Axial-vector form factors for the low lying octet baryons in the chiral quark constituent model

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(Received 20 August 2014; revised manuscript received 3 September 2014; published 1 October 2014)

We have calculated the axial-vector form factors of the low-lying octet baryons (N, Σ, Ξ, and Λ) in the chiral constituent quark model. In particular, we have studied the implications of chiral symmetry breaking and SU(3) symmetry breaking for the singlet (g_A^0) and nonsinglet (g_A^3 and g_A^8) axial-vector coupling constants expressed as combinations of the spin polarizations at zero momentum transfer. The conventional dipole form of parametrization has been used to analyze the Q^2 dependence of the axial-vector form factors [$G_A^0(Q^2)$, $G_A^3(Q^2)$, and $G_A^8(Q^2)$]. The total strange singlet and nonsinglet contents [$G_s^0(Q^2)$, $G_s^3(Q^2)$, and $G_s^8(Q^2)$] of the nucleon determining the strange quark contribution to the nucleon spin (Δs) have also been discussed.

DOI: 10.1103/PhysRevD.90.074001 PACS numbers: 12.39.Fe, 11.40.Ha, 11.30.Rd

I. INTRODUCTION

The internal structure of the baryons has been extensively studied ever since the measurements of polarized structure functions of proton in the deep inelastic scattering (DIS) experiments [1–4]. These experiments have provided the first evidence that the valence quarks of the proton carry only a small fraction of its spin, and the decomposition of the proton's spin still remains to be a major unresolved issue in high-energy spin physics. Form factors parametrized from the electromagnetic current operator as well as the isovector axial-vector current operator are important in hadron physics as they provide deep insight into understanding the internal structure. The electromagnetic Dirac and Pauli form factors are well know over a wide region of momentum transfer squared Q^2; however, the study of the axial-vector form factors has been rather limited. The measured first moment is related to the combinations of the axial-vector coupling constants that are combinations of the spin polarizations, Δu, Δd, and Δs. For example,

$$\Gamma_1^p(Q^2) = \int_0^1 g_1^p(x, Q^2)dx \\ = \frac{C_s(Q^2)}{9} g_A^0 + \frac{C_{ns}(Q^2)}{12} g_A^3 + \frac{C_{ns}(Q^2)}{36} g_A^8. \qquad (1)$$

Here, C_s and C_{ns} are the flavor singlet and nonsinglet Wilson coefficients calculable from perturbative QCD. The quantity g_A^0 corresponds to the flavor singlet component related to the total quark spin content $\Delta\Sigma$, whereas g_A^3 and g_A^8 correspond to the flavor nonsinglet components usually obtained from the neutron β decay and the semileptonic weak decays of hyperons, respectively. These axial-vector coupling constants can be related to certain well-known sum rules such as the Bjorken sum rule (BSR) [5] and Ellis–Jaffe sum rule (EJSR) [6] and derived within QCD using operator product expansion, renormalization group invariance, and isospin conservation in the DIS.

Recently, experiments measuring electromagnetic and weak form factors from the elastic scattering of electrons, for example, SAMPLE at MIT-Bates [7], G0 at JLab [8], PVA4 at MAMI [9], and HAPPEX at JLab [10] have given indications of the strangeness contribution in the nucleon. These experiments have provided considerable insight into the role played by strange quarks in the charge, current, and spin structure of the nucleon. The nucleon axial coupling constant g_A^3 has received much attention in the past and has been determined precisely from nuclear β decay [11]. It corresponds to the value of the axial form factor at zero-momentum transfer ($Q^2 = -q^2 = 0$). It is one of the fundamental parameters to test the chiral symmetry breaking effects and thereby determine the spin structure of the nucleon. Our information about the other low-lying octet baryon axial-vector form factors from experiment is also rather limited because it is difficult to measure the hyperon properties experimentally due to their short lifetimes. Even though there has been considerable progress in the past few years to determine the Q^2 dependence of axial form factors experimentally, there is no consensus regarding the various mechanisms that can contribute to it. Experiments involving elastic scattering of neutrinos and antineutrinos [12,13] and the pion electroproduction on the proton [14] have explored Q^2 dependence of axial form factors in the past, and they point out the need for additional refined data. More recently, there has been considerable refinement to measure the Q^2 dependence of the axial-vector form factor of the nucleon in the higher-energy Minerνa experiment at Fermilab [15].

The theoretical knowledge in this regard has been rather limited because of confinement, and it is still a big challenge to perform the calculations from the first principles of QCD. Even though some lattice QCD calculations of the axial charge and form factors of the nucleon have been performed [16], still a lot of refinements need to be done. The broader questions of axial charge, axial form factors, and the strange quark contribution to the axial form factors of the nucleon have also been discussed by several authors in other models recently [17]. In addition to this, the partial conservation of axial-vector current also provides important constraints on the axial exchange currents to describe the nonvalence degrees of freedom in the nucleon [18–20]. One of the most successful nonperturbative approaches, which finds its application for the quantities discussed above, is the chiral constituent quark model (χCQM) [21]. The basic idea is based on the possibility that chiral symmetry breaking takes place at a distance scale much smaller than the confinement scale. The χCQM uses the effective interaction Lagrangian approach of the strong interactions in which the effective degrees of freedom are the valence quarks and the internal Goldstone bosons (GBs) that are coupled to the valence quarks [22–25]. The χCQM successfully explains the "proton spin problem" [25], magnetic moments of octet and decuplet baryons including their transitions [26], the violation of the Gottfried sum rule [27] and Coleman–Glashow sum rule, hyperon β decay parameters [28], the strangeness content in the nucleon [29], magnetic moments of $\frac{1}{2}^-$ octet baryon resonances [30], magnetic moments of $\frac{1}{2}^-$ and $\frac{3}{2}^-$ Λ resonances [31], charge radii [32], the quadrupole moment [33], etc.. The model is successfully extended to predict the important role played by the small intrinsic charm content in the nucleon spin in the SU(4) χCQM [34] and to calculate the magnetic moment and charge radii of spin-$\frac{1}{2}^+$ and spin-$\frac{3}{2}^+$ charm baryons including their radiative decays [35,36]. The χCQM provides simultaneous unique information on the flavor and spin structure of the baryons including the heavy baryons. In view of the above developments in the χCQM, it becomes desirable to extend the model to calculate the axial-vector form factors of the low-lying octet baryons. It is widely recognized that knowledge about the axial-vector form factors of the baryons in general and the strangeness content of the nucleon in particular would undoubtedly provide vital clues to the nonperturbative aspects of QCD.

