Lett. **81B**, 110 (1979).
[6] V. Karmanov, Nucl. Phys. **B166**, 378 (1980).
[7] S. D. Głazek, Acta Phys. Polon. B **15**, 889 (1984).
[8] H. Melosh, Phys. Rev. D **9**, 1095 (1974).
[9] H. Leutwyler and J. Stern, Phys. Lett. **69B**, 207 (1977).
[10] H. Leutwyler and J. Stern, Phys. Lett. **73B**, 75 (1978).
[11] H. Leutwyler and J. Stern, Nucl. Phys. **B133**, 115 (1978).
[12] H. Leutwyler and J. Stern, Ann. Phys. (N.Y.) **112**, 94 (1978).
[13] B. Bakker, L. Kondratyuk, and M. Terentev, Nucl. Phys. **B158**, 497 (1979).
[14] T. Regge, Nuovo Cimento **14**, 951 (1959).
[15] T. Regge, Nuovo Cimento **18**, 947 (1960).
[16] K. G. Wilson, Phys. Rev. D **10**, 2445 (1974).
[17] P. Hägler, Phys. Rep. **490**, 49 (2010).
[18] J. B. Kogut and L. Susskind, Phys. Rev. D **11**, 395 (1975).
[19] W. A. Bardeen and R. Pearson, Phys. Rev. D **14**, 547 (1976).
[20] T. Appelquist and H. D. Politzer, Phys. Rev. Lett. **34**, 43 (1975).
[21] T. Appelquist, A. De Rujula, H. D. Politzer, and S. Glashow, Phys. Rev. Lett. **34**, 365 (1975).
[22] E. Eichten, K. Gottfried, T. Kinoshita, J. B. Kogut, K. Lane, and T. M. Yan, Phys. Rev. Lett. **34**, 369 (1975).
[23] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T.-M. Yan, Phys. Rev. D **17**, 3090 (1978).
[24] T. Kawanai and S. Sasaki, Phys. Rev. Lett. **107**, 091601 (2011).
[25] T. Kawanai and S. Sasaki, Phys. Rev. D **85**, 091503 (2012).
[26] International Lattice Data Grid/Japan Lattice Data Grid, http://www.jldg.org/.
[27] S. Aoki *et al.* (PACS-CS Collaboration), Phys. Rev. D **79**, 034503 (2009).
[28] Y. Koma and M. Koma, *Proc. Sci.*, LATTICE2012 140 (2014) [arXiv:1211.6795].
[29] A. Abrikosov, Sov. Phys. JETP **5**, 1174 (1957).
[30] H. B. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).
[31] G.'t Hooft, Nucl. Phys. **B72**, 461 (1974).
[32] A. A. Migdal, Phys. Rep. **102**, 199 (1983).
[33] M. Lüscher, G. Münster, and P. Weisz, Nucl. Phys. **B180**, 1 (1981).
[34] M. Lüscher, Nucl. Phys. **B180**, 317 (1981).
[35] M. Lüscher, K. Symanzik, and P. Weisz, Nucl. Phys. **B173**, 365 (1980).
[36] O. Andreev and V. I. Zakharov, Phys. Rev. D **74**, 025023 (2006).
[37] T. Goto, Prog. Theor. Phys. **46**, 1560 (1971).
[38] Y. Nambu, Phys. Rev. D **10**, 4262 (1974).
[39] J. Arvis, Phys. Lett. **127B**, 106 (1983).
[40] G. S. Bali, Phys. Rep. **343**, 1 (2001).
[41] A. Bazavov, D. Toussaint, C. Bernard, J. Laiho, C. DeTar *et al.*, Rev. Mod. Phys. **82**, 1349 (2010).
[42] H. G. Dosch and Y. Simonov, Phys. Lett. B **205**, 339 (1988).
[43] A. Di Giacomo, H. G. Dosch, V. Shevchenko, and Y. Simonov, Phys. Rep. **372**, 319 (2002).
[44] M. D'Elia, A. Di Giacomo, and E. Meggiolaro, Phys. Lett. B **408**, 315 (1997).
[45] S. D. Głazek and K. G. Wilson, Phys. Rev. D **48**, 5863 (1993).
[46] S. D. Głazek and K. G. Wilson, Phys. Rev. D **49**, 4214 (1994).
[47] K. G. Wilson, T. S. Walhout, A. Harindranath, W.-M. Zhang, R. J. Perry, and S. Głazek, Phys. Rev. D **49**, 6720 (1994).
[48] P. Ehrenfest, Z. Phys. **45**, 455 (1927).
[49] S. D. Głazek and A. P. Trawiński, Phys. Rev. D **88**, 105025 (2013).
[50] R. J. Perry, *Hadron physics 94* Gramado, Brazil, 1994 World Scientific, Gramado, (1995)
[51] M. M. Brisudova, R. J. Perry, and K. G. Wilson, Phys. Rev. Lett. **78**, 1227 (1997).
[52] M. M. Brisudova and R. Perry, Phys. Rev. D **54**, 1831 (1996).
[53] G. F. de Téramond and S. J. Brodsky, Phys. Rev. Lett. **94**, 201601 (2005).
[54] G. F. de Téramond and S. J. Brodsky, Phys. Rev. Lett. **102**, 081601 (2009).
[55] G. F. de Téramond, H. G. Dosch, and S. J. Brodsky, Phys. Rev. D **87**, 075005 (2013).
[56] A. Karch, E. Katz, D. T. Son, and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006).
[57] S. J. Brodsky and G. F. de Téramond, Phys. Rev. D **77**, 056007 (2008).
[58] J. M. Maldacena, Int. J. Theor. Phys. **38**, 1113 (1999).
[59] V. de Alfaro, S. Fubini, and G. Furlan, Nuovo Cimento **34**, 569 (1976).
[60] S. J. Brodsky, G. F. de Téramond, and H. G. Dosch, Phys. Lett. B **729**, 3 (2014).
[61] S. D. Głazek, Acta Phys. Pol., B **42**, 1933 (2011).

[62] S. J. Brodsky, T. Huang, and G. P. Lepage, Report Nos. SLAC-PUB-2540, C80-07-17-21, 1980.
[63] T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Vega, Phys. Rev. D **82**, 074022 (2010).
[64] T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Vega, Phys. Rev. D **87**, 056001 (2013).
[65] S. D. Głazek and M. Schaden, Phys. Lett. B **198**, 42 (1987).
[66] K. G. Wilson, Phys. Rev. **179**, 1499 (1969).
[67] A. Casher and L. Susskind, Phys. Rev. D **9**, 436 (1974).
[68] P. Maris and C. D. Roberts, Phys. Rev. C **56**, 3369 (1997).
[69] P. Maris, C. D. Roberts, and P. C. Tandy, Phys. Lett. B **420**, 267 (1998).
[70] S. J. Brodsky and R. Shrock, Phys. Lett. B **666**, 95 (2008).
[71] S. J. Brodsky and R. Shrock, Proc. Nat. Acad. Sci. **108**, 45 (2011).
[72] S. J. Brodsky, C. D. Roberts, R. Shrock, and P. C. Tandy, Phys. Rev. C **82**, 022201 (2010).
[73] L. Chang, C. D. Roberts, and P. C. Tandy, Phys. Rev. C **85**, 012201 (2012).
[74] M. A. Shifman, A. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).
[75] S. Narison, Phys. Lett. B **387**, 162 (1996).
[76] S. Narison, Phys. Lett. B **706**, 412 (2012).
[77] S. D. Głazek, Phys. Rev. D **69**, 065002 (2004).