The purpose of the present paper is to determine the axial-vector form factors of the low-lying octet baryons in the χCQM. In particular, we would like to phenomenologically estimate the quantities affected by chiral symmetry breaking and SU(3) symmetry breaking. We begin by computing the static properties of the axial-vector current. The singlet (g_A^0) and nonsinglet (g_A^3 and g_A^8) axial-vector coupling constants expressed as combinations of the spin polarizations at zero momentum transfer have been investigated for the cases of N, Σ, Ξ, and Λ baryons. Further, it would be significant to analyze the Q^2 dependence of the axial-vector form factors [$G_A^0(Q^2)$, $G_A^3(Q^2)$, and $G_A^8(Q^2)$] as well as their explicit flavor contributions [$G_A^u(Q^2)$, $G_A^d(Q^2)$, and $G_A^s(Q^2)$] by using a conventional dipole form of parametrization. Furthermore, it would be interesting to extend the calculations to predict the total strange singlet and nonsinglet contents [$G_s^0(Q^2)$, $G_s^3(Q^2)$, and $G_s^8(Q^2)$] of the nucleon and determine the strange quark contribution to the nucleon spin (Δs). The results can be compared with the recent available experimental observations.

II. CHIRAL CONSTITUENT QUARK MODEL

The key to understanding the structure of the baryons, in the χCQM formalism [22], is the fluctuation process

$$q^{\pm} \to \mathrm{GB} + q'^{\mp} \to (q\bar{q}') + q'^{\mp}, \qquad (2)$$

where GB represents the Goldstone boson and $q\bar{q}' + q'$ constitute the "quark sea" [22,23,25]. The effective Lagrangian describing the interaction between quarks and a nonet of GBs can be expressed as

$$\mathcal{L} = c_8 \bar{\mathbf{q}} \left(\Phi + \zeta \frac{\eta'}{\sqrt{3}} I \right) \mathbf{q} = c_8 \bar{\mathbf{q}} (\Phi') \mathbf{q}, \qquad (3)$$

where $\zeta = c_1/c_8$, c_1, and c_8 are the coupling constants for the singlet and octet GBs, respectively, and I is the 3×3 identity matrix. The matrix $\mathbf{q}$ and the GB field can be expressed in terms of the GBs and their transition probabilities as

$$\mathbf{q} = \begin{pmatrix} u \\ d \\ s \end{pmatrix},$$

$$\Phi' = \begin{pmatrix} \frac{\pi^o}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^o}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \alpha K^o \\ \alpha K^- & \alpha \bar{K}^o & -\beta \frac{2\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} \end{pmatrix}. \qquad (4)$$

If the parameter $a (= |c_8|^2)$ denotes the transition probability of chiral fluctuation of the splitting $u(d) \to d(u) + \pi^{+(-)}$, then $\alpha^2 a$, $\beta^2 a$, and $\zeta^2 a$, respectively, denote the probabilities of transitions of $u(d) \to s + K^{+(0)}$, $u(d,s) \to u(d,s) + \eta$, and $u(d,s) \to u(d,s) + \eta'$ [22,23]. These parameters provide the basis to understand the extent to which the quark sea contributes to the structure of the baryon. The symmetry breaking parameter a is introduced by considering nondegenerate quark masses $M_s > M_{u,d}$, the parameters α and β are introduced by considering nondegenerate GB masses $M_{K,\eta} > M_\pi$, and finally the parameter ζ is introduced by considering $M_{\eta'} > M_{K,\eta}$. Since the quark contributions scale as $\frac{1}{M_q^2}$, a hierarchy for the probabilities can be obtained as

$$a > a\alpha^2 \geq a\beta^2 > a\zeta^2. \tag{5}$$

Before proceeding further to calculate the axial-vector form factors, we briefly discuss the calculation of the spin structure of the baryons. Following Refs. [22,23], the quark spin polarization can be defined as

$$\Delta q = q^+ - q^-, \tag{6}$$

where $q^\pm$ can be calculated from the spin structure of a baryon,

$$\hat{B} \equiv \langle B|\mathcal{N}|B\rangle = \langle B|q^+q^-|B\rangle. \tag{7}$$

Here, $|B\rangle$ is the baryon wave function, and $\mathcal{N} = q^+q^-$ is the number operator measuring the sum of the quark numbers with spin up or down, for example,

$$\begin{aligned} q^+q^- &= \sum_{q=u,d,s} (n_{q^+}q^+ + n_{q^-}q^-) \\ &= n_{u^+}u^+ + n_{u^-}u^- + n_{d^+}d^+ + n_{d^-}d^- + n_{s^+}s^+ + n_{s^-}s^-, \end{aligned} \tag{8}$$

with the coefficients of the $q^\pm$ giving the number of $q^\pm$ quarks. The contributions of the quark sea coming from the fluctuation process in Eq. (2) can be calculated by substituting for every constituent quark

$$q^\pm \to \sum P_q q^\pm + |\psi(q^\pm)|^2, \tag{9}$$

where the transition probability of the emission of a GB from any of the q quark ($\sum P_q$) and the transition probability of the $q^\pm$ quark ($|\psi(q^\pm)|^2$) can be calculated from the Lagrangian. They are expressed as

$$\begin{aligned} \sum P_u &= a\left(\frac{9+\beta^2+2\zeta^2}{6} + \alpha^2\right) \quad \text{and} \quad |\psi(u^\pm)|^2 = \frac{a}{6}(3+\beta^2+2\zeta^2)u^\mp + ad^\mp + a\alpha^2 s^\mp, \\ \sum P_d &= a\left(\frac{9+\beta^2+2\zeta^2}{6} + \alpha^2\right) \quad \text{and} \quad |\psi(d^\pm)|^2 = au^\mp + \frac{a}{6}(3+\beta^2+2\zeta^2)d^\mp + a\alpha^2 s^\mp, \\ \sum P_s &= a\left(\frac{2\beta^2+\zeta^2}{3} + 2\alpha^2\right) \quad \text{and} \quad |\psi(s^\pm)|^2 = a\alpha^2 u^\mp + a\alpha^2 d^\mp + \frac{a}{3}(2\beta^2+\zeta^2)s^\mp. \end{aligned}$$

Spin-spin forces, known to be compatible [37–39] with the χCQM, generate configuration mixing [40–42] for the octet baryons, which effectively leads to modification of the spin distribution functions [25]. The general configuration mixing generated by the spin-spin forces has been discussed in the case of octet baryons [40,42,43]. However, it is adequate [25,42,44,45] to consider the "mixed" octet with mixing only between $|56,0^+\rangle_{N=0}$ and the $|70,0^+\rangle_{N=2}$ states, for example,

$$|B\rangle \equiv \left|8,\frac{1^+}{2}\right\rangle = \cos\phi|56,0^+\rangle_{N=0} + \sin\phi|70,0^+\rangle_{N=2}, \tag{10}$$

where ϕ represents the $|56\rangle - |70\rangle$ mixing and

$$|56,0^+\rangle_{N=0} = \frac{1}{\sqrt{2}}(\chi'\phi' + \chi''\phi'')\psi^s(0^+), \tag{11}$$

$$|70,0^+\rangle_{N=2} = \frac{1}{2}[(\phi'\chi'' + \phi''\chi')\psi'(0^+) + (\phi'\chi' - \phi''\chi'')\psi''(0^+)]. \tag{12}$$

In general, the isospin wave functions for the octet baryons (N, Σ, Ξ) of the type $B(xxy)$ are given as

$$\phi'_B = \frac{1}{\sqrt{2}}(xyx - yxx), \qquad \phi''_B = \frac{1}{\sqrt{6}}(2xxy - xyx - yxx), \tag{13}$$

whereas for $\Lambda(uds)$, they are given as

$$\begin{aligned} \phi'_\Lambda &= \frac{1}{2\sqrt{3}}(usd + sdu - sud - dsu - 2uds - 2dus), \\ \phi''_\Lambda &= \frac{1}{2}(sud + usd - sdu - dsu). \end{aligned} \tag{14}$$

The spin wave functions are expressed as

$$\begin{aligned} \chi' &= \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \\ \chi'' &= \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow). \end{aligned} \tag{15}$$

For the definition of the spatial wave functions (ψ^s, ψ', ψ'') as well as the definitions of the overlap integrals, we refer the reader to Ref. [46].

The quark polarizations can be calculated from the spin structure of a given baryon. Using Eqs. (7) and (10) of the text, the spin structure of a baryon in the mixed octet is given as

$$\hat{B} \equiv \langle B|\mathcal{N}|B\rangle = \cos^2\phi\langle 56,0^+|\mathcal{N}|56,0^+\rangle_B + \sin^2\phi\langle 70,0^+|\mathcal{N}|70,0^+\rangle_B. \quad (16)$$

For the case of N, Σ, Ξ, and Λ, using Eqs. (11) and (12), we have

$$\langle 56,0^+|\mathcal{N}|56,0^+\rangle_N = \frac{5}{3}u_+ + \frac{1}{3}u_- + \frac{1}{3}d_+ + \frac{2}{3}d_-, \quad (17)$$

$$\langle 70,0^+|\mathcal{N}|70,0^+\rangle_N = \frac{4}{3}u_+ + \frac{2}{3}u_- + \frac{2}{3}d_+ + \frac{1}{3}d_-, \quad (18)$$

$$\langle 56,0^+|\mathcal{N}|56,0^+\rangle_\Sigma = \frac{5}{3}u_+ + \frac{1}{3}u_- + \frac{1}{3}s_+ + \frac{2}{3}s_-, \quad (19)$$

$$\langle 70,0^+|\mathcal{N}|70,0^+\rangle_\Sigma = \frac{4}{3}u_+ + \frac{2}{3}u_- + \frac{2}{3}s_+ + \frac{1}{3}s_-, \quad (20)$$

$$\langle 56,0^+|\mathcal{N}|56,0^+\rangle_\Xi = \frac{5}{3}s_+ + \frac{1}{3}s_- + \frac{1}{3}u_+ + \frac{2}{3}u_-, \quad (21)$$

$$\langle 70,0^+|\mathcal{N}|70,0^+\rangle_\Xi = \frac{4}{3}s_+ + \frac{2}{3}s_- + \frac{2}{3}u_+ + \frac{1}{3}u_-, \quad (22)$$

and

$$\langle 56,0^+|\mathcal{N}|56,0^+\rangle_\Lambda = \frac{1}{2}u_+ + \frac{1}{2}u_- + \frac{1}{2}d_+ + \frac{1}{2}d_- + 1s_+ + 0s_-, \quad (23)$$

$$\langle 70,0^+|\mathcal{N}|70,0^+\rangle_\Lambda = \frac{2}{3}u_+ + \frac{1}{3}u_- + \frac{2}{3}d_+ + \frac{1}{3}d_- + \frac{2}{3}s_+ + \frac{1}{3}s_-, \quad (24)$$

respectively. Sea contributions can be included by using Eq. (9), and the results have been presented in Table I. A closer look at the expressions of these quantities reveals that the constant factors represent the naive quark model (NQM) results, which do not include the effects of chiral symmetry breaking. On the other hand, the factors with transition probability a represent the contribution from the quark sea in general [with or without SU(3) symmetry breaking].

III. AXIAL-VECTOR FORM FACTORS

The axial-vector form factors can be expressed in terms of the axial-vector current $A^{\mu,a}$ defined as $\bar{\mathbf{q}}\gamma^\mu\gamma_5\frac{\lambda^a}{2}\mathbf{q}$ through the matrix elements

$$\langle B(p')|A^{\mu,a}|B(p)\rangle = \bar{u}(p')\left[\gamma^\mu\gamma_5 G_A^i(Q^2) + \frac{q^\mu}{2M_B}\gamma_5 G_P^i(Q^2)\right]u(p), \quad (25)$$

where M_B is the baryon mass and $u(p)$ ($\bar{u}(p')$) are the Dirac spinors of the initial (final) baryon states, respectively. The four-momenta transfer is given as $Q^2 = -q^2$, where $q \equiv p - p'$. Here, λ^a ($a = 1,2,..8$) are the Gell-Mann matrices of SU(3) describing the flavor structure of the three light quarks. It is often convenient to introduce the unit matrix $\lambda^0(=\sqrt{\frac{2}{3}}I)$ in addition to these matrices. In the present context. we shall need only the matrices having diagonal representation corresponding to the flavor singlet current ($a = 0$), isovector current ($a = 3$), and hypercharge axial current ($a = 8$) [19]. The functions $G_A^i(Q^2)$ and $G_P^i(Q^2)$ ($i = 0,3,8$) are the axial and induced pseudoscalar form factors, respectively. We will ignore the induced pseudoscalar form factors as they are not relevant to the present work.

In general, the axial-vector matrix elements have implications for spin structure [25,28]. To calculate the axial charge as one of the important static properties of the form factors at zero momentum transfer, the singlet and non-singlet combinations of the spin structure can be related to the weak couplings and can be expressed in terms of the spin polarizations defined in the above section. We have

TABLE I. The quark spin polarizations for the octet baryons in the χCQM.

Baryons	Δu_B	Δd_B	Δs_B
N	$\cos^2\phi[\frac{4}{3}-\frac{a}{3}(7+4\alpha^2+\frac{4}{3}\beta^2+\frac{8}{3}\zeta^2)]$ $+\sin^2\phi[\frac{2}{3}-\frac{a}{3}(5+2\alpha^2+\frac{2}{3}\beta^2+\frac{4}{3}\zeta^2)]$	$\cos^2\phi[-\frac{1}{3}-\frac{a}{3}(2-\alpha^2-\frac{1}{3}\beta^2-\frac{2}{3}\zeta^2)]$ $+\sin^2\phi[\frac{1}{3}-\frac{a}{3}(5+2\alpha^2+\frac{2}{3}\beta^2+\frac{4}{3}\zeta^2)]$	$-a\alpha^2$
Σ	$\cos^2\phi[\frac{4}{3}-\frac{a}{3}(8+3\alpha^2+\frac{4}{3}\beta^2+\frac{8}{3}\zeta^2)]$ $+\sin^2\phi[\frac{2}{3}-\frac{a}{3}(4+3\alpha^2+\frac{2}{3}\beta^2+\frac{4}{3}\zeta^2)]$	$-\cos^2\phi[\frac{a}{3}(4-\alpha^2)]-\sin^2\phi[\frac{a}{3}(2+\alpha^2)]$	$\cos^2\phi[-\frac{1}{3}-\frac{a}{3}(2\alpha^2-\frac{4}{3}\beta^2-\frac{2}{3}\zeta^2)]$ $+\sin^2\phi[\frac{1}{3}-\frac{a}{3}(4\alpha^2+\frac{4}{3}\beta^2+\frac{2}{3}\zeta^2)]$
Ξ	$\cos^2\phi[-\frac{1}{3}-\frac{a}{3}(3\alpha^2-2-\frac{1}{3}\beta^2-\frac{2}{3}\zeta^2)]$ $+\sin^2\phi[\frac{1}{3}-\frac{a}{3}(2+3\alpha^2+\frac{1}{3}\beta^2+\frac{2}{3}\zeta^2)]$	$-\cos^2\phi[\frac{a}{3}(4\alpha^2-1)]-\sin^2\phi[\frac{a}{3}(1+2\alpha^2)]$	$\cos^2\phi[\frac{4}{3}-\frac{a}{3}(7\alpha^2+\frac{16}{3}\beta^2+\frac{8}{3}\zeta^2)]$ $+\sin^2\phi[\frac{2}{3}-\frac{a}{3}(5\alpha^2+\frac{8}{3}\beta^2+\frac{4}{3}\zeta^2)]$
Λ	$-\cos^2\phi[a\alpha^2]$ $+\sin^2\phi[\frac{1}{3}-\frac{a}{9}(9+6\alpha^2+\beta^2+2\zeta^2)]$	$-\cos^2\phi[a\alpha^2]$ $+\sin^2\phi[\frac{1}{3}-\frac{a}{9}(9+6\alpha^2+\beta^2+2\zeta^2)]$	$\cos^2\phi[1-\frac{a}{3}(6\alpha^2+4\beta^2+2\zeta^2)]$ $+\sin^2\phi[\frac{1}{3}-\frac{4}{9}a(3\alpha^2+2\beta^2+\zeta^2)]$

$$g^0_{A,B} = \langle B|u^+u^- + d^+d^- + s^+s^-|B\rangle = \Delta u_B + \Delta d_B + \Delta s_B,$$
$$g^3_{A,B} = \langle B|u^+u^- - d^+d^-|B\rangle = \Delta u_B - \Delta d_B,$$
$$g^8_{A,B} = \langle B|u^+u^- + d^+d^- + 2s^+s^-|B\rangle = \Delta u_B + \Delta d_B - 2\Delta s_B. \quad (26)$$

The axial coupling constants $g^3_{A,B}$ and $g^8_{A,B}$ basically correspond to the BSR [5] and the EJSR [6]. The axial coupling constant related to the total quark spin content $g^0_{A,B}$ reduces to the EJSR in the $\Delta s = 0$ limit.

To compare the χCQM results with the available experimental data and other model calculations, we can take the case of the quark spin polarizations and the axial coupling constants for the octet baryons at zero momentum transfer. The numerical calculation of the axial-vector coupling constants of the octet baryons at $Q^2 = 0$ involves two sets of parameters, the SU(3) symmetry breaking parameters of χCQM and the mixing angle θ. The mixing angle θ is fixed from the consideration of the neutron charge radius [41]. The χCQM parameters, a, $a\alpha^2$, $a\beta^2$, and $a\zeta^2$ represent, respectively, the probabilities of fluctuations to pions, K, η, and η'. A best fit of χCQM parameters can be obtained by carrying out a fine grained analysis of the spin and flavor distribution functions [25,29], wherein as a first step, a gross analysis was carried out to find the limits of the parameters from the well-known experimentally measurable quantities while taking into account strong physical considerations. After obtaining the limits, as a second step, a detailed and fine grained analysis was carried out to obtain the best fit. In Table II, we summarize the input parameters and their values. We would like to mention here that the positive values of ζ have also been widely used in similar calculations [19]. The sign may not be important for the case of quark spin polarizations in the present context where only ζ^2 is involved, but since this set of parameters has already been tested for a wide variety of low-energy matrix elements and is able to give a simultaneous fit to the quantities describing the proton spin and flavor structure including quark flavor distribution functions (antiquark contents, antiup and antidown quark asymmetry, fraction of quark flavors) as well as the magnetic moments of octet and decuplet baryons, etc., we use the same set here. A relative negative sign of $\zeta = c_1/c_8$ is required primarily to yield the antiquark $\bar{u} - \bar{d}$ asymmetry or the $\bar{u}/\bar{d}$ ratio [47–49] because they involve ζ. The results of the quark spin polarizations and the axial coupling constants for the octet baryons at zero momentum transfer using the parameters listed above have been presented in Table III.

TABLE II. Input parameters of the χCQM used in the analysis.

Parameter →	ϕ	a	α	β	ζ
Value	20°	0.114	0.45	0.45	−0.75

TABLE III. The χCQM results for the quark spin polarizations and the axial coupling constants for the N, Σ, Ξ, and Λ octet baryons.

Quantity	N	Σ	Ξ	Λ
Δu_B	0.904	0.881	−0.329	0.002
Δd_B	−0.362	−0.137	0.00	0.002
Δs_B	−0.023	−0.252	1.109	0.805
$g^0_{A,B}$	0.519	0.492	0.780	0.809
$g^3_{A,B}$	1.266	1.018	−0.329	0.00
$g^8_{A,B}$	0.588	1.248	−2.547	−1.606

The present experimental situation, in terms of the quark spin polarizations, Δu, Δd, and Δs for the case of N, is summarized as follows:

$$\Delta u_N^{\rm expt} = 0.85 \pm 0.05, \qquad \Delta d_N^{\rm expt} = -0.41 \pm 0.05,$$
$$\Delta s_N^{\rm expt} = -0.07 \pm 0.05, \qquad g^{0\rm expt}_{A,N} = 0.30 \pm 0.06,$$
$$g^{3\rm expt}_{A,N} = 1.267 \pm 0.0025, \qquad g^{8\rm expt}_{A,N} = 0.588 \pm 0.033[11]. \quad (27)$$

The NQM, which is quite successful in explaining a good deal of low-energy data [40–42], has the following predictions for the above-mentioned quantities:

$$\Delta u_N = 1.33, \qquad \Delta d_N = -0.33, \qquad \Delta s_N = 0,$$
$$g^0_{A,N} = 1, \qquad g^3_{A,N} = 1.66, \qquad g^8_{A,N} = 1. \quad (28)$$

The disagreement between the NQM predictions and the DIS measurements was broadly characterized as the "proton spin crisis." The results of χCQM for the case of Δu_N, Δd_N, Δs_N, $g^3_{A,N}$, and $g^8_{A,N}$ are more or less in agreement with data. This not only justifies the success of χCQM but also strengthens our conclusion regarding the qualitative and quantitative role of the quark sea in the right direction. For the case of $g^0_{A,N}$, the NQM results show that the valence quarks of the nucleon carry only about $1/3$ of the nucleon spin as obtained in the experiment. The χCQM result comes out to be 0.519, which is better than the results of NQM but still shows a large deviation from data. A detailed understanding of the deep inelastic results as well as the dynamics of the constituents of the nucleon constitute a major challenge for any model trying to explain the nonperturbative regime of QCD. In this context, it has been shown recently in a chiral constituent quark potential model that it is possible to describe the singlet axial nucleon coupling if consistent axial exchange currents are taken into account [19,50,51]. Because of angular momentum conservation, this reduction of the quark spin is compensated by the orbital angular momentum carried by the same nonvalence quark degrees of freedom.

The Q^2 dependence of the axial-vector form factors has been experimentally investigated from the quasielastic neutrino scattering [12,13] and from the pion electroproduction [14]. The dipole form of parametrization has been conventionally used to analyze the axial-vector form factors

$$G^i_{A,B}(Q^2) = \frac{g^i_{A,B}(0)}{(1+\frac{Q^2}{M_A^2})^2}, \tag{29}$$

where $g^0_A(0)$, $g^3_A(0)$, and $g^8_A(0)$ are the isovector axial-vector coupling constants at zero momentum transfer. For the axial mass M_A, a global average as extracted from neutrino scattering experiments is $M_A = (1.026 \pm 0.021)$ GeV [52]. Another recent analysis finds a slightly smaller value $M_A = (1.001 \pm 0.020)$ GeV [53]. However, in the present work, we have used the most recent value obtained by the MiniBooNE Collaboration, $M_A = 1.10^{+0.13}_{-0.15}$ GeV [54]. The axial mass can be taken as free parameter and adjusted to the experiment [20]. Since experimental data are available only for the nucleon axial coupling constants, we have used the same value of the axial mass for all the octet baryons. The axial masses corresponding to Σ, Ξ, and Λ are expected to be larger than that of the nucleon, which will in turn lead to slightly larger values of the axial-vector form factors in magnitude. The overall behavior of the form factors, however, will not be affected by this change.

After having incorporated Q^2 dependence in the axial-vector form factors, we now discuss the variation of all the Q^2-dependent quantities in the range $0 \le Q^2 \le 1$. In Fig. 1, we have presented the singlet and nonsinglet axial-vector form factors of the octet baryons N, Σ, Ξ, and Λ. From a cursory look at the plots, one can easily describe some general aspects of the sensitivity to Q^2 for the form factors. The sensitivity of the singlet and nonsinglet form factors for different baryons varies as

$$\begin{aligned} &G^0_{A,\Xi} > G^0_{A,\Lambda} > G^0_{A,N} > G^0_{A,\Sigma}, \\ &G^3_{A,N} > G^3_{A,\Sigma} > G^3_{A,\Xi} > G^3_{A,\Lambda}, \\ &G^8_{A,\Xi} > G^8_{A,\Lambda} > G^8_{A,N} > G^8_{A,\Sigma}. \end{aligned} \tag{30}$$

The behaviors of the form factors for Ξ and Λ are similar to each other. This may possibly be due to the presence of more strange quarks in the valence structure. On the other hand, the form factors for N and Σ, which have the dominance of u quarks in the valence structure, show similar variation with Q^2. This can be easily seen from Fig. 1, and this is true for $G^0_{A,B}$, $G^3_{A,B}$ and $G^8_{A,B}$. Another important observation for the case of $G^0_{A,B}$ form factors is that they fall off rapidly with the increase of Q^2 for all the octet baryons N, Σ, Ξ, and Λ. However, for the case of $G^3_{A,B}$ and $G^8_{A,B}$, the N and Σ form factors fall off with increasing Q^2, whereas the Ξ and Λ form factors increase with increasing Q^2. The case of $G^3_{A,\Lambda}$ is particularly interesting because of its flavor structure, which has equal numbers of u, d, and s quarks in its valence structure. Unlike the other octet baryons, where the form factors decrease or increase

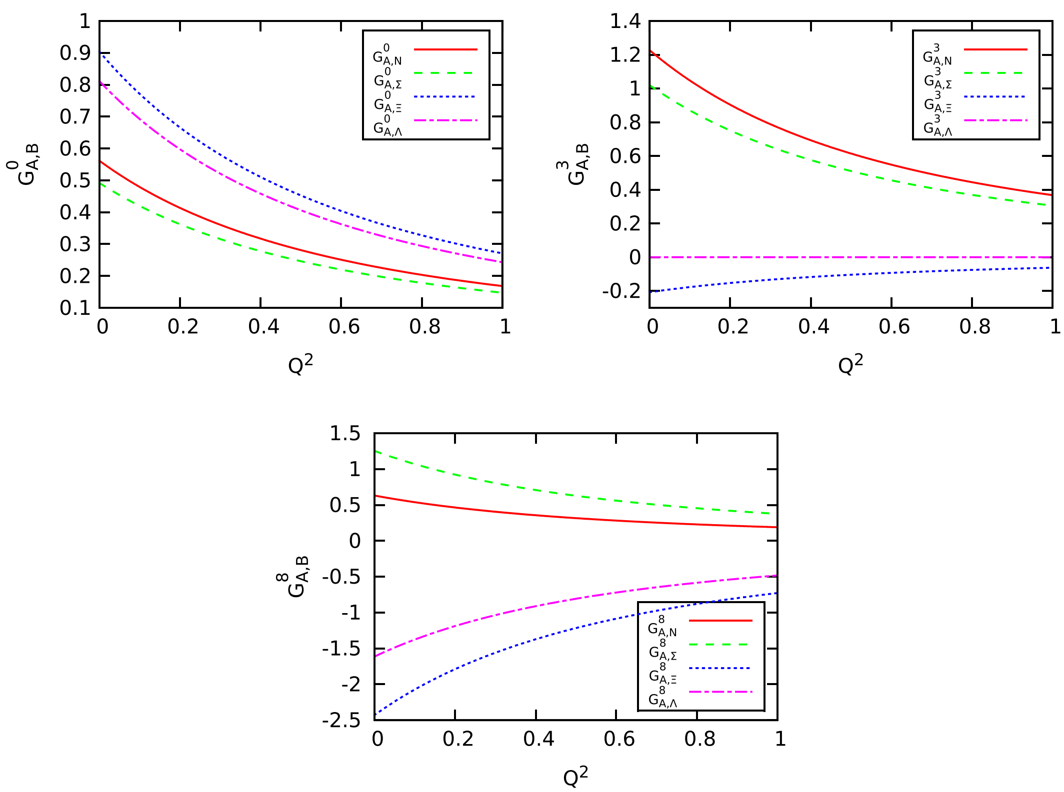

FIG. 1 (color online). Form factors for the baryons N, Σ, Ξ, and Λ plotted as a function of Q^2.

continuously with the Q^2 values, the form factor in this case has no Q^2 dependence.

Since the constituent quarks are spatially extended particles [20,55], they themselves have axial form factors. The role of nonvalence quarks in the spin structure can be studied in detail by calculating the flavor axial-vector form factors using the dipole form of parametrization [Eq. (29)]. These can be expressed in terms of the singlet and non-singlet combinations of the spin structure as follows:

$$G^u_{A,B} = \frac{1}{3}G^0_{A,B} + \frac{1}{2}G^3_{A,B} + \frac{1}{2\sqrt{3}}G^8_{A,B},$$
$$G^d_{A,B} = \frac{1}{3}G^0_{A,B} - \frac{1}{2}G^3_{A,B} + \frac{1}{2\sqrt{3}}G^8_{A,B},$$
$$G^s_{A,B} = \frac{1}{3}G^0_{A,B} - \frac{1}{\sqrt{3}}G^8_{A,B}. \tag{31}$$

In Fig. 2, we have plotted the explicit u, d, and s quark flavor contributions for each of the octet baryon axial-vector form factors. The plots clearly project out the valence quark structure of the baryon. For example, since N is dominated by the u quark, it is clear from the plot of $G^{u,d,s}_{A,N}$ that the $G^u_{A,N}$ dominates and $G^d_{A,N}$, and $G^s_{A,N}$ has a comparatively smaller contribution. The important observation in this case is the nonzero contribution of the s quarks. Even though there are no s quarks in the valence structure, the contribution of $G^s_{A,N}$ implies a presence of a quark sea, which is even more at zero momentum transfer. It is also evident from the figure that the valence quark distribution is spread over the entire Q^2 region, and as the value of Q^2 increases, the sea contributions decrease, and at even higher values of Q^2 (not presented here), the contributions should be completely dominated by the valence quarks. Further, for the case of $G^{u,d,s}_{A,\Sigma}$ and $G^{u,d,s}_{A,\Xi}$, where the valence structure is dominated by the u and s quarks, we find a significant contribution from them. In these form factors, the small but significant G^d_A can have important implications for the role of sea quarks at low Q^2. Finally, the $G^{u,d,s}_{A,\Lambda}$, even after having equal contributions from the u, d, and s quarks, does not show symmetric behavior. The $G^s_{A,\Lambda}$ clearly dominates over $G^u_{A,\Lambda}$ and $G^d_{A,\Lambda}$, which is expected because the u and d quarks also contribute toward $G^{u,d,s}_{A,\Lambda}$ through quark fluctuations. It is interesting to note that the valence and sea quark distributions contribute in the right direction to give an excellent overall fit to the axial-vector form factors where experimental data are available. This can perhaps be substantiated further by measurements for the other octet baryons.

It is well known that, for the case of the nucleon, the strange quarks contribute to the spin polarizations of u and d quarks apart from contributing to the strange spin polarization. This is because of the presence of the non-valence quark sea [Eq. (2)]. In this context, the axial-vector matrix elements will have implications for the strangeness contribution to the nucleon as well as for the effects of chiral symmetry breaking. We can calculate $G^0_s(Q^2)$, $G^3_s(Q^2)$, and $G^8_s(Q^2)$ for the case of N from Eq. (26)

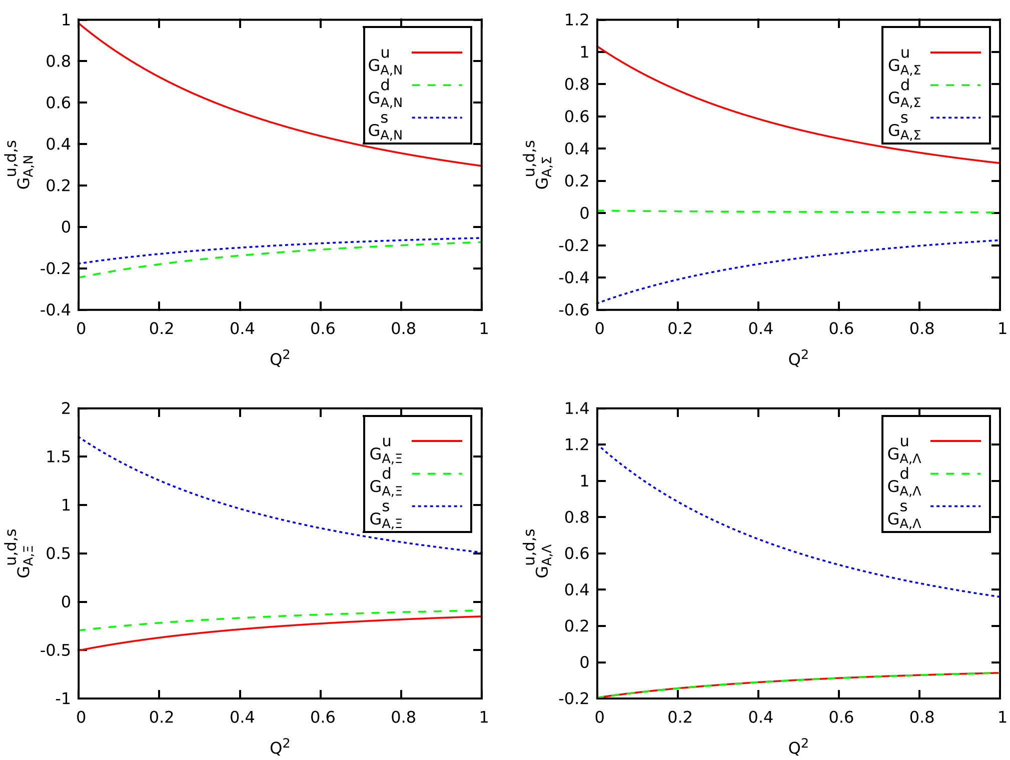

FIG. 2 (color online). The explicit flavor form factors for the baryons N, Σ, Ξ, and Λ plotted as a function of Q^2.

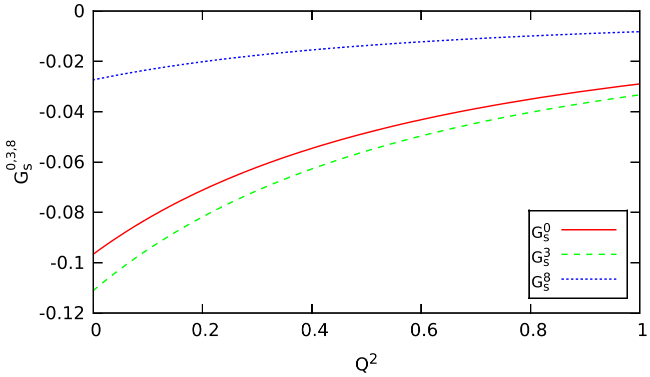

FIG. 3 (color online). The strange form factors for the nucleon plotted as a function of Q^2.

and Table I by dropping the constant factors. The factors with $a\alpha^2$, $a\beta^2$, and $a\zeta^2$ include the effects of chiral symmetry breaking as well as SU(3) symmetry breaking and give the contribution coming from the quark sea. In particular, they give the contribution of strange quarks to the nucleon spin. The explicit strangeness contribution for the other octet baryons is not so significant because of the presence of strange quarks in their valence structure. In Fig. 3, we have presented the results for $G_s^0(Q^2)$, $G_s^3(Q^2)$, and $G_s^8(Q^2)$ for the case of N. We find that the magnitudes of $G_s^0(Q^2)$ and $G_s^8(Q^2)$ fall off with the increasing value of Q^2, whereas $G_s^3(Q^2)$ has a weak Q^2 dependence. For the sake of completeness, we have also presented the numerical values of the explicit strangeness contribution to Δu, Δd, and the axial coupling constants at $Q^2 = 0$ for the case of N in Table IV. The contribution of Δs is coming purely from the quark sea and has already been presented in Table III. It is clear from the table that there is a significant contribution of nonvalence quarks in Δu_s, g_s^0, and g_s^3. These quantities not only provide a direct method to determine the presence of a significant amount of the quark sea but also impose an important constraint on a model that attempts to describe the origin of the quark sea. A small but significant contribution of strangeness in the nucleon has already been indicated by SAMPLE at MIT-Bates [7], G0 at JLab [8], PVA4 at MAMI [9], and HAPPEX at JLab [10]. A determination of G_A^s at low values of Q^2 [54] would permit a determination of strange spin polarization Δs, which is otherwise zero in the case of the nucleon. The strange quarks contribute through the quark sea generated by the chiral fluctuations, and any refinement in the case of the strangeness-dependent quantities would have important implications for the basic tenets of χCQM.

To summarize, the χCQM is able to phenomenologically estimate the quantities having implications for chiral symmetry breaking and SU(3) symmetry breaking. In particular, it provides a fairly good description of the axial-vector form factors of the low-lying octet baryons (N, Σ, Ξ, and Λ), for example, the singlet (g_A^0) and nonsinglet (g_A^3 and g_A^8) axial-vector coupling constants expressed as combinations of the spin polarizations at zero momentum transfer. To enlarge the scope of the χCQM, we have used the conventional dipole form of parametrization to analyze the Q^2 dependence of the axial-vector form factors [$G_A^0(Q^2)$, $G_A^3(Q^2)$, and $G_A^8(Q^2)$]. To understand the role of chiral symmetry breaking and the significance of nonvalence quarks in the nucleon structure, the implications of the hidden strangeness component have been studied for the strange singlet and nonsinglet contents [$G_s^0(Q^2)$, $G_s^3(Q^2)$, and $G_s^8(Q^2)$] of the nucleon. The χCQM is able to give a qualitative and quantitative description of the axial-vector form factors. The significant contribution of the strangeness is also consistent with the recent available experimental results.

TABLE IV. The NQM and χCQM results for the explicit strangeness contribution to spin polarizations and the axial coupling constants at $Q^2 = 0$ for the case of N.

Quantity →	Δu_s	Δd_s	g_s^0	g_s^3	g_s^8
NQM	0	0	0	0	0
χCQM	−0.092	0.013	−0.102	−0.105	−0.033

In conclusion, we would like to state that chiral symmetry breaking and SU(3) symmetry breaking play an important role in understanding the spin structure of the baryon and are the keys to describing the hidden strangeness content of the nucleon in the nonperturbative regime of QCD where the constituent quarks and the weakly interacting Goldstone bosons constitute the appropriate degrees of freedom at the leading order. The future experiments to measure the axial-vector form factors will not only provide a direct method to determine the presence of an appropriate amount of the quark sea but also impose an important constraint on the parity-violating asymmetries in different kinematical regions. Several groups, for example, Minerνa, are contemplating the possibility of performing the high-precision measurements over a wide Q^2 region in the near future.

ACKNOWLEDGMENTS

H. D. would like to thank Department of Science and Technology (Ref No. SB/S2/HEP-004/2013), Government of India, for financial support.

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